A MODEL OF EMPLOYMENT OUTCOMES ILLUSTRATING THE EFFECT OF THE STRUCTURE OF INFORMATION ON THE LEVEL AND DISTRIBUTION OF INCOME*

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1. Introduction

The purpose of this paper is to present a model of the way in which employers use information about workers to make salary and job placement decisions in a model in which they have imperfect information about workers' ability. In the model, job placement decisions are important because workers are more productive if they work at jobs appropriate to their ability. A skilled worker who works at an unskilled job produces less than he would if he worked at a skilled job (this is Equation (1) below). Equivalently--at least in our model--an unskilled worker is more productive at an unskilled job than a skilled worker (this is Equation (1') below). This model generalizes and synthesizes previous theoretical work, in particular that of Aigner and Cain (1977).

We built this model to emphasize the following points. First, in a world of imperfect information, the value of a credential or characteristic is composed of both a direct and an indirect effect. The direct effect is the characteristic's direct contribution to productivity. The indirect effect is the information which the characteristic conveys about other characteristics which contribute directly to productivity. These effects differ in both sign and magnitude. There seems no particularly good economic reason to distinguish between direct and indirect effects. Some courts and government agencies enforce anti-discrimination legislation--in particular, Title VII of the 1964 Civil Rights Act--by requiring employers to demonstrate that criteria on which they base hiring decisions have direct effects on productivity. If this model describes the world, such a policy violates the spirit, if not the letter, of the Act. Although the legislative history of Title VII is not without controversy, we think it is clear that Congress intended only to prohibit discriminatory behavior on the part of employers. Title VII was explicitly not supposed to interfere with or remedy competitive (but racially and sexually neutral) market processes which produce employment outcomes which are not racially and sexually neutral.[1]

Second, and more importantly, this note emphasizes what is, we believe, a major contribution of the recent work by economic theorists on the economics of information--that how an economic system works depends importantly on how it uses information--including what information it uses and what information it discards. In this model, the use of information is completely described by the partition of characteristics into those which are observable and hence used by employers and those which are unobservable and not used by employers. We show how the value of a characteristic is determined by this partition. We also show how the structure of information determines the distribution of income. In our model, there is a tradeoff between efficiency and equality. The more information used, the more smoothly the productive process runs and the more output (as measured by average wages) there is. As more information is used, dispersion of income increases. In our model the tradeoff between efficiency and equality takes a particularly simple form. Average income is a linear and increasing function of the variance of income. The structure of information determines where on this frontier the economy settles. If this model (or something like it) describes the world, an important question (both for research and for policy) is how the structure of information is determined.

II. The Model

Our model is

(1) $$Q(A,S) = \alpha + \beta A - \gamma (A - S)^2$$

where $Q(A,S)$ is the output of an employee with ability A who is placed in job level S. The interpretation of (1) is that employees work best when they work at a job requiring skills commensurate with their ability. Employers determine S on the basis of their estimates of A.

It might be argued that a specification

$$Q(A,S') = \alpha' + \beta' S' - \gamma'(A - S)^2 \tag{1'}$$

is more plausible than (1). In this story S' is the speed of the assembly line on which a worker is placed and $\alpha'(A - S)^2$ is a measure of the number of items which the worker spoils because he is forced to work at the wrong speed. It is easy to check that (1) and (1') are equivalent if $\alpha' = \alpha + \beta^2/4\gamma^2$; $S' = S - \beta/2\gamma$; $\beta' = -\beta$ and $\gamma' = \gamma$. Formula (1) emphasizes the importance of finding the right job for a man; (1') stresses the need to find the right man for a job. They are equivalent ways of looking at the same phenomenon. Since (1) leads to more transparent formulae than (1'), we use it.

The ability A of (1) is a random variable; we suppose it is determined by a worker's characteristics—a random vector $X \epsilon R^n$. In this model A is a linear function of X so that

$$A(X) = a'X = \sum_{i=1}^{n} a_i X_i \quad . \tag{2}$$

We further assume that the characteristics X are distributed among the population of workers according to a multivariate normal distribution function so that

$$X \sim N(0,\Sigma) \tag{3}$$

where 0 is a vector of n zeroes and Σ is an n-by-n variance-covariance matrix. The assumption that $EX = 0$ is a harmless normalization as non-zero means are easily absorbed in the constant term, α. Assumptions (2) and (3) imply that ability is a normal random variable with mean zero and variance $\sigma^2(A) = a'\Sigma a$.

III. The Structure of Information and the Determination of Employment Outcomes

The characteristics which determine ability are partitioned into two classes, observable and unobservable. That is, $X = (Y,Z)$ where variables in Y are observable and can be used by employers in determining wages and jobs for employees. The partition of characteristics into the observable and unobservable reflects both facts of nature and legal or social conventions. Some characteristics may be unobservable because they are really unobservable. The model is general enough to include in the variable X virtually any random influence. Other characteristics may be unobservable because it is too costly to observe them. For example, x_i might be a potential test score on an exam which is costly to give. Still others might be unobservable because the law makes it illegal for employers to use them in employment decisions. Suppose, for example, that x_i is age.

From the point of view of an employer, a worker with observable characteristics Y has ability which is a random variable $A(X|Y)$. Under our assumptions, $A(X|Y)$ is normal with a mean $E[A(X|Y)]$ and variance $V(A(X|Y))$. $E[A(X|Y])$, the conditional mean of $A(X)$ given Y, is a function of Y,

$$E[A(X|Y)] = a'E[(X|Y)] = a_Y'Y + a_z'\Sigma_{zy}\Sigma_{yy}Y \tag{4}$$

where the subscripts y and z represent the obvious partition of the vector a and the matrix Σ. Our assumption of normality, however, implies that the conditional variance of ability $V[A(X|Y)]$ is a constant independent of Y. The variance of ability <u>is</u> a function of the partition of X into observable and non-observable variables, but for a given partition π, it is a constant function of Y. We note this fact by writing

(5) $$V[A(X|Y)] = \sigma^2(A(\pi)) \quad .$$

If employers are risk neutral they will place employees in jobs which maximize expected output. It is clear from (1) that this implies that $S(Y) = E[A(X|Y)]$, where $S(Y)$ is the job assignment of a worker with observed characteristics Y. Thus, worker output is

$$Q(A(X|Y),S(Y)) = \alpha + \beta A(X|Y) - \gamma(A(X|Y) - E[A(X|Y])^2 \quad .$$

If there is competition, workers will be paid their expected output so that

(6) $$W(Y) = E\,Q(A(X|Y),S(Y)) = \alpha + \beta E[A(X|Y)] - \gamma\sigma^2(\pi) \quad .$$

IV. The Value of Characteristics

We now examine two implications of (6). First we look at its implications for the value of observed characteristics. Equation (6) is linear in Y. It is of the form

(7) $$W(Y) = \text{constant} = \sum_{j=1}^{J} d_j y_j$$

where J is the number of observable variables. The coefficient d_j is the value of the characteristic j. From (4) we know that d_j can be broken up into two parts

$$(8) \qquad d_j = \beta(a_j + f_j)$$

where a_j is the direct effect of characteristic j on productivity and f_j is the indirect information which y_j provides about the unobserved characteristics (the Z's) which influence productivity. It is worth noting that there is no particular reason to suppose that a_j and f_j (or even a_j and d_j) are of the same sign. This suggests that requiring employers to validate that those characteristics which are used for screening and placement have a direct effect on productivity--as the Equal Employment Opportunity Commission has suggested that employers who use ability tests ought to do--is a burdensome requirement which a competitive but not discriminating firm could not meet.[2]

V. The Distribution of (Wage) Income

The second implication of the wage determination formula (6) concerns the relationship between the mean and variance of wages. It is clear that in this model, the more characteristics which are observable, the higher average output will be as more information implies less misclassification. It also seems likely that the more characteristics there are, the greater the dispersion of income. This suggests that our model implies a tradeoff between the mean and the variance of income. As we shall see, this is correct. For a given partition π, mean income is

$$(9) \qquad E[\pi] = \alpha + \beta E[EA(X|Y)] - \gamma\sigma^2(A(\pi)) = \alpha - \gamma\sigma^2(A(\pi))$$

(since $E[E(A(X|Y)] = E[A(X)] = 0)$.

For the same partition, the variance of wage income is:

(10) $$\sigma^2(W(\pi)) = \beta V[E[A(X|Y)]] \quad .$$

However, since

(11) $$\sigma^2(A) = V(A(X)) = V[E(A(X|Y))] + E[V(A(X|Y))]$$

$$= V[E(A(X|Y))] + \sigma^2(A(\pi)) \quad ,$$

we see that (10) and (11) imply

$$\sigma^2(A(\pi)) = \sigma^2(A(X)) - \frac{1}{\beta}\sigma^2(W(\pi)) \quad .$$

If we substitute this into (9), we obtain

(12) $$E(\pi) = \alpha - \gamma\sigma^2(A(X)) + (\gamma/\beta)\sigma^2(W(\pi)) \quad .$$

This equation, which holds for all partitions, implies a linear tradeoff between the mean and the variance of the wage distribution.

NOTES

1. See Rothschild and Werden (1982) for a discussion of the legislative history and intent of Title VII.

2. For a discussion of this point, see Rothschild and Werden (1982). We note here that the cost minimizing firm has no reason to draw the distinction between direct and indirect effects; indeed, once the structure of information is given, the firm cannot decompose d_j as in (8).

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