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PRESENT VALUE MODELS

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The hypothesis of rational expectations (RE) has stimulated much interesting research in economics for over a decade, although many economists remain skeptical of its empirical validity. Lovell (1986) provides a survey of some evidence bearing on this issue. This paper presents further evidence on the validity of the RE hypothesis by applying it to present value models. A number of studies have attempted to estimate and test present value models under the RE assumption, including the recent studies of Campbell and Shiller (1987), Fama and French (1988), Poterba and Summers (1987), and West (1988), and the literature cited. Many of these studies have found the restrictions imposed by the RE hypothesis on present value models inconsistent with the data, but the authors usually prefer to reject the models tested while maintaining the RE hypothesis. In this paper, I employ a different method of estimation and testing from those employed in previous studies, and find one implication of the present value model under RE to be strongly rejected by the data. However, rather than searching for more complicated models and maintaining RE, I find that the simple model under adaptive expectations (AE) can explain the data very well. Such results might persuade some readers to reconsider the possible validity of the AE hypothesis as compared with the RE hypothesis in some econometric applications.

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I will begin section I by describing the models and the method used for estimation under the RE assumption. In section II present value models for stock price and long-term interest rate will be estimated and the results presented. Since the models are strongly rejected, I will examine in section III the alternative AE hypothesis and show that it explains the data much better than RE. Section IV tests the model in logarithmic form and shows that the AE version is not only preferable, but helps to explain the phenomenon of mean reversion in stock prices.

### I. Models and Method

This paper is concerned with two present value models of the form

$$y_t = \theta \sum_{i=0}^{\infty} \delta^i E_t z_{t+i} + c \quad (1)$$

where  $E_t$  denotes conditional expectation given the information  $I_t$  available to the economic agents at time  $t$ . In the stock price example,  $y_t$  denotes the price of a stock (or the total value of a number of stocks) at the beginning of period  $t$  while  $z_t$  denotes the dividend derived from the above stock(s) during period  $t$ ;  $\delta$  is the discount factor, assumed to be time invariant,  $c = 0$  and  $\theta = \delta$ . In the interest rate example,  $y_t$  denotes a long-term rate at time  $t$  while  $z_t$  is a short-term rate;  $c$  is the risk premium and  $\theta = 1 - \delta$ .

I will concentrate on testing the following implication of (1). Using (1) to evaluate  $E_t y_{t+1}$ , and subtracting  $\delta E_t y_{t+1}$  from (1) give

$$y_t = \delta E_t y_{t+1} + \theta E_t z_t + c(1 - \delta) \quad (2)$$

In the case of stock price, (2) becomes

$$y_t = \delta E_t (y_{t+1} + z_t) \quad (2a)$$

In the case of interest rates, (2) becomes

$$y_t = \delta E_t y_{t+1} + (1 - \delta)z_t + c(1 - \delta) \quad (2b)$$

Assumption (1) implies (2) but not conversely; (2) is much weaker than (1). In the stock price example, (2a) asserts only that the price of a stock (or a group of stocks) at the beginning of period  $t$  is equal to the discounted value of the expected sum of money, from selling the stock and keeping the dividend, to which the owner of the stock is entitled at the beginning of period  $t+1$ . As it allows for the effects of speculation, this arbitrage relation for any one period is much weaker than the assumption that the price of a stock is the discounted value of all future dividends. It is well known [see Sargent (1987, p. 95)] that the solution to the difference equation (2a) is (1) plus a speculative term  $\gamma_t(1/\delta)^t$ , where  $\gamma_t$  is a martingale, with  $E_t \gamma_{t+1} = \gamma_t$ . This paper is concerned mainly with hypothesis (2) under rational or adaptive expectations.

To estimate (2) under the assumption of RE of Muth (1961), we equate the subjective expectations  $E_t$  in the minds of the economic agents with the mathematical expectations generated by the econometric model used by the econometrician. Denote the information available to the econometrician at time  $t$  by  $H_t$ , which includes at least the values of the variables of the selected model up to time  $t$ .  $H_t$  is usually assumed to be a subset of  $I_t$ , the information available to the economic agents. If  $E_t$  in (2) originally refers to  $E(\cdot | I_t)$ , by applying  $E(\cdot | H_t)$  to (2), we find that (2) is valid also for  $E_t$  interpreted as  $E(\cdot | H_t)$  since  $E(E[\cdot | I_t] | H_t) = E_t(\cdot | H_t)$ . I will estimate and test (2), with  $E_t$  interpreted as  $E(\cdot | H_t)$ .

Assume that under RE  $y_{t+1}$  can be written as

$$y_{t+1} - E_t y_{t+1} = u_{t+1} \quad (3)$$

where  $u_t$  is serially independent. Using (3) to replace  $E_t y_{t+1}$  in (2), we obtain

$$y_t = \delta y_{t+1} + \theta E_t z_t + c(1-\delta) - \delta u_{t+1} \quad (4)$$

By the method of Chow (1983) we solve (4) for  $y_{t+1}$  and reduce the time subscripts of the resulting equation by one to give

$$y_t = \delta^{-1} y_{t-1} - \delta^{-1} \theta E_{t-1} z_{t-1} + c(1 - \delta^{-1}) + u_t \quad (5)$$

which is the model to be estimated. Taking expectation of (4) given  $H_t$  yields the original model (2).

To estimate equation (5) for stock price, I assume the following model for  $z_t$

$$z_t = \alpha_1 z_{t-1} + \dots + \alpha_p z_{t-p} + \gamma_0 y_t + \dots + \gamma_p y_{t-p} + b + v_t \quad (6)$$

where  $v_t$  is also assumed to be serially independent. Thus  $E_{t-1} z_{t-1}$  is a linear function of  $z_{t-2}, \dots, z_{t-p-1}, y_{t-1}, \dots, y_{t-p-1}$ . If  $E_{t-1} z_{t-1}$  in (5) is replaced by  $z_{t-1} - v_{t-1}$ , equation (5) becomes, for  $\theta = \delta$

$$y_t = \delta^{-1} y_{t-1} - z_{t-1} + u_t + v_{t-1} \quad (7)$$

Since  $z_{t-1}$  is correlated with the residual  $u_t + v_{t-1}$ , I will estimate (7) by using as instrumental variable the least squares estimate  $\hat{z}_{t-1}$  of  $E_{t-1} z_{t-1}$ . For long-term interest rate,  $E_{t-1} z_{t-1} = z_{t-1}$  and equation (5) can be estimated by least squares.

## II. Statistical Evidence

Thanks to Campbell and Shiller (1987), I have been able to use their data for studying the relations between stock price and dividends, and between long-term and short-term interest rates. The stock price and dividend series are deflated annual indices for the Standard and Poor's 500 stocks from 1871 to 1986 used by Shiller (1984) and updated to 1986. The interest rate series are U.S. Treasury 20-year yields from Solomon Brothers' Analytical Record of Yields and Yield

Spreads and the 1-month Treasury bill rate from the U.S. Treasury Bulletin.

These series are monthly from 1959,2 to 1983,11.

#### A. Stock Price

I first apply OLS to estimate equation (6) for dividends using annual data from 1874 to 1985, obtaining

$$\begin{aligned} z_t = & .0753 + 1.033 z_{t-1} - .373 z_{t-2} + .231 z_{t-3} & R^2 = .9606 & (8) \\ & (.0397) \quad (.097) \quad (.136) \quad (.095) & s = .1331 \\ & & DW = 1.988 \\ & + .0146 y_t - .0105 y_{t-1} + .0044 y_{t-2} - .0055 y_{t-3} \\ & (.0020) \quad (.0030) \quad (.0031) \quad (.0024) \end{aligned}$$

where the standard errors of the coefficients are in parentheses,  $s$  is the standard error of the regression and DW is the Durbin-Watson statistic (used only as a descriptive statistic, the first-order autocorrelation being  $1-DW/2$ ).

Introducing  $z_{t-4}$  and  $y_{t-4}$  as additional explanatory variables in (8) yields  $t$  statistics of .160 and .581 respectively, suggesting that these variables be omitted. Using the estimated  $\hat{z}_{t-1}$  from (8) as the instrumental variable for  $z_{t-1}$ , I estimate equation (7) of stock price for the sample period 1875-1986:

$$\begin{aligned} y_t = & .877 y_{t-1} + 3.088 z_{t-1} & s = 6.675 & (9) \\ & (.064) \quad (1.586) & DW = 1.793 \end{aligned}$$

The estimates of both coefficients of (9) strongly contradict the theory. The coefficient of  $y_{t-1}$  should be  $\delta^{-1} > 1$ , but is about two standard errors smaller. If we assume asymptotic normal distribution for this coefficient when  $\delta^{-1} > 1$ , the hypothesis that  $\delta^{-1} = 1.02$ , say, would be rejected at about the 2.5 percent level using a one tail test. The coefficient of  $z_{t-1}$  should be  $-1$ , but is over two and a half standard errors larger. The  $F(2, 110)$  statistic for testing the joint hypothesis that the coefficient of  $y_{t-1}$  is 1.02, say, and the coefficient of  $z_{t-1}$  is  $-1$  turns out to be 3.947, leading to rejection at the 2.21 percent level.

To check whether the above asymptotic theory is applicable to the distribution of the two coefficients of the model of equation (9) estimated by using an instrumental variable for  $z_{t-1}$ , I have performed a Monte Carlo experiment using 1000 replications. In each replication, 112 observations for  $z_t$  beginning in 1874 are generated by equation (8), with the estimated coefficients as the true values and .1331 as the residual standard deviation, using the observed data for dividend  $z_t$  from 1871 to 1873 and for stock price  $y_t$  from 1871 and 1874 as initial values. Data for  $y_t$ , beginning from 1975, are generated by  $y_t = 1.02 y_{t-1} - E_t z_{t-1}$  plus a residual with standard deviation equal to 6.675. The method of IV is applied to estimate the two coefficients of equation (9). The following distributions are found for the two standard "t" statistics and the "F(2,110)" statistic.

Theoretical one-tail probability	.010	.050	.100
Monte Carlo relative frequency:			
"t" statistic for the 1st coefficient being 1.02	.021	.068	.131
"t" statistic for the 2nd coefficient being -1	.014	.060	.122
F(2,110) statistic for both coefficients	.020	.070	.110

In particular, the t statistic for testing the first coefficient of (9) being 1.02 has theoretical left-tail probability of .025 and Monte Carlo relative frequency of .041. The F(2,110) statistic for testing the joint hypothesis of the two coefficients being 1.02 and -1 respectively has theoretical right-tail probability of .022 and Monte Carlo relative frequency of .032. Thus the above Monte Carlo experiment has demonstrated that the rejection probabilities reported by the standard t and F tests applied to the coefficients of equation (7) estimated by IV, with  $\delta^{-1}$  as low as 1.02, are reasonably accurate. In the rest of this paper, when I apply t and F tests in similar situations, I will assume the standard distribution theory to hold.

One possible shortcoming of the estimates given by equations (8) and (9) is the assumption of homoskedasticity. Since both dividends and stock price have increased substantially from 1875 to 1986, the standard deviations of the residuals of equations (8) and (9) might be expected to increase proportionally. I will assume that the variance of the residual of (8) is proportional to  $(z_{t-1})^2$  and the variance of the residual of (9) is proportional to  $(y_{t-1})^2$ . Indeed, a regression of log of the square of the residual of (9) on the log of  $(y_{t-1})^2$  has a coefficient of 1.172 with a standard error of .164. Using weighted least squares, equation (8) becomes

$$\begin{aligned}
 z_t = & .072 + .941 z_{t-1} - .240 z_{t-2} + .160 z_{t-3} & (8a) \\
 & (.034) (.100) & (.132) & (.096) \\
 & + .0188 y_t - .0133 y_{t-1} + .0053 y_{t-2} - .0064 y_{t-3} \\
 & (.0027) & (.0040) & (.0042) & (.0033)
 \end{aligned}$$

and the coefficients for  $z_{t-4}$  and  $y_{t-4}$ , if included, have t ratios of .912 and .101 respectively. Using (8a) to form the instrument  $\hat{z}_t$  and applying weighted IV, I have modified equation (9):

$$y_t = .926 y_{t-1} + 2.140 z_{t-1} \quad (9a) \\
 (.080) \quad (1.570)$$

Although the two individual coefficients are not as far from 1.02 and -1.0 respectively as are the coefficients of equation (9), the  $F(2,110)$  statistic for testing the joint hypothesis for them to equal these values turns out to be 9.131, leading to rejection at the .000214 level, much stronger than the evidence from equation (9). Since the method of weighted least squares is equivalent to ordinary least squares applied to rescaled data, the Monte Carlo experiment reported above is relevant here also.

Another useful way to look at equation (7) is to regress  $y_t + z_{t-1}$  on  $y_{t-1}$  and other variables in  $H_{t-1}$ . Hypothesis (2a) implies that  $E_{t-1}(y_t + z_{t-1})$  equals



$\delta^{-1}y_{t-1}$  and that variables other than  $y_{t-1}$  are not important in explaining  $y_t+z_{t-1}$ . If we regress  $y_t+z_{t-1}$  on  $y_{t-1}$  alone, using data from 1873 to 1986, and again assuming the residual variance to be proportional to  $(y_{t-1})^2$ , the result is

$$(y_t + z_{t-1}) = 1.082 y_{t-1} \quad (10)$$

(.017)

The estimate 1.082 for  $\delta^{-1}$  appears high but is not unreasonable. An interesting regressor to add to equation (10), suggested by (7) and (9a), is  $\hat{z}_{t-1}$  estimated from (8a). For the period 1875 to 1986, weighted least squares gives

$$(y_t + z_t) = .943 y_{t-1} + 2.787 \hat{z}_{t-1} \quad (11)$$

(.072)      (1.413)

The result contradicts (2a) strongly since the  $F(2,110)$  statistic for testing the two coefficients to equal 1.02 and zero respectively is 8.881, leading to rejection at the .000266 level. (11) is a variation of the test performed in (9a) by adding  $z_{t-1}$  to both sides and by using (weighted) least squares instead of (weighted) IV for  $z_{t-1}$  as in (9a).

To check whether the rejection of (2a) under RE might be due to a time-varying discount factor  $\delta_t$ , I have replaced the variable  $y_{t-1}$  in equation (9a) by  $\delta_t^{-1}y_{t-1}$ . According to the hypothesis (2a) with a discount factor  $\delta_t$ , the coefficient of this variable should be unity.  $\delta_t^{-1}$  is estimated by  $1+r_t$ , where  $r_t$  is the six-month commercial paper rate (Historical Statistics of the United States: Colonial Times to 1957, Series 306, p. 654, and Economic Report of the President 1986, p. 332) minus the inflation rate based on the consumer price index (Historical Statistics of the United States, series 157 and 113, pp. 125-127, and Economic Report of the President 1986, p. 318) plus a constant .061953 (risk premium) to make the mean of  $r_t$  equal to .08 as suggested by equations (10). Because data for the six-month commercial paper rate are

available only after 1890,  $r_t$  before that year is set equal to .08. The weighted regression modifying equation (9a) for the sample period 1875-1986, again using  $\hat{z}_{t-1}$  from (8a) as instrument, is

$$y_t = \underset{(.068)}{.819} (\delta_{t-1}^{-1} y_{t-1}) + \underset{(1.453)}{2.977} z_{t-1} \quad (12)$$

As in equation (9a), both coefficients contradict the theory. The first coefficient should be 1 but is about 2.7 standard errors below 1. The second should be -1 but is 2.7 standard errors larger. The F(2,110) statistic for testing the joint hypothesis that the first coefficient is 1 and the second is -1 turns out to be 3.749, leading to rejection at the 2.66 percent level. Thus introducing a time-varying discount factor does not affect our negative findings concerning (2a).

#### B. Interest Rates

To test hypothesis (2b) under RE for long-term interest rate  $y_t$ , I apply unweighted least squares to estimate equation (5) with  $E_t z_t = z_t$  and  $\theta = (1-\delta)$  using monthly data from 1959.2 to 1983.10:

$$y_t = \underset{(.0481)}{.0790} + \underset{(.0145)}{.9602} y_{t-1} + \underset{(.0137)}{.0385} z_{t-1} \quad \begin{array}{l} R^2 = .9887 \\ s = .3069 \\ DW = 1.825 \end{array} \quad (13)$$

The coefficients of  $y_{t-1}$  and  $z_{t-1}$  should be  $\delta^{-1}$  and  $(1-\delta^{-1})$  respectively, but their estimates are very different. A positive risk premium  $c$  implies a negative intercept in (14) but it turns out to be positive. To test the joint hypothesis that the coefficients of  $y_{t-1}$  and  $z_{t-1}$  sum to one and the intercept is zero, we regress  $y_t - z_{t-1}$  on  $y_{t-1} - z_{t-1}$  omitting the intercept:

$$(y_t - z_{t-1}) = \underset{(.014)}{.988} (y_{t-1} - z_{t-1}) \quad \begin{array}{l} R^2 = .9426 \\ s = .3105 \\ DW = 1.768 \end{array} \quad (14)$$

The  $F(2, 298)$  statistic for testing this joint hypothesis is 4.491, leading to rejection at about the one percent level. Furthermore the point estimate of  $\delta^{-1}$  is .988, although it is not significantly below one. If we allow a non-zero risk premium by introducing an intercept to (15), the result is

$$\begin{aligned} (y_t - z_{t-1}) &= .0706 + .9612 (y_{t-1} - z_{t-1}) & R^2 &= .9443 & (15) \\ & (.0236) (.0136) & s &= .3063 \\ & & DW &= 1.827 \end{aligned}$$

suggesting a significantly negative risk premium and a discount factor  $\delta$  significantly above one,  $\delta^{-1}$  being estimated to be .961 with a standard error of .0136. Thus model (2b) for long-term interest rate is strongly rejected.

### III. Adaptive or Rational Expectations?

Given the strong evidence against equations (2a) and (2b) under RE, one may ask, assuming (2a) or (2b) to be correct, what hypothesis concerning the formation of expectations is consistent with the data? A natural candidate is adaptive expectations (AE). We should interpret the symbol  $E_t(\cdot)$  as a subjective expectation in the minds of economic agents and not as a mathematical expectation given the information used by the econometrician. I will formulate two models under AE assuming respectively (2a) and (2b) to be correct, and compare them with the corresponding models under RE.

#### A. Stock Price

For stock price, denote  $(y_{t+1} + z_t)$  by  $x_{t+1}$ . Hypothesis (2a) asserts that  $y_t = \delta E_t x_{t+1}$ . According to the AE hypothesis

$$E_t x_{t+1} - E_{t-1} x_t = \beta (x_t - E_{t-1} x_t) + \varepsilon_t \quad (16)$$

where  $\varepsilon_t$  summarizes other factors than  $(x_t - E_{t-1} x_t)$  which may affect the change in expectations.  $\varepsilon_t$  is orthogonal to variables in  $H_t$ ; otherwise these variables should be included in (16). If (2a) is correct,  $y_t$  is a measure of the

subjective  $\delta E_t x_{t+1}$ . Multiplying (16) by  $\delta$  and using this measure of expectation, we obtain

$$y_t - y_{t-1} = \delta\beta(y_t + z_{t-1}) - \beta y_{t-1} + \delta\epsilon_t \quad (17)$$

or

$$y_t = \frac{1}{1-\delta\beta} (1-\beta)y_{t-1} + \frac{\delta\beta z_{t-1}}{1-\delta\beta} + \frac{\delta\epsilon_t}{1-\delta\beta} \quad (18)$$

When equation (18) is estimated by the stock-dividend data from 1875 to 1986, again assuming the residual variance to be proportional to  $(y_{t-1})^2$ , the result is

$$y_t = \frac{.895}{(.071)} y_{t-1} + \frac{2.757}{(1.390)} z_{t-1} \quad (19)$$

If an intercept is added to (19), its  $t$  statistic is only .468, suggesting its omission and supporting (19). Solving the coefficients of (19) for  $\delta$  and  $\beta$ , we get

$$\begin{aligned} \hat{\delta} &= .963 & \hat{\beta} &= .762 \\ &(.008) & &(.107) \end{aligned} \quad (20)$$

which are very reasonable. The standard errors in (20) are obtained by applying weighted nonlinear least squares to (18). Thus (19) is consistent with the AE hypothesis. (19) is a very reasonable result. For example, if we rewrite it as

$$(y_t + z_{t-1})/y_{t-1} = .895 + 3.757 z_{t-1}/y_{t-1}$$

it implies that the rate of return is positively affected by the dividend-price ratio. I do not claim that (2a) combined with AE is the only hypothesis consistent with (19). The point is that (19) is consistent with the above hypothesis but not with RE.

To see this, let us assume (19) to be a correct model and try to force a RE interpretation into it. We can advance its time subscript by one and take expectations given  $H_t$  to yield

$$E_t y_{t+1} = .895 y_t + 2.757 E_t z_t \quad (21)$$

The resulting equation (21) contradicts (2a), as equation (9a) did, in giving a discount factor  $\delta$  larger than one and a coefficient for  $E_t z_t$  very different from -1. Note the similarity between (19) and (9a). The coefficients of both equations make good sense under AE, but not under RE. Under RE, the coefficient of  $y_{t-1}$  in (19) would be  $\delta^{-1}$  and the coefficient of  $z_{t-1}$  would be -1. Given the restrictions  $0 < \delta < 1$  and  $0 < \beta < 1$ , the coefficient for  $y_{t-1}$  in (18) should be less than one and the coefficient for  $z_{t-1}$  should be positive under AE.

Obtaining a coefficient for  $y_{t-1}$  close to  $\delta^{-1} > 1$  and a coefficient for  $z_{t-1}$  close to -1 in (19) would contradict AE and support RE. The fact that (21), with  $E_t z_t$  added on both sides, is consistent with the regression (11) also confirms AE, for under AE we can take conditional expectation  $E_t$  of (18) for  $y_{t+1}$  and obtain (21) by using  $E_t \varepsilon_t = 0$ . Hence (11) supports AE while rejecting RE.

#### B. Interest Rates

For long-term interest  $y_t$ , I assume hypothesis (2b) to be correct and the expectation  $E_t y_{t+1}$  to be formed adaptively,

$$E_t y_{t+1} - E_{t-1} y_t = \beta (y_t - E_{t-1} y_t) + \varepsilon_t \quad (22)$$

where  $\varepsilon_t$  has the same properties as in (16). If hypothesis (2b) is true,  $\delta E_t y_{t+1}$  can be measured by

$$\delta E_t y_{t+1} = y_t - (1-\delta)z_t - c(1-\delta) \quad (23)$$

Multiplying (22) by  $\delta$  and using the right-hand side of (23) to measure  $\delta E_t y_{t+1}$  and  $\delta E_{t-1} y_t$  in the resulting equation, we obtain

$$y_t = (1-\beta)(1-\delta\beta)^{-1} y_{t-1} + (1-\delta)(1-\delta\beta)^{-1} z_t - (1-\beta)(1-\delta)(1-\delta\beta)^{-1} z_{t-1} + c\beta(1-\delta)(1-\delta\beta)^{-1} + \delta(1-\delta\beta)^{-1} \varepsilon_t \quad (24)$$

This equation will be compared with equation (5) under RE.

Equation (24) is estimated by (unweighted) least squares using monthly interest rates data from 1959,7 to 1983,10:

$$y_t = .0661 + .9441 y_{t-1} + .1956 z_t - .1364 z_{t-1} \quad R^2 = .9913 \quad (25)$$

$$\begin{array}{cccc} (.0428) & (.0128) & (.0202) & (.0217) \\ S = .2692 \\ DW = 2.160 \end{array}$$

According to (24), the three coefficients of (25) provide estimates of  $\beta$  and  $\delta$ .  $1-\delta$  can be estimated by  $.1364/.9441$  or  $.1445$ , giving  $\delta = .8555$ .  $1-\beta$  can be estimated by  $.1364/.1956$  or  $.6972$ , giving  $\beta = .3028$ . When these estimates of  $\delta$  and  $\beta$  are used to estimate the coefficient of  $z_{t-1}$ , the result is  $-.1359$ , very close to the estimate  $-.1364$ , supporting the hypothesis that the three coefficients are the functions of the only two parameters  $\delta$  and  $\beta$  according to (24). Estimation of (24) by nonlinear least squares yields

$$\delta = .8554 \quad \beta = .3003 \quad c = 1.514 \quad R^2 = .9913 \quad (26)$$

$$\begin{array}{cccc} (.0224) & (.0647) & (.2810) & S = .2689 \\ & & & DW = 2.150 \end{array}$$

The data are consistent with the adaptive expectations hypothesis with  $\delta < 1$ ,  $\beta = .30$  and a positive risk premium. Testing the nonlinear restrictions on the coefficients by computing the ratio of the additional sum of squared residuals of (26) as compared with (25), to the  $s^2$  of (25), we obtain an approximate  $F(1,288)$  statistic of only  $.361$ , strongly supporting the restrictions. However, the estimate  $.8554$  for  $\delta$  is lower than expected.

Is equation (24) or (25) consistent with the RE formulation (5)? If we take conditional expectation of (24) given  $H_{t-1}$  assuming (24) to be a correct model, we get an equation for  $E_{t-1}y_t$  having the same explanatory variables as (5) and an additional variable  $E_{t-1}z_t$ . An equation for  $y_t$  results from adding a residual to the above equation for  $E_{t-1}y_t$ . In this equation for  $y_t$ , if the formulation (5) is correct, the coefficient of  $E_{t-1}z_t$  should be zero and the remaining three

coefficients (including the intercept) should be as specified in (5). To estimate this regression equation of  $y_t$  on  $y_{t-1}$ ,  $z_{t-1}$  and  $E_{t-1}z_t$ , the method of instrumental variables is used. In other words,  $E_{t-1}z_t$  is replaced by  $z_t - v_t$  in this equation where the new residual incorporating  $v_t$  is correlated with  $z_t$ . The instrumental variable for  $z_t$  is  $\hat{z}_t$ , which is estimated by regressing  $z_t$  on  $z_{t-1}$ , ...,  $z_{t-6}$ ,  $y_{t-1}$ , ...,  $y_{t-6}$ .

The estimated equation for the sample period 1959,7 to 1983,10 is

$$y_t = .0682 + .9472 y_{t-1} + .1581 E_{t-1}z_t - .1028 z_{t-1} \quad \begin{matrix} s = .2708 \\ DW = 2.090 \end{matrix} \quad (27)$$

(.0431) (.0136)
(.0538)
(.0496)

(27) is similar to (25) and is consistent with the AE model (24). However, (27) is inconsistent with the RE model (5) in having a significantly positive coefficient for  $E_{t-1}z_t$  and an estimate of the coefficient of  $y_{t-1}$  very significantly below any reasonable value of  $\delta^{-1}$ . To test the joint hypothesis under RE that the coefficient of  $E_{t-1}z_t$  is zero and the coefficients of  $y_{t-1}$  and  $z_{t-1}$  sum to one, we use the  $F(2, 288)$  statistic which turns out to be 5.491, leading to rejection at the 0.46 percent level.

Campbell and Shiller (1987) suggest using a shorter sample 1959-1978 because of a possible structural shift after 1978, and estimating  $\hat{z}_t$  by 11 instead of 6 own lags and 11 lagged  $y$ 's. Equation (24) for the AE hypothesis is so reestimated using monthly data from 1959,12 to 1978,12 to yield

$$y_t = .0324 + .9729 y_{t-1} + .1263 z_t - .0959 z_{t-1} \quad \begin{matrix} R^2 = .9878 \\ s = .1769 \\ DW = 2.161 \end{matrix} \quad (28)$$

(.0451) (.0124)
(.0264)
(.0272)

An estimate of  $1-\delta$  is  $.0959/.9729$ , giving  $\delta = .901$ . An estimate of  $1-\beta$  is  $.0959/.1263$ , giving  $\beta = .241$ . Using the third coefficient of (24) these estimates imply a coefficient of  $.0956$  for  $z_{t-1}$ , very close to the estimate  $.0959$  in (28). Applying nonlinear least squares to estimate (24) yields

$$\begin{array}{llll}
\delta = .9006 & \beta = .2342 & c = 1.710 & R^2 = .9878 \\
(.0276) & (.0949) & (.455) & s = .1765 \\
& & & DW = 2.154
\end{array} \quad (29)$$

The data are consistent with the AE hypothesis with  $\delta < 1$ ,  $\beta = .234$  and a positive risk premium. The ratio of the difference between the sums of squared residuals of (29) and (28) to the  $s^2$  of (28) is only .1854. This approximate  $F(1,225)$  statistic strongly supports the nonlinear restrictions on the coefficients of (28).

Again, to examine the rational expectations hypothesis in the framework of (24) we estimate, for the short sample 1959,12-1978,12, a regression equation of  $y_t$  on  $y_{t-1}$ ,  $z_{t-1}$  and  $E_{t-1}z_t$ , using  $\hat{z}_t$  as instrumental variable. The result is

$$\begin{array}{llll}
y_t = .0328 + .9765 y_{t-1} + .0601 E_{t-1}z_t - .0339 z_{t-1} & R^2 = .9875 & (30) \\
(.0458) (.0129) & (.0596) & (.0570) & s = .1793 \\
& & & DW = 2.089
\end{array}$$

(30) is similar to (25) and is consistent with the AE model (24). Since the coefficient of  $E_{t-1}z_t$  is not significantly different from zero, the RE formulation (5) cannot be rejected on this ground. However, the coefficient of  $y_{t-1}$  is about two standard errors below any reasonable value of  $\delta^{-1} > 1$ . Hence the evidence still suggests rejection of (5), though it is weaker than in the case of the full sample between 1959,2 and 1983,10.

To reexamine our rejection of equation (5) as presented in Section II.B using the full sample, we have reestimated equations (13) and (15) using the shorter sample from 1959,2 to 1978,12. Corresponding to (13) is

$$\begin{array}{llll}
y_t = .0381 + .9798 y_{t-1} + .0214 z_{t-1} & R^2 = .9871 & (31) \\
(.0450) (.0127) & (.0115) & & s = .1822 \\
& & & DW = 2.034
\end{array}$$

As in (13), the coefficients of  $y_{t-1}$  and  $z_{t-1}$  are very different from the theoretical values of  $\delta^{-1}$  and  $1-\delta^{-1}$  respectively, and the positive intercept, though not significant, suggests a possibly negative risk premium. As before, we



test the joint hypothesis that the coefficients of  $y_{t-1}$  and  $z_{t-1}$  sum to one and the intercept is zero using an  $F(2,236)$  statistic, which turns out to be 3.320, leading to rejection at the 3.79 percent level. If the intercept is dropped, the hypothesis that the coefficients of  $y_{t-1}$  and  $z_{t-1}$  sum to one is rejected at the 1.56 percent level according to the  $F(1,237)$  statistic which turns out to be 5.932. Corresponding to (15) is

$$(y_t - z_{t-1}) = .0449 + .9789 (y_{t-1} - z_{t-1}) \quad R^2 = .9692 \quad (32)$$

$$(.0174) (.0113) \quad s = .1818$$

$$DW = 2.031$$

Thus, under the restriction that the coefficients of  $y_{t-1}$  and  $z_{t-1}$  in (31) sum to one, (32) suggests a significantly negative risk premium and a discount factor  $\delta$  significantly above one. Hence the model (2b) under RE is rejected as before.

#### IV. Stock Price Model in Logarithmic Form

In this section, I estimate and test hypothesis (2a) for stock price in logarithmic form under both RE and AE, and comment on the relation between the AE model and the recent literature on mean reversion of stock price.

Under RE, the hypothesis  $E_t(y_{t+1} + z_t) = \delta^{-1}y_t$  can be tested by assuming  $(y_{t+1} + z_t)$  to equal  $\delta^{-1}y_t$  plus a residual with conditional mean zero, or times a residual  $\eta_{t+1}$  with conditional mean unity. The former assumption was adopted in sections I and II. We now explore the latter assumption. By taking logarithm of the model with a multiplicative residual  $\eta_{t+1}$ , one obtains

$$\ln(y_{t+1} + z_t) - \ln y_t = \ln \delta^{-1} + \ln \eta_{t+1} = \ln \delta^{-1} + c + u_{t+1} \quad (33)$$

where  $E_t \eta_{t+1} = 1$ ,  $E_t u_{t+1} = 0$ ,  $E_t \ln \eta_{t+1} = c$ , and  $u_t$  is assumed to be serially independent and identically distributed with finite variance. Denoting stock price  $y_t$  at the beginning of period  $t$  by  $P_t$  and dividend  $z_t$  during period  $t$  by  $D_t$ , and denoting  $\ln P_t$  and  $\ln D_t$  by  $p_t$  and  $d_t$  respectively, one can approximate

$\ln(P_{t+1} + D_t)$ , as in Campbell and Shiller (1988), by a linear function of  $p_{t+1}$  and  $d_t$  assuming the variable  $\delta_{t+1} \equiv d_t - p_{t+1}$  is near its mean  $\delta$ :

$$\begin{aligned}
 \ln(P_{t+1} + D_t) &= \ln(\exp(p_{t+1}) + \exp(d_t)) = \ln(\exp(p_{t+1})(1 + \exp(\delta_{t+1}))) \\
 &= p_{t+1} + \ln(1 + \exp(\delta_{t+1})) \\
 &\approx p_{t+1} + \ln(1 + \exp(\delta)) + (1 + \exp(\delta))^{-1} \exp(\delta) (\delta_{t+1} - \delta) \\
 &= \rho p_{t+1} + (1 - \rho) d_t + k
 \end{aligned} \tag{34}$$

where  $\rho = 1/(1 + \exp(\delta))$  and  $k = \ln(1 + \exp(\delta)) - \delta \exp(\delta)/(1 + \exp(\delta))$ . By this approximation, equation (33) can be approximated by

$$\rho p_{t+1} + (1 - \rho) d_t + k - p_t = \ln \delta^{-1} + c + u_{t+1} \tag{35}$$

which corresponds to equation (5), and can therefore be estimated and tested by equations corresponding to (6) and (7).

Equation (35) implies

$$p_{t+1} = \rho^{-1} p_t - \rho^{-1} (1 - \rho) d_t + \rho^{-1} (\ln \delta^{-1} + c - k) + \rho^{-1} u_{t+1} \tag{36}$$

which corresponds to (7). Since  $d_t$  is correlated with  $u_{t+1}$ , we apply IV to estimate (36). Using an estimated regression of  $d_t$  on  $d_{t-1}, \dots, d_{t-3}, p_t, \dots, p_{t-3}$  as instrument for  $d_t$  (if added,  $d_{t-4}$  and  $p_{t-4}$  having  $t$  ratios of .57 and .20), I have estimated, for the period 1875 to 1986,

$$\begin{aligned}
 p_{t+1} &= .929 p_t + .027 d_t + .246 \\
 &\quad (.092) \quad (.126) \quad (.269)
 \end{aligned} \tag{37}$$

For this sample  $\rho$  is found to be .936, as in Campbell and Shiller (1988). The  $F(2, 109)$  statistic for testing the hypothesis that the two coefficients of (37) are respectively  $\rho^{-1} = 1.0684$  and  $-\rho^{-1}(1 - \rho) = .0684$  is 3.441, significant at the 3.55 percent level.

To examine how well the hypothesis of AE can explain the data, we assume  $E_t \ln(P_{t+1} + D_t)$  to be formed adaptively as in (16):

$$E_t \ln(P_{t+1} + D_t) - E_{t-1} \ln(P_t + D_{t-1}) = \beta [\ln(P_t + D_{t-1}) - E_{t-1} \ln(P_t + D_{t-1})] + \varepsilon_t \quad (38)$$

By hypothesis,  $E_t \ln(P_{t+1} + D_t) = \ln P_{t+1} + (\ln \delta^{-1} + c) = p_{t+h}$ . On substituting  $p_{t+h}$  for  $E_t \ln(P_{t+1} + D_t)$ , and the linear approximation (34) for  $\ln(P_t + D_{t-1})$ , (38) becomes

$$p_t - p_{t-1} = \beta [\rho p_t + (1-\rho) d_{t-1} + k - p_{t-1} - h] + \varepsilon_t$$

$$p_t = \frac{1-\beta}{1-\beta\rho} p_{t-1} + \frac{\beta(1-\rho)}{1-\beta\rho} d_{t-1} + \frac{\beta(k-h)}{1-\beta\rho} + \frac{\varepsilon_t}{1-\beta\rho} \quad (39)$$

Estimation of (39) by least squares for the period 1875 to 1986 yields

$$p_t = .877 p_{t-1} + .103 d_{t-1} + .396 \quad R^2 = .9066 \quad (40)$$

(.077)
(.104)
(.229)
s = .1779
DW = 1.776

implying  $\rho = .927$  and  $\beta = .618$ , which are reasonable results. Furthermore, the two coefficients of (40) sum to one, as the AE theory implies.

Equation (40), as equation (19), is consistent with AE but inconsistent with RE. If we assume the AE model (39) to be correct, and take expectation of (40) conditional on  $H_{t-1}$ , we find

$$E_t p_t = .877 p_{t-1} + .103 E_t d_{t-1} + .396 \quad (41)$$

Equation (41), just like (37), is inconsistent with RE given by (36), for the two coefficients should be  $\rho^{-1} = 1.0684$  and  $-\rho^{-1}(1-\rho) = -.0684$  respectively. The fact that (40) estimated by LS is very similar to (37) estimated by IV for  $d_{t-1}$  supports the assumption of (38) and (39) that  $E_{t-1} \varepsilon_t = 0$ . Hence the evidence confirms AE while it rejects RE.

Furthermore, equation (39) or (40) is capable of explaining the negative relation between the log of one-period return  $\ln((P_t + D_{t-1})/P_{t-1})$  and log price

$p_{t-1}$  found in recent studies including Fama and French (1988), Poterba and Summers (1988), and Campbell and Shiller (1988). Multiplying (39) by  $\rho$  and adding  $(1-\rho)d_{t-1}-p_{t-1}+k$  to both sides of the resulting equation, we obtain an equation for log one-period return

$$\rho p_t + (1-\rho)d_{t-1} + k - p_{t-1} = \frac{1-\rho}{1-\beta\rho} p_{t-1} + \frac{1-\rho}{1-\beta\rho} d_{t-1} + \frac{k-\beta\rho h}{1-\beta\rho} + \frac{\rho\varepsilon_t}{1-\beta\rho} \quad (42)$$

Hence, the log of one-period return is related negatively to  $p_{t-1}$  and positively to  $d_{t-1}$ , with coefficients of the same absolute value. Assuming (42) to be the true model and taking expectation conditional on  $H_{t-1}$ , we derive

$$E_{t-1}(\rho p_t + (1-\rho)d_{t-1} + k - p_{t-1}) = \frac{1-\rho}{1-\beta\rho} p_{t-1} + \frac{1-\rho}{1-\beta\rho} E_{t-1}d_{t-1} + \frac{k-\beta\rho h}{1-\beta\rho} \quad (43)$$

(43) is inconsistent with hypothesis (2a) or (35) under RE, for the latter hypothesis implies zero coefficients for both  $p_{t-1}$  and  $E_{t-1}d_{t-1}$ . Estimating equation (42) by least squares for the period 1875 to 1986, with the dependent variable denoted by  $\xi_{1,t-1}$  as in Campbell and Shiller (1988), yields

$$\xi_{1,t-1} = \begin{matrix} -.179 & + & .161 & + & .608 \\ (.072) & & (.097) & & (.214) \end{matrix} p_{t-1} \quad d_{t-1} \quad (44)$$

The result supports the AE hypothesis, with the two coefficients having the correct signs and almost equal in absolute value. Replacing the two regressors by the difference  $d_{t-1}-p_{t-1}$  yields a coefficient .193 with a standard error of .065, implying  $\beta = .672$  if  $\rho = .927$  as estimated by (40). Estimating (43) by LS with the regressor  $E_{t-1}d_{t-1}$  replaced by  $\hat{d}_{t-1}$ , the instrumental variable used in (37), for the same period 1875 to 1986 gives

$$\xi_{1,t-1} = \begin{matrix} -.130 & + & .090 & + & .468 \\ (.086) & & (.119) & & (.254) \end{matrix} p_{t-1} \quad E_{t-1}d_{t-1} \quad (45)$$

(45) is not very different from (44) and supports the AE assumption of  $E_{t-1}\epsilon_t = 0$  in (42) while it rejects the RE assumption of two zero coefficients, the  $F(2,109)$  statistic being 3.391, significant at the 3.73 percent level.

It is of interest to note that the model (43) is consistent with, and provides an explanation for, the regression equation for  $\xi_{1,t-1}$  reported in Campbell and Shiller (1988, Table 3):

$$\xi_{1,t-1} = \begin{matrix} .008 & \delta_{t-1} & + & .137 & \Delta d_{t-2} & + & .126 & \epsilon_{t-1}^{30} & + & \text{constant} & & R^2 = .086 \\ (.125) & & & (.155) & & & (.085) & & & & & \end{matrix}$$

where  $\epsilon_{t-1}^{30} = ((e_{t-2} + \dots + e_{t-31})/30) - p_{t-1}$ ,  $e_t$  being real earnings. In this regression, the main explanatory variable is  $\epsilon_{t-1}^{30}$ , with coefficient .126. If we interpret the 30-year average of earnings as a good estimate of expected dividend  $E_{t-1}d_{t-1}$ , equation (43) or (45) provides an explanation of the above regression. It is also of interest to compare hypothesis (43) with the hypotheses of Fama and French (1988), and Poterba and Summers (1988) which also imply a negative effect of  $p_{t-1}$  on log one-period return, but this topic is beyond the scope of the present paper.

#### V. Conclusion

Using data on stock price and dividends, and on long and short-term interest rates, I have tested hypothesis (2) on present value, which is an important implication of the present value model (1). Combined with RE, hypothesis (2) is strongly rejected. Combined with AE, it is accepted. The AE formulation, if accepted, implies a model which is inconsistent with the RE formulation. The data supports the former while they reject the latter. The former model is also capable of explaining the observed negative relation between the rate of return and stock price, but how well it does, as compared with alternative models which maintain rational expectations, remains to be studied. The acceptance of hypothesis (2) under AE does not imply the acceptance of the present-value model

(1). Hence the finding that stock price is too volatile to be consistent with (1) under RE does not affect the validity of hypothesis (2) under AE. How the latter hypothesis explains the volatility of stock price is another topic deserving further study.

This paper has illustrated a well-known fact, that incorrectly imposing the assumption of rational expectations on an otherwise correct model can lead to unreasonable estimates of important parameters. Assuming hypothesis (2) under adaptive expectations to be correct, we have seen that taking conditional expectations of equations (19), (25) and (39) would lead to models which, if a rational expectations assumption is forced on them, will yield unreasonable parameter estimates. Hence the assumption of rational expectations should be used with caution. This paper also suggests that the assumption of adaptive expectations can sometimes be a useful working hypothesis in econometric practice.

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