THE CONSISTENCY OF WELFARE JUDGEMENTS WITH A REPRESENTATIVE CONSUMER

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ABSTRACT: This paper is about economies with a representative consumer. In general a representative consumer need not exist, although there are several well known sets of sufficient conditions under which one will. It is common practice, however, to use the representative consumer hypothesis without specifically assuming any of these. We show, firstly, that it is possible for the utility of the representative consumer to increase when every actual consumer is made worse off. This shows a serious shortcoming of welfare judgements based on the representative consumer. Secondly, in economies where this does not occur, there exists a social welfare function, which we construct, which is consistent with welfare judgements based on the utility of the representative consumer. Finally we provide a converse to Samuelson's 1956 representative consumer result, which relates it to Scitovsky's community indifference curves.

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1. INTRODUCTION

This paper is about economies where a representative consumer exists, i.e. where the demand functions for goods, aggregated over consumers, are generated by some utility function. Equivalently, there exist preferences such that, if all consumers had these preferences and had equal shares of total income, the aggregate demand would be the same as with the actual preferences and income shares.

In general, a representative consumer need not exist. The hypothesis that individual agents are utility maximizers imposes no restrictions on aggregate demand other than budget balance, homogeneity of degree zero in prices and income, and continuity. (See Shafer and Sonnenschein [13].) On the other hand, there are several sets of conditions under which a representative consumer does exist. If preferences are identical and homothetic (Antonelli [1], Gorman [6], Nataf [10]), if preferences are homothetic and the income distribution is fixed (Eisenberg [5], and Chipman and Moore [2]) or if the income distribution is optimal according to some welfare function (Samuelson [11], and Chipman and Moore [2]) then the result holds.

It is common practice however, in economic theory, for example in the theory of international trade, to use the representative consumer hypothesis without specifically assuming any of these conditions, and to make welfare judgements based on the utility of the representative consumer. The purpose of this paper is to investigate the validity of this practice. Is it possible, for instance, that the representative consumer shows an increase in utility when, in fact, every consumer has been made worse off? Or on the other hand, might it be that an improvement for the representative consumer is equivalent to an improvement in social welfare for some social welfare function? This would represent a high degree of consistency in making welfare judgements, while the former possibility would indicate a lack of consistency. We think that most economists would guess that an intermediate case is the norm; namely that not all judgements should be consistent with one specific welfare function, but that an improvement for the representative consumer should imply that at least one consumer has been made better off. We show that the opposite is true.

The organization of this paper is as follows. Section 2 explains the notation used in this paper and states existing results which are referred to in subsequent sections. Section 3 contains an example which shows that the case of an absence of consistency in welfare judgements is possible. In section 4 we show that if this inconsistency does not occur, then there is a welfare function, which we exhibit, that rationalizes the welfare judgements. In section 5 we introduce a stronger consistency requirement, and thereby prove a converse of Samuelson's theorem [11]. Section 6 is the summary of the results.

2. NOTATION AND KNOWN RESULTS

There are n consumers and ℓ goods. The preferences of the consumers are represented by continuous, locally nonsatiated utility functions $U^i:R^\ell_+ \longrightarrow R$ $(i=1,\ldots,n)$ with corresponding indirect utility functions $V^i:R^\ell_{++}\times R_+\longrightarrow R$. The demand correspondence of the ith. consumer is ξ^i: $R^\ell_{++}\times R_+ \longrightarrow R^\ell_+$.

Given prices p and total income I, $\delta:R^\ell_{++}\times R_+\longrightarrow S^{n-1}$ gives the distribution of income. For example when income shares are fixed δ is a constant function. When incomes are given by individual endowments ω^i we have $\delta^i(p,p.\Sigma\omega^j) =$ $= p.\omega^i / p.\Sigma\omega^j$. Our analysis does not require that δ be single-valued. It may be a correspondence: for instance in Antonelli's case, where aggregate demand is independent of the income distribution, we have $\delta(p,I)=S^{n-1}$. This complicates the notation however, so throughout the paper we will treat δ as a function. (The obvious modifications required for the general case are explained in remark 5.6.) It is natural to assume that δ is homogeneous of degree zero.

We will define $\bar{V}^i(p,I) = V^i(p,\delta^i(p,I)I)$ since this expression will be used frequently. This is the utility of the consumer as a function of the prices and total income, given income shares δ.

Aggregate demand is $\xi(p,I) = \Sigma\ \xi^i(p,\delta^i(p,I)I)$. This is also known as community demand, market demand or total demand. The hypothesis that a representative consumer exists is translated into formal terms by saying that ξ is generated by a utility function $U:R^\ell_+\longrightarrow R$. This hypothesis will be maintained throughout. The associated indirect utility function is $V:R^\ell_{++}\times R_+\longrightarrow R$.

A social welfare function of the Bergson form depends only on individual utility levels. We say that W is increasing if

$$U^i \geq \bar{U}^i \text{ for all } i \Rightarrow W(U^1,\ldots,U^n) \geq W(\bar{U}^1,\ldots,\bar{U}^n)$$

and $U^i > \bar{U}^i$ for all $i \Rightarrow W(U^1,\ldots,U^n) > W(\bar{U}^1,\ldots,\bar{U}^n)$.

2.1. Definition. Scitovsky Indifference Curves. (See Scitovsky [12].) The Scitovsky upper contour set for an allocation among consumers $(\bar{x}^1,\ldots,\bar{x}^n)$ is the sum (over all i) of the individual upper contour sets

$$\{x^i \in R^\ell_+ \mid U^i(x^i) \geq U^i(\bar{x}^i)\}.$$

The Scitovsky indifference curve is the set of $x \in R^\ell_+$ such that there exists $(x^1,\ldots,x^n) \in R^{\ell n}_+$ with $\Sigma x^i = x$ and, $\forall i$: $U^i(x^i) = U^i(\bar{x}^i)$.

Two different allocations which sum to the same aggregate quantities will, in general, have different associated Scitovsky indifference curves.

2.2. Theorem. Samuelson [11]. If there exists an increasing welfare function W such that

$$\delta(p,I) \in \underset{(d^1,\ldots,d^n)\in S^{n-1}}{\operatorname{argmax}} W(V^1(p,d^1I),\ldots,V^n(p,d^nI))$$

then aggregate demand is generated by the utility function

$$U(x) = \max W(U^1(x^1),\ldots,U^n(x^n)) \text{ s.t. } \Sigma x^i = x.$$

For the proof of this result refer to Samuelson [11], Chipman and Moore [2] and Dow and Sonnenschein [4]. This theorem has a corollary which says that whenever a representative consumer is a consequence of Samuelson's theorem, it is always the case that the Scitovsky upper contour set for the optimal allocation is contained in the upper contour set of U at the aggregate bundle. The corollary holds for every selection $\delta(p,I)$ of the argmax.

2.3. Corollary. Samuelson [11]. Under the same hypotheses, for $x \in \xi(p,I)$, the Scitovsky upper contour set of W through

$$(\xi^1(p,\delta^1(p,I)I),\ldots, \xi^n(p,\delta^n(p,I)I))$$

is contained in the upper contour set of U through the point x.

Proof. We have to show that $\forall(x^1,\ldots,x^n)$, $\forall(p,I)$,

$$(\forall i : U^i(x^i) \geq \bar{V}^i(p,I)) \Rightarrow U(\Sigma x^j) \geq V(p,I).$$

By the theorem we have $U(\Sigma x^j) = \max\limits_{\Sigma y^j = \Sigma x^j} W(U^1(y^1),\ldots, U^n(y^n))$.

In particular $U(\Sigma x^j) \geq W(U^1(x^1),\ldots, U^n(x^n))$. But W is increasing, and thus $U^i(x^i) \geq \bar{V}^i(p,I) \Rightarrow W(U^1(x^1),\ldots,U^n(x^n)) \geq W(\bar{V}^1(p,I),\ldots,\bar{V}^n(p,I)) = V(p,I)$.

Q.E.D.

3. A PERVERSE EXAMPLE

A minimal consistency requirement that could be imposed on welfare judgements is the following:

3.1. Definition. Pareto Property.

$$\bar{V}^i(p,I) \geq \bar{V}^i(p',I') \text{ for all } i \Rightarrow V(p,I) \geq V(p',I')$$

and $$\bar{V}^i(p,I) > \bar{V}^i(p',I') \text{ for all } i \Rightarrow V(p,I) > V(p',I').$$

In this section we present a graphical example showing that this property is not implied by the representative consumer hypothesis. The example is motivated by the following reasoning. If $U(x') > U(x)$, and x' is in the range of ξ, then there exist budgets (p^1,I^1), (p^2,I^2) through (p^k,I^k) such that x' is revealed preferred to x, i.e.

$$x' \in \xi(p^1,I^1)$$
$$p^1.x' \geq p^1.\xi(p^2,I^2)$$
$$p^2.\xi(p^2,I^2) \geq p^2.\xi(p^3,I^3)$$
$$\ldots$$
$$p^k.\xi(p^k,I^k) \geq p^k.x$$

In case $k=1$, x' is directly revealed preferred to x; if $k>1$ x' is indirectly revealed preferred. In the former case we therefore have

$$x' \in \xi(p^1,I^1)$$
$$p^1.x' \geq p^1.x$$

It is straightforward to show that in this case, there is no allocation of x among the consumers which is pareto superior to the allocation of x' which occurs when all consumers maximize utility at prices p^1 and incomes $\delta^1(p^1,I^1),\ldots,\delta^n(p^1,I^1)$. In other words, when national income increases, valued at the original prices, at least one consumer must be made better off. (See Hicks [7] and Kaldor [9].) Thus there can be no violation of the Pareto property in cases where

$$p'.\xi(p',I') \geq p'.\xi(p,I).$$

In order to find a violation, we must look to the case of indirect revealed preference. If $x' \epsilon \xi(p',I')$ is indirectly revealed preferred to x, is it true that no allocation of x is Pareto superior to

$$(\xi^1(p',\delta^1(p',I')I'),\ldots,\xi^n(p',\delta^n(p',I')I'))?$$

If so then the the Pareto property is always satisfied. The example below shows that it is not so and, by choosing an appropriate income distribution, we provide a direct example of an economy violating the Pareto property.

<u>3.2. Example. Violation of the Pareto Property.</u> There are two goods and two consumers. We consider three budget sets, (p,I), (p',I'), and (p",I") at which consumer 1 has 2/3 of the income and consumer 2 has 1/3.

Figure 1 shows these three aggregate budget sets, and the corresponding individual budget sets. We have

$$\bar{V}^1(p,I) > \bar{V}^1(p',I') > \bar{V}^1(p'',I'')$$

This ranking does not follow unambiguously (i.e. by revealed preference) but the preferences of the first consumer can be chosen to satisfy these inequalities (as drawn in Figure 1). For the second consumer we have, by revealed preference, that

$$\bar{V}^2(p'',I'') > \bar{V}^2(p,I) > \bar{V}^2(p',I').$$

Thus,

$$\bar{V}^1(p,I) > \bar{V}^1(p',I')$$

and $\bar{V}^2(p,I) > \bar{V}^2(p',I')$

and yet $x' = \xi(p',I')$ is indirectly revealed preferred to x.

We can complete the example to show a violation of the Pareto property by introducing a third consumer, who has zero income at budgets (p,I), (p',I') and (p",I") but has all the income at all other budgets. The preferences of this consumer satisfy $x' = \xi^3(p',I')$ and $x'' = \xi^3(p'',I'')$. We now have an economy with three consumers and utility generated aggregate demand; the representative preferences are the same as the preferences of the third consumer. The Pareto property is violated because

$$\bar{V}^1(p,I) > \bar{V}^1(p',I')$$

$$\bar{V}^2(p,I) > \bar{V}^2(p',I')$$

$$\bar{V}^3(p,I) \geq \bar{V}^3(p',I')$$

and $V(p,I) < V(p',I')$.

This example has serious implications for economic analysis where the representative consumer is used as a basis for welfare judgements. As it stands, however, it suffers from the defect that the income distribution, δ, is badly behaved (in particular, it is discontinuous)[1]. It would be desirable to know whether violation of the Pareto property is the norm or whether it is pathological.

4. WELFARE JUDGEMENTS WITH THE PARETO PROPERTY

In this section we construct an increasing welfare function which allows judgements consistent with the welfare of the representative consumer. This construction is motivated by the Pareto property (3.1). The welfare function can always be defined, but if the property holds it has desirable properties. This does not mean that judgements based on the representative consumer are justified: only if the welfare function implicit in these judgements is normatively appropriate can they be justified. That is why we stress that we are concerned merely with the consistency of welfare judgements.

4.1. Definition of the Welfare Function.

$$W(U^1,\dots,U^n) = \sup_{\{(p,I)\ |\ \forall i:\ \bar{V}^i(p,I) \le U^i\}} V(p,I)$$

W is clearly nondecreasing. The following result clarifies the definition.

4.2. Proposition. If the Pareto property holds then

$$W(\bar{V}^1(p,I),\dots,\bar{V}^n(p,I)) = V(p,I)$$

Proof. By definition of W we have $W(\bar{V}^1(p,I),\dots,\bar{V}^n(p,I)) \ge V(p,I)$. Suppose that $W(\bar{V}^1(p,I),\dots,\bar{V}^n(p,I)) > V(p,I)$. Then there exists (p',I') so that $\forall i$, $\bar{V}^i(p,I) \ge \bar{V}^i(p',I')$ and $V(p',I') > V(p,I)$, a violation of the Pareto property.

Q.E.D.

4.3. Theorem. If the Pareto property holds then

$$V(p,I) \ge V(p',I') \Rightarrow W(\bar{V}^1(p,I),\dots,\bar{V}^n(p,I)) \ge W(\bar{V}^1(p',I'),\dots,\bar{V}^n(p',I'))$$

$$V(p,I) > V(p',I') \Rightarrow W(\bar{V}^1(p,I),\dots,\bar{V}^n(p,I)) > W(\bar{V}^1(p',I'),\dots,\bar{V}^n(p',I')).$$

Proof. This is an immediate corollary of 4.2.

Q.E.D.

In general, W will be nondecreasing. However, if we restrict attention to the range of utilities attained, we have:

<u>4.4. Proposition.</u> W is increasing on $\bar{V}[R^{\ell}_{++}\times R_{+}]$, where $\bar{V}:R^{\ell}_{++}\times R_{+}\longrightarrow R^{n}$ is defined by $\bar{V}(p,I)=(\bar{V}^{1}(p,I),\ldots,\bar{V}^{n}(p,I))$.

<u>Proof.</u> Let $(U^{1},\ldots,U^{n})$ and $(\bar{U}^{1},\ldots,\bar{U}^{n})$ be elements of $\bar{V}[R^{\ell}_{++}\times R_{+}]$, with $U^{i}>\bar{U}^{i}$ $\forall i$. Then $\exists$ (p,I) and $(\bar{p},\bar{I})$ so $U^{i}=\bar{V}^{i}(p,I)$ and $\bar{U}^{i} = \bar{V}^{i}(\bar{p},\bar{I})$. The result follows immediately from 4.2. and the Pareto property.

<u>Q.E.D.</u>

Thus, when making welfare judgements using the representative consumer, the implied welfare function W will be increasing in the relevant range.

The idea behind the definition is to ensure that the property described in 4.3 holds. An obvious way to acheive this is to define W on $\bar{V}[R^{\ell}_{++}\times R_{+}]$, using 4.2 as a definition, and then to extend it to the whole of R^{n} in a way consistent with W being nondecreasing. Definition 4.1 is based on this reasoning. Note that W as defined in 4.1 is unique in the sense that the restriction of any other welfare function satisfying 4.3 to $\bar{V}[R^{\ell}_{++}\times R_{+}]$, must be a monotonic transformation of the restriction of W to that domain.

Theorem 4.3 has economic importance, as it states that using V as a welfare criterion is equivalent to using the social welfare function defined in 4.1. Thus, if the Pareto property holds, welfare judgements based on the representative consumer are made according to an implicit social welfare function.

Suppose that in an economy where the Pareto property holds, the government were to choose income to maximize W. It is not necessarily the case that $\delta(p,I)$ is the optimal income distribution for W or for any other welfare function, as the following example shows.

<u>4.5. Example.</u> There are two individuals with $U^1(x^1,y^1) = \min\{x^1,y^1\}$ and $U^2(x^2,y^2) = \sqrt{(x^2y^2)}$. Define $\delta^1(p_x,p_y,I) = 1$ except when $p_x = p_y$ and $\delta^1(p_x,p_y,I) = 0$ for $p_x = p_y$ $(\delta^2 = 1-\delta^1)$. This economy has aggregate demand generated by $U(x,y) = \min\{x,y\}$. It can easily be verified that the Pareto property is satisfied. Using Definition 4.1, we have a welfare function $W(U^1,U^2) = \max\{U^1,U^2\}$. Optimal income shares for this welfare function are $\delta^1(p_x,p_y,I) = 0$ for $p_x \neq p_y$ and any shares are optimal for $p_x = p_y$. Furthermore, the actual income shares are suboptimal for any Bergson welfare function. The reason is as follows: if not, the Scitovsky indifference curve for the allocation $(\xi^1(p,\delta^1(p,I)I),\xi^2(p,\delta^2(p,I)I))$ would be contained in the upper contour set of $U(x)$ through $\xi(p,I)$ for all (p,I), as stated in Corollary 2.4. For $p_x \neq p_y$, these indifference curves coincide, hence do not violate this property. However, at $p_x = p_y$ we have the situation depicted in Figure 2. The Scitovsky upper contour set at the allocation $((0,0),(1,1))$ (corresponding to prices $p_x = p_y = 1$ and income $I = 2$) is the sum of the two sets $\{(x,y)\in R^2_+ \mid xy\geq 1\}$ and R^2_+ (which are the upper contour sets of the first and the second persons, respectively). Thus the Scitovsky curve through $(1,1)$, for that allocation, is $xy = 1$. On the other hand, the indifference curve of $U(x)$ through $(1,1)$ is given by $\min\{x,y\} = 1$.

This example therefore shows that a representative consumer may exist in an economy which neither violates the Pareto property, nor has optimal income shares for any welfare function.

5. THE CONVERSE OF SAMUELSON'S THEOREM

As shown in the last section, whenever the Pareto property holds we can define a welfare function that respects welfare judgements based on the representative consumer. However, the example in that section shows that the Pareto property is not sufficient to give a converse to Samuelson's theorem. Here we show that if we add the hypothesis that the Scitovsky upper contour sets through the market allocation lie inside the upper contour sets of the representative consumer at the market aggregate allocation, then Samuelson's condition on optimal income shares will follow. As (by 2.3) this additional hypothesis is a consequence of Samuelson's result, we are provided with a complete converse of his theorem. First we see how the Pareto property and the above hypothesis about the Scitovsky curves can be interpreted as a stronger Pareto property. Then we go on to prove that W, as defined in the last section, is finite everywhere. Finally we prove our main result.

5.1. Definition. Strong Pareto Property. We say that the strong Pareto property holds when

$$\forall(x^1,\ldots,x^n),\ \forall(p,I),$$

$$U^i(x^i) \geq \overline{V}^i(p,I)) \text{ for all } i \Rightarrow U(\Sigma x^j) \geq V(p,I)$$

$$U^i(x^i) > \overline{V}^i(p,I)) \text{ for all } i \Rightarrow U(\Sigma x^j) > V(p,I).$$

This property implies the Pareto property. The statement with weak inequalities means that the Scitovsky upper contour sets, through the appropriate market allocations, are contained in the upper contour sets of the aggregate utility function. The stronger statement with strict inequality corresponds to W being increasing (It is implied by the former if U is continuous). It can be weakened by requiring it to hold only for market allocations, as in 3.1, without affecting the proof of 5.4.

5.2 Proposition. The strong Pareto property implies that

$$\forall(x^1,\ldots,x^n): \qquad W(U^1(x^1),\ldots,U^n(x^n)) < \infty.$$

Proof. If not, there exists $\{(p_k,I_k)\}_{k=1}^{\infty}$ with $V(p_k,I_k) \longrightarrow \infty$ as $k \longrightarrow \infty$ and also $\bar{V}^i(p_k,I_k) \le U^i(x^i)$ for all i and all k. By the strong Pareto property we have $U(\Sigma x^j) \ge V(p_k,I_k)$ which provides an upper bound for V, hence a contradiction.

Q.E.D.

5.3. Theorem. The Converse of Samuelson's Theorem. If the strong Pareto property holds, then the income shares δ are optimal for some welfare function. In particular, they are optimal for W as defined in 4.1, i.e.

$$\forall(p,I),\ \forall i:\ \delta^i(p,I) \in \underset{(d^1,\ldots,d^n)}{\operatorname{argmax}}\ W(U^1(x^1),\ldots,U^n(x^n)) \quad \text{s.t.}\quad x^i \in \xi^i(p,d^iI) \quad \text{and} \quad \Sigma d^i = 1.$$

Proof. If not, we have some (p,I) and $(d^1,\ldots,d^n) \in S^{n-1}$ so that $W(V^1(p,d^1I),\ldots,V^n(p,d^nI)) > W(V^1(p,\delta^1(p,I)I),\ldots,V^n(p,\delta^n(p,I)I))$, which in turn is equal to $V(p,I)$ by 4.2. Therefore, by the definition of $W(V^1(p,d^1I),\ldots,V^n(p,d^nI))$, there is (p',I') such that $V(p',I') > V(p,I)$, and $\forall i$: $\bar{V}^i(p',I') \le V^i(p,d^iI)$. The strong Pareto property implies that $U(\Sigma\ \xi^i(p,d^iI)) \ge V(p',I')$. Thus $U(\Sigma\ \xi^i(p,d^iI)) > V(p,I)$. On the other hand, since $\xi(p,I)$ is generated by U, and since $p.\Sigma\ \xi^i(p,d^iI) \le I$, we have $U(\Sigma\ \xi^i(p,d^iI)) \le V(p,I)$, a contradiction.

Q.E.D.

The following two examples illustrate the result.

5.4. Example. Eisenberg's aggregation result. (Chipman and Moore [2].) Suppose $\forall i$: U^i is homogeneous of degree one, and δ^i are given constants for all prices and income. We know that in this case the individual demands aggregate to a market demand which is generated by

$$U(x) = \max_{\Sigma x^i = x} \Pi_{i=1}^{n} [U^i(x^i)^{\delta^i}]$$

One can easily see that the strong Pareto property holds. Applying the definition of W, as above, to this case one gets $W(U^1,\ldots,U^n)=\Pi(U^i)^{\delta^i}$. It is easy to verify that δ^i are the optimal income shares.

5.5. Example. Antonelli's aggregation result. (Dow [3].) This is only one side of the implications of Antonelli's theorem. For every $i=1,\ldots,n$, let U^i be homogeneous of degree one and identical. Let $\delta = (\delta^1,\ldots,\delta^n) = S^{n-1}$. This is an example where the δ is a correspondence, as our result allows. We know these consumers aggregate to a market demand generated by the same utility function as the consumers'. As before, it is easy to see that the strong Pareto property holds. Using the method described one gets $W(U^1,\ldots,U^n) = \Sigma U^i$. Since the value of W is independent of the income distribution δ is the optimal income distribution.

5.6. Remark. When δ is a correspondence some slight modifications are needed througout. Let $d(p,I)$ be a selection of $\delta(p,I)$. So that the problem makes sense, we must have that the choice of a different selection $d(p,I)$ does not affect the market demand $\xi(p,I)$. This means: $\xi(p,I) = \Sigma\ \xi^i(p,d^i(p,I)I)$ for any given $d(p,I)$. $\bar{V}^i$ is now a function of the selection d. We write $\bar{V}^i(p,I,d)$. Regarding the Pareto property: one should add the words 'for every selection d such that ...'. Then, in the definition of W (4.1) one should have: sup V(p,I)

$$\{ (p,I) \mid \exists d \in \delta \text{ s.t. } \forall i: \bar{V}^i(p,I,d) \le U^i \}.$$

Proposition 4.2 will then read: $\forall d \in \delta$, $W(\bar{V}^1(p,I,d),\ldots, \bar{V}^n(p,I,d)) = V(p,I)$. From then on the rest of the paper follows.

6. SUMMARY.

We have investigated the validity of welfare judgements in economies where there is a representative consumer. The most important conclusions are:

(i) It is possible that welfare judgements based on the welfare of the representative consumer can be totally erroneous (section 3).

(ii) If this is not the case, then, on the contrary, these welfare judgements are highly consistent (section 4). However, there is no reason why the welfare function implied by this consistency should be ethically desirable. If the welfare function function judged suitable for such comparisons does not coincide with W as defined in section 4 then welfare statements based on the representative consumer will not, in general, be correct.

(iii) Finally, in section 5, we have derived the converse of Samuelson's 1956 result. The hypothesis that a representative consumer exists and that, in addition, the Scitovsky indifference curves lie above those of the representative consumer are equivalent to the hypothesis that the income distribution is optimal according to a Bergson welfare function.

We do not provide an answer to the question of whether representative consumer welfare analysis is justifiable. On balance it seems that caution is indicated since, even if the Pareto property were violated only in rare or peculiar cases, the welfare function implicit in the judgements may not be ethically desirable. In future research we intend to investigate whether the Pareto property can be shown to be abnormal, or, to the contrary, whether it is the norm.

<u>FOOTNOTE.</u> [1]Professor Deaton drew our attention to a recent paper by Jerison [8], which contains an example violating the Pareto property for an economy with fixed income shares and Stone-Geary preferences.

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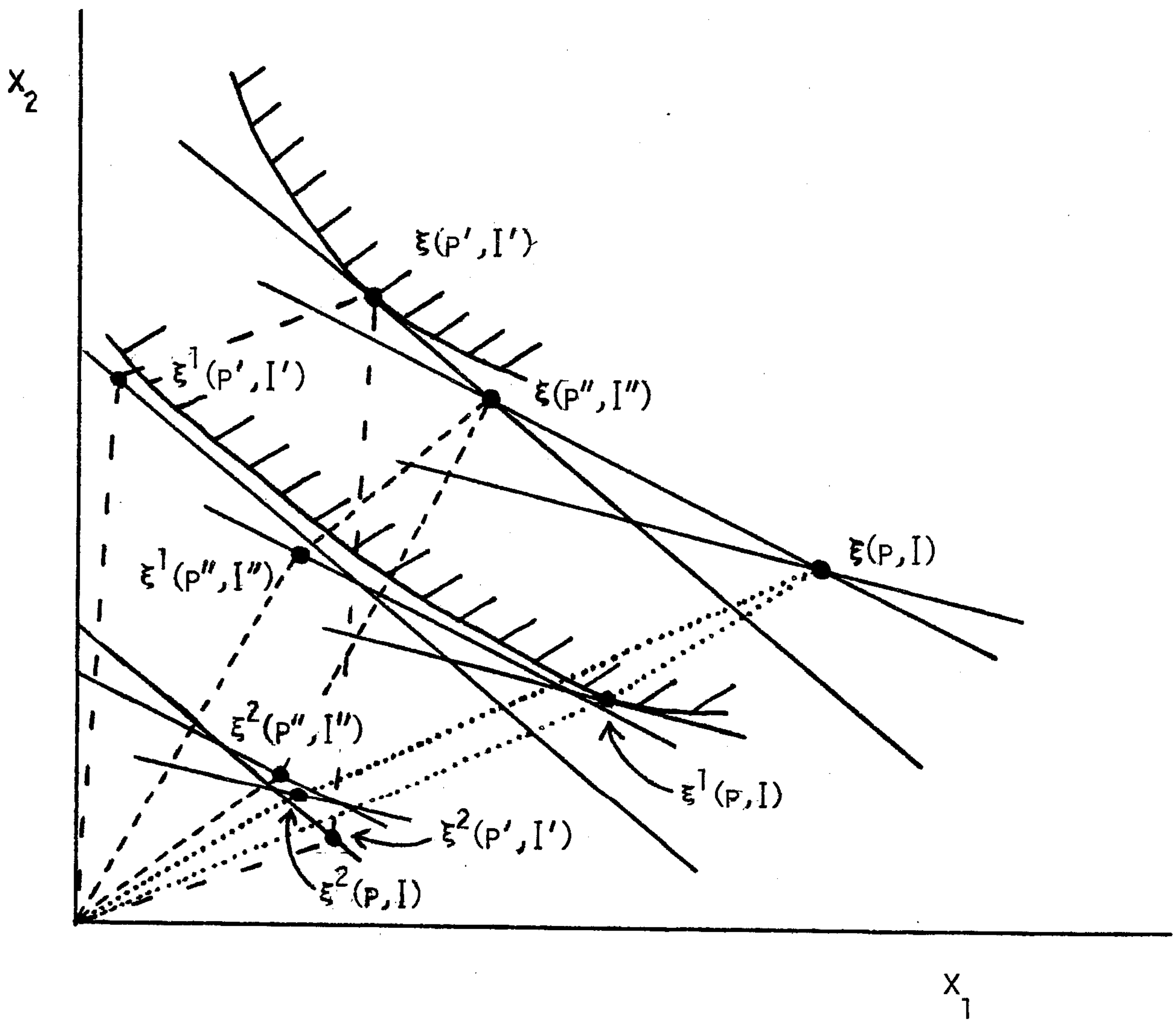
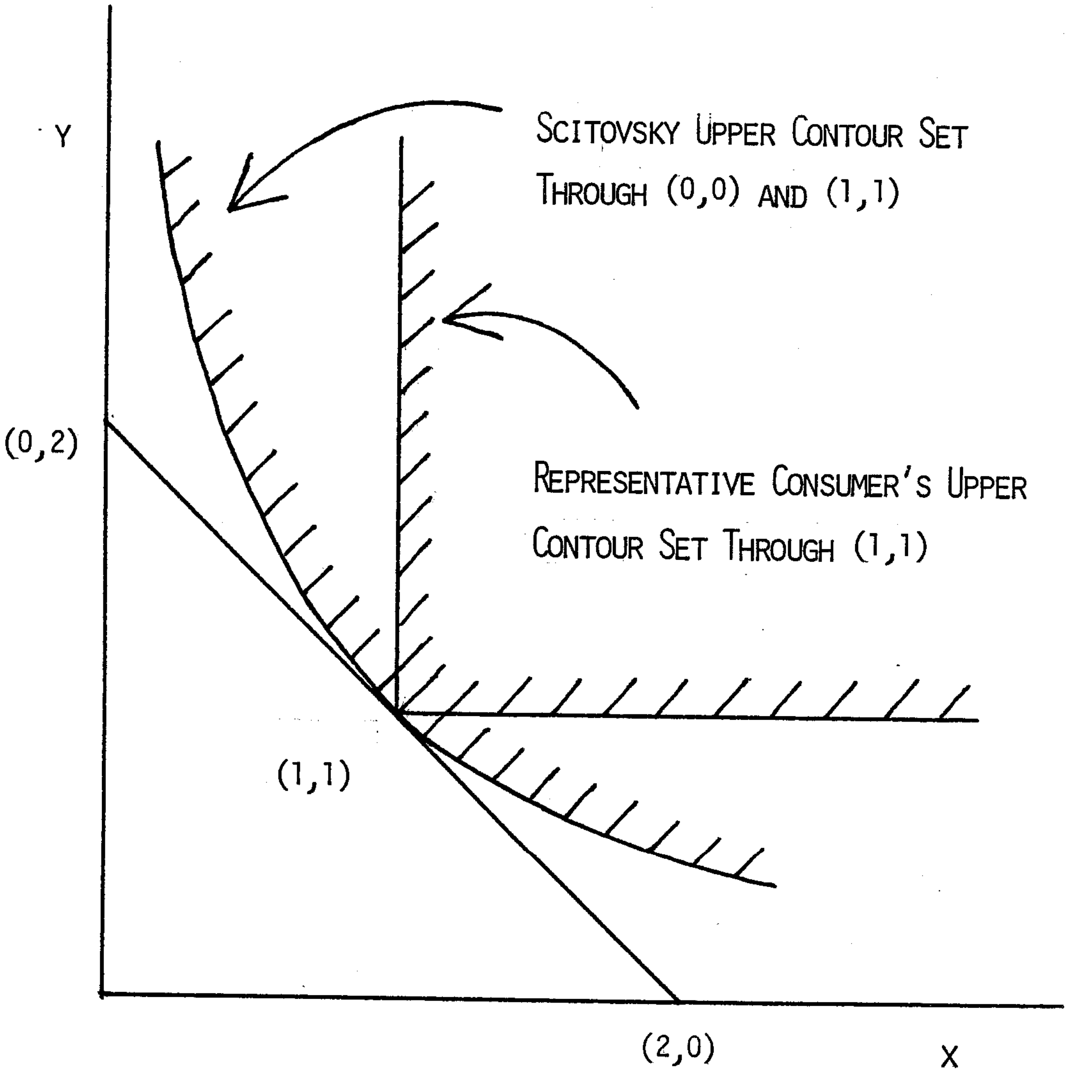


X_2
ξ(P′,I′)
ξ(P″,I″)
ξ(P,I)
ξ¹(P′,I′)
ξ¹(P″,I″)
ξ¹(P,I)
ξ²(P″,I″)
ξ²(P′,I′)
ξ²(P,I)
X_1


Y
SCITOVSKY UPPER CONTOUR SET
THROUGH (0,0) AND (1,1)
(0,2)
REPRESENTATIVE CONSUMER'S UPPER
CONTOUR SET THROUGH (1,1)
(1,1)
(2,0)
X