

TOWARDS A MORE GENERAL  
THEORY OF MONOPOLISTIC COMPETITION

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1. Introduction

The publication in 1933 of the books by Chamberlain and Robinson on monopolistic and imperfect competition was heralded as the beginning of a revolution which would replace the clearly unsatisfactory perfectly competitive equilibrium theory of classical economics. Their theories were to micro-economics what Keynes' theory was to macro-economics. Yet twenty five years later, the theory of monopolistic competition had been relegated to a "special topic" within the typical graduate micro-economics course: the perfectly competitive model still occupied the central place. It was seemingly a revolution which failed.<sup>1</sup>

Yet, some forty years later, there appears to be a sudden revival of interest in the theory of monopolistic competition, with recent papers by Stern (1972), Eaton and Lipsey (1975, 1984), Dixit and Stiglitz

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<sup>1</sup> One can speculate on the reasons for the failure: it was the Keynesian revolution, though based on imperfectly articulated theory of imperfect competition, which dominated the scene in the immediate pre and post war periods. The perfectly competitive model was more amenable to mathematical analysis, or at least it appeared so, and less thought and knowledge about economic structures was required to model it than to model an imperfectly competitive economy.

(1977), Salop (1979), Spence (1976), Stiglitz (1976), Lancaster (1975), Hart (1979), Willig (1973) as well as a book length treatment of the subject by Lancaster.<sup>2</sup> Here, I wish to set out why I think the development of a better theory of monopolistic competition is important, to describe some of the major results obtained so far, to relate the alternative approaches to one another, and to indicate some directions for future research.

Though in the analysis below, I identify a large number of factors which affect the nature of the equilibrium and its welfare properties -- including the ability of firms to price discriminate and the existence of rent differentials to compensate for differences in transport costs -- probably the most important factors are those that affect the degree of competitiveness in the market: the presence of sunk costs (as opposed to just fixed costs) and the presence of a large number of "neighbors" (as opposed to just two, as in the one dimensional locational models.)

The assessment of the implications of sunk costs requires a precise definition of equilibrium. We develop below several alternative definitions. One definition involving the concept of latent firms provides a resolution of the stability problems noted by Salop (1979), and leads to zero profits, as suggested by standard contestability theory (Baumol) (1982); with sunk costs, we show, however, that the most natural definition of equilibrium is consistent with positive profits. Whether profits are driven to zero or not, market equilibrium is not, in general, pareto efficient.

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<sup>2</sup>This list is not meant to be exhaustive. Other studies will be noted below.

The fact that in the Dixit-Stiglitz-Spence model, each firm has a large number of competitors is the reason that we would argue that it more adequately captures the spirit of the Chamberlinian analysis of monopolistic competition than do the simple locational models which have been the center of analysis to date.

We also provide here a new model, a multi-dimensional locational model, which shares the virtue of the Dixit-Stiglitz-Spence model that each firm has many competitors, without having (what may be) the vice of all firms being equi-distant from each other. The central welfare propositions are shown to be critically dependent on the dimensionality of the "characteristics" space.

Though most of our analysis focuses on a comparison of market allocations with socially efficient allocations, we also present a comparison with the monopoly equilibrium. We show that under plausible conditions there is a tendency for a monopolist to locate his stores even closer than they are under monopolistic competition.<sup>3</sup>

#### Part I: The Central Issues

##### 2. On the Importance of Monopolistic Competition

The theory of monopolistic competition begins with two basic observations:

1. Most firms are price setters; if they raise their price by a small amount, they will not lose all their customers, as predicted by the perfectly competitive paradigm. Firms thus have some degree of

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<sup>3</sup>We investigate this comparison for only one specific model; the validity of the conclusions for other models remains a subject for future research.

monopoly power.

The limitations in competition may result either because only one or two firms produce a given commodity, or because transport costs make some firms (stores) more accessible than others<sup>4</sup>; or because individuals are uninformed.

2. In most industries, there is not a natural monopoly; if there are profits, the profits will attract entry. Firms are thus embedded in a competitive environment.

These two observations would, I think, generate little dissent. What is not so clear, however, is the conclusion to be drawn from these two observations that, in many market situations, the perfectly competitive paradigm is inadequate and inappropriate. It is inadequate because it leaves unanswered some of the most important questions, e.g. concerning how many and what commodities get produced; it is inappropriate because the welfare judgments (the Pareto optimality of the market allocation) and predictions (e.g. concerning the effects of a change in the size of the market or the effects of the imposition of a tax) may be misleading and incorrect.

### 3. The Basic Issues and Insights

There are four important classes of theorems concerning monopolistic competition:

(a) Welfare theorems: how can we describe the optimal allocation of resources, and how does it compare with that of the market economy?

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<sup>4</sup>From a general equilibrium perspective, this is just a special case of the more general case of a limited number of firms producing a commodity, where the commodity's description includes its location.

(b) Existence and stability theorems: under what conditions does the monopolistically competitive equilibrium exist, and when is it stable?

(c) Limit theorems: as the number of firms increases (say as a result of a reduction in fixed costs), does the economy converge to the competitive equilibrium?

(d) Characterization theorems: what is the effect of a change in, say, taxes or population, on the market equilibrium?

Underlying all the analyses is the assumption that there are important returns to scale, which limit the set of commodities which can be produced within the economy, and force the economy to make important decisions concerning the variety of commodities to be offered.<sup>5,6</sup>

In all of the issues, there appears to be some controversy: while Hotelling (1929) and others have claimed there is too little product variety, the general presumption (see Kaldor) has been that the market provides too much product variety. By contrast, Dixit-Stiglitz have recently provided a model in which they have argued that in a central case the market provides the optimal amount of product diversity.

While Chamberlain and Dixit-Stiglitz have claimed that as the number

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<sup>5</sup> Thus, analyses of differentiated competition which assume constant or diminishing returns to scale in production miss the essence of the issue with which we are concerned here. (cf. Mas Collé (1975)).

<sup>6</sup> The existence of some degree of increasing returns implies that the standard Arrow-Debreu model is not directly applicable. There have been some attempts to extend the standard model to situations with some degree of increasing returns. (Dasgupta - Ushio, Scnnenschein and Novshek; Grossman [1979], Baumol, Panzer and Willig [1982], and Baumol [1980]). One of our objectives is to delineate situations where increasing returns gives rise to imperfect competition, from those where it does not.

of firms in the economy becomes large, the economy may not converge to the competitive equilibrium, Hart has claimed that it will.

To those who are not specialists in the field, the variety of models with their contrasting results may seem bewildering: where are the underlying principles, the basic economic insights? How can we account for these differences?

There are, I think, two basic insights that pervade all the models that have been formulated.

First, if variety is valued, either because individual tastes differ, or because each individual values variety, then both the optimal and market allocations will entail production at outputs lower than those which minimize average cost: it is worth paying something for variety. Thus, the naive argument that monopolistic competition is inefficient simply because production does not occur at the minimum point on the average cost curve, is incorrect.

Secondly, in a market economy, firms enter when there are profits to be made; from a social point of view, introducing an additional firm is desirable if the "surplus"<sup>7</sup> created thereby is positive. The profits of a marginal firm and the surplus he creates are not in general identical, and it is therefore not surprising that in general, the market solution is not optimal. The relationship between the two, however, is not simple, and this is the reason that different parameterizations appear to yield different results.

There are two factors that we can identify: (1) If there are

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<sup>7</sup>In the formal analyses below, we do not use consumer or producer surplus; we are using it here only as a simple heuristic.

profits, competition will result in entry driving these profits down to zero.<sup>8</sup> This dissipation of rents through entry tends to result in there being too many firms. In particular, the profit of the firm does not, in general, correspond to the net change in profits in the economy as a whole: some of the customers (and the associated profits) are captured away from other firms. (In a perfectly competitive economy where price equals marginal costs, these interactions have no consequences; here, in general, price exceeds marginal costs, so these effects can be important.)<sup>9</sup> (2) On the other hand, a new entrant -- increased product variety -- usually raises the welfare of some consumers, since the firm is an imperfectly discriminating monopolist. Since the firm cannot appropriate all the consumer surplus associated with the new product, there will be, on this account, insufficient incentives for product diversification.

These different effects may operate, with different strengths, in different parts of the product spectrum. Thus, it is conceivable that there may be too few firms producing "speciality products" and at the same time too many firms catering to the mass markets. Much of the subsequent formal analysis is an attempt to capture these intuitions and to identify those characteristics of the market equilibrium which are important in determining whether there is likely to be an under or over supply of variety.<sup>10</sup> In particular, we identify three factors which are

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<sup>8</sup> We discuss below some circumstances in which profits are not driven to zero.

<sup>9</sup> Readers familiar with the public finance literature on the incidence of a tax on one sector in an economy with taxes (or distortions) in other sectors, will see the parallel. (cf. Atkinson and Stiglitz (1980), Harberger (1974)) and Greenwald and Stiglitz (1984).)



critical:

(a) The nature of competition among the firms, e.g. does each firm have one or two close competitors (with possibly a larger number of more distant competitors), or does each firm have a large number of firms with which it is competing directly.

The number of close competitors is important for two reasons. When there are only two neighboring competitors, the assumption that the firm's competitors will not react to his actions is less plausible than when the firm has a large number of competitors, from each of whom he can draw a few customers if he lowers his prices. Secondly, the elasticity of demand may be larger if there are a large number of firms with which the firm is competing directly, and as we shall see, the magnitude of the elasticity of demand is an important determinant of whether there is an under or over supply of variety.

(b) The ability of firms to price discriminate. Though we had expected that the greater this ability, the less likely there is to be an undersupply of firms, the effects of an improved ability to discriminate turn out to be more subtle and complicated.

(c) The nature of fixed costs -- whether they are "sunk" or not. When they are sunk, early entrants into an industry can take a position (a location, a product niche) which ensures to them a profit, but which can still deter entry; if they are not sunk, profits will be driven to zero, and hence there is a greater likelihood of an oversupply of firms.

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<sup>10</sup> As we shall see, some of the popular parameterizations in the literature implicitly make assumptions concerning the relative strength of the two effects which we identified above as giving rise to an over or under supply of variety.

#### 4. The Problems in Modelling

There is no theory of monopolistic competition which has the apparent (but deceptive) generality of the pure competition model.

There are three critical problems in developing a good model:

a. Characterizing the Product Space. One of the central issues, as we have noted, is what commodities are, and what commodities should be, produced. If one had a completely detailed specification of tastes and technology, a well defined welfare criterion, and a well formulated equilibrium theory, one could calculate the welfare optimum and contrast it with the market equilibrium. One could then list the commodities which are not produced, but should be, or are produced and should not be. Economic theory, however, must be concerned with more abstract characterizations of the commodity space, and it is here that we run into difficulty: how do we describe the set of all possible commodities, and on what characteristics should we focus our analysis?

Commodities differ both with respect to the technologies with which they are produced -- the importance of fixed versus variable costs, and of sunk costs -- and demand characteristics. Three approaches to modelling demand characteristics have been taken in the literature: the location model (Hotelling); the characteristics approach (Lancaster); and the general utility approach (Spence, Dixit-Stiglitz).

Unfortunately, it turns out that the kinds of results obtained depend critically on the particular parameterization chosen.<sup>11</sup> Much of our discussion later will focus on the special properties of each of these

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<sup>11</sup> In a recent Oxford B. Phil thesis, T. Omori has provided a framework which includes all of these as special cases.

models.

b. The Nature of Market Equilibrium. The essential feature of the perfectly competitive model is that firms believe that the prices at which they can sell their goods will not be affected by the quantity they produce. The essential feature of monopolistically competitive models is that firms believe that they face downward sloping demand schedule for their commodities.

On the other hand, we explicitly ignore here strategic interactions. This is what differentiates monopolistic competitive models from oligopoly models. (Whether the assumption of no strategic interactions is plausible in simple models, such as the circle location model, where each firm has only two neighbors is debatable. This is one of the marked advantages of the Dixit-Stiglitz-Spence approach, as well as the multi-dimensional location model described below.)

What is critical for the nature of the equilibrium is firms' beliefs (perceptions) concerning the demand curves. We would, of course, like these beliefs to be "reasonable". One property, that all analysts agree upon, is that firms believe that their demand curves pass through the actual {price, quantity} combination they realize. There is some controversy, however, about what are reasonable hypotheses concerning the elasticity of the demand curve. As Chamberlain recognized, this depends critically on how other firms respond to the action of the given firm. Two classes of models have been extensively investigated: those in which firms are price setters and those in which they are quantity setters.<sup>12</sup> (Some (Dixit-Stiglitz) have focused on the limiting case

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<sup>12</sup>Since normally, if the rival fixes his quantity, as the competitors  
(Footnote continued)

where there are sufficiently large numbers of firms that not only does each firm believe that it has no effect on the other firms actions but also that there are so many firms that the income effect of its price change can be ignored.) There is, of course, no reason to limit ourselves to the extremes of price or quantity setting. Firms could set supply functions, specifying how much they would supply at each value of the market price. (See Grossman [1979].) Koenker and Perry have examined the case where the action of the firm gives rise to some conjectural variation on the part of the other participants. All of these approaches are consistent and there are market situations where they may provide a good description. What I find less persuasive are mixed cases, for instance where it is assumed that there are sufficiently few firms that income effects cannot be ignored in the calculation of demand curves, yet sufficiently many firms that each assumes it has no effect on the output or price of other firms<sup>13</sup>

It is, of course, not only the number of firms which are important, but also their "competitive relations." It is in this respect that the Hotelling-Lancaster models differ markedly from the Chamberlain-Dixit-Stiglitz-Spence models; in the former, each firm has only two direct competitors (although there may be a large number of other possible competitors), while in the latter, each firm faces a

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<sup>12</sup>(continued)

sales increase, the rivals price will fall, perceived demand curves will be more elastic in Nash-price equilibria than in Nash-quantity equilibria.

<sup>13</sup>Shortly after Chamberlain's book was published, Triffin commented on this unsatisfactory aspect of Chamberlain's analysis for the small group case.

large number of competitors. This should have some important behavioral implications, which unfortunately have not been pursued in the literature. Moreover, it should be noted that while in the Lancaster-Hotelling, and Chamberlain models, goods can only be substitutes, in the Spence and Dixit-Stiglitz models, complementarity is also possible.

There is a second characteristic by which market equilibrium may differ, which has received only limited attention: most of the models assume that firms can only charge a single price. But various forms of price discrimination (including non-linear price schedules, commodity bundling, etc.) may be possible (particularly for location models.)<sup>14</sup> Many of the qualitative propositions concerning the comparisons between market equilibrium and welfare optimum are a consequence of an implicit, but unpersuasive, assumption that the former cannot discriminate, but the latter can.

c. The Welfare Criterion. Several approaches have been taken, with differing assumptions concerning (i) the government's knowledge, (ii) the instruments at its disposal; and (iii) the criterion to be used in evaluating alternative allocations.

(i) Knowledge. Lancaster, for instance, assumes that the government has full knowledge of tastes, and allocates resources making a full use of this knowledge. Thus, if there is a change in product variety which makes someone worse off, he can and, in Lancaster's analysis, will be

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<sup>14</sup>The critical role of the form of the price schedule charged by firms was made clear in Stern's analysis using the location model.

identified and compensated. Lancaster can thus consider the set of all Pareto optimal allocations.<sup>15</sup>

Although this seems possible in those situations where individuals differ only in location (and so it is only transport costs with which we need concern ourselves), when individuals differ in tastes this seems an irrelevant criterion.<sup>16</sup> All individuals would claim to be those who need to be compensated. Appropriate criteria should make use only of (a) statistical information about characteristics of the population as a whole; and (b) individual information which can be revealed by the actions taken by the individuals (e.g. purchases they make). The analyses presently in the literature do not, unfortunately, take this approach. The extent to which this would alter the kinds of conclusions obtained is thus an open question. (To call this kind of welfare analysis second best seems inappropriate. Costs of information are no more a reflection of an imperfection than is the fact that it takes inputs to produce outputs.)

(ii) Instruments and Constraints. In the previous subsection, we noted the critical role played in the analysis of the market allocation by the

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<sup>15</sup> He focuses on one special allocation, the uniform utility level; he unfortunately does not show to what extent this results are dependent on this assumption, and does not discuss the meaning of the implicit cardinalization of utility which he employs.

Lancaster also considers some of the "second best" allocations to be described below.

<sup>16</sup> Some readers have suggested that the full information (compensation) comparison is the appropriate one for assessing the validity of the claim that the market provides firms with the incentives to provide an efficient resource allocation; Adam Smith's invisible hand is supposed to lead firms to acquire the requisite information for an efficient resource allocation.

assumption that firms must charge a single price and cannot discriminate. Similar remarks are relevant here.

Dixit-Stiglitz and Spence have considered welfare optima where the government faces an additional constraint: it cannot provide lump sum subsidies to firms. This is meant to capture the notion that in a decentralized economy it is difficult to ascertain what a firm is; one can provide subsidies to inputs or to outputs. But if each production unit were to receive, say, a fixed subsidy, then a firm would have an incentive to divide, and call itself two firms.

In all of this, the objective is to engage in fair and appropriate comparisons: if it is argued that the government can ascertain differences in tastes, then it is reasonable to assume that the private sector can. If it is argued that the private sector must charge a uniform price, it is plausible that the government faces a similar constraint. Many of the comparisons found in the literature appear to be, at best, misleading on that account.

(iii) Objective Function. Most of the literature has not been concerned with pareto optimality, but has evaluated alternative allocations using a utilitarian (or social welfare) approach. This approach has failed to emphasize the important distributional effects of alternative choices of product mixes.<sup>17</sup> Dixit and Stiglitz and Spence and Stiglitz (1974)

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<sup>17</sup> There are some cases where at least one version of utilitarianism and Pareto optimality coincide: consider, for instance, the case where individual tastes for each commodity group are evenly distributed over an infinite line. Then, if the location of products is to be determined randomly, the utilitarian allocation maximizes ex ante expected utility provided the location of the "center" commodity is determined randomly. If there are a large number of commodity groups, with individual

(Footnote continued)

avoid these difficulties by positing that all individuals are identical; but whereas in the characteristics and location models, each individual consumes a single commodity, in the Dixit-Stiglitz-S:ence analysis, individuals consume a variety of commodities.

Second Best Comparisons. Even after deciding on the relevant welfare criterion, it is not clear how the comparisons between the optimum allocation and the market solution should be interpreted. In some situations, for instance, the market will charge too high a price for the goods in the monopolistically competitive sector. This reduces, of course, the demand for such goods. Given the lower level of demand, there may be, not surprisingly, fewer firms (and a narrower range of product diversity). Is it more meaningful simply to compare the total number of firms in the monopolistically competitive sector in the two allocations, or to attempt, in the comparison, to take into account the different levels of aggregate expenditure in the sector, e.g. by asking, given the level of expenditure on the monopolistically competitive sector, are there the correct number of firms?

## Part II

### The Basic Models

There are four basic models of monopolistic competition.<sup>18</sup> Although

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<sup>17</sup> (continued)

preferences being symmetric, then not only does the utilitarian rule maximize ex ante expected utility, it may also be ex post pareto efficient. These are situations where, because of our strong assumptions, we can ignore the distributional implications of product choice; but if some individuals are always "outliers" then this may be a less satisfactory approach.



the mathematical details of each are complicated, the basic structure of the models may easily be presented; this will enable us to compare the relative strengths and limitations of the alternative approaches.

5. The One-Dimensional Location Model: Finite Line, Fixed Number of Firms

We begin our discussion with the oldest of the models, the one-dimensional spatial (location) model first analyzed by Hotelling.<sup>19</sup> We present the model in a slightly more general form than analyzed by Hotelling. Assume individuals are located at different points along a line; each individual has a demand curve  $D(q)$  where  $q$  is the price he pays. We assume that transport costs from the place of sale to "home" are proportional to the quantity purchased, so if he has to transport the goods a distance of  $t$ , and the transport cost function is  $\psi(t)$ ,  $\psi' > 0$  then

$$q = p + \psi(t)$$

where  $p$  is the price charged by the firm.<sup>20</sup>

We do not impose any additional restrictions on the transport cost function  $\psi$ ; it is tempting to assume convexity<sup>21</sup> ( $\psi'' > 0$ ). But there

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<sup>18</sup> A fifth model, the imperfect information monopolistic competition model, is referred to briefly below in section 11. For a survey of some of these models, see Salop (1976) and Stiglitz (1979).

<sup>19</sup> Thus the theory of monopolistic competition pre-dated Chamberlain and Robinson.

<sup>20</sup> This is obviously not the most general form: we could have postulated  $q = q(p,t)$ . This makes sense, for instance, if part of the costs of transportation includes spoilage of the purchased product as it is transported. If the only transport costs are from spoilage,

$$q = q \psi(t)$$

<sup>21</sup> If  $\psi'' \leq 0$ , a standard (non-mixed strategy) equilibrium may not exist, (Footnote continued)

is no a priori reason to impose this restriction. For instance, if there are a number of different modes of transportation, with each mode characterized by a fixed cost and a constant variable costs per unit distance, then the transport cost function will be concave. (See figure 1). When later we come to the commodities interpretation of this model, since there are really no natural units for measuring distance in commodity space, this point is even more important. Assume, for instance, that commodities differ in their color, and we measure color along the spectrum. Individuals' favorite color may be uniformly distributed along the spectrum, but there is no reason to believe that the utility they receive from colors which are away from their favorite color in the spectrum is convex in the wave length of light. (cf. Stigler (1968)).

We assume further that the distribution of individuals along the line is uniform, and the line is of fixed length (say 2). Finally, we assume there are two firms. Some of these assumptions are crucial, some are not, as we shall shortly see.

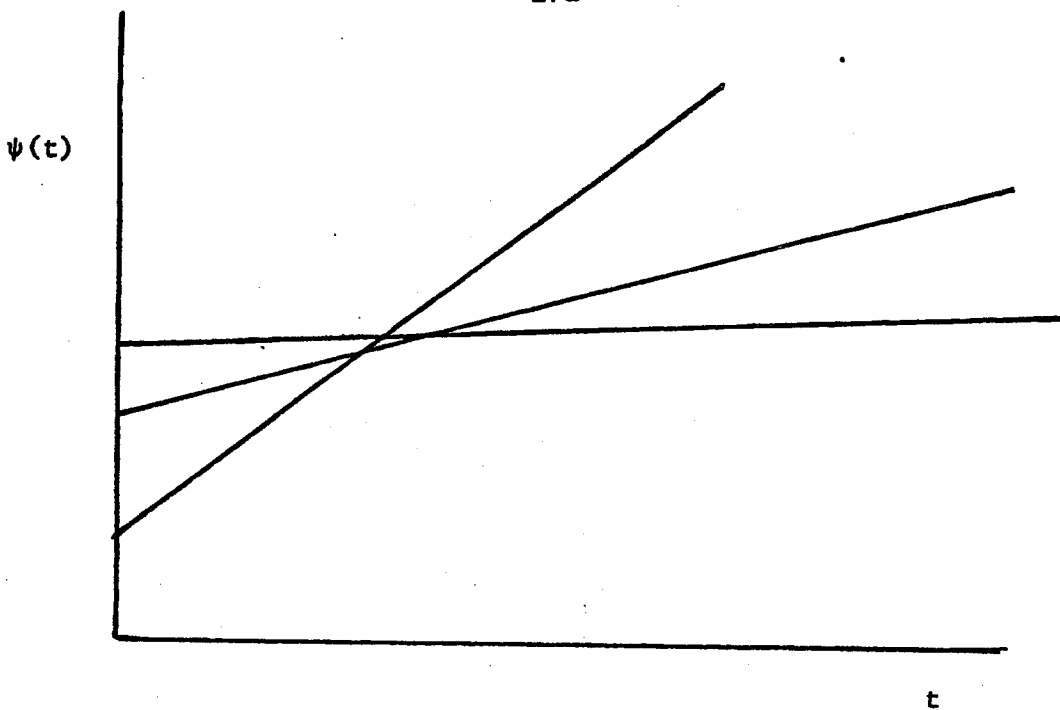
With this model, we first ask, is there enough product diversity, i.e. will the two competitive firms choose the correct locations?

### 5.1 Location Competition

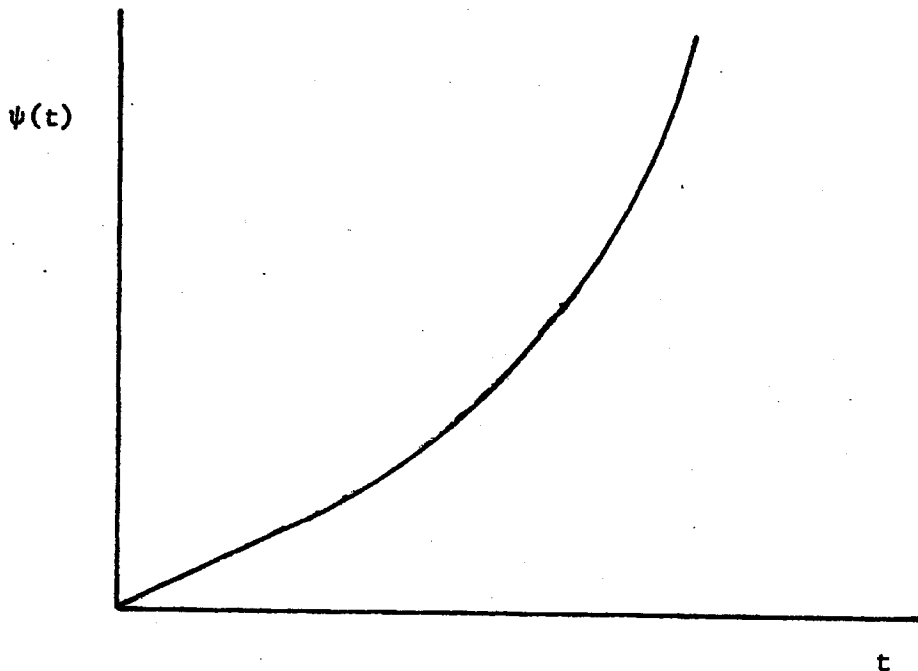
To answer this, let us make two further (crucial) assumptions: prices are fixed (and identical) and demand is inelastic. Each individual buys one unit, provided the price is less than or equal to the reservation price  $u$ . Clearly, individuals will go to the firm which

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<sup>21</sup> (continued)  
as we shall comment later.



Concave transportation cost function: transportation costs are lower envelope of several alternative modes of transport



Convex transportation cost function: marginal cost of transporting goods an extra unit increases with distance

Figure 1

is closest to them. Given any location of one's rival, the optimal location of the firm is half way between that point and the center point. It is immediate then that the only equilibrium is for both firms to be located in the center: there is insufficient diversity. Given that all firms are located at the center, social optimality entails only a single firm producing. Given the market's failure to provide adequate diversity, there are too many firms.<sup>22</sup>

### 5.2 Location - Price Competition With No Sunk Costs

Now assume that firms can compete on price and location. Let  $l^*$  be the location chosen by the first firm,  $l^{**}$  the location chosen by the second, and  $\hat{l}$  the location of the individual who is indifferent between the two stores, as shown in Figure 2. Let  $p^*$  be the price at the first store,  $p^{**}$  at the second. Then  $\hat{l}$  is determined by

$$(1) \quad p^{**} + \psi(l^{**} - \hat{l}) = p^* + \psi(\hat{l} - l^*)$$

### 5.3 Non-existence of Equilibrium

Let us now assume that transport costs are a linear function of distance. It is immediate that there is a discontinuity in the demand for the product as price is lowered for any arbitrarily specified set of locations. If  $p^*$  is sufficiently low that at  $l^{**}$ ,  $p^* + \psi(l^{**} - l^*) < p^{**}$ , clearly everyone will purchase from him; while if at  $l^*$ ,  $p^{**} + \psi(l^{**} - l^*) < p^*$ , everyone will purchase from his rival. The demand curves appear as in Figure 3. It is clear that generally there will

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<sup>22</sup> There is a sense in which there is no pure strategy equilibrium with three firms: each tries to be the outside firm, just "next" to the center.

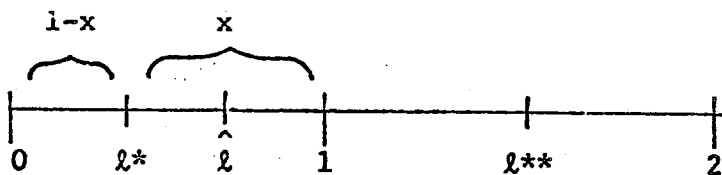


Figure 2

The Basic Location Model

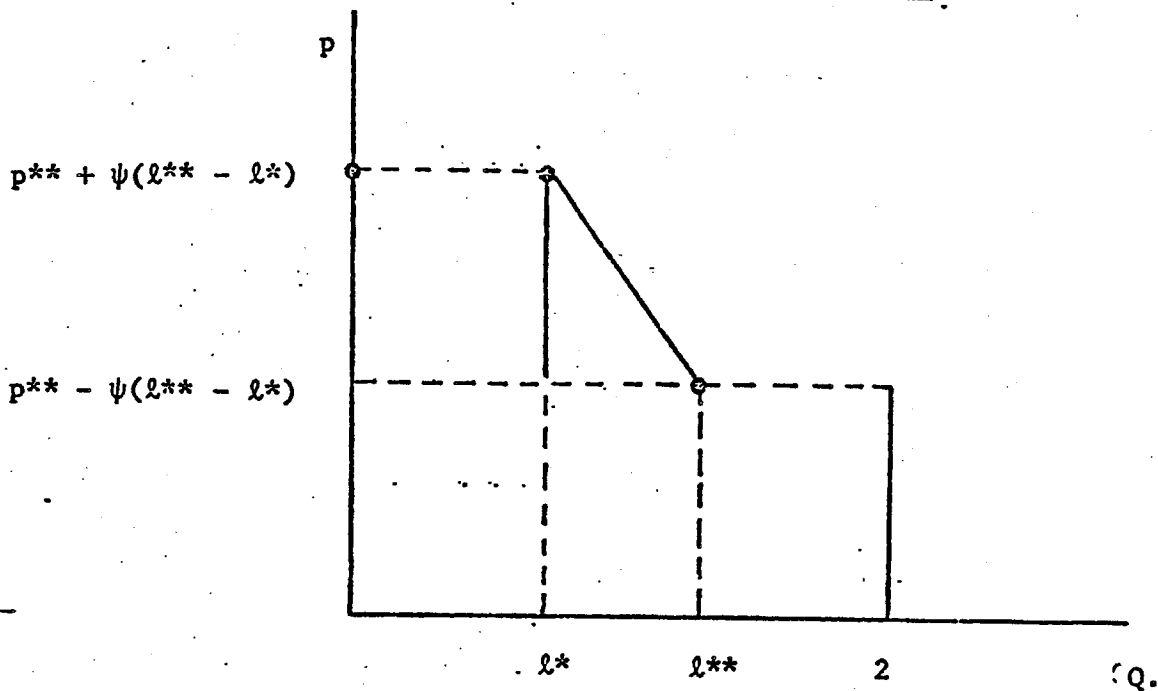


Figure 3

Demand Curves for Fixed Location

exist no Nash equilibrium. Assume, for instance, that they both locate in the center as before. So long as there are positive profits, it pays one of the firms to undercut the other, capturing the entire market. But when prices have been lowered to the point where there is zero profit, the other firm will not produce; but if the other firm does not produce, the first firm will raise its price. There is thus no price setting Nash equilibrium (in pure strategies). The reason for this, as we shall see shortly, is that, under the assumption of linear transport costs, the two commodities become essentially perfect substitutes.<sup>23</sup>

#### 5.4 Alternative Equilibrium Concept: Mixed Strategy Equilibrium

First, however, we should note there is an alternative equilibrium concept for which an equilibrium exists: there will in general exist a mixed strategy equilibrium. (See Dasgupta and Maskin). Assume, for simplicity, that the marginal cost of production is zero. Assume, moreover, that each firm locates at the center. Assume that one firm uses a price distribution  $F(p)$ . Then the expected profits of the second firm if it charges price  $p$  is

$$p(1 - F(p)),$$

since he will have the entire market if his price is lower, and none of the market if his price is higher. Thus, if

$$1 - F(p) = k/p$$

where  $p \geq k$ , the firm is indifferent between all prices greater than or equal to  $k$ .

If there are  $n$  firms, then the firm obtains all sales if none of

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<sup>23</sup>The non-existence of equilibrium with price competition among competitors with fixed capacity was noted long ago by Edgeworth.

the other firms offer the good at a lower price, i.e. expected profits are

$$p(1-F(p))^{n-1}$$

so the price distribution is

$$1 - F(p) = (k/p)^{1/n-1}$$

Assuming unlimited entry with costs of entry being  $K$ , equilibrium entails  $k = K$ .<sup>24</sup> Thus, there is an indeterminacy of equilibrium: equilibrium is consistent with their being both a large number and a small number of firms. But the probability distribution of the lowest price

$$G(p,n) = 1 - (1-F(p))^n = 1 - (K/p)^{n/n-1}$$

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<sup>24</sup> we assume risk neutrality and ignore the discreteness of  $n$ .

is unambiguously adversely affected by an increase in  $n$

$dG$

$$\frac{dG}{dn} = 2n(K/p) (K/p)^{n/n-1} / (n-1)^2 < 0$$

$dn$ .

(since  $p > k$ ). Competition unambiguously results in higher prices.<sup>25,26</sup>

### 5.5 Critical Assumptions

In the construction of the basic location model described above, there are several critical assumptions, to which we now call attention.<sup>27</sup>

#### 5.5.1 Elastic Demand Curves

Two of these assumptions are easily dealt with: the inelasticity of

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<sup>25</sup> This particular formulation has the unattractive feature that it requires that there be no reservation price on the part of consumers. But a slight extension of the model is consistent with there being a reservation price. In particular, assume each firm has a capacity of  $\alpha$  and total sales are normalized at unity. Then expected profits are  $\alpha p(1-F(p)) + (1-\alpha)pF(p) = k$

so

$$F = (k - \alpha p) / (1-2\alpha)p$$

and we require

$$k/\alpha \leq p \leq k / (1-\alpha)$$

<sup>26</sup> This analysis takes location as given, and addresses the question of equilibrium prices. A full analysis, with prices and location determined simultaneously is beyond the scope of this paper. We would argue that the appropriate model entails determining location first, and then prices; a simple version of such a sequential model is discussed

(Footnote continued)



demand and linearity of transport costs both result in the demand curves having a number of peculiar properties. In particular, if demand curves are downward sloping, then as the firm moves towards the center, the "delivered" price at the fringe increases, and this reduces the demand on the fringe. It is thus possible that there exist equilibria in which the two firms do not locate precisely in the center.

### 5.5.2 Convex Transport Costs

This effect is reinforced if transport costs are convex, i.e. by placing his firm in the middle of his market area, average transport costs are reduced even more than with linear transport, and hence demand is increased. More importantly, the discontinuity in demand which arises when one firm undercuts the other disappears when the transport cost function is strictly convex. If it is convex enough, an equilibrium can be assured. With elastic demand some of the gains from reducing transport costs are appropriated by the firm, and the market demand curve facing the firm is more elastic than if market area were fixed. Hence, for sufficiently elastic demand curves and convex transport cost functions a "diversified" equilibrium exists.<sup>28</sup>

Though the specific parameterizations have a significant effect on the nature of the equilibrium, the most critical assumption is the absence of entry.

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<sup>26</sup> (continued)  
briefly below.

<sup>27</sup> A critical assumption, to be discussed later, is the one dimensional nature of the formulation. See below and Nalebuff.

<sup>28</sup> See Appendix B.

## 6.0 Linear Location Model, Fixed Boundaries, Endogenous Number of Firms

The analysis so far has followed Hotelling in assuming a fixed number of firms. An essential aspect of monopolistic competition theory, however, is that there may be entry in response to profitable opportunities. The question is, whether the threat of entry serves as a sufficiently strong discipline on the market place to ensure that there will be zero profits; and if profits are driven to zero, is it still the case that locations and prices will be chosen, in some sense, optimally? The answer depends, in part, on the notion of equilibrium and whether there are sunk costs.

### 6.1 Contestability

In the argument for nonexistence, when the firm raised its price above the competitive level, it ignored the obvious incentive that provided for firms to enter; and when firms entered, they ignored the obvious incentive that they provided for the existing firm to lower its price. These assumptions about the failure of potential rivals to respond are clearly unsatisfactory, in the particular circumstances being considered. There are a number of possible alternative hypotheses which are at least as persuasive, particularly if the fixed costs are not sunk costs. Any firm which attempted to sell at a price which generated revenues in excess of average costs would quickly be met with entry. Though this suggests that the only equilibria entail zero profits, there may be many zero profit price-location pairs; which of these (if any) are equilibria, depend on the precise definition of equilibrium.

Consider the following definition.

An equilibrium is defined as a set of prices and locations such that there is no incentive for a new entrant to enter (if the entrant assumes these prices and locations will remain unchanged), and such that it does not pay any firm to change its price location pair, given the price and locations of all other firms. An immediate implication of this definition is that each firm must be making zero profits (if there are positive profits, there is an incentive for an entrant to enter at the same location, and undercut the price.)<sup>29</sup>

With this definition, it is possible to show that there are circumstances in which there would be positive resource savings from the establishment of a second (optimally located) firm, and yet the market will not do so; and there are other circumstances, in which fixed costs are relatively high, in which the fixed costs of establishing a second firm exceed the savings in transport costs, yet, the market equilibrium entails at least two firms. When a second firm does enter, however, it will not enter "on top" of an existing firm, as in Hotelling's analysis. (See Appendix A).<sup>30</sup>

## 6.2 Competition with Sunk Costs

The implicit assumption that locations are easily changed is clearly

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<sup>29</sup>This equilibrium concept has the unattractive property of postulating "reactions" for potential entrants, but not for existing firms. There is also something slightly unsatisfactory in the assumption that firms anticipate entry, but entrants do not anticipate the departure of unprofitable firms in response to their entry (recall there are no sunk costs) or a change in price or location of existing firms. This equilibrium concept is an attempt to model dynamic reactions within a framework which is not explicitly dynamic. Developing an explicit dynamic framework would take us beyond this paper.

not valid for many commodities. (Hay, Prescott and Visscher). Assume that once a firm's location is fixed, it is prohibitively expensive to move. (This corresponds, in the product space interpretation of the model, that there are sunk costs in engineering, making dies, product design, etc. in deciding to bring out a product.)

The first firm to enter a market area will not -- if it is thinking strategically -- enter at the center point; for it will attempt to anticipate where the next firm will enter. Rather, it will locate towards a boundary; and at a distance from the boundary which is just great enough that no other firm can enter between it and the boundary, and still make a profit.

Just how far it locates from the boundary depends on the nature of post entry competition. Thus, if the entrant believes that the incumbent firm will continue charging the same price, then he is more likely to enter than if he believes that the incumbent will react to his entry, by engaging in, say, Bertrand price competition. Thus, under the postulate that the firm will keep his price unchanged (cf. our discussion above on contestability) profits will be zero, even with sunk

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<sup>30</sup> Under the contestability hypothesis, there may exist multiple equilibria, with differing degrees of diversity. Thus, if a single firm locates in the center and charges price equal to average costs, entry may not be desirable. But there may exist another equilibrium in which two firms operate, each at a distance somewhat greater than .5 from the boundary (in our line of length 2). If each firm were located .5 from the boundary, a slight shift towards the center would shift their demand curve out. Under the definition of equilibrium given above, however, the firm has no incentive to adjust its location, since it knows that it cannot take advantage of it: it will be forced, at the new location, to lower its price to average costs; the firm also knows that at its existing location, no entrant has an incentive to enter; hence the threat of entry does not motivate him to alter his location.

costs. It would pay a firm to enter on top of (or next to) an existing firm making positive profits, lower its price by  $G$ , and steal all the sales.

But this postulate of no price reaction does not seem to make any sense in the context of sunk costs, unless somehow firms are committed to charging their current prices. Prices are not state variables.<sup>31</sup> But then, with any sunk cost, if a firm locates too near an existing firm, the price competition which results will drive prices to marginal costs of production. This has two immediate implications. Equilibrium is consistent with positive profits, and if entry occurs, it will occur at some distance from an existing firm.<sup>32</sup>

6.3 Monopoly: multi-store firms. The analysis so far has assumed that each firm owns a single store. There is nothing, however, to stop a single firm from opening up several stores. Assume that there are no

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<sup>31</sup> Except under certain limited situations. See Stiglitz (1981)).

<sup>32</sup> Consider the following simple dynamic model. In the first period, the first firm picks a location. In the second period, the second firm can enter. It chooses a location (the expenditures are sunk.) If entry occurs, in the third period, firms play a Bertrand price competition game. In making their locational decision, firms know the nature of the post entry game.

Assume the first firm locates in the center, while the second firm locates at a distance  $l^*$  from the boundary. We let  $\psi = zt$ . Then, if the first firm charges  $p^*$ , the second,  $\hat{p}$ , the marginal individual buying at the first firm is located at a distance  $y$  from the center where

$$p^* + zy = \hat{p} + z(1-l-y)$$

or

$$y = \frac{\hat{p} - p^* + z(1-l)}{2z}$$

Straightforward but tedious calculations confirm that there is no  
(Footnote continued)

diseconomies of scale. Then, the first store to enter will construct stores at distances apart which are such as to deter further entry. (Assume that it paid an entrant to enter at a location, given the price competition that would result; clearly, if the firm were to establish another store at that location, it could obtain greater profits than the entrant, since it would not engage in the subsequent price competition.)

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It is worth noting, however that (with sunk costs) locations may be closer together with a monopolist and, as a result, average prices higher. The reason for this is that an entrant entering into a market with several different firms will calculate the Bertrand equilibrium that will result. He knows that a reduction of the price at one location will induce a lowering of the price at adjacent locations; and these lowered prices will, in turn lead to lower prices at locations still further away. The lowered prices lower the entrant's return. A monopolist, controlling all stores, simply chooses the prices at each store optimally. He knows that lowering the price at one location lowers profits at adjacent locations, and takes this into account. This

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<sup>32</sup> (continued)

location at which there exists a pure strategy equilibrium. (For each location, we calculate the Bertrand price equilibrium, assuming that each firm does not undercut the other's price by enough to steal the entire market; we can then show that at every location, it pays at least one of the two firms to attempt to steal the market.) For each location, we can calculate the mixed strategy equilibrium. At locations other than the center, this will in general entail positive profits to the entrant. The entrant will locate at that location at which his (expected) profits are highest.

<sup>33</sup> The argument is the standard pre-emption argument, first formalized in the context of technical change. See Salop (1979) Dasgupta and Stiglitz (1980), Gilbert and Newbery (1982), and Gilbert, (1985).

induces him not to lower his prices as much in response to entry; thus, to make entry less attractive, he must place his stores closer together. Choosing store locations is the only form of pre-commitment which the firm can engage in.<sup>34</sup>

### 6.3 Summary

Hotelling's basic insight that competition among two firms along a line of finite length would lead to insufficient diversification (each firm occupying the center position) has been shown not to be robust. Some degree of diversification (product differentiation) may occur if (a) demand curves are elastic; or (b) transport cost functions are convex. The assumptions of linear transport cost functions and inelastic demand curves also give rise to existence problems in pure strategies, when there is both price and location competition, though there exists a mixed strategy equilibrium.

Two alternative equilibrium notions were considered. A version of "contestability" lead to the usual result that the only equilibrium entails zero profits, but there may be one firm, when the fixed costs of establishing a second were less than the savings in transportation costs that would result, and there might be more than one firm, when the fixed costs of the second firm exceeded the savings in transport costs. There may also be multiple equilibria.

On the other hand, with sunk costs, with firms assuming that after

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<sup>34</sup> If the monopolist could delegate the responsibility for the management of each store to a different individual, and could pre-commit himself not to intervene, to co-ordinate their actions, then it would pay for him to do so.

entry there would be Bertrand price competition, equilibrium was consistent with the persistence of positive profits. In both this and the previous case, however, when entry occurs, it does not occur next to existing firms: there is always some degree of diversification.

There is, however, always an incentive for a single firm to establish "stores" at different locations, in such a way as to deter entry. The threat of potential competition may lead the monopolist to locate his stores closer than he otherwise would and indeed closer than under monopolistic competition, but does not induce him to charge lower prices.

#### 7. Single Dimensional Locational Model: No Boundaries

The finite market with a linear structure has one special property; each firm has only two neighbors, and, in particular, the firms at the boundary have only one competitor. The question naturally arises which, if any, of the properties of the model discussed in the preceding section are due to the lack of competition at the boundary.

This problem has been addressed (within the location model) in two different ways. One is to assume an infinite line; the other that the market is described by a circle. In either case, every firm has a competitor on both sides. But now, under the usual symmetric conditions which are imposed, the issue of product diversity and the number of firms become identical.

Given the location of his two neighbors, both of whom are choosing the same price it is obvious that he would locate precisely half way between. If he assumes his rivals will not change either their prices or their location, we can easily calculate his optimal price, and determine whether entry is desirable. (It is, of course, entirely



possible that existing firms be making strictly positive profits, and yet the maximized value of the profits of the entrant be negative, in which case entry would not occur. Most of the discussion has, however, focused on the special case where those within the industry are making zero profit.) The question now is, how far apart will firms locate in equilibrium?

### 7.1 Efficient Location

First, however, we address the question of how far apart should firms be located? As we emphasized, one has to be careful about which welfare criterion to employ. A convenient reference point, however, is the allocation which minimizes the sum of transport costs plus fixed costs,  $c$ ,

$$(2) \min [2 \int_0^{l^*} \psi(l) dl + c] / 2l^*$$

34a

$$(3) [c/2 + \int_0^{l^*} \psi(l) dl] / l^* = \psi(l^*)$$

(Equation (3) is plotted in Fig. 4.)

This just says at the optimum, average costs equal marginal transport costs.

Hence if

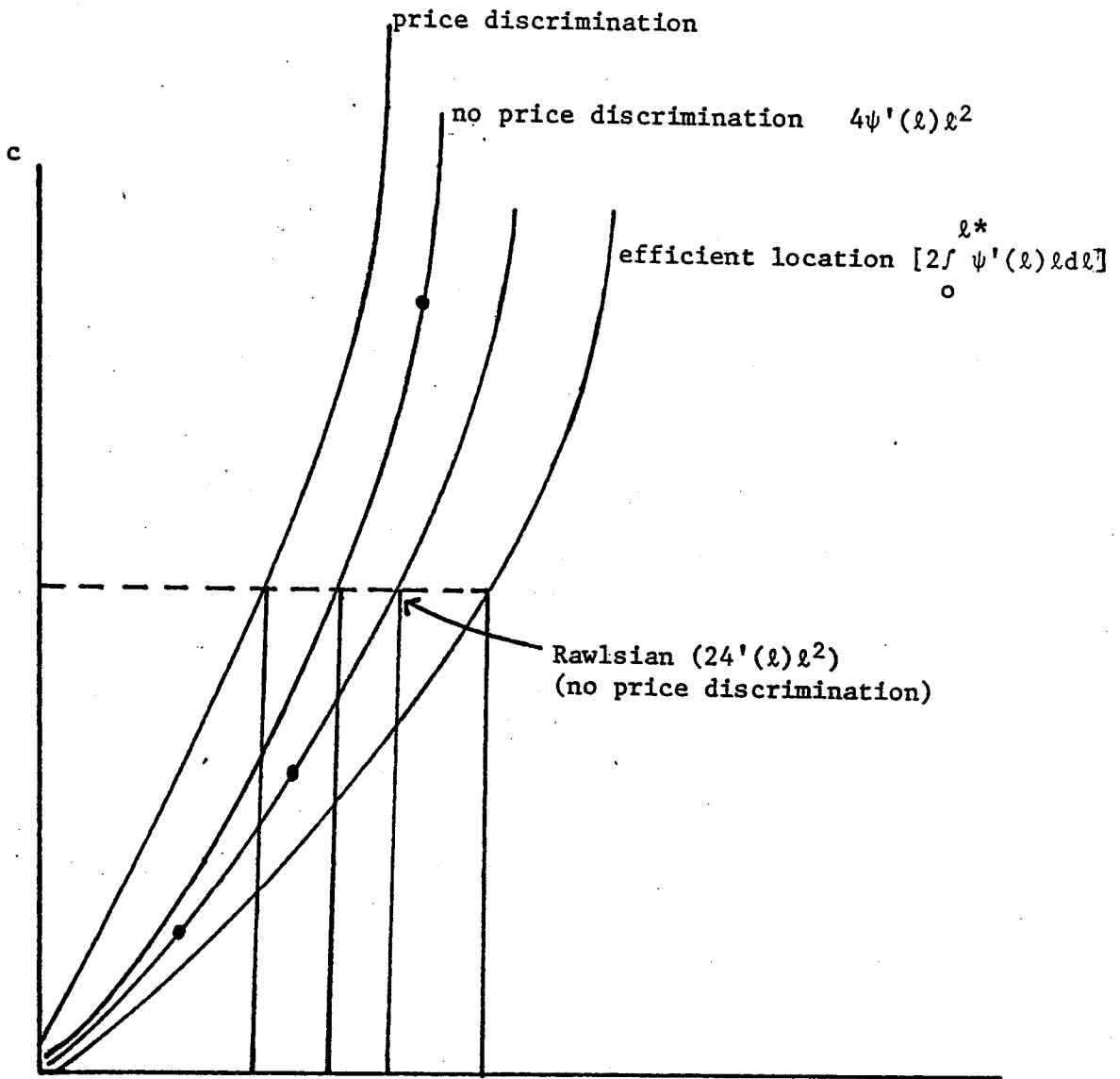
$$(4) \psi = l^* \alpha / a,$$

$$(l \alpha / a) - (l^* \alpha / a(1+a)) = c / 2l^*$$

or

34a Integrating by parts, we obtain

$$c = 2 \int_0^{l^*} \psi'(l) l dl = 2 \left[ \frac{\psi'(l^*) l^{*2}}{2} - \int_0^{l^*} \frac{\psi'' l^2 dl}{2} \right] < \psi'(l^*) l^{*2} \text{ as } \psi'' \geq 0.$$



Comparison of Market Areas

Figure 4

$$(5) \quad l^* = [(1+\alpha) c/2]^{1/(1+\alpha)}$$

If  $\alpha = 2$  (quadratic transport cost function)

$$(6) \quad l^* = (3c/2)^{1/3}$$

## 7.2 No Price Discrimination

Consider first the traditional formulation, where the monopolist is not allowed to discriminate; he charges a uniform price and individuals must pay their own transport costs. (This interpretation is particularly cogent for the case of "tastes"). We assume each firm takes the price and location decisions of others as given.

From the basic condition (1) it is clear that as the firm lowers the price, it attracts more customers. The slope of his demand curve is simply found by differentiation (assuming, as before that each consumer purchases one unit, and the density of individual is one per unit length and locations and prices of other firms are fixed). At  $p=p^*$ ,

$$(7) \quad dl/dp = -1/2\psi'(l)$$

A firm maximizes  $2(p-m)l$ , where  $m$  is the marginal cost of production or

$$(8) \quad l = (p-m)/2\psi'(l)$$

Zero profit equilibrium requires, in addition, that

$$(9) \quad 2(p-m)l = c$$

so

$$(10) \quad c = 4 \psi'(l)l^2$$

This is also plotted in Figure 4.

Thus, if  $\psi = l^\alpha/\alpha$

$$(11) \quad l = (c/4)^{1/(1+\alpha)}$$

In the quadratic example,

$$(12) \quad l = (c/4)^{1/3}$$

### 3.3 Comparison with Optimal Spacing

In making our comparison with the optimal allocation, it seems only appropriate to make the comparison under the assumption that the government cannot price discriminate either.

Since the government is not allowed to subsidize transport costs, the welfare of the consumer is proportional to

$$(13) \quad p + \psi(l)$$

Where  $l$  is his distance from the firm. We shall consider, in particular, the individual at the margin. Assume the government could impose lump sum taxes on everyone (with the proceeds used to finance fixed costs), but that, in keeping with our non-discrimination rules, it

must be uniform. Let  $T$  be the amount of the tax. The budget constraint for the firm must satisfy

$$(14) \quad 2l^*(T+p-m) = c$$

It is clear that (in our example with an inelastic demand) the possibility of levying lump sum taxes does not alter the nature of the solution. We thus

$$(15) \quad \min p + \psi(c/2(p-m))$$

obtaining

$$1 = \psi' l^* / (p-m)$$

or

$$(16) \quad c = 2\psi'(l^*)l^{*2}.$$

This is plotted in Fig.

With convex transport cost functions, the market area which maximizes the welfare of the worst off individual is larger than the market equilibrium. Infra-marginal individuals are unambiguously made better off by increasing market area. The market equilibrium is unambiguously pareto inefficient, with too many firms, provided the transport cost function is convex.<sup>35</sup> The magnitude of the inefficiency depends on the transport cost function.

### 7.3 Discriminating Monopolist

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<sup>35</sup>Or even not too concave. (See Fn. 34a.)

As we remarked in the previous section, the relationship between the optimal and market allocation depends critically on the kinds of pricing schedules we allow.

Consider now a discriminating monopolist. He cannot perfectly price discriminate because of the pressure of competition. Assume he charges an individual at location  $l$  a fixed amount  $h(l)$  plus a price equal to marginal cost  $m$ . The magnitude of the fixed fee is determined by the competition from neighboring stores; in particular, it is obvious that the individuals who are at the boundary between the market areas of two stores pay no fixed fee. (Otherwise, one of the stores would lower its fixed fee, and attract the customer away, and make a profit.) We denote this individual with a caret, so  $h(\hat{l}) = 0$ . (Stores are located, in equilibrium, at a distance  $2\hat{l}$  from each other.) For other individuals,  $h$  is determined as the maximal value of the fixed fee which leaves the individual indifferent to shopping at the neighboring store (and paying the larger transport costs). (In addition, the individual has to prefer to buy some of the good; we assume the consumer surplus associated with consuming the good is sufficiently large that this latter constraint is never binding.) We thus obtain

$$(17) \quad h(l) + \psi(l) = \psi(2\hat{l} - l)$$

Since

$$(18) \quad -h' = \psi'(l) - \psi'(2\hat{l} - l);$$

as  $l$  increases, the fixed fee decreases faster than transport costs increase. Free entry with zero profits entails

$$(19) \quad 2 \int_0^l h(l) = c.$$

i.e.

$$(20) \quad 2 \int_0^l [\psi(2l-l) - \psi(l)] dl = c.$$

For instance, for the constant elasticity function  $\psi = l\alpha/\alpha$

$$(21) \quad l = \left[ \frac{c}{2} \frac{\alpha(1+\alpha)}{2^{1+\alpha}-2} \right]^{1/1+\alpha}$$

For the quadratic function  
This is plotted in Fig.4.

$$l = \left[ \frac{c}{2} \right]^{1/3}$$

Contrasting (21) and (10), we immediately see that if  $\alpha < 5$ , then market areas with discrimination are larger than without it, and conversely if  $\alpha > 5$ .<sup>36</sup> Though one might have thought that the possibility of discrimination unambiguously resulted in higher profits, with any given market area and therefore more entry and therefore smaller equilibrium market areas, this is not quite correct: what is critical in the absence of discrimination is the change in the market area for any, given change in price; here, what is critical is how much the price charged to each individual can be increased without inducing the individual to switch.

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<sup>36</sup>We need to evaluate

$$1 - [(2\alpha - 1)/\alpha(1+\alpha)]$$

This is positive if  $\alpha < 5$ , negative if  $\alpha > 5$ .

If the government has the ability to discriminate (as it might plausibly, under the same conditions under which a firm could discriminate) then clearly it will wish to minimize resource expenditures (transportation costs plus fixed costs of firms). For the case of constant elasticity convex transport cost functions, this entails a larger market area. Though a utilitarian solution would entail charging all individuals the same price (inclusive of transport costs) when demands are inelastic,<sup>37</sup> the utilitarian solution will not, in general, represent a pareto improvement over the market solution; the individual at the boundary, who pays only marginal costs, is better off under the market allocation. On the other hand, there exist price schedules which the government could impose which would be a pareto improvement.

Under the Rawlsian allocation, goods are allocated (prices charged) so that the utility of individuals at all locations is the same. This

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<sup>37</sup> Implicitly, our representative individual's utility function is of the form

$$U = uC + M \quad \text{for } C < 1$$

$$= u + M \quad \text{for } C > 1$$

where  $C$  is his consumption of the good produced by the monopolistically competitive sector and  $M$  is his consumption of other goods. If the individual had the more conventional utility function of the form

$$U = u(C) + v(M)$$

then individuals further out would receive fewer  $C$ - goods (since the cost of delivering goods to them would be greater, at the margin the marginal utility of consumption would exceed that to individuals living near the firm; but they would receive the same amount of  $M$ -goods (under the hypothesis that there was no transport costs associated with them). Thus, utilitarianism entails some inequality (cf. Stiglitz, 1982); but unlike the monopolistically competitive market with discrimination, it is individuals further away from the firm who are worse off, rather than those closer to it.



is the same result that emerges from a location model in which there are land rents, as we shall now see.

#### 7.4 Rental Markets for Land

Slight changes in the assumptions yield marked changes in results. In the previous analysis, we assumed that individuals were fixed in location. Now, let us assume that there is a rental market for land; for simplicity, we assume each individual consumes a single unit of land.<sup>38</sup>

To avoid distributional considerations, we assume that each individual owns a proportionate share of the land at each location. The rent gradient will adjust to reflect differences in transport costs and prices charged at different locations. Since competition is more keen at the boundary, rents will increase with distance from the firm. Thus, if all individuals are identical, (using ( 11 )),

$$h(l) + R(l) + \psi(l) = h(0) = \psi(2l^*),$$

where  $R(l)$  is the rent at distance  $l$  from the firm, and where it is still the case that

$$h(l) + \psi(l) = \psi(2l^* - l)$$

Market equilibrium is unchanged. But now when all individuals are identical, the Rawlsian solution described earlier is, in fact, the Pareto optimal allocation; it is also<sup>39</sup> clear that all individuals are

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<sup>38</sup>These results can easily be generalized, using the kinds of techniques employed by Arnott and Stiglitz (1980).

<sup>39</sup>Under the assumption that the individual consumes only one unit of the commodity and the utility function is separable; in this case the utilitarian solution and the Rawlsian solution are identical. But see footnote 37.

worse off in the market equilibrium than they would be in the social optimum.<sup>40</sup>

In this case, the optimal allocation has an interesting interpretation. Assume the firm charged all individuals just  $m$ , the marginal cost of production. Then aggregate rents must equal the fixed costs of production, as illustrated in Figure , (making use of (3)). This is just another application of the Generalized Henry George Theorem.<sup>41</sup>

#### 7.4 Zero Profits?

In the previous discussion, we have not fully addressed the question of why profits should be zero in a Nash equilibrium. Assume that firms are located sufficiently far apart that each firm is earning a positive profit. The conventional argument is that this will attract an entrant. Assume, as we argued earlier, that he entered half way between two incumbents. There may be no price which he can charge which, if the other firms do not change their locations and prices, will allow him to break even.<sup>42</sup>

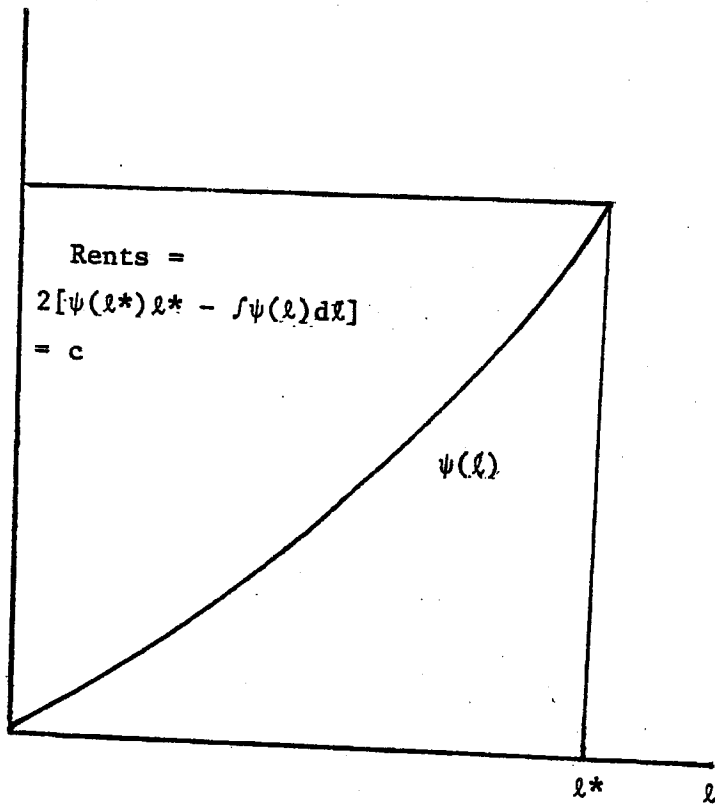
Thus, the presumption of zero profit equilibrium is based on an assumption that the existing firms will react to the new entrant by

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<sup>40</sup> If the firm can discriminate among customers, but must face each customer with a single price, there is an additional distortion associated with the market equilibrium: in its attempt to exploit each consumer, it reduces consumption.

<sup>41</sup> See Flatters, Henderson, Meiskowski (1974), Stiglitz (1977) and Vickrey (1977).

<sup>42</sup> This is a peculiar property of the spatial equilibrium model (shared by the characteristics model which we discuss in the next section); any new firm must enter between two existing firms. This is not true of the Spence-Dixit-Stiglitz Model.



With optimal market area, fixed costs equal rents

Figure 5

choosing new locations; it is based, in other words, on some kind of reaction function equilibrium.

But, as we noted earlier, matters are even worse: there never exists a price-location Nash equilibrium (in pure strategies) in the location or Lancaster characteristics model. For assume that there is an equilibrium; for simplicity, we focus on the zero profit equilibrium. Then any single firm could, by locating at precisely the location of his neighbor, and undercutting his price by  $\epsilon$ , increase his profits, by an amount which may exceed 50 percent of fixed costs ( $c$ ). (He would obtain all of his rival's customers, and still retain 50 percent of his own, if he simply lowered his price by  $\epsilon$ ; but this will not necessarily be optimal.) On the other hand, in the Dixit-Stiglitz-Spence model (as well as in the  $n$ -dimensional location model) the increase in profits from following this policy will, if there are enough firms, be arbitrarily small, since he is able to attract only a few customers from each of the other competitors. This simply serves to emphasize the critical role played by the assumptions concerning the nature of the competitive interactions, which are implicit in the specification of the model.

There are, within the location model, at least three alternative solutions to this problem. The first is to look for an alternative Nash equilibrium concept, e.g. a quantity-location setting Nash equilibrium. With a quantity setting Nash equilibrium, an entrant will always locate midway between existing firms. On the other hand, in equilibrium there may be sizeable profits, if demand curves are relatively inelastic. The second approach is to recognize that, when each firm has only two direct competitors, the assumptions underlying a Nash equilibrium are not very

plausible. The firms may either collude, or there will be strategic interactions. In any case, the entrant recognizes that when he enters, existing firms will change their prices; whether it still pays him to enter will depend, then, on his expectations concerning the nature of the equilibrium which emerges after he enters. Again, there is no presumption for zero profits in equilibrium.

### 7.5 Latent Firms

The third approach is to employ the notion of latent firms. Assume at every location we install a firm which does not produce, stands willing to produce at an announced price above the established firm. Then, so long as there are no fixed costs associated with existing as a latent firm (only fixed costs associated with production), this firm will be making zero profits. Consider our zero profit Nash price equilibrium. There is no other location or price at which either a latent or actual firm could increase its profits. If a firm attempted to jump on top of another existing firm (as described earlier), his profits would not increase; the latent firm at his previous location would then begin to operate, and he would only obtain the same market that the firm which he has attempted to upstage had; but that firm had only zero profits.<sup>43</sup>

### 7.6 Summary

The nature of the equilibrium depends critically on the instruments

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<sup>43</sup>The notion of latent firms appears to be an alternative way of formalizing the concept of contestability within the Nash price setting equilibrium framework.

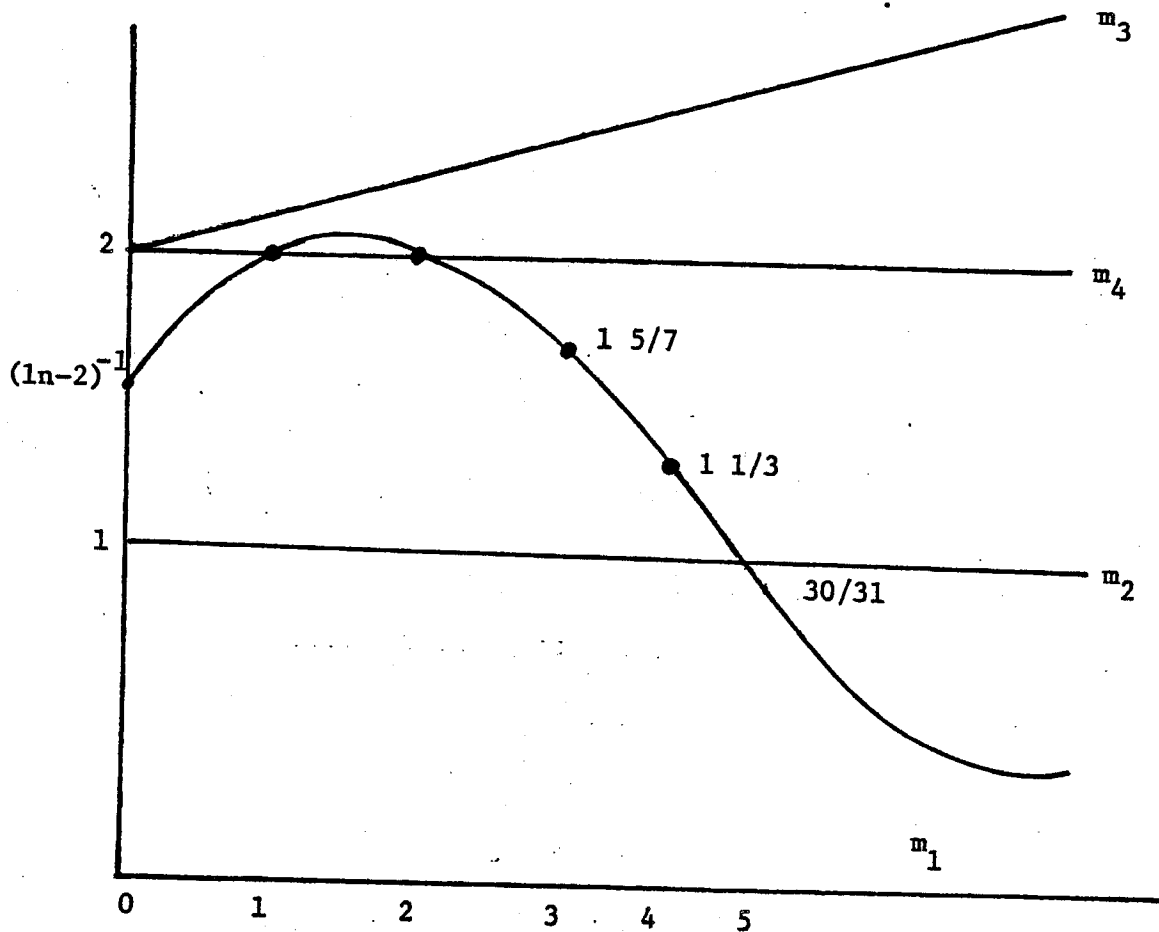
available to the firm. If it cannot price discriminate, the market equilibrium entails too many firms. All individuals could be made better off.

Though a perfectly discriminating monopolist would obviously have greater profits than a non-discriminating monopolist, the question of whether, under monopolistic competition, the ability to discriminate results in larger or smaller market areas is much more subtle: the profits any firm can extract depend on the actions of other firms. With constant elasticity transport cost curves, market areas with discrimination are larger if the elasticity of cost with respect to distance is small, but if it is large, just the opposite results; on the other hand, so long as costs increase with distance, market areas with discrimination are smaller than the resource cost minimizing solution. The contrasting market areas are depicted graphically in Figure . . .

If the government can discriminate, then the optimal allocation (under both Rawlsian or utilitarian criterion) entails minimizing the sum of transport costs plus fixed costs of production; if the firm charged individuals just the marginal cost of production, aggregate land rents would equal the fixed costs of production.

While a discriminating monopolist would charge those furthest from him a lower price (since effective competition here is keenest), a sufficiently lower price that the welfare of those furthest from the firm is actually higher (the lower price more than compensates for the differential transport costs), a utilitarian or a Rawlsian would have prices decreasing with distance to just offset the increased transport costs. When there is a competitive land market, rents will have the same effect.<sup>44</sup>

These results are predicated on the assumption of zero profits.



$$l = \left[ \frac{c}{4} m_1 \right]^{1/1 + \alpha}$$

$$m_1 = \frac{\alpha(1+\alpha)}{2^\alpha - 1} \quad \text{price discrimination}^1$$

$$m_2 = 1 \quad \text{no price discrimination}$$

$$m_3 = 2(1+\alpha) \quad \text{efficient}$$

$$m_4 = 2 \quad \text{Rawlsian}$$

Efficient markets are always larger than monopolistic competitive markets.  
 Market area with discrimination larger than without for  $\alpha < 5$ .  
 Market area with discrimination larger than Rawlsian for  $1 < \alpha < 2$ .

Figure 6

Market areas for constant elasticity transport cost functions

$$\lim_{\alpha \rightarrow 0} \frac{1}{m_1} = \frac{\alpha(1+\alpha)}{2^\alpha - 1} = \frac{1}{\ln 2}$$

While we presented an alternative equilibrium notion, making use of the concept of latent firms, under which (in the absence of sunk costs) equilibrium will be characterized by zero profits, we argue that in the one dimensional locational models, in which each firm has at most two rivals, the assumption of the absence of strategic interactions seems unpersuasive. It is this which provides the motivation for the alternative approaches sketched in the following sections.

#### 8. The Chamberlain Multi-dimensional Model

The models described in this and the next section are significantly different from that represented by the Hotelling-Lancaster approach.

While the previous models implicitly assumed that there was a single characteristic (location along a line) by which commodities could differ, in fact, of course, there are many dimensions, and this "higher dimensionality" makes the nature of competition far more complex. In two-dimensional space, for instance, with transportation costs a function of Euclidean distance (i.e. the length of the straight line to the production location) with an infinite plane, market areas will be hexagonal (Stern, 1972) (also Mills and Lave, 1964). (Figure 7.) Thus, each firm has not two, but six competitors. If transport costs are a function of the distances along both coordinates (see Figure 8), as might be the case with a grid transportation network, then market areas are squares, and each firm has four competitors. As the number of characteristics increase, the number of competitors for each firm increases. Not

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<sup>44</sup>The non-existence of compensating rents is a critical difference between the location theory model and the "characteristics" interpretation of that model.



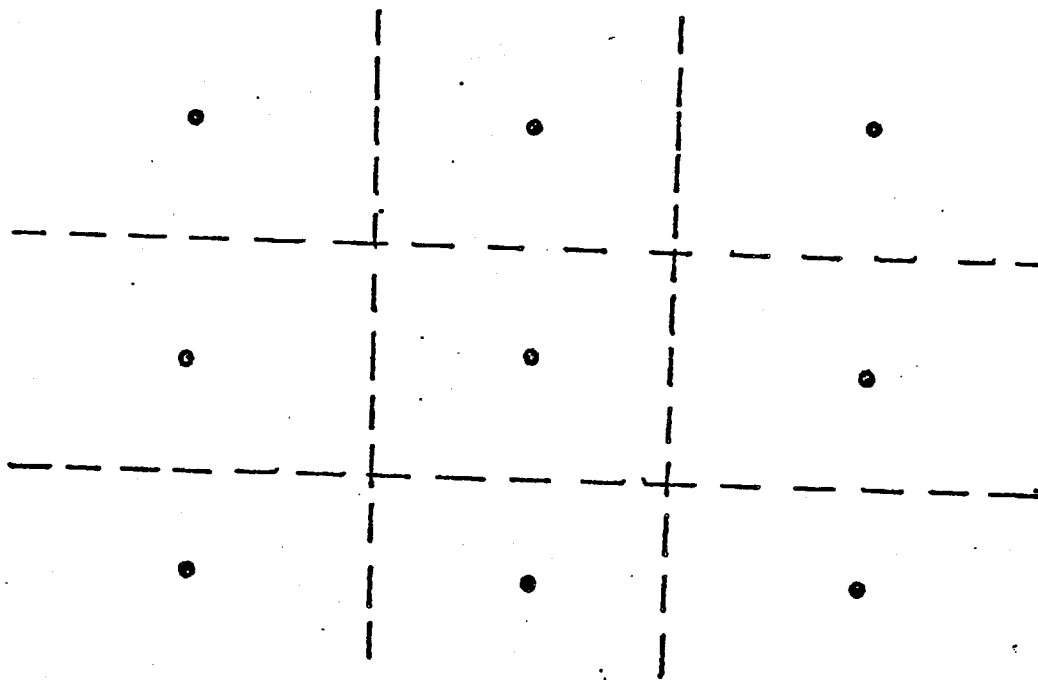


Figure 7

Market Areas with Transport Grid

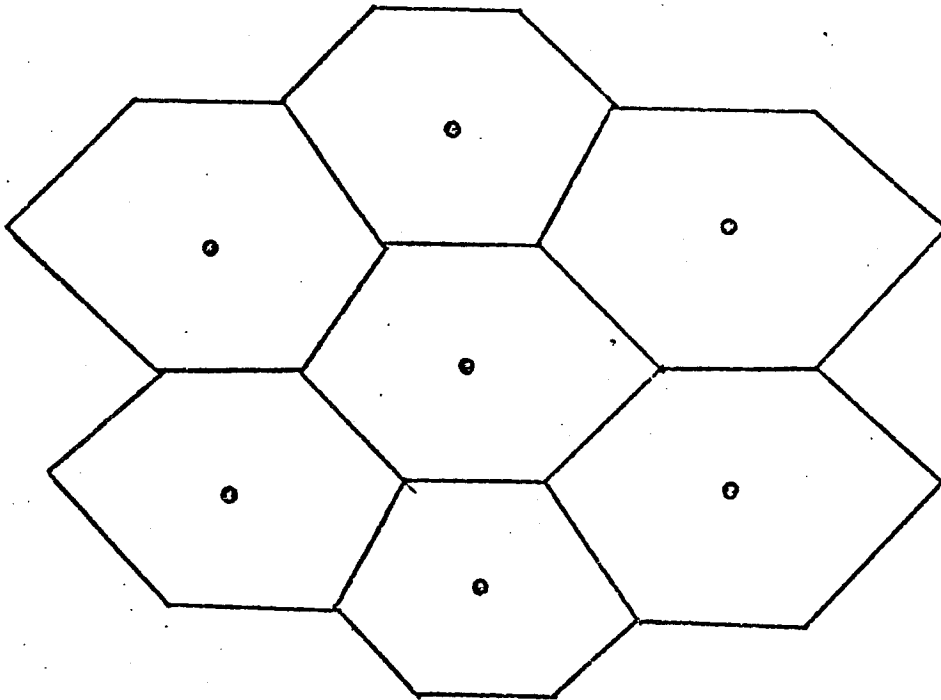


Figure 8

Market Areas with Euclidean Transport Costs :

Hexagons

surprisingly, results on the comparison between market and optimal allocations will thus depend critically on the number of independent characteristics (as well as the nature of the "transport cost" function.)<sup>45</sup>

Assume as before that marginal costs of production are  $m$  and that there are fixed costs  $c$ . There is a uniform density of the population in the infinite hypercube and we assume that transport costs to a point  $(x_1, \dots, x_n)$  from the origin is simply  $\Sigma \psi(x_i)$ .<sup>46</sup> If the market area is an  $n$ -dimensional cube of length  $2\hat{l}$ , total sales are

$$(22) \quad Q = (2\hat{l})^n$$

In equilibrium, we can show as before, that

$$(23) \quad \frac{\partial \hat{l}}{\partial p} = \frac{-1}{2\psi'(\hat{l})}$$

We focus on the case where

$$(24) \quad \psi = \hat{l}^\alpha / \alpha$$

Thus we can calculate the elasticity of demand,  $E$ , as

$$(25) \quad E = \frac{-n p \hat{l}^{2-\alpha}}{2} .$$

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<sup>45</sup>This section has benefited greatly from discussions with C. von Weizacker.

<sup>46</sup>This is not necessarily the most plausible parameterization in the strict location interpretation of the model; but in the "characteristics" interpretation it corresponds to the utility function being separable in characteristics. The parameterization is chosen for analytical simplicity. It includes, as a special case, linear transport costs.

Thus, in equilibrium

$$(26) \quad (p-m) \frac{\partial Q}{\partial p} + Q = (p-m)Q \frac{\partial Q}{\partial p} + Q^2 = 0$$

or since,

$$(p-m) Q = c$$

$$(27) \quad \frac{c}{Q^2} \frac{\partial Q}{\partial p} = -1$$

or

$$(28) \quad \frac{Q^{1+\alpha/n}}{n2^{\alpha-1}} = c$$

This can be contrasted with the cost minimizing market area. For travel in any orthogonal direction, a fraction  $\hat{l}/l$  live within a distance  $l$  from the firm. Thus the mean transport costs in any direction are

$$(29) \quad \frac{1}{\alpha \hat{l}} \int_0^{\hat{l}} l^{\alpha} dl = \frac{\hat{l}^{\alpha}}{(1+\alpha)\alpha}$$

Total mean transport costs for an n-dimensional characteristics (location) model are

$$\frac{\hat{l}^{\alpha}}{(1+\alpha)\alpha}$$

Thus total costs per capita are (using (2))

$$(30) \quad \frac{n2^{-\alpha} Q^{\alpha/n}}{\alpha(1+\alpha)} + \frac{c}{Q}$$

and these are minimized when

$$(31) \quad \frac{1}{1+\alpha} 2^{-\alpha} Q^{(\alpha/n)-1} = \frac{c}{Q^2}$$

or

$$(32) \quad \frac{Q^{1+\alpha/n} 2^{-\alpha}}{1+\alpha} = c$$

Hence, denoting by  $Q_0$  the optimal value of  $Q$  and by  $Q_m$  the value in market equilibrium,

$$(33) \quad Q_0 <> Q_m \text{ as } 2(1+\alpha) <> n$$

As  $n$  goes up, the elasticity of demand increases, leading to lower prices and larger market areas. (33) says that the greater is  $n$  the more likely is the market area to exceed the optimal market size, i.e. there are too few firms. The critical boundary between having too few and too many firms depends on the rate at which transport costs increase with distance.<sup>47</sup> At  $\alpha = 2$

$$Q_0 <> Q_m \text{ as } n <> 6.$$

Not only does increasing the dimensionality of the characteristics space change the nature of competition, and hence the relationship between the market equilibrium and the optimal number of firms, but it also has important effects on the stability of equilibrium. Consider, for instance, competition between two firms in a plane with transport costs being a linear function of Euclidean distance, as illustrated in figure 9. This is the case which gave rise to problems in the one-dimensional location model. Now, market area is described by

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<sup>47</sup> For  $n = 1$ , (33) becomes

$$Q_0 <> Q_m \text{ as } 2(1+\alpha) <> 1$$

The market area is always too small.

For convex transport cost functions, the minimal number of dimensions required for market areas to be too large is 5.

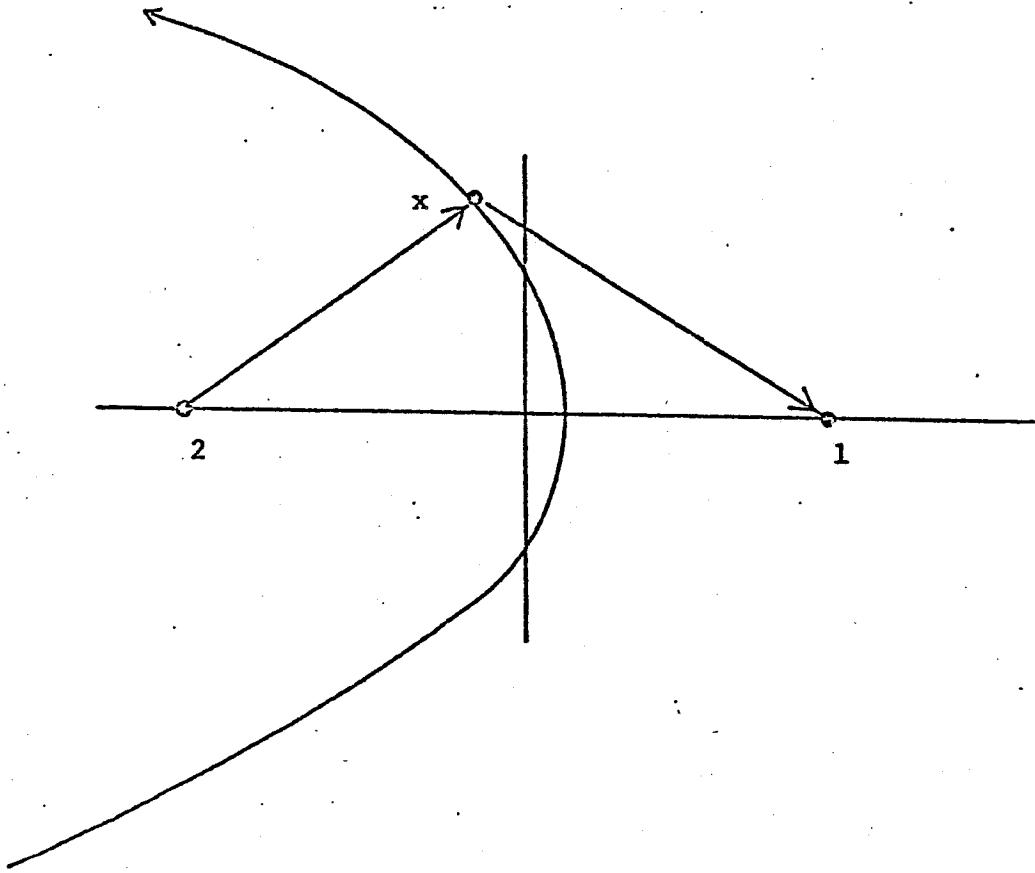


Figure 9

With transportation costs a linear function of Euclidean distance, market area is a continuous function of price

$$p_1 - p_2 = \left\| \|x - l_2\| - \|x - l_1\| \right\|$$

$$= \sqrt{(x_1 - l_{21})^2 + (x_2 - l_{22})^2} - \sqrt{(x_1 - l_{11})^2 + (x_2 - l_{12})^2}$$

where  $x = (x_1, x_2)$ ,  $l_2 \equiv (l_{21}, l_{22})$ ,  $l_1 \equiv (l_{11}, l_{12})$ .

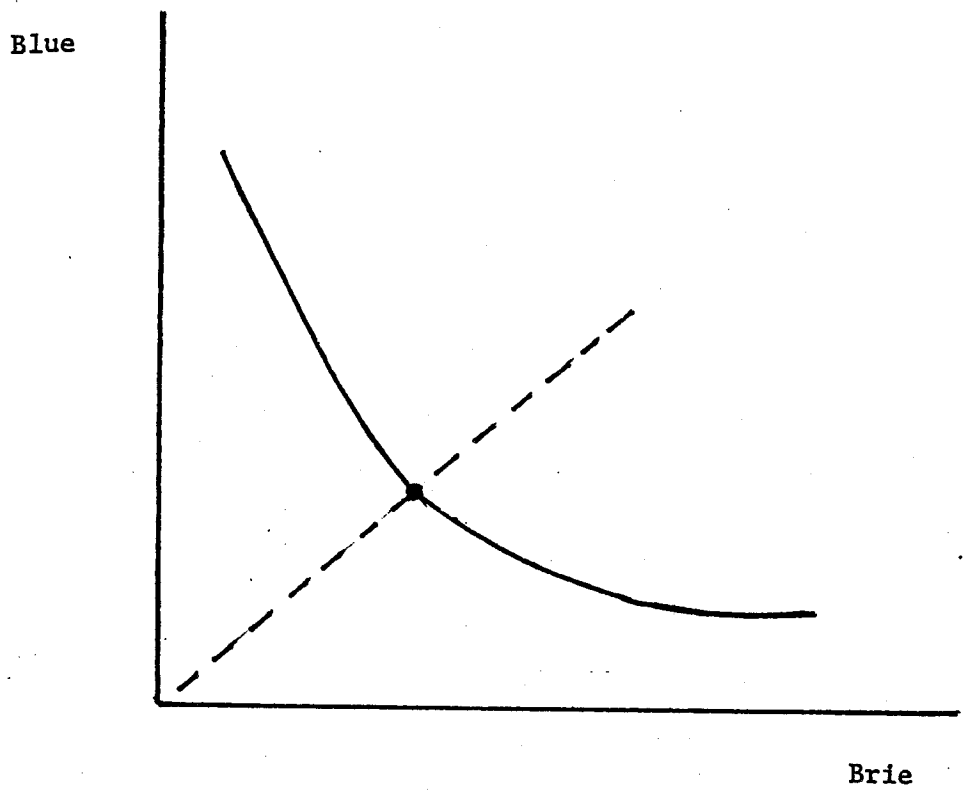
Thus, market area increases continuously as price is lowered, rather than discretely, as in the 2-dimensional location model. It is thus possible that there exists a stable equilibrium under weaker conditions than in the one dimensional model.

### 9. The Spence-Dixit-Stiglitz Models

The approach taken by Spence, Dixit, and Stiglitz is fundamentally different from that represented by the models described so far. While the latter assumes that each individual has a single preferred commodity, the former recognizes that individuals value variety itself. Indeed, one of the earliest applications of this approach was to the traditional portfolio problem, where, by diversifying among a large number of securities, the individual could reduce the risk he faced. (Stiglitz, 1974). More generally, the fact that individuals indifference curves are quasi-concave means that individuals would be willing to sacrifice some reduction in total cheese consumption if they can consume some blue cheese and some brie.<sup>48</sup> (See Figure 10).

<sup>48</sup> The question of whether one can add up units of blue cheese and units of brie cheese, to talk about total cheese consumption, is, to mix a metaphor, a red herring: assume both cheeses could be produced by labor alone. Choose units so that one unit of each kind of cheese requires a unit of labor. The quasi-concavity of the indifference curve means that, if both kinds of cheeses are provided, the total resources required to attain a given level of utility are less than if only one

(Footnote continued)



Quasi-concave indifference curves mean individuals prefer variety

Figure 10



Moreover, while in the Hotelling-Lancaster approach, every firm has precisely two competitors, in the Spence, Dixit-Stiglitz as in the Chamberlain approach, each firm may have many competitors.

The nature of the equilibrium depends, again, critically on the ability of firms to discriminate. If we assume that firms must charge a uniform price, then, since profits are less than consumer surplus, one might have thought that there were too few firms. At the same time, the introduction of a new commodity has an effect on the demand for other commodities; since price exceeds marginal costs for those commodities, there is a loss of profits (producer surplus) if, as one might expect, demand for these other commodities is reduced. Thus the increment in welfare of the representative individual will depend critically on (a) the relationship between the size of his own profits and consumer surplus; and (b) the interrelations between his demand and the demand for other commodities.

In the central case where we do not allow lump sum taxes (but, as before, allow commodity and franchise taxes and subsidies) with additive constant elasticity utility functions, we obtain the result that the market equilibrium is pareto optimal. If the elasticity of the demand curve increases with quantity, then the ratio of consumer surplus to profits is smaller than for the constant elasticity case, and not surprisingly, the market equilibrium entails too many commodities; conversly if the elasticity of the demand curve decreases with quantity.

There is thus no general presumption concerning the direction of

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48  
(continued)  
kind of cheese is provided.

bias.

These results depended on the assumption that there were a large number of firms and that each firm took the other's price as given. Recently, Perry and Koenker have explored the implications of alternative conjectural variations. Clearly, if firms expect their rivals to react, they will perceive themselves as facing a more elastic demand curve; hence price will be a smaller markup over marginal cost, and, accordingly, in equilibrium, there will be fewer firms. There is thus a greater likelihood that market equilibrium will be characterized by too few firms.

### Part III

#### Extensions, Applications, and Implications

##### 10. The Lancaster Characteristics Approach

Lancaster has made use of his "characteristics" approach to the study of consumption demand for the analysis of monopolistically competitive equilibrium. Individuals receive utility from characteristics; goods bundle these characteristics in particular ways. A good is thus a point in characteristics space. Let us consider two goods, each of which costs the same amount to produce (this is a choice of units). Then if individuals have homothetic indifference maps, we can ask how many units of one good yields the same utility as one unit of the other. If we have a transformation function that describes all the possible commodities that can be produced for the same cost, we can define the individuals' preferred commodity; associated with any other commodity there will be a compensation function describing, as we have

said, how much of that commodity yields the same utility as one unit of the preferred commodity.

What Lancaster seems to have done, then, is cleverly convert his characteristics model into a location model: the compensation function looks almost like a transport cost function. Almost, but not quite.<sup>49</sup>

There is one critical difference. In the previous section, we assumed transport used up "income" -- the numeraire -- but not the good purchased. If, however, we had assumed that the primary cost was spoilage, then the transport costs (relative to our numeraire) would be proportional to the price of the good. Thus changes in the price of the good also change the transport cost function. Not surprisingly, the market equilibrium may change as a result, but so too will the optimum. Within this wider perspective, then, of location models, there seems nothing intrinsically different between location models and Lancaster's approach.<sup>50</sup>

There is one critical assumption which does require some comment, that concerning the relationship between the number of characteristics and the number of commodities and the combinability of commodities to produce the desired mixed number of characteristics. If goods are combinable, then if the commodities "span" the characteristics space,

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<sup>49</sup>Lancaster makes much of the difference. His comparison is, however, unfair. He compares a particular variant of the transport cost model with his model, e.g. one which assumes linearity of transport costs and thus has strictly positive marginal transport costs (even at zero). But, as our previous discussion has made clear, these are clearly not intrinsic properties of the location model.

<sup>50</sup>The difference in results have as much to do with differences in assumptions concerning the instruments which are available to the government and to the private firm as they do with differences in the specification of technology.

any new entrant will face a perfectly horizontal demand function. He will have to act as a price taker, and the conventional competitive equilibrium model obtains. If there are fewer commodities than characteristics, then, if goods are combinable, there will be a range of tastes in which individuals purchase several commodities, and the analysis appears much like that of the Spence-Dixit-Stiglitz models.

#### 11. Generalizations and Mixed Models

While in the Dixit-Stiglitz-Spence Models, all firms are, in effect, equi-distant from one another,<sup>51</sup> in the multi-dimensional Chamberlinian model, some firms are "nearer" than others. In the latter, an increase in the price of one firm effects only "neighboring" firms; in the former, it effects all firms.

Assume we have a one-dimensional circle model, but individuals have imperfect information. They search sequentially, stopping when they get "reasonably" close to their preferred commodity. (Their optimal stopping rule will specify a price, at a given distance from their preferred location; this price, in turn, will be a function of beliefs about prices and location of other stores.) If a store lowers its price, it will induce some individuals who would have continued to search to stop. Those who are induced to stop searching would have eventually wound up at a number of different stores. In this model, then, when a store lowers its price, it attracts its customers from a large number of other stores (even though there is only a single dimension.) Still, however not all stores are equally close. Some

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<sup>51</sup> Though their analysis encompasses more general cases as well.

stores are affected, others are not. (Salop and Stiglitz, 1977)).

In a multi-dimensional model, it is easy for symmetry to be introduced. Assume there are  $N$  commodities. These can be ranked by different individuals in different ways. Consider the symmetric equilibrium in which all commodities cost the same to produce, and in which all rankings are equally likely. Assume the compensation required for an individual to go from his first to second ranked commodity is fixed (this is the transportation cost associated with going to the next nearest commodity.) Then by lowering his price by that amount, the firm induces individuals to switch from their first to their second most preferred commodity; but since all rankings were assumed equally likely,  $1/N-1$  of the customers come from each of the other commodities.<sup>5253</sup>

(Indeed, all that is required is that each individual be interested in two commodities, but the two commodities in which he is interested in differ.)<sup>54</sup>

## 12. Comparative Statics: The Effect of Taxes

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<sup>52</sup>In the circle model, if we arbitrarily assign numbers to firm sequentially, in a clockwise manner, choosing any firm arbitrarily as firm 0, the only rankings are 1,2,3,4..... $N$ ; 2,3,4,..... $N$ ,1; etc. Thus, as we have repeatedly noted, firm 1 has only two neighbors, two firms which are ranked second to it,  $i-1$ , and  $i+1$ .

<sup>53</sup>It is straightforward to use the location model interpretation to calculate the magnitude of this "compensation" in terms of the distances between commodities.

<sup>54</sup>A version of this model has recently been developed by Hart. These models are closely related to the Salop-Perloff model, where there is no product diversity, but stores may differ in the price they charge. They assume a fixed sample size. Thus, as a store raises its price, it loses customers when its price switches from being the lowest to the second lowest.

The previous discussion should have made clear both that there is no clear presumption with monopolistic competition that the market equilibrium is Pareto optimal, and that there is no clear presumption about the nature of the biases introduced by the market. There is no simple formula for government intervention.

Skeptical readers may, at this point, say that they never ascribed much importance to the Fundamental Theorem of Welfare Economics anyway. The real question is, are there important predictions of the monopolistically competitive model which differ from a naive perfectly competitive model. The answer is, yes. To illustrate this, we consider three kinds of changes.

First, consider the effect of an imposition of a specific tax on a commodity group (say toothpastes). Assume that there are no resources which are specific to the production of that commodity, but each production unit (producing a different variety) has a U-shaped cost curve. Then, competitive equilibrium theory would predict that the tax would be entirely borne by consumers in the form of higher prices; prices would rise precisely by the amount of the tax. The monopolistically competitive model would predict that price would rise by more than the tax, by an amount which is inversely related to the elasticity of demand.<sup>55</sup> On the other hand, while with the competitive

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<sup>55</sup> The relevant elasticity of demand is the partial elasticity -- given that all other firms were to keep their prices constant. What is critical is that they increase their market share by taking away a little from each of a large number of competitors. In contrast, in the Hotelling-Lancaster formulation, they increase their market share by taking away customers from the nearest (two) competitors. With respect to its effect on price, the sector acts like a monopolist but a monopolist which perceives itself facing a more elastic demand curve than the true sectoral demand curve.

model, there will be a decrease in the number of firms, with the monopolistically competitive model, there will be an increase in the number of firms, if the sectoral elasticity of demand is less than unity.

On the other hand, a franchise tax (a fixed tax per producing unit) will be passed on in the purely competitive model, but prices will rise by less than the tax per unit and before tax output; in the monopolistically competitive model, if there were constant marginal costs, it would have no effect at all on price, but simply decrease the product variety; with marginal costs increasing with output, it would lead to some increase in price, while if marginal costs decrease with output, it could actually lead to a decrease in price.

### 13. Comparative Statics: the Effects of Trade

The models have important and interesting differences in their predictions concerning the consequences of opening up trade between two otherwise identical countries. Assume that there are some (arbitrarily small) transport costs. Then the competitive equilibrium model predicts that, even with arbitrarily small transport costs, there should be no trade between countries with identical endowments and tastes. The monopolistic competition models predict that 50% of GNP would be traded.<sup>56</sup>

More interestingly, the competitive equilibrium model predicts that the opening of trade has no effect on prices, on the number of products produced and consumed, on the scale of production, etc.

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<sup>56</sup> Obviously, this exceeds actual trade, because for many goods transport costs are significant. Although the modern trade theory has focused on the special case of constant (or diminishing) returns technologies, there is a long tradition of "monopolistically competitive" models; cf. Ohlin (1933)).

The various versions of the monopolistically competitive model predict that these will change, but the models differ in the estimate of the precise amount. For simplicity, consider the case where each firm has a fixed cost plus a constant marginal cost  $m$ . Then price is a markup over marginal cost; the size of the mark-up depends on the perceived elasticity of demand. The characteristics model predicts that firms will come closer together. If "transport costs" are a convex function of distance, the elasticity of demand increases as the market area decreases. Hence, the price will decrease. The Spence-Dixit-Stiglitz models give less clear predictions. In the central case with constant elasticity demand curves, price remains unchanged, and product variety exactly doubles. But this is a special case. Clearly, the lower scale at which each commodity is consumed could be associated with either a smaller or greater elasticity, although there is some presumption that (as in the location model) the elasticity will increase and price will fall. In that case, opening of trade will reduce the total number of production units.

But Spence and Dixit-Stiglitz have also emphasized that the nature of the product mix may change dramatically, in a way which is not adequately captured by symmetric models. Products which require high fixed costs ("mass consumption commodities") but which have high elasticities of demand (little consumer surplus) may become viable when the product market doubles in size. This may, at the same time, make "specialty products" for which there is a large consumer surplus no longer viable (if, as one might expect, there are interactions in the demands). As a result, Dixit and Stiglitz have shown not only that the opening of trade may be disadvantageous to some groups within the



population, but that it is possible that all groups may be disadvantaged: free trade may be pareto inferior to no trade.

It is worth noting that the traditional argument for the gains to specialization implicitly (or explicitly) assumes that there is an important element of returns to scale. Smith's observation that specialization is limited by the extent of the market is equivalent to the observation that individuals (firms) are operating on the downward sloping portion of their cost curves.

#### 14. Limit Theorems

The effects of opening up trade are equivalent, in the monopolistically competitive models, to the effects of lowering the fixed costs and/or increasing the density of population. The various versions of the monopolistically competitive model thus differ in their predictions concerning what happens as the number of firms increases. Over the years, there has been particular interest in the questions of whether the monopolistically competitive model converges to the competitive equilibrium model.

In some models (described below) it can be shown that, as the number of firms increases (as a result of a reduction of fixed costs or an increase in market size), the equilibrium converges to the competitive equilibrium. This result has been used by some to suggest that the competitive model is, after all, the only relevant model to study. This conclusion is not warranted, for two reasons. First, the economy we live in is an economy with a finite number of firms. The empirical question of whether firms perceive themselves as facing downward sloping demand schedules simply cannot be addressed by the theoretical

observation that, under certain circumstances, if there were "enough" firms, they would perceive themselves as facing horizontal demand schedules. Indeed, I would argue that the reason we are interested in studying the limiting case is for its analytical convenience. There is little doubt that, at least in some sectors of the economy, firms perceive themselves as facing downward sloping demand schedules. The question then is how do we model these sectors? What we can say, for instance about the extent of product diversity? The "limiting" model of monopolistic competition is of use only to the extent that it can throw light on these questions.

Secondly, the fact that a particular limit converges to the competitive equilibrium does not imply that with some alternative equally plausible limit might not converge to an imperfectly competitive equilibrium. We shall provide an example of this below.

Consider the standard location model, as fixed costs become small, with a fixed number of characteristics (dimensions).<sup>57</sup> There is "crowding": commodities become located next to each other, and hence, for every commodity, there exists a large number of almost perfect substitutes. Thus, in the limit, these models behave much like perfectly competitive models; by contrast, in the Spence-Dixit-Stiglitz models there are an infinite number of possible characteristics.

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<sup>57</sup>This is, of course, not the only possible limit that one could focus on; one could, for instance, allow the range of tastes or the dimensionality of the characteristics space to increase in some appropriate way we lower the fixed costs and increase the number of firms. There is not a priori "correct" limit. The question is, which kind of limit provide better insights into the actual behavior of those sectors of the economy where we believe there is the kind of product diversification with which we are concerned here. See also Salop (1979) and Samuelson (1967).

Commodities need not crowd each other. Since, in fact, firms do seem to face downward sloping demand curves, this means that the limiting version of the Lancaster-Hotelling model cannot be used to study monopolistically competitive economies.<sup>58</sup>

It is important to observe that even if in the limit as fixed costs go to zero and the number of firms increases, the market approaches the perfectly competitive equilibrium, the approach may not be monotonic. That is, the ratio of price to marginal cost (the "profit margin") may not decrease monotonically. Whether it does or not, for instance, within the location model depends critically on the convexity of the transport cost function.

#### 15. Imperfect Markets and Imperfect Competition

It has become customary among economists to describe situations in which there is only one (or a few) firms(s) producing a given commodity, or in which there is imperfect information about the set of commodities produced, as having market imperfections, suggesting that these are

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<sup>58</sup>Hart provided a general set of conditions under which the limit of a monopolistically competitive economy was pareto efficient. A number of critical conditions were required. The set of commodities that could be produced had to be compact. This ensured that crowding had to occur somewhere, as the number of firms increased. The production set of each firm was bounded. This ensures that in fact at (or near) each location there will be an indefinitely large number of firms. And each individual's demand curve for the commodity must hit the axis at a finite price. The preceding assumptions imply that as the size of the economy increases, either there are an indefinitely large number of firms producing a given commodity, or the amount consumed by any individual goes to zero. But in the latter case, the individuals demand elasticity goes to infinity, and hence the market demand elasticity goes to infinity: the firm has no monopoly power. Dixit-Stiglitz violate all three of these assumptions, but is it possible to show that if anyone of the assumptions is violated, the results on the convergence to competitiveness may not obtain.

blemishes which would be corrected, in the better and more perfect world for which we are all striving. But the fact that only one firm produces a given commodity is not an imperfection of the market. It is a characteristic of any optimal resource allocation in which there are fixed production costs in any society which values variety. Similarly, there is no more reason to view the costs of information as "imperfections" than the fact that outputs require inputs. Any society in which resources are required to produce and convey information should be characterized by imperfect information. The models we have described so far have emphasized the role of fixed costs of production in giving rise to monopolistic competition. But imperfect information may give rise to monopolistic competition even without there being product variety. A firm which raises its price may lose few of its customers to rivals, if there are (even arbitrarily small) search costs. These models have recently been surveyed by Salop (1976) and Stiglitz (1979), and so we shall only make one remark here: imperfect information has an important effect both on the optimal amount of product variety and on the extent of variety in the market equilibrium. In particular, if we view individuals as having a "reservation" quality (characteristic) analogous to a reservation price, then imperfect information will result in each firm having a wider market area (some individuals purchase at the firm which is not closest to their tastes); as a result, the elasticity of demand may be lower, and prices higher. At the same time, the social return to increasing product variety when individuals are imperfectly matched may be much smaller than when they are. Increasing the number of commodities increases the number of searches to find the best match; in one limiting case, Salop and Stiglitz have shown that

when reservation quality is adjusted optimally, increasing product variety may have a negligible effect on welfare even though, if individuals were perfectly matched, it would have a significantly beneficial effect.

16. Concluding Comments: On the Nature of Competition

One of the reasons that the kinds of developments I have described here are so important is that they serve to remind us that the competitive process is far richer and more complicated than can adequately be represented by the traditional models of price competition. We have focused here on product competition, and the interactions which result between product and price competition. The technology and tastes are taken here as given. But one of the most important aspects of the competitive process is the development of new products. Thus the models developed here can be thought of as a prelude to a more complete theory of competition, involving product, price, and R&D competition. The extent to which the propositions and insights of the pure price competition theory will remain valid in this more general theory remains a moot question.<sup>59</sup>

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<sup>59</sup> Some progress towards a development of these more general "neoSchumpeterian" theories has recently been made in the work of Nelson and Winter (1978) and Dasgupta and Stiglitz (1980a, 1980b) Dasgupta, Gilbert and Stiglitz (1982, 1983), and Gilbert and Stiglitz (1979).

Appendix A

Equilibrium Market Areas Under Contestability

Assume marginal costs of production are zero. If there were a single firm, it would be located at the center, charging a price  $c/2$ , where  $c$  is the fixed cost. We now ask, when is entry feasible. Assume the entering firm charges a price just high enough to capture the market to its "left", (this can be shown to be the optimal strategy) i.e. it sets

$$p = p^* + z (1-l^*)$$

where  $p^*$  is the price of the "center" firm, and

$$\psi (t) = zt, z \text{ is the marginal cost of transportation.}$$

Then, (assuming at  $p$  all individual's to the left of  $l^*$  purchase one unit) revenue,  $R$ , is

$$R = [p^* + z (1 - l^*)] l^*$$

This is maximized at

$$l^* = [z+p^*] / 2z^*$$

so

$$\begin{aligned} R^* &= [p^* + z - (z + p^*)/2 ] (z + p^*/2z) \\ &= ((p^* + z) /2)^2 / z \end{aligned}$$

But  $p^* = c/2$ . We require, for entry

$$\frac{(c/2 + z)^2}{4z} > c$$

or

$$(c/2z) + 1)^2 > 4c/z$$

or

$$c/z \leq .4 \text{ or } c/z \geq 11.6$$

The resource savings from having two (optimally located) firms rather than one is

$$z/2 - c$$

This is positive if

$$c/z \leq .5$$

Hence if

$$.4 \leq c/z \leq .5$$

entry does not occur, when it "should", while if  $c/z \geq 11.6$ , entry occurs, when it should not.

Appendix B

Symmetric Equilibrium with Elastic Demands and Non-Linear Transport Costs

The symmetric equilibrium is simply described. From (1) we calculate the market areas elasticities.

$$(B1) \quad \frac{d\hat{l}}{dl^*} = \frac{\psi'(\hat{l} - l^*)}{\psi'(l - l^*) + \psi'(l^{**} - \hat{l})} = 1/2$$

$$(B2) \quad \frac{d\hat{l}}{dp^*} = - \frac{1}{\psi'(\hat{l} - l^*) + \psi'(l^{**} - \hat{l})} = - \frac{1}{2\psi'(x)}$$

where

$$(B3) \quad x = \hat{l} - l^* = l^{**} - \hat{l}$$

The firm

$$(B4) \quad \max_{\{l^*, p^*\}} (p^* - m) \int_0^{\hat{l}} D(p^* + \psi(|l - l^*|)) dl$$

where  $D(p + \psi)$  is the demand curve and  $m$  is the marginal cost of production. Thus it sets

$$(B5) \quad (p^* - m) \left[ \frac{D}{2} + \int_0^{l^*} D'\psi' dl - \int_{l^*}^{\hat{l}} D'\psi' dl \right] = 0$$

or

$$\int_0^{1-2x} \left( \frac{\psi'}{p^* + \psi} \right) D \epsilon dl = \frac{D(p^* + \psi(x))}{2}$$

where  $\epsilon$  is the elasticity of demand, and

$$(B6) \quad \int_0^{\hat{l}} D dl + (p^* - m) \left[ - \frac{D}{2\psi'(x)} + \int_0^{\hat{l}} D' dl \right] = 0$$

To see that  $l^* \neq l^{**}$ , note, from (5) that at  $l^* = l^{**}$ ,

$$\begin{aligned} \frac{D(p^*)}{2} + \int_0^1 D'\psi' dl &= \frac{D(p^*)}{2} - \int_0^1 \frac{dD}{d\hat{l}} d\hat{l} \\ &= \frac{D(p^*)}{2} - (D(p^*) - D(p^* + \psi(l^*))) \\ &= D(p^* + \psi(l^*)) - \frac{D(p^*)}{2} < 0 \end{aligned}$$



provided  $D$  is sufficiently elastic and  $\psi$  is sufficiently convex. Consider, as an example, the quadratic function

$$\psi = 1/2(\ell - \ell^*)^2$$

and the linear demand curves

$$D = a - bq.$$

Then we obtain as our first order conditions

$$\begin{aligned} \int_0^{1-2x} (\ell^* - \ell)b \, d\ell &= b[\ell^*(1-2x) - \frac{(1-2x)^2}{2}] \\ (B7) \qquad \qquad \qquad &= \frac{b[1-2x]}{2} = \frac{a}{2} - \frac{b}{2}(p^* + \frac{x^2}{2}) \end{aligned}$$

and

$$(B8) \quad a - bp^* - \frac{b(1-3x+3x^2)}{6} = (p^* - m) \left[ (a - bp^* - \frac{bx^2}{2}) \frac{1}{2x} + b \right]$$

Substituting ( 8 ) into ( 9 ), we obtain

$$\frac{bx^2}{2} + b(1-2x) - \frac{b(1-3x+3x^2)}{6} = (p^* - m) \left[ \frac{b(1-2x)}{2x} + b \right]$$

or

$$(B9) \quad p^* - m = \frac{(\frac{5}{6} - \frac{3}{2}x)}{1/2x} = \frac{5}{3}x - 3x^2$$

( B7 ) and ( B9 ) are plotted in Figure 11. There exists a unique value of  $x > 0$  satisfying these equations for  $m > a/b - 1$ .

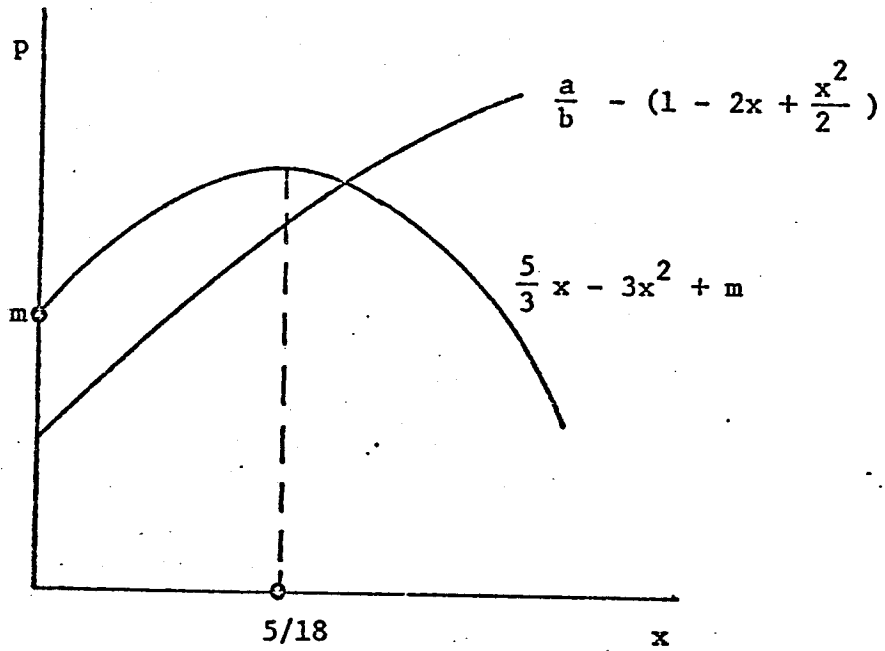


Figure 11

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