

MORAL HAZARD, INCENTIVE FLEXIBILITY AND RISK:  
COST SHARING ARRANGEMENTS UNDER SHARECROPPING \*

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Econometric Research Program  
Research Memorandum No. 298

June 1982

\* This paper appears simultaneously as a World Bank, Development Economic Department, Public Finance Division discussion paper. The views presented in this paper are solely those of the authors and do not necessarily reflect the official opinions or views of The World Bank. We thank Robin Lindsey and two anonymous referees for extremely helpful comments.

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## ABSTRACT

Contracts between landlords and tenants are often characterized by output sharing as well as input cost sharing arrangements. The conventional wisdom among agricultural economists was that the efficient contract would entail the landlord paying the same share of the cost of the input that he received of the output. In a first-best world, this rule implies an efficient application of inputs. However, in a first-best world there is no need to resort to either sharecropping or cost sharing contracts. Such contracts are often observed in rural economies of L.C.D.s because of the absence of certain important markets. In particular as uncertainty, moral hazard and incentive flexibility problems are crucial in such environments, various cost sharing rules will emerge as optimal contracts. The incentive flexibility property of cost sharing rules becomes the critical factor in providing the rationale for such rules. This theoretical attempt may provide insight into explaining the observed phenomena of a wider variety of cost sharing rules.

## I. INTRODUCTION

Quite often the agricultural output in less developed countries is produced by peasants working under some form of sharecropping. There has been a long tradition of concern that with such contracts, not only will peasants have insufficient incentives to work, and thus effort (labor supply) will be below the optimal level, but they will also have insufficient incentives to supply other inputs, such as fertilizer.

A number of years ago, Heady [1947] pointed out that the distortions associated with these other inputs could easily be corrected: if the landlord and tenant share in costs to the same extent that they share in output, the standard marginal cost equal marginal benefit conditions will be satisfied. (This argument was subsequently formalized by Adams and Rask [1968]). For then, although the tenant only receives a fraction of the product he pays only the same fraction of the cost.

In the past decade, a considerable amount of empirical research has, in fact, confirmed that tenancy contracts generally do have provisions for cost sharing (e.g. see I. Singh [1982], chapter 10). At the same time, these studies have shown that there are frequent departures from the simple rule of setting the cost share equal to the output share. These departures are surprising, since the rule of setting cost share equal to output share has both the virtue of simplicity, and, if Heady's argument were correct, of efficiency.

The purpose of this paper is to analyze the equilibrium cost sharing rules under sharecropping. Our analysis suggests that there are several factors which the simple peasants in the less developed countries may have taken into account, which the more sophisticated analysis of Heady [1947] and

Adams and Rask [1968] ignored, which explain why cost shares may differ from output shares.

In a first best world, the equal share rule does imply an efficient application of inputs. But the very presence of sharecropping itself represents a significant departure from a first best world; and as always, it is not obvious, given one important departure from the standard optimality conditions, that there should not be other offsetting departures. There are, in fact, two important departures from first best optimality in sharecropping:

(i) the tenant's allocation of effort is not at its first best level and

(ii) the tenant must absorb more risk than he would, were there a perfect set of risk markets.

Since the level of effort cannot be specified (at least perfectly) in the contract, and since increases in the level of effort will increase the landlord's profits, the landlord will seek to induce the worker to take greater effort. If fertilizer (or other inputs) are strongly complementary to labor, the landlord may seek to increase the level of effort by increasing the supply of these inputs, i.e. by lowering the cost share of the tenant. The implications of imperfect risk sharing are somewhat more complicated; the effects depend critically on the set of instruments available for risk sharing. Thus, if the landlord can charge a fixed fee for the use of his land (in addition to collecting a share), and if this fixed fee can vary to compensate for changes in the cost sharing rule, then so long as a compensated reduction in the cost share leads to an increase in effort (the lowering of the tenant's cost share increases the use of fertilizer, and if fertilizer and effort are complementary, this will lead to an increase in effort) the cost share will be less than the output share 1/

Thus, the nature of the cost sharing arrangement between the landlord and the tenant will depend on the institutional environment which is envisaged. We consider here three polar cases:

(a) In Section II, we consider the "standard" sharecropping model; the sharecropping contract specifies a share of output,  $\alpha$ , received by the tenant, and a share  $\beta$  of costs of inputs which he must bear. The terms of the contract are, however, assumed to be variable; they are chosen to maximize the expected profits of the landlord, given a particular level of expected utility of tenants.<sup>2/</sup>

(b) The empirical literature on sharecropping contracts suggests that, in fact, in the long run the terms of the contract are variable; there are marked changes in contracts in response to changes in technology. These changes, however, often take a considerable length of time to occur. Thus there is some interest in analyzing briefly the optimal cost share given a fixed output share. This we do in Section III.

(c) To some readers of an earlier version of this paper, the contracts analyzed in Section II seemed unnecessarily restrictive. Hence, in the last Sections we show that the general result, that the cost share may differ markedly from the output share, is valid under considerably more general conditions than those analyzed there, although the particular value of the cost share will, clearly, be altered if we consider more general contract structures.

To demonstrate this last point we first follow Stiglitz (1974) in analyzing the set of linear contracts <sup>3/</sup> (i.e. we allow fixed fees for the use of land). In that case, of course, if tenants are risk neutral, the optimal contract will entail a pure rental contract, and the incentive problems with which this paper is concerned disappear.<sup>4/</sup>

Secondly, we address the question of whether the contract can specify the quantity of the input; if a cost sharing rule is used, it must be the case that the quantity of input is observable. If that is the case, cannot a contract just as well specify the level of input as attempt to induce that level of input through some cost sharing rule? We show in Section IV that if, in fact, linear contracts can be employed (i.e. contracts which include a fixed fee) then the equilibrium that emerges with the optimal cost sharing rule is equivalent to one in which the quantities of fertilizer (or other inputs) are prespecified.

This analysis, however, makes one crucial, and unrealistic assumption—that the "optimal" level of input can be known at the time the contract is designed. But more generally the optimal level of input needs to be changed in response to variations in, say, weather. If these changes in circumstances are observable to the tenant but not to the landlord 5/ then he wishes to design a contract which induces the tenant to adjust his input in response to these changes in circumstances. A cost sharing arrangement does this; a contract which specifies a fixed quantity does not. In Section V we show that only minor changes are required to our earlier analysis to incorporate the variability of inputs in response to changes in the environment.

Elsewhere, Nalebuff and Stiglitz [1982] have referred to this property of contracts, that they induce changes in behavior in response to environmental changes, as that of flexibility. Although our formal analysis is limited to an examination of "flexibility" of the contract within the period, e.g. to changes in weather, it is important to note that there is a broader issue of flexibility in response to larger, more significant changes,

e.g. the availability of new seeds or fertilizers, the construction of a dam, or a marked change in prices. The full implications of these changes are seldom understood at the time they first occur. A contract which specified the amount of fertilizer of one type to be used would have to be renegotiated as soon as slightly different fertilizers became available: how many units of the new fertilizer are equivalent to one unit of the old fertilizer. A cost sharing contract does not require these elaborate negotiations. Eventually, as technological changes occur and information about the consequences of new technologies becomes available as these new technologies are employed, if the terms of the contract come to differ significantly from the new "optimal" terms, there will, of course, be a readjustment in the terms of the contract.

## II. THE BASIC MODEL

### II.1 Tenant's Problem

All tenants are assumed to be identical. They lease from the landlord a plot of land whose size is assumed to be technologically fixed. The tenant determines his labor effort input,  $e$ , and fertilizer input,  $\frac{6}{x}$ , taking into account the terms of the contract as given. The terms of the contract include his output share,  $\alpha$ , and his cost share,  $\beta$ . The production function is concave in the two factors of production, effort and fertilizer. Output is uncertain due to the changing states of nature. For simplicity we model uncertainty in multiplicative form. Hence, the tenant's income,  $Y$ , is:

$$Y = \alpha g f(e, x) - \beta P x$$

where  $g$  denotes the non-negative, multiplicative uncertainty factor

distributed according to  $h(g)$ , with mean  $E(g) = 1$ , and  $P$  denotes the fertilizer market price; the output price is normalized to 1. The tenant maximizes his expected utility of income and labor effort, i.e.,

$$\max_{\{e,x\}} EU [Y(e, x), e] \equiv V(\alpha, \beta) \tag{1}$$

The two first-order conditions can be rewritten as:

$$f_x = \frac{\beta}{\alpha} P / \frac{EU_1 g}{EU_1} = \frac{\beta P}{\alpha \rho} \tag{2}$$

and

$$f_e = -\frac{1}{\alpha} / \frac{EU_1 g}{EU_2} = -\frac{1}{\alpha \rho} \cdot \frac{EU_2}{EU_1} \tag{3}$$

where  $\rho \equiv EU_1 g / EU_1$  is the risk premium factor satisfying  $0 < \rho < 1$  since  $COV U_1(g)g < 0$ .<sup>7/</sup> For risk neutral tenants,  $\rho = 1$ , and for infinitely risk averse tenants  $\rho = \min g$ .

Conditions (2) and (3) imply the fertilizer and effort supply functions  $x(\alpha, \beta)$ ,  $e(\alpha, \beta)$ . By substituting  $x(\alpha, \beta)$  and  $e(\alpha, \beta)$  into (1) at the optimum we obtain the tenant's indirect utility function  $EU\{Y[e(\alpha, \beta), x(\alpha, \beta)], e(\alpha, \beta)\} \equiv V(\alpha, \beta)$ .

We assume that the tenant has an alternative occupation yielding a given expected utility level  $\bar{V}$ . Thus, in order to accept a tenancy position, he requires that  $V(\alpha, \beta) > \bar{V}$ . For the remainder of the discussion, we shall assume that the utility constraint is always binding. (In general, this need not be the case, and there are straightforward modifications to our analysis).



## II.2 The Landlord's Problem

When the utility constraint is binding the landlord's problem is to search over the set of contracts (here, defined by  $\alpha$  and  $\beta$ ) which yield the same  $\bar{V}$ , for the contract which maximizes his expected utility. In this maximization the landlord takes into account the responses of the tenant-in terms of effort and other inputs-to changes in the contract <sup>8/</sup> For simplicity we assume that the landlord is risk-neutral and, therefore, maximizes his expected profits. Since  $Eg = 1$ , we can write the landlord's problem as:

$$\max_{(\alpha, \beta)} \Pi = (1 - \alpha)f(e, x) - (1 - \beta)Px \quad (4)$$

subject to

$$V(\alpha, \beta) = \bar{V} . \quad (4a)$$

The landlord's controls are the output share,  $\alpha$ , and the cost share,  $\beta$ . From the utility equivalence constraint, we can derive the relation  $\alpha(\beta)$  which implies the pairs of output share and cost share, maintaining the tenant on his iso-utility  $\bar{V}$ . Substituting  $\alpha(\beta)$  into (4) we obtain:

$$\max_{\beta} \Pi(\beta) = \{1 - \alpha(\beta)\} f\{e(\beta), x(\beta)\} - (1 - \beta) Px(\beta) \quad (5)$$

From the first order condition of (5) we can derive the cost-sharing rules.

We assume that  $\Pi_{\beta\beta} < 0$  for the existence of the maximum.

### II.3 Cost-Sharing Rules

#### Case 1: The Equal Shares Rule: Neither Risk Sharing nor Incentive Effects

Consider first the case where there is no effort incentive effect, i.e., effort supply is perfectly inelastic at  $e = \bar{e}$  and there is no uncertainty i.e.,  $\rho = 1$ . Under such a structure, the tenant's first order condition (2) becomes

$$f_x = \frac{\beta}{\alpha} P. \quad (2')$$

Deriving the first order condition from (5) where  $e = \bar{e}$ , we obtain:

$$\Pi_\beta = [(1 - \alpha)f_x - (1 - \beta)P] x_\beta \Big|_{\bar{v}} - f \frac{d\alpha}{d\beta} \Big|_{\bar{v}} + Px = 0 \quad (6)$$

Total differentiation of  $V(\alpha, \beta) = \bar{v}$  implies  $\frac{d\alpha}{d\beta} \Big|_{\bar{v}} = \frac{Px}{f}$ . Substituting this relation and (2') into (6) we obtain:

$$\frac{d\Pi}{d\beta} = \left\{ \frac{(1 - \alpha)\beta}{\alpha} - (1 - \beta) \right\} Px_\beta \Big|_{\bar{v}} = 0 \quad (7)$$

Equation (7) implies that the optimal solution is  $\alpha = \beta$ . Clearly the first order condition (2') then becomes  $f_x = P$  which is the first-best rule for the application of fertilizers. This rule states that the tenant's internal price for fertilizer,  $\frac{\alpha}{\beta} P$ , equals the external market price  $P$ . Hence, the following proposition (first derived by Heady [1947]):

Proposition 1: In the absence of both incentive and risk-sharing effects, the landlord will choose his cost share in the fertilizer input to equal his output share, i.e.,  $\alpha = \beta$ .

Case 2: No Incentive Effects: Only Risk-Sharing Effects

Consider now the objective function (5) with the absence of incentive effects, i.e.,  $e = \bar{e}$ , but with the presence of output uncertainty  $g$ . The tenant's first order condition with respect to fertilizer includes the risk premium factor as specified in (2). In addition,  $\frac{d\alpha}{d\beta}$  also includes the risk premium factor  $\rho$ , i.e.,  $\frac{d\alpha}{d\beta}\bigg|_{\bar{v}} = \frac{1}{\rho} \frac{Px}{F}$ . Evaluating the first order condition at the point  $\alpha = \beta$  and collecting terms, we obtain:

$$\Pi_{\beta}\bigg|_{\alpha=\beta} = \left(\frac{1}{\rho} - 1\right) [(1 - \alpha)Px_{\beta}\bigg|_{\bar{v}} - Px] \quad (8)$$

Since  $\rho < 1$ , a sufficient condition for  $\Pi_{\beta} < 0$  is for  $x_{\beta}\bigg|_{\bar{v}} < 0$ .

Now since  $x(\beta) = x\{\alpha(\beta), \beta\}$ ,

$$x_{\beta} = \frac{\partial x}{\partial \alpha} \frac{d\alpha}{d\beta}\bigg|_{\bar{v}} + \frac{\partial x}{\partial \beta} \quad (9)$$

(This is a compensated change in the input share).

It is clear that  $\frac{\partial x}{\partial \beta}$ , the direct effect of an increase in the tenant's cost share on his fertilizer input, is negative; increasing the internal price of fertilizer will reduce fertilizer usage by the concavity of the production function. The indirect effect,  $\frac{\partial x}{\partial \alpha} \frac{d\alpha}{d\beta}$ , can go in the other direction since  $\frac{d\alpha}{d\beta}\bigg|_{\bar{v}}$  is clearly positive, and if fertilizers and effort are complements in production an increase in the output share  $\alpha$  may increase the fertilizer input. However, as long as the direct effect is stronger than the indirect effect,  $x_{\beta}\bigg|_{\bar{v}} < 0$ . Thus, under this condition  $\Pi_{\beta}\bigg|_{\beta=\alpha} < 0$ . Since  $\Pi_{\beta\beta} < 0$ , for  $\Pi_{\beta} = 0$  to be satisfied, the tenant's cost share  $\beta$  must be less than his output share,  $\alpha$ . Hence:

Proposition 2: In the presence of uncertainty with a perfectly in elastic supply of effort (labor) by the tenant, (i.e., no incentive effects in the effort supply), and when the tenant is risk-averse while the landlord is risk-neutral, the landlord will choose the tenant's cost-share  $\beta$  to be lower than the tenant's output share  $\alpha$ , i.e.,  $\beta < \alpha$ , provided a compensated increase in input share leads to a reduction in the input.

Remark: The internal price of fertilizer is therefore  $\frac{\beta}{\alpha} P < P$ . The risk-neutral landlord subsidizes the tenant's fertilizer price in order to compensate for the under supply of fertilizer due to the risk aversion of the tenant.

In the appendix we derive several sufficient conditions for  $x_{\beta} < 0$ .

- (i) Provided risk is not too large, inputs are always reduced.
- (ii) If the individual has constant absolute risk aversion, inputs are reduced.

In addition, for the case of constant relative risk aversion we obtain the stronger result that as risk aversion increases,  $\beta/\alpha$  is monotonically reduced, i.e., the landlord increases his share of the cost relative to his output share.

Case 3: Only Incentive Effects

In this case, since there is no uncertainty,  $\rho = 1$ . However, the landlords cannot monitor or enforce the tenant's effort supply. Hence, the first order condition of (5) evaluated at  $\alpha = \beta$  becomes

$$\Pi_{\beta} \Big|_{\alpha=\beta} = (1 - \alpha) f_{e_{\beta}} \Big|_{\bar{v}} \quad (10)$$

Hence, the following proposition:

Proposition 3: In the absence of uncertainty, but where effort supply is elastic (i.e., there is an incentive effect) the landlord will choose the tenant's cost share  $\beta$  to be lower (equal, bigger) than the tenant's output share,  $\alpha$ , i.e.,  $\beta \begin{matrix} < \\ > \end{matrix} \alpha$ , as  $e_{\beta|\bar{v}} \begin{matrix} < \\ > \end{matrix} 0$ .

$$e_{\beta|\bar{v}} = \frac{\partial e}{\partial \alpha} \frac{d\alpha}{d\beta} \Big|_{\bar{v}} + \frac{\partial e}{\partial \beta} \quad (11)$$

Again we have to evaluate the direct and indirect effects, but now there is no clear presumption concerning the sign. The compensated increase in  $\alpha$  does tend to increase effort; but the increase in the cost share reduces the inputs, and the reduction in these will (if  $x$  and  $e$  are complements) tend to reduce effort. Indeed, in the special case of a Cobb-Douglas production function effort is left unchanged, so  $\alpha = \beta$ . For the case where there is a low elasticity of substitution (high degree of complementarity),  $e_{\beta|\bar{v}} < 0$  so  $\beta < \alpha$ . (See Appendix).

Case 4: Presence of Both Risk-Sharing and Incentive Effects

In this case, the first order condition of (5) evaluated at  $\alpha = \beta$  becomes

$$\Pi_{\beta} \Big|_{\alpha=\beta} = \left(\frac{1}{\rho} - 1\right) [(1 - \alpha)Px_{\beta} \Big|_{\bar{v}} - Px] + (1 - \alpha)f_e e_{\beta} \Big|_{\bar{v}}. \quad (12)$$

From the preceding discussion arises the following proposition:

Proposition 4: In the presence of both uncertainty and an elastic effort supply,  $\beta < \alpha$  only if the incentive effects are such that (a)  $e_{\beta|\bar{v}} < 0$  or (b) the positive incentive effects are dominated by the negative risk-sharing effects.

### III. FIXED OUTPUT SHARE

The previous section analyzed the equilibrium cost sharing arrangement in the long run, when the output share can be adjusted to compensate for changes in the cost share. In some circumstances, the output share may be fixed by social norms or legislation; or even if variable, it may adjust slowly. In this section, we ask, if the output share is fixed, what cost sharing rule maximizes landlords expected income?

Formally, we simply

$$\max_{\{\beta\}} \Pi$$

This differs from our earlier analysis in two respects: there, we had two controls,  $\alpha$  and  $\beta$ , and we had a utility constraint (4a); now we have a single control,  $\beta$ , and ignore the utility constraint <sup>9/</sup>. The solution to this problem is straightforward; we require

$$\frac{\partial \Pi}{\partial \beta} = (1 - \alpha) \left( f_e \frac{\partial e}{\partial \beta} + f_x \frac{\partial x}{\partial \beta} \right) - P(1 - \beta) \frac{\partial x}{\partial \beta} + Px = 0$$

At  $\alpha = \beta$  we have, as before, (eq. (2))

$$f_x = \frac{P}{\rho}$$

so that

$$\frac{\partial \Pi}{\partial \beta} \Big|_{\alpha=\beta} = (1 - \alpha) \left[ f_e \frac{\partial e}{\partial \beta} + \left( \frac{1}{\rho} - 1 \right) \frac{\partial x}{\partial \beta} \right] + Px = 0 \quad (13)$$

An increase in  $\beta$  normally will reduce  $x$ , and hence  $e$ , so the bracketed term of (13) is negative, while the second term is positive. It is clear that

although there is a strong presumption that  $\alpha \neq \beta$ , one cannot, without further assumptions determine whether  $\alpha \gtrless \beta$ . If, for instance  $\rho = 1$  (risk neutrality), and effort is relatively inelastic ( $\frac{\partial e}{\partial \beta}$  is small), then  $\beta > \alpha$ . Thus, in this case even though the landlord could lower  $\beta$ , without losing his tenant, it is the incentive effects which keep the landlord from reducing the cost share.

#### IV. A BASIC EQUIVALENCE RESULT

In this section we establish two results. First we show that the analysis of section II can be extended to rather more general contractual arrangements, without changing qualitatively the kinds of results obtained (although the precise value of  $\beta$ , the cost sharing rule, will, in general be different). Secondly, we establish an equivalence result similar to that established in the earlier literature on risk and sharecropping (Stiglitz [1974]). There it was shown that by mixing rental and wage contracts, tenants could reduce the risk they faced in exactly the same way that a sharecropping contract reduced risk: sharecropping contracts were thus shown to be unnecessary. Only when incentive considerations were introduced could sharecropping be explained.<sup>10/</sup> Here, we show that one can construct contracts which specify inputs and contracts with cost sharing which are equivalent; within the context of the model analyzed here, cost sharing is completely unnecessary. The resolution of this paradox is provided in Section V.

##### IV.1 Linear Sharecropping Contracts

We assume now that a sharecropping contract is specified by  $\alpha$ , the share of output received by the tenant,  $\beta$  the share of costs of inputs which he must pay, and  $\gamma$ , a fixed payment from the tenant to the landlord (may be positive or negative).

The expected income of the landlord is now

$$\Pi = (1 - \alpha)f - P(1 - \beta)x + \gamma \quad (14)$$

while the expected utility of the tenant is now

$$\max_{\{e, x\}} EU[(\alpha gf - \beta Px - \gamma), e] \equiv V(\alpha, \beta, \gamma) \quad (15)$$

The contract is chosen to

$$\max_{(\alpha, \beta, \gamma)} \Pi \quad (16)$$

subject to

$$V(\alpha, \beta, \gamma) \geq \bar{V} \quad (17)$$

This problem is a straightforward extension of that analyzed in section II and yields the following first order conditions:

$$\frac{\partial \Pi / \partial \alpha}{\partial \Pi / \partial \gamma} = \frac{-f + (1 - \alpha)(f_e \frac{\partial e}{\partial \alpha} + f_x \frac{\partial x}{\partial \alpha}) - P(1 - \beta) \frac{\partial x}{\partial \alpha}}{1 + (1 - \alpha)(f_e \frac{\partial e}{\partial \gamma} + f_x \frac{\partial x}{\partial \gamma}) - P(1 - \beta) \frac{\partial x}{\partial \gamma}} = \frac{EU_1 f g}{EU_1} = - \frac{\partial V / \partial \alpha}{\partial V / \partial \gamma} \quad (18)$$

$$\frac{\partial \Pi / \partial \beta}{\partial \Pi / \partial \gamma} = \frac{Px + (1 - \alpha)(f_e \frac{\partial e}{\partial \beta} + f_x \frac{\partial x}{\partial \beta}) - P(1 - \beta) \frac{\partial x}{\partial \beta}}{1 + (1 - \alpha)(f_e \frac{\partial e}{\partial \gamma} + f_x \frac{\partial x}{\partial \gamma}) - P(1 - \beta) \frac{\partial x}{\partial \gamma}} = Px = \frac{\partial V / \partial \beta}{\partial V / \partial \gamma} \quad (19)$$

At  $\alpha = \beta$ , the tenant sets

$$f_x = \frac{P}{\rho} \quad (20)$$



Hence

, (19) can be rewritten as

$$f_e \frac{de}{d\beta} \bar{v} + f_x \frac{dx}{d\beta} \bar{v} (1 - \rho) = 0 \quad (21)$$

The normal presumption is that  $\frac{dx}{d\beta} \bar{v} < 0$ , while  $\frac{de}{d\beta} \bar{v}$  can be of either sign. If  $\frac{de}{d\beta} \bar{v} < 0$  then clearly  $\beta > \alpha$ ; if  $\frac{de}{d\beta} \bar{v} > 0$  and  $\rho$  is near unity, then  $\beta < \alpha$ . Clearly, it is much more likely that  $\beta > \alpha$  than in our earlier analyses.

#### IV.2 Direct Control of Fertilizers' Input

Consider now the problem of the landlord who can control  $x$  directly. He then

$$\max_{\{\alpha, x, \gamma\}} \Pi = (1 - \alpha)f - Px + \gamma \quad (22)$$

$$\text{s.t. } EU(\alpha f - \gamma, e) > \bar{v}$$

where the landlord takes into account (as before) the response of effort,  $e$ , to the terms of the contract (eq. (3), above). Thus, the first order conditions are:

$$\frac{\partial \Pi}{\partial \alpha} / \frac{\partial \Pi}{\partial \gamma} = \frac{-f + (1 - \alpha) f_e \frac{\partial e}{\partial \alpha}}{1 + (1 - \alpha) f_e \frac{\partial e}{\partial \gamma}} = - \frac{EU_1 f g}{EU_1} = \frac{\partial V / \partial \alpha}{\partial V / \partial \gamma} \quad (23)$$

$$\frac{\partial \Pi}{\partial x} / \frac{\partial \Pi}{\partial \gamma} = \frac{(1 - \alpha) f_x - P}{1 + (1 - \alpha) f_e \frac{\partial e}{\partial \gamma}} = \frac{\alpha EU_1 f_x g}{EU_1} = \frac{\partial V / \partial x}{\partial V / \partial \gamma} \quad (24)$$

It is easy to show that in fact the solutions to (16) and (22) are equivalent in the sense that for a given level of  $\bar{V}$ ,  $\bar{\Pi} = \max \Pi$  is the same. Moreover, incomes of tenants and of landlords in each state of nature, effort level and inputs are all the same under the two problems. To see that, denote by carets the solution to (22) and by asteriks the solution to (16). Let

$$\alpha^* = \hat{\alpha} \tag{25a}$$

$$\gamma^* = \hat{\gamma} - \beta^* \hat{x} P \tag{25b}$$

$$\beta^* = \frac{\hat{\alpha} \hat{p} \hat{f}}{P} \tag{25c}$$

Recall the tenant's first order conditions (2) and (3) and rewrite them with asteriks, i.e.

$$\alpha^* \rho^* f^*_{x} = P \beta^* \tag{26a}$$

and

$$\alpha^* \rho^* f^*_{e} = - \frac{EU_2}{EU_1} \tag{26b}$$

To establish our result, all we need to do is to show that if

$$x^* = \hat{x} \tag{27a}$$

and

$$e^* = \hat{e} \tag{27b}$$

both (26a) and (26b) are satisfied; i.e. the contract  $(\alpha^*, \beta^*, \gamma^*)$  generates the same level of fertilizers and effort as the contract which specified the fertilizer input,  $\hat{x}$ , directly, in addition to  $(\hat{\alpha}, \hat{\gamma})$ . But this result is

immediate; if  $\hat{x} = x^*$ ,  $\hat{e} = e^*$  then by using (25a) and (25b) we obtain

$$\hat{Y} = Y^* \quad \text{for every state of nature} \quad (28a)$$

From this follows

$$\hat{\rho} = \rho^* \quad (28b)$$

Hence

$$\frac{\alpha^* \rho^* f_x^*}{P} = \frac{\hat{\alpha} \hat{\rho} \hat{f}_x}{P} = \beta^* \quad (29)$$

The first equality follows from (25a), (28b) and (27). The second equality follows from (25c). Thus (26a), the first order condition for fertilizers is clearly satisfied. It is then immediate that (26b) will be satisfied. Given that  $e^* = \hat{e}$  and  $x^* = \hat{x}$ , it is immediate that the landlord's profit under the contract described by equations (25) is identical to that with the optimal cost sharing rule in each state of nature. Thus, indirect control of inputs through cost sharing is just as good as direct control, provided the other terms of the contract can be adjusted appropriately.

#### V. COST SHARING AND INCENTIVE FLEXIBILITY

The previous section made it clear that the issues of cost sharing cannot really be adequately addressed within the simple framework which we have employed thus far. Just as sharecropping cannot be understood without taking into account consideration of incentives, so too here: an important issue in cost sharing is not only the level of incentives but their

flexibility, their ability to allow for adjustments in behavior response to changes in the environment. Thus if the landlord and the tenant have identical information we established that it made no difference whether the landlord specified the level of inputs or adopted the appropriate cost sharing arrangement. Assume now that there is some aspect of the technology which varies, say, with the weather which affects the productivity of an input such as fertilizer. Thus our production function is now

$$Q = f(e, ux)g \quad (30)$$

where  $u$  is a random variable (normalized to have mean of unity) which is observable to the tenant but not to the landlord. Moreover the tenant can observe  $u$  before making his decision concerning the input of fertilizer. In the first best world of perfect information, and perfect insurance, (risk neutrality)

$$uf_x = P \quad (31)$$

However, with cost sharing and sharecropping under risk aversion, (31) becomes

$$\alpha u p f_x = \beta P \quad (32)$$

Thus,  $f_x$  will still vary with  $u$ . When  $u$  is higher,  $x$  will normally be higher. It is easy to modify our earlier formula to show that in general  $\alpha$  is still not equal to  $\beta$ . We can also show that in general, at least some form of cost sharing (as opposed to direct controls) is desirable. Assume that the landlord provides input  $x_0$ , but sets  $\beta = 1$  for purchases in excess of  $x_0$ .

Let us then calculate the compensated derivative with respect to  $\beta$

at  $\beta = 1$ . Denoting by  $\tilde{x}$  the private (non specified) purchase of the input, we obtain

$$\frac{\frac{\partial \Pi}{\partial \beta}}{\frac{\partial \Pi}{\partial \gamma}} = \frac{E[\tilde{x} P + (1 - \alpha) (f_e \frac{\partial e}{\partial \beta} + u f_x \frac{\partial \tilde{x}}{\partial \beta})]}{E[1 + (1 - \alpha) (f_e \frac{\partial e}{\partial \gamma} + u f_x \frac{\partial \tilde{x}}{\partial \gamma})]} < E\tilde{x}P, \quad (33)$$

provided  $\frac{\partial e}{\partial \beta} \left| \frac{f_e}{V} + \frac{\partial \tilde{x}}{\partial \beta} \right| \frac{f_x}{V} < 0$  and provided for each  $u \tilde{x} > 0$

while

$$\frac{\frac{\partial V}{\partial \beta}}{\frac{\partial V}{\partial \gamma}} = E\tilde{x}P \quad (34)$$

Hence with perfect information (no variability in  $u$ )  $\tilde{x} = 0$ , and (33) and (34) will be equal. But with private purchases (for  $u$  large enough) it always pays the landlord to lower  $\beta$  below unity to induce the tenant to purchase more fertilizers.

## VI. CONCLUDING REMARKS

In recent years there has been considerable interest in the analysis of incentive problems, situations in which one individual (the landlord, lender, employer), generally referred to as the principal, seeks to affect the actions of another (the tenant, borrower, employee), generally referred to as the agent, by appropriately choosing the terms of the contract between them. The interests of the two are assumed to differ and it is assumed prohibitively expensive for the principal to monitor directly the actions of the agent. 11/

Thus the principal must base his compensation scheme on the outputs (or other observable variables) of the agent.

The prototype of this kind of relationship is that between the landlord and his tenant. In this "new view" sharecropping is not seen just as an inefficient anachronism (this last characterization is unjustifiably attributed to Marshall <sup>12/</sup>). Rather, sharecropping contracts are viewed as playing an important role both in sharing risks and providing incentives. (e.g. see Stiglitz [1974]).

Most of the earlier analysis of the principal agent relationships have focused on situations where the agent has a single variable under his control (in the sharecropping model, effort). But in most situations, the tenant may have several variables -- effort, choice of technique, level of other inputs -- which he can determine. The terms of the contract affect all of these decisions. Analysis focusing on a single decision may accordingly be very misleading. In addition, the landlord may consider the impact of other contracts on the tenant's control variables. This may lead to the interlinking of agrarian contracts. (See Bardhan [1980]), Bell-Zusman [1980], Braverman-Srinivasan [1981], Braverman-Stiglitz [1982] and Mitra [1982]).

In addition the recent literature on principle agent problems has stressed the importance not only of the risk and incentive properties of alternative contractual arrangements, but also their flexibility, their ability to adapt to changes in the environment (Nalebuff and Stiglitz [1982]). The discussion of this paper can be viewed as an important application of all these general principles of principal-agent problems.

Our analysis has been directed towards two questions: why is cost sharing used at all; if cost sharing is used, the input must be observable, and if it is observable, why does not the contract simply specify the level of

input? And why, if it is used, are such a variety of cost sharing rules employed? Why, in other words, is not the simple rule of equating the cost share to the output share, which would seem to equate the marginal benefit of the input to the marginal cost, always employed.

In answering these questions we conduct our analysis in two parts. In the first part we allow only proportional sharing rules, while in the second part the contract includes also fixed fee component. We know from contract theory that mere optimality considerations (ignoring transaction costs) will always imply that contracts be non-linear. Clearly, it is also the case that linear contracts will be preferred to proportional contracts. However, there is substantial evidence (see Singh [1982], chapter 2) that often in LDC's rental contracts between landlords and tenants are either of the form of pure fixed rents or pure share rents. In the environments characterized by sharecropping, there is also a wide empirical documentation regarding arrangements of costs sharing. e.g. Marshall [1920, p. 645] notes the existence as well as the usefulness of the practice of sharing inputs in the United States and France. Studies by Ladejinsky [1977] and Rao [1965] report a 50:50 equal share rule to be prevalent in India, while Ashok Rudra [1975] documented a wider variety of cost-sharing arrangements in West Bengal. Therefore, in the first part of the paper we provide some insights into why such variety of cost sharing rules are observed.

In particular we show that when there is cost sharing, setting the cost share equal to the output share will be optimal only if (a) there are no incentive effects associated with the use of the input; otherwise an increase in the input of fertilizer may lead to an increase in the supply of effort; because of the sharecropping contract, individuals are supplying too little effort, and hence it may be desirable to subsidize fertilizer to partially

offset the distortion with respect to the labor supply, i.e.  $\beta < \alpha$ ; and (b) if tenants have the same degree or risk aversion as landlords. However, if landlords are risk neutral, but tenants risk averse then by increasing the cost share one can write a contract which generates the same expected utility to the tenant, but increases the expected profit of the landlord.

In the second part of the paper we allow fee landlord both to specify the level of input and to use a fixed fee component in the contract.<sup>13/</sup> Even in such a framework some degree of cost sharing (as opposed to complete specification of inputs by the landlord) is preferable, so long as there is some variability in the productivity of the inputs, and so long as the tenant has better information concerning the productivity than does the landlord. If the landlord and tenant have identical information,<sup>14/</sup> then the appropriately designed cost sharing arrangement is equivalent to complete specification of the inputs by the landlord.

In concluding we should restate that we have not provided in this paper an explanation for the two more puzzling aspects of the prevalent forms of sharecropping—that this contracts involve simple proportional sharing rules (i.e. they are not non-linear and they do not include fixed fees). one explanation for the absence of fixed fees may be that quite often tenancy contracts are interlinked with credit contracts, and the fixed interest component of the interlinked loan contract is equivalent to a fixed fee in the rental contract, and thus make such additional fixed fee unnecessary.



APPENDIX

Effects of Cost Shares on Effort and Inputs

The two first order conditions for the problem (1) are:

$$EU_1 [\alpha g f - \beta P x, e] (\alpha g f_x - \beta P) = 0 \quad (A.1)$$

$$EU_1 [\alpha g f - \beta P x, e] \alpha g f_e + EU_2 [\alpha g f - \beta P x, e] = 0 \quad (A.2)$$

Taking the total differential of (1) and (2), we obtain (letting  $\alpha = \alpha(\beta)$ , by the utility equivalence relation)

$$\begin{aligned} & \left[ \begin{array}{cc} EU_{11} Y_x^2 + EU_1 Y_{xx} & E\{U_{11} Y_e Y_x + U_1 Y_{ex} + U_{12} Y_x\} \\ E\{U_{11} Y_x Y_e + U_1 Y_{ex} + U_{21} Y_x\} & E\{U_{11} Y_e^2 + U_1 Y_{ee} + U_{12} Y_e + U_{22}\} \end{array} \right] \begin{array}{l} dx \\ de \end{array} = \\ & - \left[ \begin{array}{c} x \left( \frac{EU_{11} g Y_x}{EU_1 g} EU_1 - EU_{11} Y_x \right) + EU_1 \left( \frac{f_x}{f} - 1 \right) \\ x \left( \frac{EU_{11} g Y_e}{EU_1 g} EU_1 - EU_{11} Y_e \right) + \frac{EU_1 f_e x}{f} + \left( \frac{EU_{12} g EU_1}{EU_1 g} - EU_{12} \right) \end{array} \right] \cdot (Pd\beta) \end{aligned}$$

Two special cases:

(a) No effort elasticity. Then

$$\frac{dx}{d\beta} \Big|_{\bar{V}} \geq 0 \text{ as } \left[ \frac{EU_{11} g Y_x x}{EU_1 g} - \frac{EU_{11} Y_x x}{EU_1} \right] + \left( \frac{f_x}{f} - 1 \right) \geq 0$$

The second term is always negative. In the absence of uncertainty, the first

term is zero. The first term is negative with constant absolute risk aversion if the variance of  $g$  is small, since  $EU_{11}Y_x = -AEU_1Y_x = 0$  where  $A = -U_{11}/U_1$  is the measure of absolute risk aversions and  $EU_{11}gY_x = -AEgU_1Y_x < 0$

Thus, provided risk is not too great or risk aversion does not change too rapidly,

$$\frac{dx}{d\beta} < 0.$$

With constant relative risk aversion and separable utility functions, we can write (A.1) as

$$E\left(\frac{\alpha}{\beta} gf - Px\right)^{-R} \left(\frac{\alpha}{\beta} gf_x - P\right) = 0.$$

Taking the derivative with respect to  $R$ , we obtain

$$- E\left(\frac{\alpha}{\beta} gf - Px\right)^{-R} \left(\frac{\alpha}{\beta} gf_x - P\right) \ln \left(\frac{\alpha}{\beta} gf - Px\right) < 0,$$

since

$$\frac{d\left(\frac{\alpha}{\beta} gf - Px\right)}{dg} = \frac{\alpha}{\beta} f > 0. \quad \text{Hence } \frac{d\alpha}{dR} < 0.$$

(b) No risk. Then at  $\alpha = \beta$

$$\frac{de}{d\beta} \lesseqgtr 0 \quad \text{as } \alpha \left[ f_{xx} \frac{f_{ex}}{f} - \left( \frac{f_{xx}}{f} - 1 \right) f_{ex} \right] \gtrless 0.$$

Recall the definition of the elasticity of substitution

$$\sigma = \frac{f_x}{f} \frac{f}{f_{xx}},$$

and let  $S_x$  = share of factor  $x = \frac{f_x x}{f}$ , and

$$\eta = - \frac{f_x}{f_{xx} x}, \text{ the price elasticity of the demand for factor } x.$$

Thus

$$\frac{de}{d\beta} \gtrless 0 \text{ as } \frac{\eta}{\sigma} (1 - S_x) \gtrless 1$$

For a Cobb-Douglas production function  $f = x^{S_x} e^{S_e}$ . Hence,

$$\eta = \frac{1}{1 - S_x}$$

and

$$\frac{de}{d\beta} = 0.$$

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Footnotes

- 1/ There is, effectively, no risk impact of a compensated change in the cost share. However, if landlords do not charge any fixed fee on the use of their land, then even when there is no incentive effect, the cost share may be less than the output share, because by lowering the tenant's cost share, the tenant's output share can be lowered (keeping the individual at the same level of expected utility) and this will reduce the risks that he faces.
- 2/ Thus, the problem here can be viewed as that of characterizing the constrained pareto efficiency contracts, where we take the form of the contract as given. The fact that the contracts are constrained pareto efficient, in this particular sense, does not, of course imply anything about the constrained pareto efficiency of the economy as a whole. If there is more than one commodity, the market equilibrium will not in general be constrained efficient. See, for instance, Hart [1975], Stiglitz [1982] and Newbery-Stiglitz [1982].
- 3/ Clearly if rents are paid at the end of the period, and there is some probability of default, then there are still important incentive problems. See, Stiglitz and Weiss [1981].
- 4/ One could, of course, consider the more general case of non-linear incentive contracts. We do not do so here for two reasons: first, most of the contracts observed do seem to be of the simple linear form investigated here; secondly, to do so would simply complicate the analysis, without altering the basic qualitative conclusions. For a general discussion of non-linear incentive contracts, see Stiglitz [Forthcoming].

- 5/ All that is really required is that there be differential information between the two. Alternatively, we could have assumed that the costs of writing a contract which specified the level of input corresponding to each state of nature are prohibitive.
- 6/ Henceforth fertilizers will be used as an example of raw material input.
- 7/ The larger  $g$  is, the larger is output  $gf$ ; hence, the larger is the tenant's income, and by the concavity of  $U$ , the smaller is the marginal utility of income.
- 8/ Such an equilibrium (explored in Stiglitz [1974]) is accordingly sometimes referred to as a utility equivalent contract equilibrium. See Braverman-Stiglitz [1982] and Braverman-Srinivasan [1981] for further discussions of both utility equivalent and non-equivalent contract equilibria in rural developing economies.
- 9/ It should be obvious that there are a large number of other possible variations around this theme. Both here and in the previous section, we have taken plot size as given. This too could be taken as a variable. See, for instance, Braverman-Stiglitz [1982]. Here, and in the preceding section, we have taken the fixed rent component as invariant (as set, for simplicity, at zero). In the next section we allow this too to be a control variable.
- 10/ Transaction cost differences provide an alternative possible explanation, see e.g. Cheung [1969].
- 11/ It is in this sense that these problems differ from the team theory problems.
- 12/ Marshall [1920] recognized the importance of share contracts in a world dominated by market imperfections and the absence of certain markets. The so-called "Marshallian School" of sharecropping was originated out

of a technical footnote rather than the main text. (See Bliss-Stern [1981], Chapter 3, and Jaynes [1982] on this point).

13/ For completeness, we should, perhaps, have dealt with the case of proportional contract with direct specification of inputs by the landlord. However, the analysis of this case is straightforward given the framework provided in the main text.

14/ They both know the state of nature. We require, in addition, that it must be possible to write a contract which is contingent on the state of nature. This may not be possible, not only because of the inherent complexity of the contracts which would result. To write contracts with the terms of the contract contingent on the state of nature, this state must not only be observable, it must be verifiable. The landlord and the tenant may both "know" that the weather has been good, but the tenant may claim that it has not been good; for the contract to be enforceable, it must be possible for there to be third party verification. Alternatively, the contract can be designed so that informed parties to the contract have the incentive to reveal the information, i.e. they satisfy certain self-selection or incentive compatibility constraints. These considerations imply that even if there were symmetric information, cost sharing may still be desirable.