DUOPOLIES ARE MORE COMPETITIVE
THAN ATOMISTIC MARKETS[1]

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How many firms are needed to make a market function competitively?
The standard theorist's answer is that, so long as there are a finite number
of firms, each should take into account its effect on the market price, and
hence firms will not behave in a price-taking manner. For the assumption
of price-taking behavior to be plausible, so it is argued, there must be an
infinite number of firms.

On the other hand, if search is costly for all consumers, it is easy
to establish that if there are an infinite number of firms, the market equi-
librium entails all firms charging the monopoly price. The object of this
paper is to show that, when search is costly, markets with two firms may
behave more competitively (have lower prices) than such atomistic markets.

The reason for this is simple: when search is costly, when there are
few firms, each believes it can affect search behavior; when there are many
firms, each believes it will have no effect on search. As a result, when
there are few firms, each perceives its elasticity of demand as being greater
than when there are an arbitrarily large number of firms, and hence each will
charge a lower price.

One important corollary to our analysis is a clarification of the con-
ditions under which market equilibrium will be characterized by a price

distribution or by a single price, at the same level that a monopolist would charge. If <u>all</u> individuals have strictly positive search costs, and there is a <u>continuum</u> of firms, then the only equilibrium entails all firms charging the monopoly price; there cannot be a price distribution. Assume there is some firm charging a price strictly less than that of all other firms. With strictly positive search costs, all individuals will have a reservation price strictly greater than that firm's price; hence it will not lose any customers as it would if it raised its price slightly. Both of these assumptions turn out to be critical. When there is a probability distribution of search costs with strictly positive density in an interval around zero search costs, this argument is no longer valid; for by raising its price slightly, it will lose a few customers. We can show that there <u>may</u> exist an equilibrium price distribution.

Similarly, when there are a finite number of firms (no matter how large), though any store's lowering its price induces very few individuals who would have purchased at <u>each</u> of these other firms to search, the number of customers gained is the <u>product</u> of this number times the number of stores from which the given store succeeds in recruiting customers.

This product may increase or decrease with the number of firms, depending on the nature of the search process. Indeed, in one case, we show that the elasticity of demand (and hence the market equilibrium (price)) remains independent of the number of firms.

There is a fundamental difference between how we calculate the elasticity of demand for increases in prices and decreases in prices. In the symmetric price equilibrium, note that no one searches. Thus, increasing

one's price only affects search by those who arrive at your store: a few
of the low search cost individuals decide to go to one more store (where,
with probability one, they will find the item at a lower price). On the
other hand, when one decreases one's price, to increase one's sales, one
must induce some of those who arrive at other stores to continue to look
for you. If there are only two stores, the required search is relatively
little, but if there are many stores, it may be considerable. There appears
to be no a priori reason why the amount of additional sales from lowering
the price should just equal the loss of sales from raising the price.
Though we establish that in one central case, with duopoly, the two are
equal, in general, they are not. There is a kink in the demand curve.
This has profound implications for the nature of equilibrium.

In some cases, the kink is such that the percentage loss from in-
creasing prices is greater than the percentage gain from reducing prices.
This results in an indeterminacy of equilibrium, and in the equilibrium
price and quantity being invariant to changes in marginal costs (within a
region).

On the other hand, there are important instances where the kink takes
on the other form: the percentage gain from a reduction in price exceeds
the percentage loss from an increase in price (under the hypothesis that
all firms charge the same price). This implies that there does not exist
(in these circumstances) a single price equilibrium. The only equilibrium
entails a price distribution. We construct one such example.

When the density function of search costs near zero is zero, there is a discontinuity: if there are fewer than a critical number of stores, no pure strategy equilibrium exists; the only possible equilibria entail mixed strategies (price distributions).

Mixed strategy solutions are of particular interest in the analysis of product markets in which there is imperfect information about which stores charge which price, but in which there is perfect information concerning the price distribution. When all firms are pursuing mixed strategies, these assumptions concerning what consumers know have a much greater degree of plausibility than they do when a given firm always charges the same price, but different firms charge different prices. In this case, all the prices are below the monopoly price which prevails when there is a continuum of firms. Again, duopoly is more competitive than atomistic markets.

Whether there exists a single price equilibrium or not, with a fixed, finite number of firms there can easily exist a price distribution; we illustrate this for the case of a duopoly. Again, there is some presumption that the maximum price in the price distribution is below the single price symmetric equilibrium for a finite number of firms, and a fortiori, below the equilibrium which emerges with a continuum of firms.

A final interesting property of the equilibrium we derive is that when the density near zero is zero, there is always a kink in the demand curve when there is a continuum of firms,[1] while when there are a finite number of firms, the demand curve may not be kinked at the equilibrium point.[2]

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1. A similar property was noted by Braverman, in the slightly different context of the Salop-Stiglitz model of search.

2. Too much stress should not be placed on this result, since in more general search models, even with a finite number of firms, the demand curves are kinked. See Salop and Stiglitz (1976) or Stiglitz (1979).

This again reverses our normal presumptions: we are used to thinking of kinked demand curves as associated with oligopolistic markets, not markets with a continuum of firms. The implications of the presence of a kinked demand curve are, of course, the same: the firm does not respond to changes in costs, either by changing output or prices. This has important implications for macro-economic analysis, which we do not pursue here.[1]

I.   The Model

We consider a market in which, for simplicity, all individuals have identical demand functions for the given commodity

$$(1) \qquad x = x(p) \ ,$$

but they differ in their search costs. Also for simplicity, we shall assume that the first search is free, but all subsequent searches have a cost s .[2] There is a distribution of search costs across the population given by $F(s)$. The density function $f(s) \equiv F'(s)$,[3] a special case that has played an important role in the search literature, is that where

$$(2) \qquad x(p) = 1 \quad \text{for} \quad p \le u \ ,$$
$$\qquad\qquad\quad = 0 \quad \text{for} \quad p > u \ .$$

u is the maximum price that the individual is willing to pay for the commodity (Figure 1). We refer to this as the monopoly price, since it is the

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1. For a brief discussion, see Stiglitz (1978).

2. This is to avoid the problems of nonexistence which frequently arise when the first search is costly. See Salop and Stiglitz (1982) and Stiglitz (1979).

3. We assume, for simplicity, that in general F is differentiable, but most of the results do not depend on this.
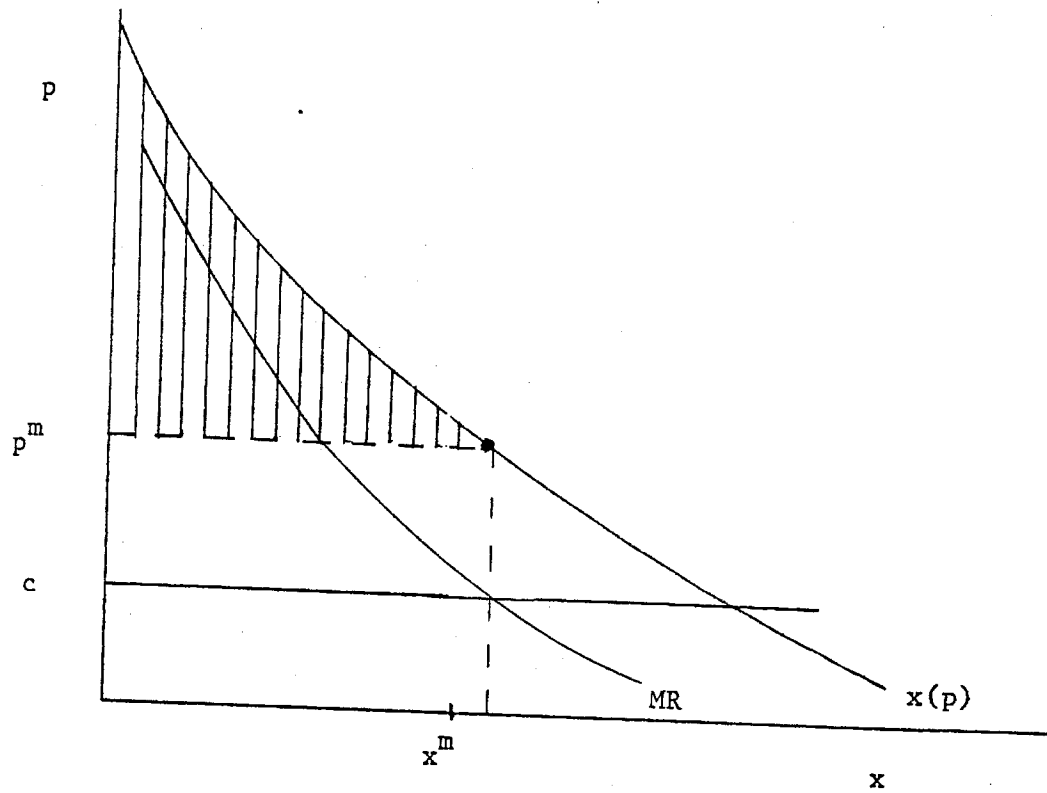
Figure 1a

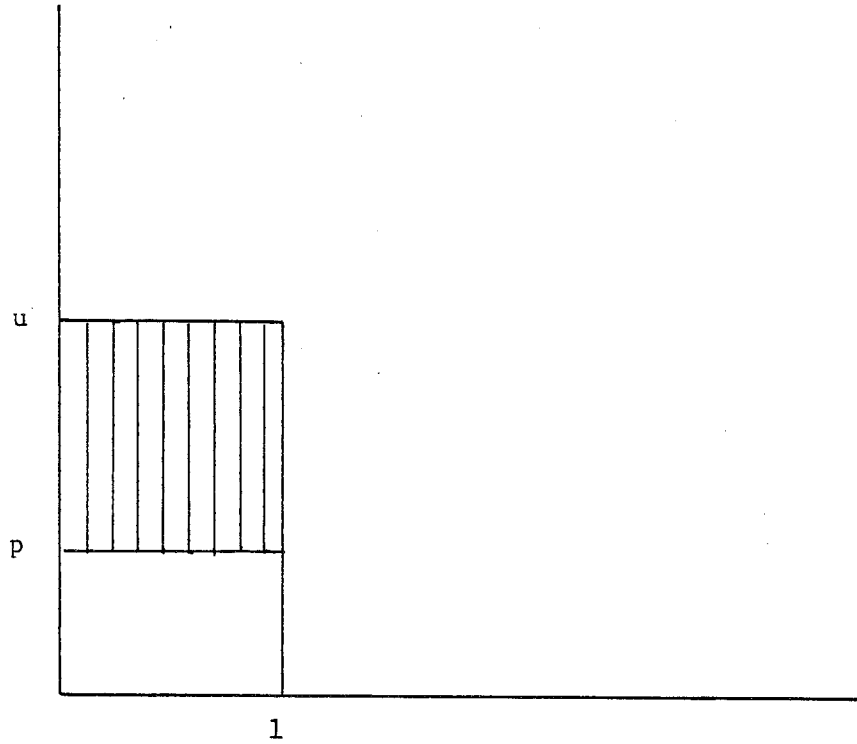Monopoly price equates marginal revenue (MR) to marginal costs. Consumer surplus ($v(p)$) is shaded area.

Figure 1b

Special demand function commonly employed
in search literature. Monopoly price is u ,
and consumer surplus is u-p .

price that a monopolist would charge. More generally, we define the monopoly
price p^m as that value of p for which

(3) $$\max_{\{p\}} \{p-c\}x$$

where c is the constant marginal cost of production of the commodity, i.e.,

(4) $$x(p) = -(p-c)x'(p)$$

or

(5) $$\frac{p-c}{p} = 1/E \ ,$$

where

$$E = \frac{d \ln x}{d \ln p} \ ,$$

the elasticity of demand. This is, of course, the familier formula relating
the monopolist's markup over marginal cost to the elasticity of demand.

A.   Search

Assume that in this market there are L individuals and M firms.
All individuals are assumed to know the probability distribution of prices,
but not which store charges which price. Customers randomly search among
firms; each individual continues to search until he finds a store which
charges a price at or below his reservation price. We assume that the in-
dividual's indirect utility function can be written as

(6) $$U = v(p) + Y \ ,$$

where Y is the individual's income. Thus, by Roy's formula,

(7) $x = -v'$,

Individuals have a simple search rule: purchase if $p \leq \hat{p}$, do not if $p > \hat{p}$. $\hat{p}$ is the reservation price. It is the price such that the utility an individual obtains from purchasing at the store he is presently at must be equal to the expected utility he obtains from continuing search. How the expected utility from continuing search is calculated depends on the nature of the search process, e.g., whether search is with or without re-placement. In either case, given our assumption of constant search costs, if it pays the individual to continue searching when he samples a price p today, it pays him to continue searching if he subsequently samples the same price at another store.

B.   Sampling with Replacement

If he samples with replacement (he forgets immediately which stores he has already visited), then if $G(p)$ is the probability distribution of stores, the expected number of searches required until he obtains a store with price below $\hat{p}$ is $1/G(\hat{p})$, and the expected utility he obtains is

(8) $$\frac{\int_{0}^{p} v(p)\,dG(p)}{G(\hat{p})} .$$

This is maximized with respect to $\hat{p}$, i.e.,

(9) $$v(\hat{p}) = \frac{\int_{0}^{\hat{p}} v(p)\,dG(p) - s}{G(\hat{p})} .$$

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1. We discuss sampling without replacement later.

In the special case (2), we obtain as our reservation price rule.

$$(10) \qquad u - \hat{p} = \frac{\int_0^{\hat{p}} (u-p) dG(p) - s}{G(\hat{p})}$$

or

$$(11) \qquad \hat{p} = \frac{\int_0^{\hat{p}} p \, dG(p)}{G(\hat{p})} + \frac{s}{G(\hat{p})} .$$

The reservation price is such that the expected price paid if one continues searching plus the expected search costs to find a lower price store must be equal to the reservation price.

II. The Symmetric Equilibrium with a Continuum of Firms

We can now solve for the reservation price for each individual as a function of the price distribution, and from this the equilibrium for the market. In the case of an infinite number of firms, no firm believes that it affects the reservation prices. Thus, if all firms but a measure zero are charging the price p^* , any individual knows that if he samples a store with a price in excess of p^* , with probability 1 on the next search he will obtain price p^* . Thus, in this limiting case, if a store charges more than p^* , the individual will continue searching (will purchase) if

$$(12) \qquad v(p^*) - s > (<) v(\hat{p}) .$$

On the other hand, if the firm lowers its price below p^* , then it induces no additional search, since the probability of anyone finding the store is zero, i.e., the expected number of searches is infinite. Thus the demand
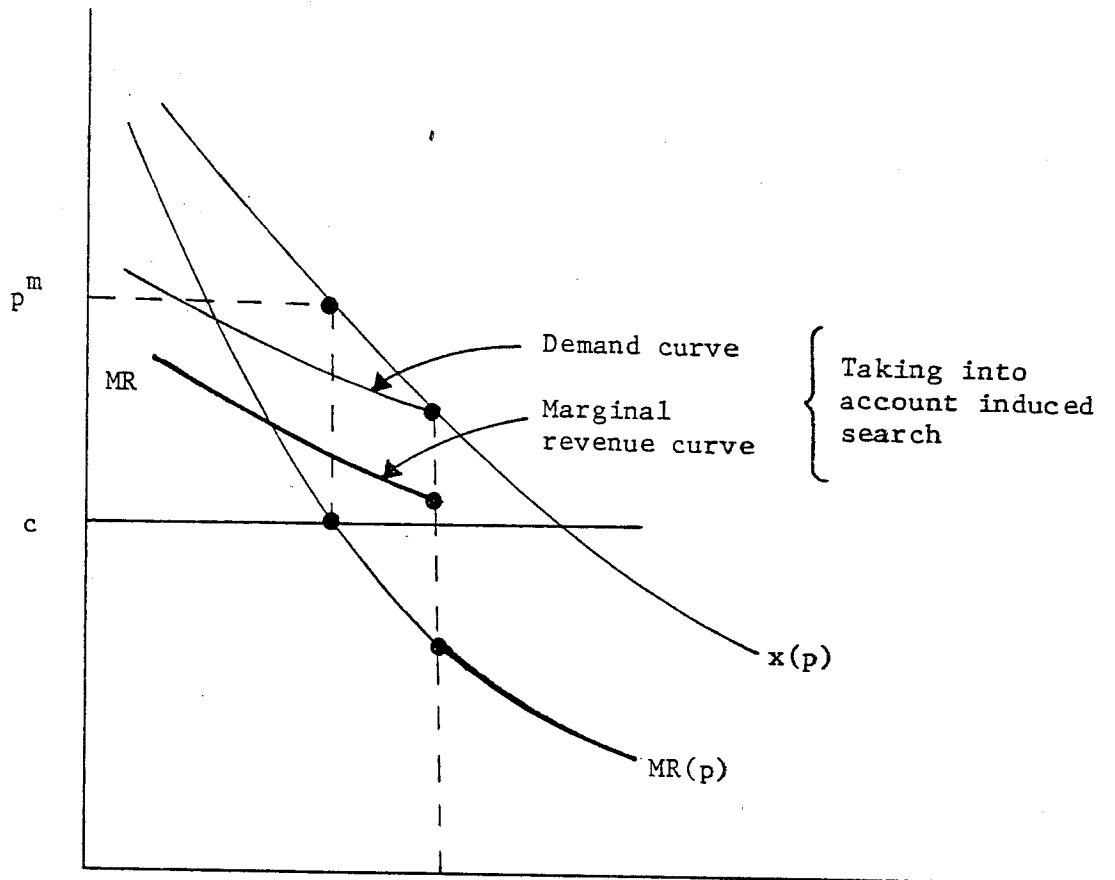
Figure 2a

With search and $f(0) > 0$, the demand curve is kinked:
lowering prices induces no search, but raising prices
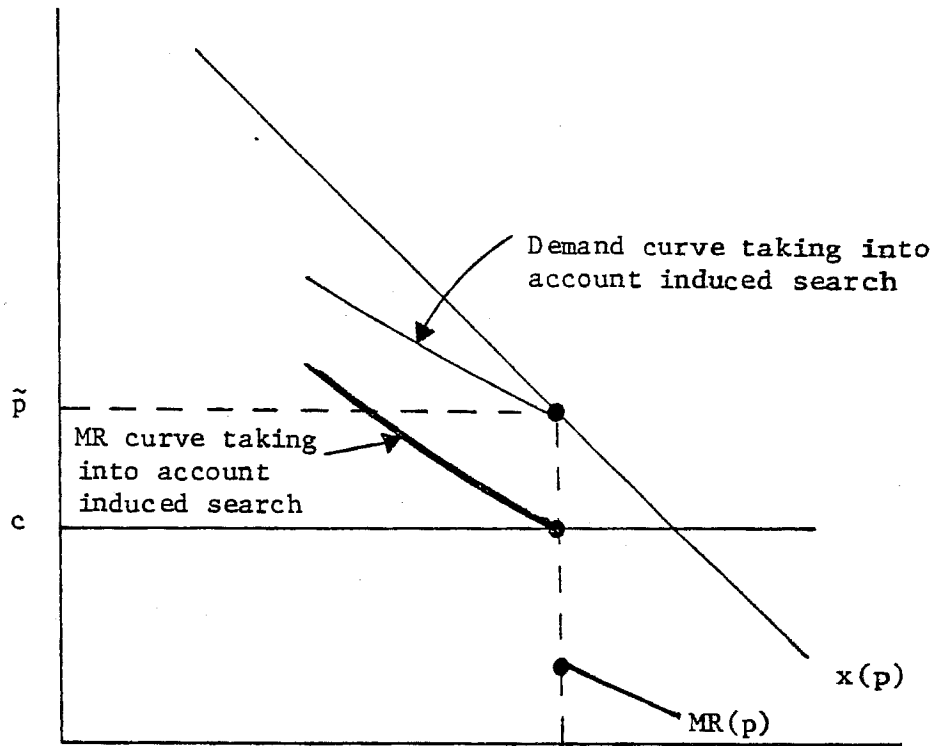induces current customers to leave.

Figure 2b

Equilibrium with a continuum of firms and $f(0) > 0$: the
equilibrium is indeterminate ; the price must be less than
or equal to the monopoly price, but greater than or equal
to $\tilde{p}$.

xurve takes on one of two forms, depending on whether $f(0) > 0$, or $f(0) = 0$:

    1. <u>$f(0) > 0$</u>. The revenues of the store are given by

(13) $R \equiv Npx$

where N is the number of customers who purchase:

(14) $N = \dfrac{L}{M} H(p)$

where $H(p)$ is the fraction of customers who have reservation prices below p :

(15) $H(p) = F(\hat{s}(p))$

where $\hat{s}$ is defined by (16):

(16) $\hat{s}(p) \equiv v(p^*) - v(p)$,

so

(17) $\dfrac{d\hat{s}}{dp} = -v'(p) = x(p)$,

using Roy's formula. Hence

(18) $H' = fx.$

Thus, at p^* , the elasticity of the demand curve is

(19) $E - \dfrac{N'p}{N} = E + f(0)x(p)p$,

where E is the elasticity of the individual demand curves.

There is thus a kink in the demand curve at $p = p*$.

There is also a fundamental indeterminacy of equilibrium; any price between the monopoly price and p defined by

$$(20) \qquad \frac{\tilde{p}-c}{\tilde{p}} = 1/E(\tilde{p}) + f(0)x(\tilde{p})\tilde{p}$$

is a symmetric equilibrium.

Notice that the symmetric equilibrium has the important property that a change in the costs will not, in general, result in any change either in price or output.

2. $\underline{f(0) = 0}$. This is the case on which most of the search literature has focused.[1] Then, at the symmetric equilibrium the demand elasticity, both for increases and decreases in price, is just E. There is a kink in the demand curve (unless at s_{min}, the minumum search cost, $f'(s_{min}) = 0$). But this has no bearing on the equilibrium: the only symmetric equilibrium occurs at the monopoly price (Figure 3).

III. <u>Finite Numbers Case $(f(0) > 0)$</u>

The analysis is markedly different if there are a finite number of firms (no matter how large). To help fix our ideas, let us assume that there are two firms.

A. <u>Duopoly</u>

Now, when the first firm changes its prices, it affects whether those who happen to go to the other store purchase there or continue to search.
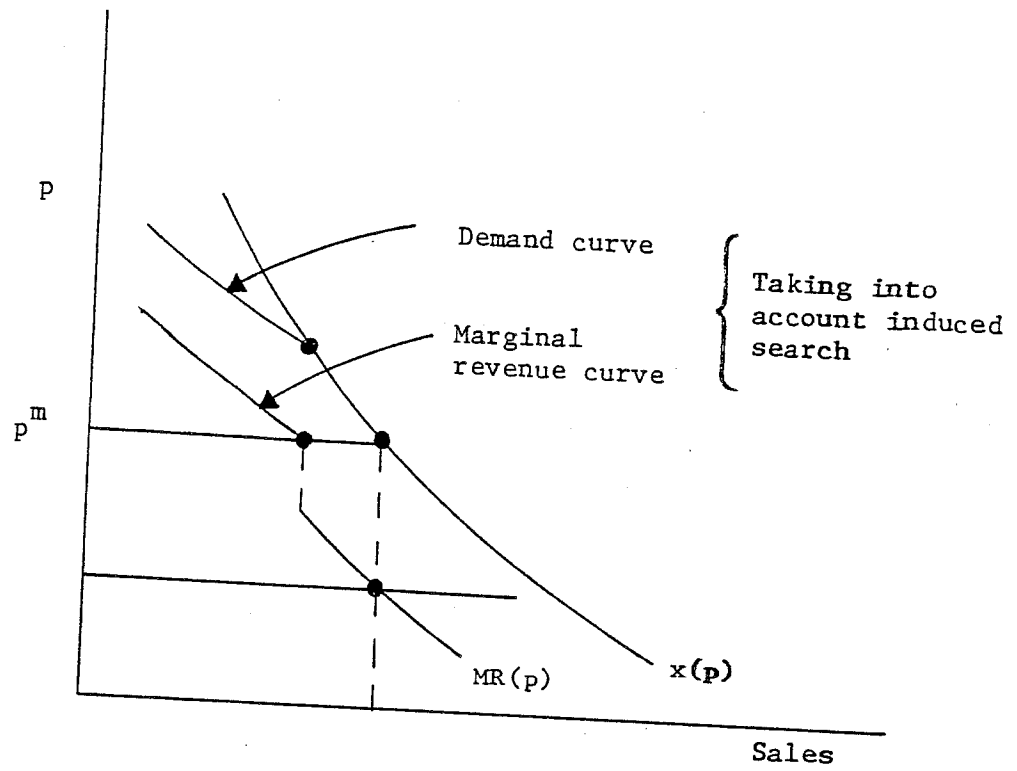
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1. See Diamond (1971).

Figure 3

If $f(0) = 0$, there is no kink at $p = p^m = p^*$, and the only equilibrium is the monopoly price.

If the second store is charging $p_2 < p_1$, then an individual who arrives at the first store will be indifferent between purchasing or continuing to search if

(21) $\qquad v(p_1) = v(p_2) - 2s$

(where we have assumed that the individual searches with replacement, i.e., he either forgets where he searched last time, or the two stores are pursuing mixed strategies so that the information about the stores' current price is of no value in structuring future search strategies).

Thus, we can write

(22) $\qquad s = s(p_1, p_2)$,

and we can define s_2 in a symmetric way;

(23) $\qquad s_2 = s_2(p_1 \cdot p_2)$,

where

(24) $\qquad 2\, \dfrac{\partial s_1}{\partial p_1} = -v'(p_1) = x(p_1), \qquad 2\, \dfrac{\partial s_2}{\partial p_1} = -v'(p_1) = -x(p_1)$,

(25) $\qquad 2\, \dfrac{\partial s_1}{\partial p_2} = -v'(p_2) = -x(p_2), \qquad 2\, \dfrac{\partial s_2}{\partial p_2} = -v'(p_2) = x(p_2)$.

When the first store increases its price, it induces more of those who arrive at it to continue searching; and when the second store raises its price, it induces more of those who arrive at the first store not to continue searching.

There are again two cases to consider: that where $f(0) > 0$, and that where $f(0) = 0$.

Now, when the firm <u>raises</u> its price, it is aware that some of its potential customers will refuse to purchase, but go to the other store:

$$(26) \qquad \frac{\partial \ln N}{\partial \ln p_1^+} = \frac{f(0)}{2} x(p^*)p .$$

In the previous case, when it <u>lowered</u> its price, it did not gain any other customers; now, however, it is aware that some of the other store's customers will be induced to search; indeed, from (25)

$$(27) \qquad \frac{\partial \ln N}{\partial \ln p_1^-} = \frac{f(0)}{2} x(p_2)p_2 = \frac{f(0)}{2} x(p^*)p^* .$$

<u>At the symmetric equilibrium where $p_1 = p_2 = p^*$ there is no kink in the demand curve</u>. The increased sales from lowering the price from induced search from the other firm is exactly equal to the loss in sales from raising one's prices. There is a unique symmetric equilibrium where

$$(28) \qquad \frac{p^*-c}{p^*} = 1/E(p^*) + \frac{f(0)}{2} x(p^*)p^* .$$

The unique symmetric equilibrium is at a price below the monopoly price.

On the other hand, contrasting (28) and (20), it is clear that the lowest possible price with a continuum of firms is lower than the unique duopoly equilibrium.

In the case where there is sampling without replacement, it takes the individual only one search to find the other store, so the reservation price rule is simply

(29) $\qquad v(p_1) = v(p_2) - s$.

Hence again there is no kink in the demand curve and

(30) $\qquad \dfrac{p^* - c}{p^*} = 1/E(p^*) + \dfrac{f(0)}{2} x(p^*)p^*$.

<u>The equilibrium price is at the lower bound of the set of possible symmetric equilibrium prices which emerge when there is a continuum of firms. There is an unambiguous sense in which markets with duopoly are more competitive than atomistic markets. Contrasting (28) and (29), we see that the market equioibrium price depends critically on the nature of the search process.</u>

B. <u>Three Firms</u>

Consider how what happens when there are three firms. Lowering the price induces less search than with two firms, and it is this observation which has led to the presumption that with a large but finite number of firms the equilibrium will be the monopoly price. But though fewer individuals are induced to search from each store, there are more stores. The two effects at least partially cancel. The equilibrium now depends rather sensitively on the nature of the search process. We first consider the equilibrium when there is search with replacement. We continue to focus on the symmetric equilibrium.

1. <u>Search With Replacement</u>

Then, if one store raises its prices, the expected number of searches for an individual who arrives at that store to find a low-price store[1] is 3/2. Since sales are proportional to $1 - F(2_1)/3$ where $F(s_1)$ is the

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1. There is a probability of 2/3 that he finds a low-price store on each search. Thus the probability that:

one search is required is 2/3;
two searches are required is 1/3 x 2/3 = 2/9;
three searches are required is 1/9 x 2/3 = 2/27;
N searches are required is $2/3^N$

$$2/3 + 2 \times 2/9 + 3 \times 2/27 \ldots = \frac{2/3}{1-1/3} + \frac{2/9}{1-1/3} + \ldots$$

$$= \frac{2/3}{2/3} \left[1 + 1/3 + 1/9 + \ldots \right]$$

$$= \frac{1}{1-1/3} = 3/2 .$$

fraction of individuals who arrive at the store who feel it is worthwhile to search further,

(30a) $\quad -\dfrac{d \ln N}{d \ln p^+} = 2/3\ f(0)px(p)$.

When a store lowers its price, the expected number of searches for someone who arrives at one of the two other (high-price) stores to find the low-price store[1] is three. The sales at the low-price store are now proportional to

$$F + \frac{1-F}{3} \ ,$$

where F is the proportion of the population who find it profitable to search, i.e., the low-price store gets all of the sales of the searchers, and a proportionate share of the nonsearchers. Hence

(30b) $\quad -\dfrac{d \ln N}{d \ln p^-} = 2/3\ f(0)px(p)$.

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1. There is a probability of 1/3 that he finds the low-price store in each search. Hence, the probability that:
   one search is required is 1/3;
   two searches are required is $1/3 \times 2/3 = 2/9$;
   three searches are required is $1/3 \times 4/9 = 4/27$;
   four searches are required is $1/3 \times 8/27 = 8/81$;
   five searches are required is $1/3 \times 16/81 = 16/243$;
   N searches are required $= 2^N - 1/3^N$.
   Hence the expected number of searches is
   $$1/3 \left[\frac{1}{1-2/3}\right] + 2/9 \left[\frac{1}{1-2/3}\right] + 4/27 \left[\frac{1}{1-2/3}\right] \ . \ . \ .$$
   $$= 3 \left[\frac{1}{1-2/3}\right] 1/3 = 3 \ .$$

Contrasting (30a) and (30b), with three firms and search with replacement, there is no kink in the demand curve.

Comparing (30) with (28), it is clear that <u>with more firms the demand elasticity is smaller and hence the price is higher</u>.

2. <u>Search Without Replacement</u>

Equally interesting, however, are the consequences of searching without replacement. Consider a firm lowering its prices. When search is without replacement, the probability that an individual at the high-price store finds the low-price store on the first search os 0.5 and on the second search is 0.5. Thus, the expected number of searches is 1.5. Sales are again $F + 1-F/3$, so lowering the price increases its sales by

$$\frac{2f}{3} \frac{ds}{dp_1} = 2/3 \ f \ \cdot \ 2/3 \ \cdot \ x(p)$$

and

$$- \frac{d \ \ln N}{d \ \ln p^-} = \frac{4xf(0)}{3} \ px(p) \ .$$

When a store raises its prices, its sales are proportional to $1-F/3$. The number of searches to find a lower-price store is just one. Hence,

$$- \frac{d \ \ln N}{d \ \ln p^+} = f \ px(p) \ .$$

Thus, there is a kink, but it is of the opposite kind to that discussed earlier. As Figure 4 illustrates, when there are three firms and search without replacement, there cannot exist a single price equilibrium.
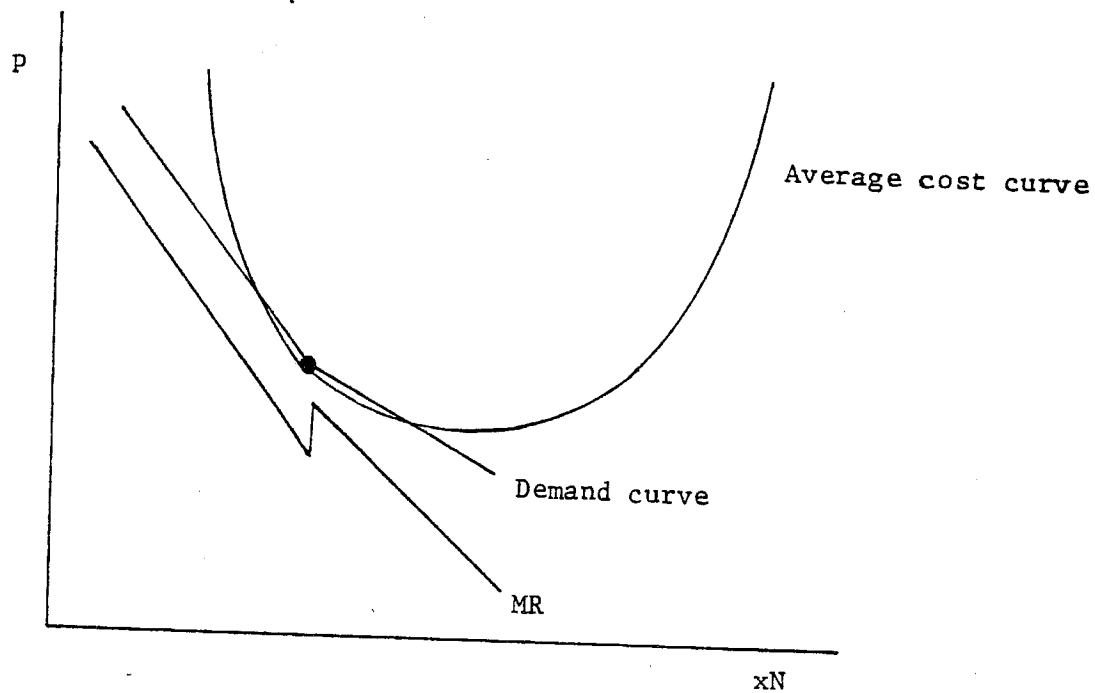
Figure 4

With three stores and search without replacement there
is a kink at p^* , so that there cannot exist a single
price equilibrium.

C.   Multiple Price Equilibria

There will, however, exist (frequently multiple) multiple price equilibria. There may exist, for instance, equilibria in which two stores charge the high price p_1 and one store charges the low price p_2, with a fraction $F(\hat{s})$ of those who arrive at a higher-price store searching, where

$$v(p_2) - v(p_{1-} = \frac{3\hat{s}}{2}$$

(since it takes, on average, 3/2 searches to find a low-price store). Sales at a high-price store are $1-F(\hat{s})/3$, and at the low-price store $1 + 2F(\hat{s})/3$.

If one of the high-price stores raises its price by Δ, it loses some customers. But if it pays someone to search with a 50/50 chance of saving Δ or $p_1 - p_2$, it surely pays him to search if there is a probability of one of saving $p_1 - p_2$ (per unit purchased).[1] Hence, such an individual searches if $s < \hat{\hat{s}}$, where

$$\frac{3\hat{\hat{s}}}{2} = v(p_2) - v(p_1+\Delta) .$$

If the firm lowers its price, by the same reasoning any individual who is willing to search from the highest-price firm to the second is willing to search until he finds the lowest-price firm. Hence, his sales are still $1-F(\hat{s})/3$, with

$$3/2 \ \hat{\hat{s}} = v(p_2) - v(p_1-\Delta) .$$

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1. This would not necessarily be true if search costs were no constant.

Thus, the demand elasticity is

$$E + \frac{2f(\hat{s})xp}{3(1-F(\hat{s}))}$$

and

(31) $$\frac{p_1-c}{p_1} = 1/E + \frac{2f(\hat{s})xp_1}{3(1-F(\hat{s}))} \ .$$

Similar calculations establish that the demand elasticity at the low-price store is

$$E + \frac{4f(\hat{s})xp}{3(1+2F(\hat{s}))}$$

so

$$\frac{p_2-c}{p_2} = 1/E + \frac{4f(\hat{s})xp_2}{3(1+2F(\hat{s}))} \ .$$

With constant elasticity demand curves, if $p_1 > p_2$, this implies that

$$F(\hat{s}) < 1/4 \ .$$

Comparing (31) and (29), it is clear that the high price with three firms is lower than the price with duopoly of $F < 1/3$; but since $F < 1/4$, it immediately follows that both prices in the two-price distribution are lower than the symmetric duopoly price (which is lower than the monopoly price.)

It is easy to construct examples satisfying these relations. Thus, if $E = 0$ (the demand function (2)), $x = 1$, $c = 1$,

there exists an equilibrium with

$$P_1 = \frac{11}{8}, \quad P_2 = \frac{10}{8}, \quad \hat{s} = \frac{2}{3} \times \frac{1}{8} = \frac{1}{12}, \quad f(\hat{s}) = \frac{18}{5}$$

$$F(\hat{s}) = \frac{1}{10} .$$

It should be noted that in constructing this example, only local properties of the search cost distribution have played a role; in particular, $f(\hat{s})$ and $F(\hat{s})$. Hence, the analysis applies equally to the case of $f(0) = 0$.

IV. Finite Number of Firms: $f(0) = 0$

For simplicity, we focus our analysis on the case where the distribution of search costs is degenerate at s; that is, all individuals have the same level of strictly positive search costs; and on the case where there is unitary demand for price less than u. By the usual argument, if there exists a symmetric equilibrium, it must be the monopoly price; for at any other price it would pay any firm to raise its price by a small amount, sufficiently small so as not to induce any search. But with two firms, this will not, in general, be an equilibrium; for at the symmetric equilibrium, each firm enjoys half the market. By lowering its price by $2s$ (if search occurs without replacement, or $3s$ if search occurs with replacement), the firm can get the entire market. The increment in its profits is

$$2(u-s-c) - (u-c) = u-c-2s$$

which is positive, provided

$$2 < \frac{u-c}{2} .$$

More generally, if there are M firms, the increment in profits from any firm lowering its price enough to induce search is

$$(M-1)(u-c)-sM\Phi(M)$$

where $\Phi(M)$ is the expected number of searches to find the low-price store when there are M stores. It is thus apparent that there is a critical M, M^*, such that for $M > M^*$ (where $M^*\Phi(M^*)/M^*-1 = (u-c)/s$)) the monopoly price is the equilibrium.

When the monopoly price is not an equilibrium, the only symmetric equilibrium is a mixed strategy equilibrium; for the duopoly the probability distribution of price satisfies

$$(p-c)\, 1-G(p+s)\ + (p-c)\left[\frac{G(p+s)\ -\ G(p-s)}{2}\right] = k\ .$$

All prices in the price distribution are less than or equal to the marginal price; again, markets with fewer firms behave more competitively.

V. Asymmetric Equilibrium and Equilibrium Price Distributions

The analysis so far has been confined mainly to an examination of the symmetric equilibrium. We now consider the possibility of asymmetric equilibria, and ascertain whether there is a sense in which, when such equilibria exist, competition is more effective when there are few firms.

A. Continuum of Firms: $f(0) = 0$

In the limiting case of a continuum of firms, when $f(0) = 0$, it is easy to establish that there cannot exist an equilibrium price distribution. For consider the lowest-price firm. Clearly, the reservation price of all

individuals must be above that firm; hence, unless that price is the monopoly price, it pays the firm to raise its price by a small amount. The only possible equilibrium entails the lowest price (and hence all prices) equaling the monopoly price.

B.   Continuum of Firms: $f(0) > 0$

If there exists an equilibrium price distribution, revenues at all prices charged must be the same. To show that there may exist an equilibrium price distribution, and to derive the differential equation which it must satisfy, we focus on the limiting case where production costs are zero, and all individuals purchase one unit, provided price is less than u . If $G(p)$ is the price distribution, recall that reservation price for an individual with search costs s is given by (11):

$$\hat{p} = \hat{p}(s) .$$

Thus, those with search costs $\hat{s}$ will allocate themselves evenly among all the lower-price stores, i.e., each such store will get a number of such individuals which is proportional to

$$f(s)/G(\hat{p}(s)) .$$

The total sales at any store, then, is

$$\int_{\hat{s}(p)}^{\infty} [f(s)/G(\hat{p}(s))]ds ,$$

where $\hat{s}(p)$ is the search cost of the individual for whom p is the reservation price. We require then that

(32) $\quad \pi = p \int_{s(\hat{p})}^{\infty} \frac{pf(s)}{G(\hat{p}(s))} \, ds = k$

for all prices in the distribution, and

$$\pi = p \int_{s(\hat{p})}^{\infty} \frac{f(s)}{G(\hat{p}(s))} \, ds \leq k$$

for all other prices.

Integrating (11) by parts

(33) $\quad s = \hat{p}G + \int_{0}^{\hat{p}} G(p) \, dp - pG \big|_{0}^{\hat{p}}$

$$= \int_{0}^{\hat{p}} G(p) \, dp \equiv H(\hat{p}) \, ,$$

and

$$\frac{ds}{d\hat{p}} = G(\hat{p}) = H' \equiv h(p)$$

Thus, we can reqrite (32)

(32') $\quad \pi = \int_{p}^{p_{max}} pf(s(p)) \, dp + [1 - F(s(p_{max}))]p$

and

$$\frac{d\pi}{dp}\bigg|_{\hat{p}} = \int_{s(\hat{p})}^{\infty} \frac{f}{G} \, ds - \frac{\hat{p}f}{G} \frac{d\hat{s}}{dp}$$

$$= \frac{k}{\hat{p}} - \hat{p}f = 0 \quad .$$

Hence

$$\hat{p}^2 = \frac{k}{f(\hat{s})} \quad .$$

Since

$$\frac{ds}{dp} = \frac{2f}{f'p} = G > 0 \ , \quad f' < 0$$

$$\frac{d^2s}{dp^2} = \frac{2f}{f'p^2} + \left(\frac{2}{p} - \frac{2f}{f'^2}\frac{f''}{p}\right)\frac{2f}{f'p}$$

$$= \frac{2f}{f'p^2}\left(3 - \frac{2f}{f'^2}f''\right) = g > 0 \ ,$$

so

$$\frac{ff''}{f'^2} > \frac{3}{2} \quad .$$

For instance, assume

$$g = \frac{1}{c_1} \ , \quad 1 \le p \le k + 1 \ ;$$

then

$$G = \frac{p}{c_1} + c_2 \ .$$

Using (33),

$$\hat{s} = \frac{\hat{p}^2}{2c_1} + c_2\hat{p} + c_3 \ .$$

Let $c_2 = 0$ (i.e., $G(1) = 1/c_1$; there is a mass point at $p = 1$), then, if

$$f = \frac{1}{a+s}$$

where

$$a = -c_3 = \frac{1}{2c_1}$$

(since at $p = 1$, $\hat{s} = 0$).

Hence,

$$\int_p^{k+1} f(s(p))dp = 2c_1 \int_p^{k+1} \frac{dp}{p^2} = -\frac{2c_1}{p}\Big|_p^{k+1}$$

$$= \frac{2c_1}{p} - \frac{2c_1}{k+1} \quad . \quad .$$

In addition, each store gets its share of those who don't search

$$\int_0^{s_{max}} f(s)ds = \ln \frac{a + s_{max}}{a} = \ln[1 + (1+k)^2 - 1] = 2\ln(1+k)$$

where

$$s_{max} = \frac{1}{2c_1} [(1+k)^2 - 1] \quad .$$

Hence

$$\frac{2c_1}{1+k} = 2\ln(1+k)$$

or

$$c_1 = (1+k)\ln(1+k) \ .$$

We require $c_1 > 1$, i.e., $k \geq k^*$, where k^* is the unique solution to

$$\ln(1+k) = \frac{1}{1+k} \ .$$

In general, the maximum price of the distribution is below the monopoly price; for if it is to raise the same revenues as other stores do, if there are not mass points to the distribution F , then the highest-price store must serve an interval of individuals, i.e., all individuals with search costs greater than or equal to $\overset{\vee}{s}$. But then, there is some group, those with $s = \overset{\vee}{s}$, who are indifferent to searching or not, and hence the elasticity of demand of the highest-price store is greater than the elasticity of demand of the individual's demand curve. (The maximum price may be greater or smaller than the symmetric equilibrium price, since $f(\overset{\vee}{s})/1-F(\overset{\vee}{s}) \gtrless f(0)$.)

C.   Multiple Price Equilibrium With Finite Number of Firms

When there are a finite number of firms, we have already noted that there are some circumstances when the only equilibria entail a price distribution. There are other cases, where there may exist both a single price equilibrium and a multiple price equilibrium. The reason for this is simple: the firm sets marginal revenue equal to marginal cost, but the marginal revenue depends on the actions of his rivals. In the analysis, we have assumed that each firm is a price setter, and investigated the Nash equilibrium, where each firm takes the price of his rivals as given. This

is the case which, without search costs, has been investigated by Bertrand and Edgeworth. The solution (without capacity constraints) entails price equaling marginal cost.

With search costs, we obtain markedly different results. While in the Bertrand-Edgeworth analysis at the symmetric equilibrium lowering price by any amount, no matter how small, results in the firm capturing the entire market (the elasticity of demand is infinite), in our model--and we would argue in reality as well--the firm captures only a fraction of his rival's market.

The analysis is complicated by the fact that the profit function is not, in general, concave; hence, local analysis is not, in general, suffi-cient. Indeed, we can show that <u>in a duopoly</u>, though there may be asymmetric solutions to $MR(p_1, p_2) = MC$ (where MR = marginal revenue, MC = marginal cost), <u>there never exists a price distribution</u>. Assume $p_1 > p_2$. The high-price firm's current profits are

$$x(p_1)(p_1 - c_1)(1-F);$$

if the high-price firm imitates the low, its profits are

$$x(p_2)(p_2-c) .$$

We require

$$x(p_1)(p_1-c)(1-f) > x(p_2)(p_2-c) .$$

Conversely,

$$x(p_2)(p_2-c)(1+F) > x(p_1)(p_1-c) .$$

Thus, we require that $1-F > 1/1+F$; but this cannot be satisfied for any f.

It is possible to show that with more than two firms asymmetric equilibria exist, as the example of Section III.C illustrates.

VI. Robustness of Results

This paper has shown that the one model of sequential search which has been extensively investigated in the liturature, that where there are strictly positive search costs and a continuum of firms, is indeed special. In that case, the standard argument has it that the unique equilibrium entails a single price, at the monopoly level. In contrast, we have argued that if either assumption is removed, prices may be below the monopoly level, and there may exist a price distribution.

There are two reasons that it is important to investigate models which generate price distributions. First, there is considerable evidence that product markets are frequently characterized by essentially identical commodities being sold at markedly different prices,[1] and a model of the product market should at least admit this as a possible outcome. Second, one of the striking consequences of sequential search models in which there is a strictly positive cost to entering the market (going to each store) is that if the demand function is of the form (2) (though the reservation price

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1. Those who believe in the Law of the Single Price may claim that the products sold at different prices differ in some important way, e.g., location, service, etc. This may be true in some instances, but in other instances the magnitude of the price differences is sufficiently large to suggest that these "quality" differences cannot fully account for the observed price differences. In those cases where these quality differences are important, the appropriate model for analyzing the market is a differentiated commodity market with search costs; such markets will, in general, be characterized by price distributions in the natural sense that, even if the technologies with which the goods are produced are identical, they will sell for different prices. See Salop and Stiglitz (1976).

and search costs can differ among individuals) or, even if the demand function is of the more general form (1), provided firms can use non-linear price schedules, there exists no equilibrium in the market (see Salop and Stiglitz (1977) and Stiglitz (1979)). This seeming paradox is resolved if there is at least some probability that the price charged is below the monopoly price.

Thus, the models we have constructed have an internal consistency which was lacking from the models of product markets in which all firms have strictly positive search costs and there is a continuum of firms.[1]

We also believe that our results are reasonably robust to changes in assumptions concerning, e.g., the information acquisition technology. In Salop and Stiglitz (1977), for instance, we considered a model in which individuals, by paying a given amount, obtained complete information concerning the market, and in Salop and Stiglitz (1983), we considered a dynamic model in which individuals could make only one search per period, but could, at any date, decide to store for future consumption. Both of these yielded equilibrium price distributions.

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1. Formally, we avoided these nonexistence difficulties in the standard way by assuming that the first search cost was free (see above, p. 5). But this is clearly an unsatisfactory assumption; if we assumed that the first search costs the same as subsequent searches, our analysis would be essentially unaffected provided the average equilibrium price is low enough. (Otherwise, we would again encounter problems of existence.) For instance, in the case of the symmetric equilibrium with $f(0) > 0$, and with demand functions of the form (2), using (5) and (19),

$$P_{min} [c + 1/f(0), u] .$$

Thus, a market exists provided

$$f(0) > 1/u-c$$

and all of those with search costs $s > u-c-1/f(0)$ drop out of the market.

Similarly, it is clear that in markets with differentiated commodities, or in markets in which individuals are imperfectly informed concerning the price distribution, and have (as we would expect) different priorities concerning it, when firms lower their prices, they will not only sell more to each customer, but they will find that there are more customers willing to purchase. It was the absence of the <u>extensive</u> margin which resulted in the market equilibrium price with strictly positive search costs and a continuum of firms being the monopoly price; the general case, which we have depicted in this paper, is that where there is an extensive margin, and hence where the highest price is lower than the monopoly price.

The result on the possible presence of a kink in the demand curve we also believe is relatively robust. Indeed, we have investigated multi-period models in which the kink is more pronounced.

In the stationary equilibrium, individuals search only in the first period of their life. A store which lowers its price finds more individuals who stop at the store willing to purchase, and each purchases a greater quantity. But a single store lowering its price may induce relatively few individuals who are at other stores to search. On the other hand, when the firm raises its price, not only do fewer individuals who stop at the store purchase, and those who do purchase, purchase less; but also some of the existing <u>stock</u> of customers, those who had decided to stop searching, now decide to start searching again. The natural asymmetry of information (one knows more about the price of the store that one normally purchases at than about other stores) gives rise to a kink in the demand schedule.

VII. Conclusions

It is surprising that, in spite of the long recognition of the importance of search costs, its full implications for the nature of market equilibrium have, until now, been so little investigated. The one case which has been studied in the literature, that where there is a continuum of firms and the density function of search costs at zero is zero, has been shown to be indeed special.

We have shown that markets with fewer firms may behave more competitively than markets with a continuum of firms. Each firm then believes it can have an effect on search behavior, and therefore perceives itself as facing a larger elasticity of demand.

We have also shown that market equilibrium may be--as in fact they are--characterized by price distributions. In some cases there are multiple equilibria, a single price equilibrium, and (one or more) multiple price equilibria. In other cases the only equilibrium entails a price distribution.

Finally, we have shown that there are instances in which the demand curve facing a firm has a kink, resulting in the kinds of rigidities in behavior, lack of responsiveness to changes in, say, costs, which we normally associated with macro-economic phenomena. In fact, such kinks in the demand curves in seemingly competitive environments occur much more generally than the analysis here suggests. In a sequel to this paper, we plan to show this, and to explore its consequences.

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