Credibility and Commitment: Some New Methods for the Design and Evaluation of Policy in Continuous Time Rational Expectations Models

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Econometric Research Program Research Memorandum No. 332

October 1987

* Graduate School of Business Administration, New York University. This paper is the third essay of my Princeton Ph.D. dissertation, Asset Markets and Macroeconomic Policy. I am very grateful to Alan Blinder, John Campbell, Gregory Chow, and Peter Hartley for their comments and suggestions. All remaining errors are my own responsibility.

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Abstract

The aim of the paper is to extend the techniques used for policy evaluation and design in continuous time rational expectations models. Following the recent literature, the optimal policy is designed in the context of a dynamic game, played between the government and the public. Using this we address the problem of time inconsistency that arises in the absence of a commitment by the government to follow the originally announced policy.

Employing the concept of incentive compatibilty it is shown that the government's commitment to its original policy can be determined endogenously. This allows a set of partially credible policies to be identified from which the optimal policy can be chosen.

The technique is applied to a simple inflation model to illustrate the how the range of policies available to the government are restricted by the requirement of incentive compatibility. It also allows the optimal policy to be examined in some detail.

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I Introduction

In the past decade the conventional theory of economic policy has undergone considerable revision. Following the work of Kydland and Prescott (1977) and Calvo (1978), the "Rational Expectations Revolution" has prompted renewed interest in the problem of time inconsistency. Kydland and Prescott argued that when current behavior in the private sector is partially determined by forward-looking expectations, the government may find it optimal to depart from pre-announced plans at some future date. A number of authors [Chow (1981), Lucas and Sargent (1981), Buiter (1983), and Miller and Salmon (1984), for example] have argued that this ability places the government in a position analogous to a Stackelberg leader in a dynamic game. Here the problem of time inconsistency arises because there is an incentive for the government to renege on the originally announced policy.

The problem of choosing an optimal policy is considerably complicated by the presence of time inconsistency. Once the public recognizes the government's incentive to renege on its originally announced policy, the announcement itself loses credibility. As a result, the economy may diverge form the path originally predicted, so that the government is left following an inferior policy. To avoid this possibility, the government must look for the optimal policy within the set of "credible policies". Unfortunately this poses a complex problem. A number of authors [Buiter (1983), Miller and Salmon (1984), Cohen and Michel (1984) and Currie and Levine (1985)] have pointed out that the credibility of the policy can be established if the government decides to eschew its ability to influence the economy via policy announcements altogether. Essentially this obviates the incentive to renege, thereby making the policy time consistent. 2 It is unclear however, why such policies should be chosen. If the government can convince the public that it will adhere to its original plan, a credible policy with this feature will always be preferred. Thus the choice of policy is influenced by the government's ability to convince the public that it will adhere to its originally announced plan. In short, the choice of policy depends on the government's reputation.

A number of models have recently been developed addressing this issue [see Backus and Driffill (1984,1985) and Barro (1985)]. These analyze situations in which the private sector has incomplete information about the government's preferences. They show that there exists an incentive for the government to misrepresent its preferences in an attempt to convince the

public that the optimal policy is time consistent. This is interpreted as "building a reputation".

Although the timing of a policy change is determined endogenously in these models, relatively little attention is paid to the nature of the chosen policies. In general, one would expect the form of the new policy to be influenced by the state of the economy when the government reneges. For example, if the government reneges on an announced anti-inflationary policy by expanding monetary growth, the extent of the expansion will be influenced by the state of say, unemployment and inflation. However, as the current state of inflation and unemployment in part reflect the original policy, the degree of monetary expansion will depend on the old monetary policy. In this way the policies adopted under a time inconsistent regime are interdependent.

These observations suggest the following questions: (a) Under what conditions will the government adopt a time inconsistent policy? (b) If the government chooses a time inconsistent regime, how will policy be designed in view of the interdependence between the originally announced policy and the one chosen when it reneges?

The aim of this paper is to answer, at least partially, both of these questions. The analysis begins by examining the question of policy design in a time inconsistent regime. I shall consider a situation in which the public expects the government to renege on its announced policy at a hypothetical date. Given the structure of the economy and its policy objectives, the government can find the optimal set of policies (one that is initially followed and one that is adopted when the government reneges) conditioned on this expected period of commitment (i.e. from the present to that date). Next, I check whether the public's arbitrarily held expectations are consistent with the proposed set of policies. This is done by examining whether the government would adhere to the original policy for the period expected by the public. If this is the case, the set of optimal policies conditioned on this period of commitment are termed incentive compatible. The set of incentive compatible policies is identified by repeating this process for a range of hypothetical dates. Since this set includes both time consistent and inconsistent policies, the technique allows us to examine the design of a far wider range of policies than are usually considered.

The analysis must be taken a stage further in order to examine the conditions under which a time inconsistent regime is established. In the view of the remarks above, it should not be surprising to find that time

consistent policies are incentive compatible. In fact, an incentive compatible policy with a zero period of commitment is time consistent. Thus finding the "optimal credible policy" by optimizing among the set of incentive compatible policies implicitly requires the government to decide whether or not to adopt a time inconsistent policy. In general the government's choice will depend on both the characteristics of the alternative policies and its ability to convince the public of their incentive compatibility. Unfortunately this turns out to be an extremely complex problem. In this paper a partial solution is proposed with the aim of highlighting the issues that need to be addressed in future research.

Although the techniques presented can be applied to a wide variety of models, I shall illustrate their use in the context of the familiar open economy version of the IS/LM framework. Here the differences between the "optimal credible" and other anti-inflation policies are illustrated. Although this model was chosen primarily to provide an economic interpretation for the analysis, some of the results are of intrinsic interest. In particular, the government's concern for the behavior of the exchange rate is found to be crucial in establishing the credibility of monetary policy.

The remainder of the paper is divided into four sections. In the first I introduce the specific example and show why, in general, the optimal policy is time inconsistent. The analysis in section III begins by examining how changes in the expected length of the commitment affect the behavior of the economy. Next, I show how a proposed policy can be checked for incentive compatibility before illustrating how to find the "optimal credible policy". In section IV, these techniques are applied to the specific example. The paper ends with a summary of the results and some comments on the direction of future research.

II:The Model

Throughout the paper I shall refer to the well-known open economy model of Dornbusch (1976,1983) and Buiter and Miller (1981,1982). This continuous time model is comprised of the following equations:

$$m(t) - p(t) = \alpha_1 y(t) - \alpha_2 r(t)$$
 $\alpha_1, \alpha_2 > 0$ (1.1)

$$y(t) = -\beta_1[r(t) - E_tDp(t)] + \beta_2[i(t) - p(t)]$$
 $\beta_1, \beta_2 > 0$ (1.2)

$$Dp(t) = \sigma y(t) + Dm(t) \qquad \qquad \sigma > 0 \qquad (1.3)$$

$$E_{t}Di(t) = r(t)$$
 (1.4)

Here $\rm E_t$ denotes the public's expectation using information available at time t. I assume that both the structure of the model and the path of the exogenous variable Dm(t), are included in this information set. This makes (1.1)-(1.4) a perfect foresight model. 4 y(t), p(t), m(t) and i(t) are respectively the logs of domestic output, the price level, the nominal money stock and the nominal exchange rate. r(t) is the domestic nominal interest rate. Following Buiter and Miller (1981,1982) the effect of the exchange rate on domestic prices is ignored so that the LM equation (1.1) can be written in its familiar form. Equation (1.2) describes equilibrium in the goods market. Here output is demand determined and depends upon the real interest and exchange rates. In (1.3) inflation Dp(t), is generated by a Phillips curve augmented by monetary growth Dm(t). The dynamics of the exchange rate are governed by the uncovered interest parity condition (1.4) in which the foreign nominal interest rate is normalized to zero.

The dynamics of the economy can be summarized by considering the behavior of real balances $l(t) \equiv m(t)-p(t)$, and the real exchange rate $e(t) \equiv i(t)-p(t)$. [I will also refer to this as the level of competitiveness.] Substituting for output in (1.1) and (1.3) using (1.2), and for the nominal interest rate using (1.4), yields;

$$\begin{bmatrix} DI(t) \\ E_t De(t) \end{bmatrix} = 1/d \begin{bmatrix} \alpha \beta_1 & \alpha \beta_2 \alpha_2 \\ 1 & \beta_2 (\alpha \alpha_2 - \alpha_1) \end{bmatrix} + 1/d \begin{bmatrix} \alpha \alpha_2 \beta_1 \\ \alpha_2 \end{bmatrix} Dm(t).$$

$$d \equiv \beta_1 (\alpha \alpha_2 - \alpha_1) - \alpha_2$$
(1.5)

Before analyzing the dynamics of this system, it is important to distinguish between the predetermined and the non-predetermined variables. At any point in time the money stock m(t), and the price level p(t), are predetermined by the past history of the economy. By contrast, the exchange rate is free to jump at any instant in response to news concerning previously unanticipated changes in current and future exogenous variables. As such, the real exchange rate e(t) is a non-predetermined variable.

It can be shown [see Buiter (1982)] that a unique saddle-path converging to the long run equilibrium will exist provided there is one stable and one unstable characteristic root to the system. I assume that the sufficient condition for this holds, so that the real exchange rate always attains a level that places the economy on the saddle-path. Formally this requires

$$\lim_{s\to\infty} \exp(-bs) E_t e(s) = 0 \qquad \text{all } b > 0, t \ge 0, \tag{1.6a}$$

which rules out the possibility of the economy being placed on an unstable trajectory. The dynamics of the economy are further restricted by the initial value of the predetermined variable,

$$1(0) = 1$$
 (1.6b)

(1.6a), (1.6b) and the saddle-path structure of (1.5) ensure that the economy will follow a unique trajectory given the path of monetary growth Dm(t).

The sole policy instrument available to the government is the growth of the money stock Dm(t). This is chosen to minimize the present value of a quadratic cost function C(t) of the state variables over an infinite horizon;

$$V(0) \quad 1/2 \int_{0}^{\infty} \exp[-\delta t]C(t) dt$$
(1.7)

Policy is designed by treating the system of equations in (1.5) as the reaction function of the "follower" in a Stackelberg game where the government takes the position of the "leader". Thus the optimal path for Dm(t) is found by minimizing (1.7) subject to (1.5). Using Pontryagin's maximum principle, the behavior of the economy under the optimum policy can be represented by an expanded set of differential equations, known as the Adjoint system [see Miller and Salmon (1984)]:

$$\begin{bmatrix} Dl(t) \\ E_tDe(t) \\ E_tD\mu_L(t) \\ D\mu_e(t) \end{bmatrix} = A \begin{bmatrix} l(t) \\ e(t) \\ \mu_L(t) \\ \mu_e(t) \end{bmatrix}$$
(1.8)

Here the matrix A is a function of the parameters of (1.5) and the cost function C(t). $\mu_{\rm l}(t)$ and $\mu_{\rm e}(t)$ are the current value co-state variables corresponding to l(t) and e(t). These indicate the sensitivity of the optimal value of the cost function V(t) to changes in the state variables l(t) and e(t) discounted back to the present. 10

In addition to the boundary conditions (1.6a) and (1.6b), the choice of an optimal policy requires the imposition of two transversality conditions.

$$\lim_{s\to\infty} \exp(-bs) \ \mathsf{E}_{\mathsf{t}} \ \mu_{\mathsf{l}}(s) \, \mathsf{l}(s) = 0 \qquad \text{all } b > 0, \quad \mathsf{t} \geq 0 \tag{1.9a}$$

$$\mu_{a}(0) = 0.$$
 (1.9b)

While (1.9a) is a familiar sufficiency condition for optimality [see Buiter (1984) and Miller and Salmon (1984)], (1.9b) plays a crucial role in the analysis. The non-predetermined variable e(t) is free to "jump" in response to the policy announced at t=0. Thus if policy is set optimally, the marginal contribution of e(0) to cost V(0) should be zero. This is implied by condition (1.9b) [see Calvo (1978)].

The source of time inconsistency in the optimal policy can now be readily identified. If the government were to re-optimize at some later date [t>0], this would entail re-setting policy so that $\mu_{\bullet}(t)=0$. In general, because $\mu_{\bullet}(t)$ would be non-zero under the original policy, re-optimization will therefore result in the falsification of private sector expectations. Since this will be foreseen by intelligent agents endowed with perfect foresight, the original policy will lack credibility.

On way to avoid this problem is to restrict the analysis to those policies that ensure $\mu_{\bullet}(t)=0$ throughout their course. By ensuring that $\mu_{\bullet}(t)=0$ for the duration of the policy, the government essentially eschews the ability to influence the behavior of the private sector through the design of future policy. Consequently, only the current and past actions of the government have any bearing on the behavior of the economy. While such policies may be optimal under extreme circumstances [see below], there is little reason, a priori, to believe that no others can be sustained. What is required is a mechanism to commit the government to its original policy despite the fact that $\mu_{\bullet}(t)$ is non-zero.

III:Identifying the Set of Incentive Compatible Policies and the "Optimalb Credible Policy"; General Technique

In this section, the techniques used to identify the set of incentive compatible policies and the "optimal credible policy" are described. The analysis proceeds in three stages. I begin in subsection (A) by considering how the economy will respond to an announced policy if the government reneges at an arbitrary hypothetical date that coincides with the public's expectations. In subsection (B) I check whether the public's expectations are consistent with the proposed set of policies by examining whether an incentive exist for the government renege before or after the hypothetical date. This allows the incentive compatibility of the set of policies to be established. As a final step, in subsection (C) I describe how the "optimal credible policy" is chosen from the set of incentive compatible policies.

III:A

Suppose the public expects the government to be committed to follow its announced policy over the interval $[0,h^{\bullet})$. After this period the government is expected to renege at every opportunity on any subsequently adopted policies i.e. policy announcements made from $t=h^{\bullet}$ onwards have no credibility. On the assumption that the government recognizes the existence of these beliefs, how will it formulate policy?

Consider the policy choice at t = h $^{\circ}$. The government can either continue with the original policy, or renege. If it reneges, the newly announced policy will have no credibility. Faced with a complete loss of credibility, the government should expect the behavior of the economy to diverge from the path predicted under the propose policy unless $\mu_{\bullet}(t) = 0$ over the interval $[h^{\circ} \ \omega)$. As a result, it is forced to adopt a time consistent policy. This means that the first order condition in (1.8) governing the evolution of $\mu_{\bullet}(t)$ becomes irrelevant in determining the dynamics of economy. Instead $D\mu_{\bullet}(t) = 0$ for $t \ge h^{\circ}$ where $\mu_{\bullet}(h^{\circ}) = 0$. By imposing this restriction on the set of differential equations in (1.8), the anticipated behavior of the economy for $t \ge h^{\circ}$ is described by the "reduced" adjoint system,

$$\begin{bmatrix} D1(t) \\ E_t De(t) \\ E_t D\mu_L(t) \end{bmatrix} = A \begin{bmatrix} 1(t) \\ e(t) \\ \mu_L(t) \end{bmatrix}$$

$$(2.1)$$

with boundary conditions;

$$l(h^{\bullet}) = l_{h} \tag{2.2a}$$

$$\lim_{s\to\infty} \exp(-bs) E_t e(s) = 0 \qquad \text{all } b > 0 \qquad s \ge h^{\bullet} \qquad (2.2b)$$

$$\lim_{s\to\infty} \exp(-bs) E_t \mu_l(s) I(s) = 0 \qquad \text{all } b > 0 \qquad s \ge h^{\bullet}. \tag{2.2c}$$

Condition (2.2a) reflects the fact that $l(h^{\circ})$ must be treated as a predetermined variable with respect to policies pursued over $t \geq h^{\circ}$. As such it represents the legacy of the previously adopted policy. Equations (2.1) and (2.2) represent a two point boundary problem, which, under the assumed saddle-path structure of the model, has a unique solution. This has the general form [see, for example, Miller and Salmon (1984)],

$$\begin{bmatrix} l(t) \\ e(t) \\ \mu_{l}(t) \end{bmatrix} = \begin{bmatrix} \alpha \\ A \\ a \times a \end{bmatrix} \begin{bmatrix} \pi_{11} \\ \pi_{21} \\ \pi_{34} \end{bmatrix}$$

where

and $\phi_{\rm s}$, $\phi_{\rm ul}$ and $\phi_{\rm u2}$ are respectively the stable and unstable characteristic roots of $\tilde{\rm A}$. This system of equations describe the anticipated behavior of the economy following a loss of credibility conditioned on the level of real balances $l(h^{\rm e})$. Since there is clearly no opportunity to renege here, I shall refer to the policy accompanying these dynamics as strictly time consistent.

How does the government calculate the public's response to the policy announced at t=0? During the (hypothetical) expected period of commitment $[0,h^{\bullet})$, the government retains its ability to influence the behavior of the private sector through the design of future policy. Thus the exchange rate e(t) is not treated as a predetermined variable when policy is formulated. As before, the optimal policy is found by minimizing (1.7) subject to (1.5) using Pontryagin's maximum principle. Hence the behavior

of the economy depends on the full adjoint system.

$$\begin{bmatrix} DI(t) \\ E_tDe(t) \\ E_tD\mu_1(t) \\ D\mu_2(t) \end{bmatrix} = A \begin{bmatrix} I(t) \\ e(t) \\ \mu_1(t) \\ \mu_2(t) \end{bmatrix}$$
(1.8)

As before, the dynamics of the economy are restricted by the initial level of real balances

$$1(0) = 1$$
 , $(1.6b)$

and the necessary condition for an initially optimal choice of policy

$$\mu_{a}(0) = 0.$$
 (1.9b)

The final two boundary conditions depend on how the public responds to the announced policy. Since h^{\bullet} is assumed to be finite, the usual boundary conditions on e(t) (1.6a), and $\mu_{l}(t)$ (1.9a), no longer have any direct bearing on the behavior of the economy over $[0,h^{\bullet})$. Rather, the relevant boundary conditions should reflect the absence of anticipated "jumps" in the exchange rate accompanying the switch in policy. If this were not the case, the anticipated response to the policy announcement implies that a profitable trading opportunity in the currency market remains unexploited. Since this is inconsistent with private sector behavior, the possibility of such "jumps" in the exchange rate must be excluded when calculating the public's anticipated response to the policy announcement. The relevant condition for the exchange rate is therefore found by setting $t = h^{\bullet}$ in (2.3) and taking expectations. This yields;

$$E_e(h^e) = \pi_{12}[\pi_{11}]^{-1}E_h \qquad 0 \le s \le h^e \qquad (2.4a)$$

Once anticipated "jumps" in the non-predetermined variable are ruled out, $\mu_{\rm L}({\rm t})$ must become a continuous function of time. As a result, the final boundary condition is obtained from (2.3) in an identical manner

$$E_{\mathfrak{sl}}(h^{e}) = \pi_{\mathfrak{sl}}[\pi_{\mathfrak{sl}}]^{-1} E_{\mathfrak{sl}} \quad 0 \leq s \leq he.$$
 (2.4b)

Hence, for an anticipated period of commitment $[0,h^{\bullet})$, the relevant boundary conditions governing the behavior of the economy are (1.6a), (1.9b), (2.4a) and (2.4b).

The impact of these conditions on e(0) and fl(0) can be calculated by solving (1.8). Using the method found in Miller and Salmon (1984) the solution has the following general form,

$$\begin{bmatrix} l(t) \\ e(t) \\ \mu_{l}(t) \\ \mu_{e}(t) \end{bmatrix} = \Gamma \begin{bmatrix} \exp[z\,t] & 0 & 0 & 0 \\ 0 & \exp[z_{2}t] & 0 & 0 \\ 0 & 0 & \exp[z_{3}t] & 0 \\ 0 & 0 & \exp[z_{4}t] \end{bmatrix} \Gamma^{-1} \begin{bmatrix} l(0) \\ e(0) \\ \mu_{l}(0) \\ \mu_{e}(0) \end{bmatrix}^{-1}$$
(2.5)

Here $\Phi(t)$ is the state transition matrix, z_i are the characteristic roots of A and Γ is the matrix of their associated vectors. Setting $t=h^{\bullet}$, taking expectations at t=0 and substituting the boundary values for l(0), $\mu_{\bullet}(0)$, $e(h^{\bullet})$ and $\mu_{l}(h^{\bullet})$ using (1.6a), (1.9b), (2.4a) and (2.4b), yields;

$$\begin{bmatrix} E_{0} | (h^{e}) \\ \pi_{2i} [\pi_{ij}]^{-1} E_{0} | (h^{e}) \\ \pi_{gi} [\pi_{ij}]^{-1} E_{0} | (h^{e}) \\ \mu_{e} (h^{e}) \end{bmatrix} = \Phi(h^{e}) \begin{bmatrix} \frac{1}{e(0)} \\ e(0) \\ \mu_{i} (0) \\ 0 \end{bmatrix}$$

or, after some manipulation,

$$\begin{bmatrix} 1 & -\Phi_{12} & -\Phi_{13} & 0 \\ \pi_{21} [\pi_{11}]^{-1} & -\Phi_{22} & -\Phi_{23} & 0 \\ \pi_{31} [\pi_{11}]^{-1} & -\Phi_{32} & -\Phi_{33} & 0 \\ 0 & -\Phi_{42} & -\Phi_{43} & 1 \end{bmatrix} \begin{bmatrix} E_{0}! (h^{\bullet}) \\ e(0) \\ \mu_{1} (0) \\ \mu_{1} (h^{\bullet}) \end{bmatrix} = \begin{bmatrix} \Phi_{11} \\ \Phi_{21} \\ \Phi_{31} \\ \Phi_{41} \end{bmatrix}$$
(2.6)

The public's response to the announced policy and its content can be calculated by solving these equations for e(0) and $\mu_{l}(0)$. The former is simply given by the solution for e(0). The latter is found by substituting

both e(0) and $\mu_l(0)$ in (2.5) to calculate the trajectories of e(t) and l(t) over $[0,h^e]$. This shows how the economy behaves under the policy. The exact behavior of the money supply can then be found using (1.5).

The procedure described above has several noteworthy features: First, the government takes account of the anticipated loss of credibility when designing the new policy. Equation (2.3) shows that this has precise implications for the relation between real balances and the exchange rate. Second, the anticipated continuity of the exchange rate, required by the perfect foresight equilibria in the currency market, implies that the expected consequences of the switch in policy influence current behavior. In particular, over the interval $[0,h^{\bullet})$ the economy will be driven by both the stable and unstable roots of the matrix A. It is here that the influence of the public's beliefs will be most clearly manifest. Third, the government initially adopts an optimal policy. Notice however, that the characteristics of this policy are affected by the public's beliefs, insofar as they influence $\mu_{\downarrow}(0)$. Thus beliefs about the government's commitment to a policy affect its content. Finally, all these implications are reflected in the initial level of the exchange rate, e(0).

III:B

So far I have shown how optimal policies can be identified when the government's commitment to the original policy coincides with the arbitrary expectations of the public. Now, in order to show that public's beliefs are consistent with the government's equilibrium strategy, we must examine whether an incentive exists for the government to renege before or after $t = h^{\bullet}$.

First, consider whether the government would continue with the original policy beyond $t = h^{\bullet}$. Conceptually, it is useful to imagine that the government re-optimizes at $t = h^{\bullet}$ among a set of policies that includes the continuation of the original policy. The question is, would the government always choose the strictly time consistent policy from within this set? Posed in this form the answer is clear. Given the legacy of the original policy l_h [see (2.2a)] and the beliefs of the public, the strictly time consistent policy must be the optimal choice. Thus, the government will never continue with the original policy for longer than [0 he).

Next, consider whether the government would renege on the original policy before t = h^{*}? The answer will, in part, depend upon the anticipated reaction of the public. So far I have assumed that the government

anticipates loosing all credibility at $t = h^{\Theta}$. Thus it seems natural for the government to anticipate a similar reaction if it reneges prior to $t = h^{\Theta}$.

Consider the anticipated future costs facing the government under the alternative policy choices available at t = τ , given the expected duration of their commitment $[0,h^{\bullet})$, where $0 \le \tau \le h^{\bullet}$. Let $C_1(t)$ and $C_2(t)$ be the instantaneous cost associated with the first and second policies adopted. If the government continues with the original policy, the anticipated present value of future costs are,

$$V_{1}(\tau; h^{\bullet}) = (1/2) \int_{\tau}^{(h^{\bullet})^{-}} C_{1}(t) \exp[-\delta(t-\tau)] dt$$

$$(1/2) \int_{h^{\bullet}}^{\infty} C_{2}(t) \exp[-\delta(t-\tau)] dt$$
(2.7)

where $(h^e)^- \equiv \sup [0,h^e)$. If the government considers reneging at τ , prior to the end of the expected period of commitment, it should anticipate incurring the costs associated with a strictly time consistent policy immediately. These are,

$$V_{\mathbf{z}}(\tau; \mathbf{h}^{\mathbf{e}}) = (1/2) \int_{\tau}^{\infty} C_{\mathbf{z}}(t) \exp[-\delta(t-\tau)] dt \qquad (2.8)$$

Hence the government will not anticipate reneging on its original policy before the end of the expected commitment if,

$$\Omega(\tau; h^{\bullet}) \equiv V_2(\tau; h^{\bullet}) - V_1(\tau; h^{\bullet}) > 0, \quad \text{for all } \tau < h^{\bullet}.$$
 (2.9)

Although this condition can be examined by direct calculation of the costs, the complexity of the dynamics make this approach relatively unattractive. Fortunately it is often possible to avoid this problem by exploiting the interpretation of the co-state variables and the general properties of $V_4(t;h^{\bullet})$ and $V_2(t;h^{\bullet})$.

Consider a situation in which the government anticipates that it will be profitable to renege before the end of the expected period of commitment. In particular, suppose that the costs $V_1(\tau;h^{\bullet})$ and $V_2(\tau;h^{\bullet})$ are as shown in

figure I. Here condition (2.9) fails to hold over the interval (h^*,h°) . It should also be noted that $V_1(\tau;h^\circ)$ and $V_2(\tau;h^\circ)$ are drawn as convex functions of τ . This important feature of linear – quadratic control problems can be derived from the properties of the associated Riccati matrix. As a result, $\Omega(\tau;h^\circ)$ is negative for any $\tau\in(h^*,h^\circ)$ allowing an approximation to the anticipated cost differential $\Omega(\tau;h^\circ)$ to be made.

Let τ^* (τ^* < h^{\bullet}) be a point arbitrarily close to the (hypothetical) date at which the public expects the switch. Taking a linear approximation to $V_2(\tau^*;h^{\bullet})$ about $V_4(\tau^*;h^{\bullet})$ yields

$$V_{2}(\tau^{*};h^{e}) \approx V_{1}(\tau^{*};h^{e}) + \frac{\partial V_{1}(\tau^{*};h^{e})}{\partial e(\tau)} \det(\tau^{*}) + \frac{\partial V_{2}(\tau^{*};h^{e})}{\partial l(\tau)} \det(\tau^{*})$$
 or,

$$\Omega(\tau^*; h^{\bullet}) \approx \frac{\partial V_1(\tau^*; h^{\bullet})}{\partial e(\tau)} de(\tau^*) + \frac{\partial V_2(\tau^*; h^{\bullet})}{\partial I(\tau)} dI(\tau^*)$$
 (2.10)

expression can be simplified in two important respects. First, the appropriate co-state variables may be substituted for the partial derivatives. Second, since I(t) is a predetermined variable, instantaneously it will not be affected by the change in policy. Hence in this application $dI(\tau^*) = 0$. However, since the change in policy would take the public by surprise, the real exchange rate should "jump" at τ^* . This is represented by $de(\tau^*) = e_2(\tau^*) - e_1(\tau^*)$ where $e_2(\tau^*)$ and $e_1(\tau^*)$ are the real exchange rates under the new an old polices respectively. Once these simplifications are made (2.10) becomes,

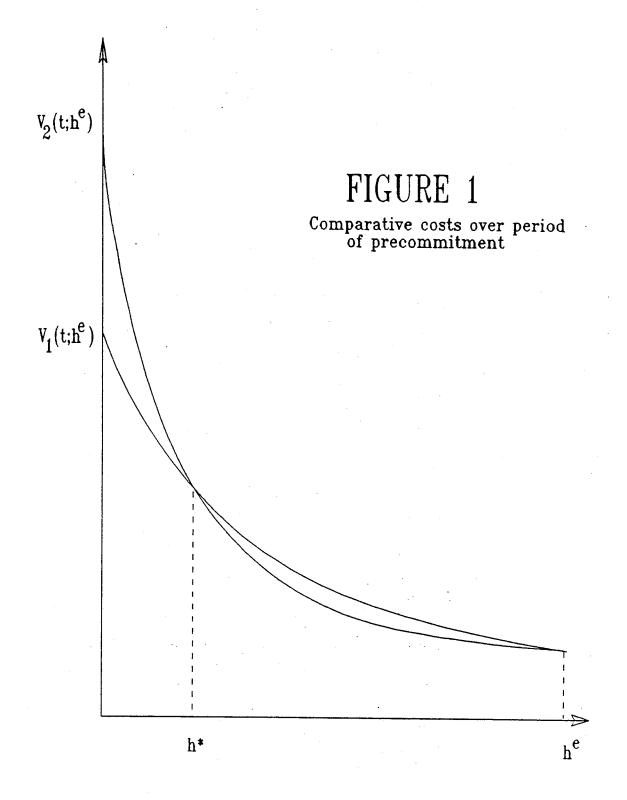
$$\Omega(\tau^*; h^*) \approx \exp[-\delta \tau^*] \mu_{\bullet}(\tau^*) [e_2(\tau^*) - e_1(\tau^*)].$$
 (2.11)

Since $\Omega(\tau^*; h^e)$ is negative for any τ in the (h^*, h^e) , only this approximation needs to be evaluated to check condition (2.9) and thereby establish that the behavior of the government is consistent with the expected period of commitment.

The set of incentive compatible policies can now be identified. To begin with, define the first point at which the government would renege:

h
$$\stackrel{\pm}{=}$$
 sup [Ω(τ^* ;h $^{\bullet}$) > 0 for all g s t.] (2.12) (t:0

The set of incentive compatible commitments can therefore be identified by H



where.

$$H = \{ h^{\circ} : h^{\circ} = h^{+} \}.$$
 (2.13)

For each expected commitment in this set there corresponds an optimal policy, which if announced, would enjoy the same degree of credibility as the government anticipated in its design.

Although a detailed examination of the relation between the degree of credibility and the content of the policy requires both the structure of the economy and the preferences of the government to be specified (see section IV), some general observations can be made here. The first concerns the relation between h^{\dagger} and h^{\bullet} .

Proposition:

If
$$e_1(0) \neq e_2(0)$$
, then $h^+ > 0$.

Proof:

If $e_1(0) \neq e_2(0)$ then $h^{\bullet} > 0$. Furthermore, because the policy adopted under a strictly time consistent policy is always available, $\Omega(0;h^{\bullet}) > 0$. The result follows from the continuity of $\Omega(t;h^{\bullet})$ in t and the definition (2.12).

Thus, provided the government expects the announcement of the policy to have some effect on the initial level of the non-predetermined variable, it will anticipate continuing with its original policy over $[0,h^{\dagger})$ despite the absence of any formal commitment. This contradicts the presumption that the government will continually re-optimize while its policy retains some credibility.

The source of this contradiction is the government's concern for its reputation: Suppose (hypothetically) that the government anticipates being able to retain its credibility over the interval $[h^{\dagger},h^{\bullet})$ after reneging on the original policy. If the cost associated with this policy are $V_3(\tau;h^{\bullet})$, we may identify the additional cost due to the loss of credibility as,

$$\Omega(\tau; h^{\bullet})_{2,3} \equiv V_2(\tau; h^{\bullet}) - V_3(\tau; h^{\bullet}) \geq 0. \qquad (2.14)$$

Against these additional costs, the government must weigh the benefits of

deceiving the public. These are

$$\Omega(\tau; h^e)_{1,3} \equiv V_1(\tau; h^e) - V_3(\tau; h^e) \ge 0.$$
 (2.15)

In the absence of any loss of credibility, $\Omega(\tau;h^e)_{2,3}=0$, so that the government will always find it profitable to renege. On the other hand, when there is no opportunity to deceive the public $\Omega(\tau;h^e)_{1,3}=0$. Here the original policy remains optimal. For $\tau\in[0,h^e)$ however, neither of these conditions hold. Instead the dynamic evolution of these cost and benefits determine whether the government will renege at any point. When the policy is announced (t=0), the cost of a loss of credibility must be as least as great as the benefits from immediately reneging since a policy requiring no credibility can always be chosen. Hence the original policy will be followed over a finite period.

The dynamic evolution of these costs and benefits also affects the characteristics of the set of incentive compatible commitments H. In particular, since $\Omega(t;0)_{1,3}=0$ for all $t\geq 0$, $0\in H$. Of course this simply corresponds to the set of policies requiring no credibility, which by design are incentive compatible. The question of whether other incentive compatible commitments exist is far more substantive. It will depend upon the extent to which the public's beliefs affect the dynamics of the economy over $[0,h^{\circ})$ and thereby the behavior of $\Omega(t;0)_{2,3}$ and $\Omega(t;0)_{1,3}$. These effects will be studied using the open economy model in the next section.

III:C

Having identified the set of incentive compatible policies, we are left to consider how the government will choose among them. In general, policies consistent with longer expected periods of commitment will be less costly to implement. So, if at t = 0 the government is in a position to convince the public that it should be trusted to adhere to any incentive compatible policy, the "optimal credible policy" would be the incentive compatible policy consistent with the longest period of commitment.

Unfortunately, the government's ability in this respect may be limited by its reputation. For example, the government may be unable to convince the public of its commitment to a policy beyond the horizon h^{e^*} because of a reputation for periodically changing its policy objectives. Under these circumstances, the choice of policies available to the government may be severely restricted. In particular, if $h^e \not \subseteq H$ for $0 < h^e \le h^{e^*}$, a time

consistent policy must be adopted at t=0. On the other hand, if $h^{\bullet}\in H$ for some $0< h^{\bullet} \le h^{\bullet*}$, the government will be able to pursue a partially credible policy throughout $[0,h^{\bullet})$ before reneging.

In a general policy model the government's reputation would be derived endogenously. Specifically, the government would be allowed to enhance its reputation through the choice of policy. Here, for the sake of tractability, I ignore this possibility. Rather it is assumed that the government enjoys a pre-specified reputation allowing it to convince the public of its commitment to a policy up to a certain horizon.

Although this approach simplifies the choice of the "optimal credible policy" considerably, the influence of the government's reputation on the choice between time consistent and inconsistent policies can still be examined. In particular, to continuing with the example above, the "optimal credible policy" will have a period of commitment [0,h') where $h' = \sup[H \cap [0,h^{e^*})]$.

IV: An Application; Credible Anti-inflation Policy in an Open Economy

I now apply these techniques to the open economy model presented in section II. The model is assumed to have the following structure:

$$m(t) - p(t) = 0.5y(t) - 0.2r(t)$$
 (3.1)

$$y(t) = -0.2[r(t) - E_t Dp(t)] + 0.5[i(t) - p(t)]$$
 (3.2)

$$Dp(t) = 0.3y(t) + Dm(t)$$
 (3.3)

$$E_{t}Di(t) = r(t)$$
 (3.4)

In order to determine the optimal path for monetary growth Dm(t), the cost function facing the government also needs to be specified:

$$V(0) = 1/2 \int_{0}^{\infty} \exp[-\delta t] [\theta e(t)^{2} + Dp(t)^{2} + y(t)^{2}] dt$$
 (3.5)

Here deviations in inflation, output and the real exchange rate from the irrespective target levels (taken as zero for convenience) are penalized. One unusual feature of this specification is the presence of the real

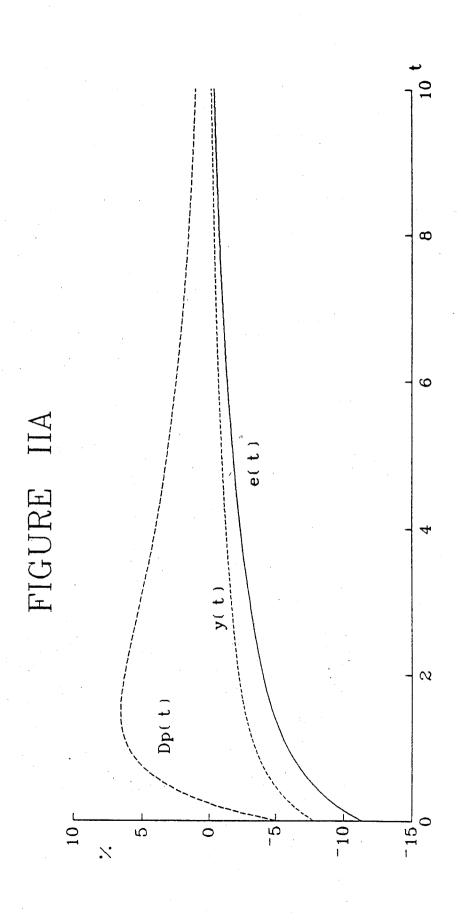
exchange rate e(t), since it implies that the government is concerned with its behavior aside from its influence on inflation and output. The reason for this is based on the credibility of the policy adopted under different values for θ . This is discussed at length below.

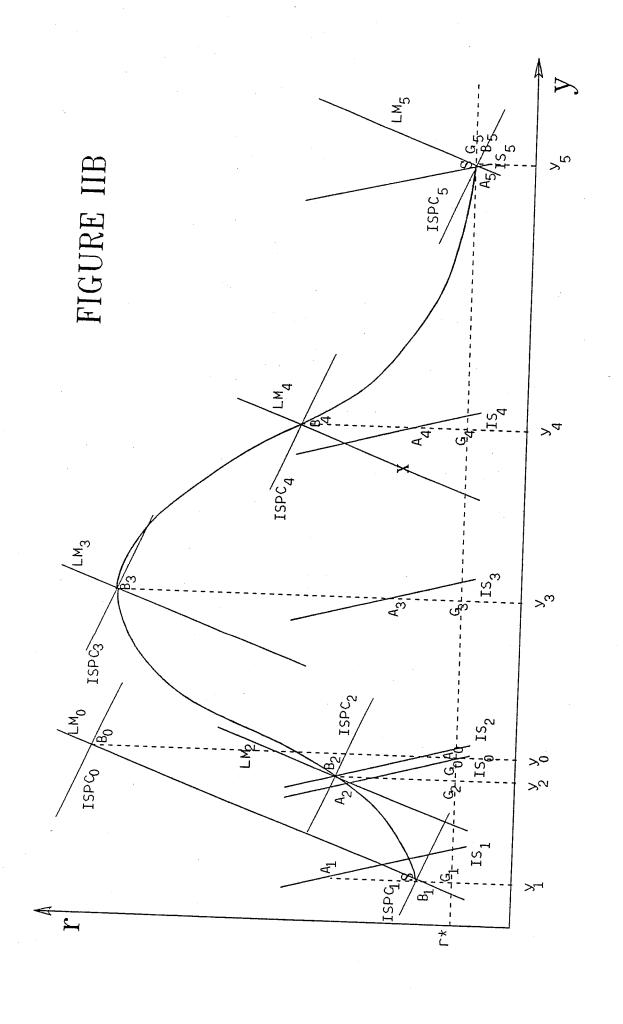
I begin by examining a benchmark case where the expected period of commitment is infinite and the government continues indefinitely with its original policy. Although this is not an incentive compatible policy, for convenience I shall refer to this as completely credible. In this and every other example, the government is assumed to inherit an inflation rate of 12.5% and a level of output 5% below the natural level.

Even in this relatively simple model, the dynamics are complex. In figure IIA the paths of inflation, output and the real exchange rate are shown under monetary policy designed to minimize (3.5) with $\theta = 5$. Here the economy is initially subjected to a large deflationary stimulus which is reflected in the negative rate of inflation and the low level of output. This is, in the main, due to the instantaneous appreciation of the exchange rate e(t), required to maintain (uncovered) parity between domestic and foreign interest rates. After this inial shock, inflation quickly rebounds, peeking at approximately 7.5% before finally falling to zero. By contrast, output and competitiveness steadily improve.

A clearer understanding of the forces responsible for this behavior can be gained by studying figure IIB. This shows the saddle-path SS of the economy in terms of output and the nominal interest rate. The initial equilibrium is at point B_o . Here the LM curve intersects with the Phillips augmented IS curve, ISPC $_o$. The latter combines the IS curve (drawn as IS $_o$ with respect to the real interest rate) with the Phillips curve (1.3). Hence the vertical distance between B_o and A_o represents the initial rate of inflation. The expected depreciation of the real exchange rate $E_iDe(t)$ can also be identified. Since $E_iDi(t) = r(t) - r^*$ (where r^* is the constant foreign nominal interest rate), the rate of depreciation of the nominal exchange rate is represented by the distance B_oG_o , so that the difference A_oG_o indicates $E_iDe(t)$.

The stages of the policy can be traced by considering the sequence of points $\{B_0, B_1, B_2, B_3, B_4, B_5\}$. On the announcement of the policy there is a large reduction in the growth of the money stock shifting the Phillips augmented IS curve to ISPC₁. The nominal exchange rate appreciates in anticipation of the depreciation expected over the duration of the policy. In the presence of price inertia this causes the level of competitiveness

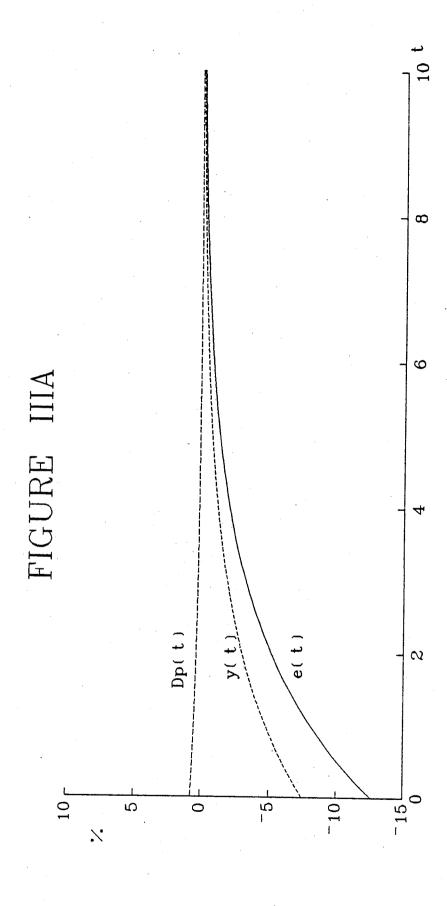


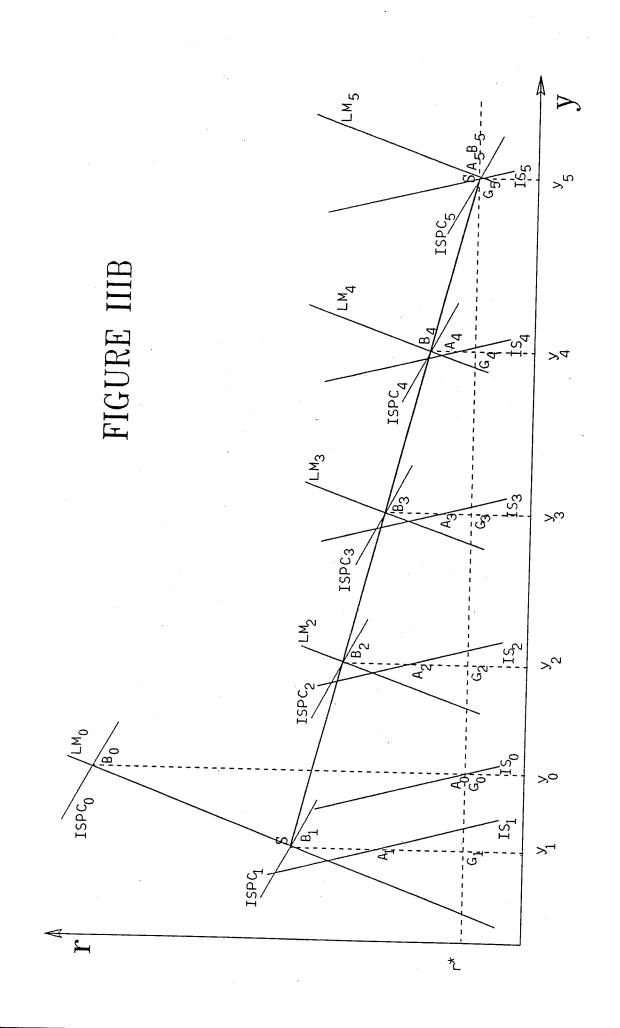


e(t) to deteriorate, pushing the IS curve to $\mathrm{IS_1}$. Hence, since the level of real balances is predetermined, the impact of the policy pushes the economy to $\mathrm{B_1}$ where there is a negative rate of inflation $(\mathrm{B_1A_1})$ and a large expected depreciation of the real exchange rate $(\mathrm{A_1G_1})$. Hereafter the level of real balances begins to rise as shown by the sequence $\{\mathrm{LM_2}, \mathrm{LM_3}, \mathrm{LM_4}, \mathrm{LM_5}\}$. Initially this can only be achieved at the expense of inflation which peeks at $\mathrm{B_3A_3}$ before falling to zero at $\mathrm{B_5A_5}$. In spite of this, output rises over the duration of the policy spurred on by the improvement in competitiveness and the falling real interest rate. This is represented by the sequence $\{\mathrm{IS_1,IS_2}, \mathrm{IS_3}, \mathrm{IS_4}, \mathrm{IS_5}\}$.

In the light of these observations, it appears that there are two main elements to the government's policy. The first relies upon the forward looking behavior in the exchange market to assess the long term implications of the policy. Although this produces a large initial loss of competitiveness, the subsequent improvement is the vital force in pushing output towards its targeted level. The second element concerns the short and medium-term use of monetary policy. Initially monetary growth is stimulated to counteract the immediate consequences of the loss of competitiveness on output. Once inflation begins to accelerate however, this policy must be reversed to avoid putting the ultimate policy goals in jeopardy. Thus the effects of medium and long term monetary growth are used in opposition to simultaneously reduce inflation and increase output in the later phases of the policy.

If the government has no reputation a far less sophisticated policy must be pursued since no credibility could have been anticipated in its design ($h^{\bullet}=0$). The results of this are shown in figure IIIA. Here, in contrast to figure IIA, the rate of inflation declines over the whole course of the policy. Moreover, at comparable points, both output and the real exchange rate are further from their target levels. The reason for these differences can be appreciated with the aid of figure IIIB. This shows that the initial impact of the policy is less dramatic. Although there is a loss of competitiveness, the expected rate of depreciation De(t) (represented by AIGI) is smaller than under the policy described above. This can be directly attributed to the policy's lack of credibility. Here only the implications of current monetary policy are incorporated into the exchange rate. Thus even if more dramatic movements in monetary growth were announced for the medium – term, they would not influence e(0).





Within these restrictions the government must adopt a far more conservative policy, comprising a gradual reduction in monetary growth. This is represented by the sequence $\{\mathrm{ISPC}_1,\ \mathrm{ISPC}_2,\ \mathrm{ISPC}_3,\ \mathrm{ISPC}_4,\ \mathrm{ISPC}_5\}$. As a result, the legacy of the initial loss in competitiveness must be endured for a longer period than when the policy enjoyed complete credibility.

With this preliminary analysis complete, the behavior of the economy under partially credible policies can be considered. Following the procedure described in the previous section, elements of the set of incentive compatible policies are calculated for a variety of different cost functions paramaterized by θ . The results of these calculations are reported in Table 1.

There are two important findings: First, the expected period of commitment must be of a significant length if it is to be incentive compatible. In fact, the interventionist phase of the policy is over before the majority of these horizons are reached (see figure IIA). This suggests that the government's reputation must be very good if the optimal policy is to retain any credibility. Second, the impact of θ on h^{8} implies that the quality of the minimum reputation necessary to retain some credibility can be influenced by the design of the proposed policy. In particular, policies taking explicit account of the behavior of the real exchange rate, may, ceteris paribus, suffer less of a "credibility problem". Table I also reports the cost of adopting the alternative policies. This shows that as θ rises the percentage saving form adopting a partially credible rather than a non-credible policy tend to increase.

In order to interpret these findings, consider the incentive to renege on the original policy. Suppose the government decides to renege when the economy reaches B_4 in figure IIB. What changes in policy will take place? Since the level of real balances is predetermined, the locus of possible equilibria are represented by LM $_4$. This means that any immediate gains in reduced inflationcan only be achieved at the cost of lower output. The long run trade off however, depends upon the dynamic properties of the economy. In particular, since the current level of e(t) implies that the real exchange rate must depreciate over the remainder of the policy, the loss of output accompanying a reduction in inflation will be comparatively short lived. Viewed from this perspective, the reduction of inflation is attractive, so that the government reneges by announcing an unexpectedly low rate of monetary growth. This will shift the ISPC curve to the left so that the economy "jumps" from B_4 to X. Of course this account ignores the

Table 1 Simulation Results

θ	h ^a	V(0;∞)	V(0;0)	%(h ^s)
0.0	11.3	60.47	60.65	0.3
0.5	10.5	143.60	147.16	3.1
1.0	10.1	222.83	233.66	4.6
5.0	7.2	757.25	925.73	16.6
10.0	6.0	1274.18	1790.80	21.5
15.0	5.6	1694.84	2655.88	21.1
20.0	4.5	2052.28	3520.96	14.0

Notes: i: coefficient on e(t)² in cost function.

[0,hs): the shortest (non-zero) period of expected commitment consistent with an incentive compatible policy.

V(0;1): Cost of policy enjoying complete credibility.

V(0;0): Cost of policy enjoying no credibility.

%(hs): Percentage saving of adopting a partially credible policy with expected commitment $h^{\mathbf{s}}$.

longer-term gains and losses associated with reneging. To examine these, the effects of the anticipated change in policy must be examined.

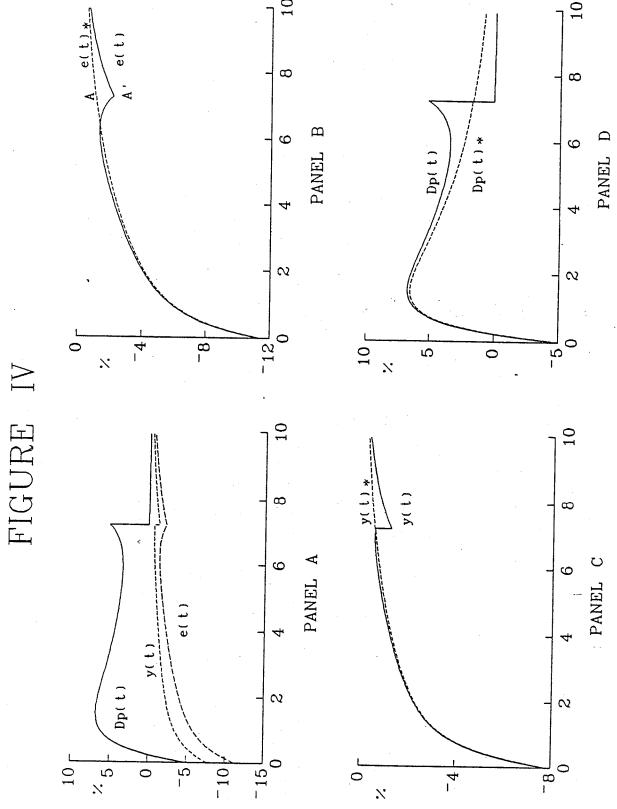
Assume that, given the government's reputation and preferences $[\theta=5]$, the "optimal credible policy" has some credibility $[h^s=7.2]$. The behavior of the economy under this policy is shown in figure IV panel A. Here all three variables [Dp(t), y(t), e(t)] mirror the behavior seen in figure IIA over the initial phase of the policy. However once the anticipated switch in policy draws near, significant differences appear. These are emphasized in panels B C and D which directly compare the trajectories of the exchange rate, output and inflation under the two policies. In contrast to the steady rise (decline) in output (inflation) found during the later phase of the completely credibility policy, the economy experiences a short period of stagflation immediately prior to the change in policies. Here output falls and inflation rises at an ever increasing rate. This can, in the main, be attributed to the dramatic appreciation of the real exchange rate shown in panel B.

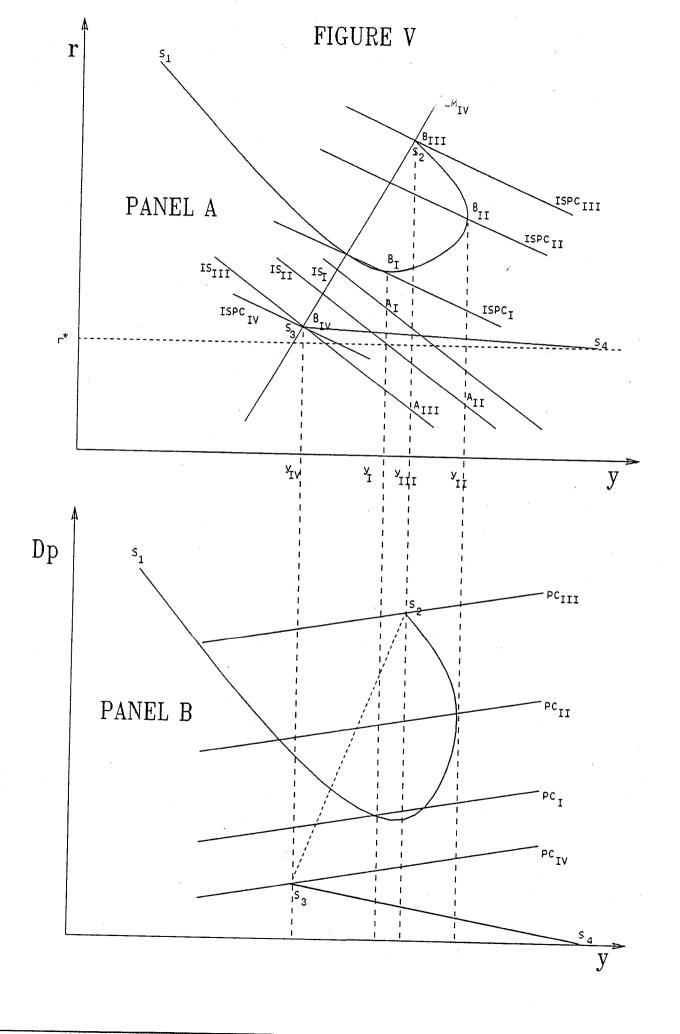
The appreciation of the exchange rate reflects the anticipated consequences of the change in policy. In particular, since the rate of depreciation, De(t) must "jump" upward when the switch to a lower rate of monetary growth is made, the level of competitiveness e(t) must be below that associated with a continuation of the original policy. Hence, because anticipated "jumps" in e(t) are inconsistent with market equilibrium, the real exchange rate must appreciate prior to the change in policy.

If the original policy was formulated only with regard to inflation, one would expect the loss of competitiveness to have a serious impact on output. However panel C of figure IV indicates that the fall in output is relatively small. This suggests that monetary policy must be far more active.

The role played by policy can be analyzed with the aid of figure V. Panel A depicts the behavior of interest rates and output. Below this movements in the Phillips curve are shown. In both panels the (dis-continuous) saddle-path is drawn as $S_1S_2S_3S_4$.

The loss of competitiveness prior to the switch in policy is represented by the sequence $\{\mathrm{IS_{II}}, \mathrm{IS_{III}}\}$ in panel A. In the absence of a change in policy this would cause a significant reduction in output. However, this tendency is countered by a rapid increase in monetary growth, depicted by the sequences $\{\mathrm{ISPC_{II}}, \mathrm{ISPC_{III}}\}$ and $\{\mathrm{PC_{II}}, \mathrm{PC_{III}}\}$ in





panels A and B respectively. While this is initially effective in maintaining the growth of output, after point B_{II} the large loss of competitiveness prevails. The cost of "forestalling the inevitable" is a rapid increase in inflation [see {B_{I}A_{I}}, B_{II}A_{II}]. Consequently, the economy suffers rapidly worsening stagflation as the government strives to counter the influence of the foreign exchange market.

It will not be optimal for the government to continue with this policy indefinitely, even though reneging results in a loss of credibility. In fact it will continue until the benefits of retaining some credibility are outweighed by the cost associated with the deteriorating state of inflation, output and competitiveness. When this point is reached (S_2) there is dramatic reduction in monetary growth pushing the economy (instantaneously) to S_3 . For the case analyzed in figure IV, because the policy is incentive compatible, this is precisely the response expected by the public.

Can this analysis shed any light on the findings reported in table 1? In most respects the crucial aspect of the anticipated change in policy is the "jump" in the rate of depreciation of the real exchange rate. Without this there is no loss of competitiveness, and thus no incentive for the government to stimulate the economy. This suggests that the influence of different expected commitments on the nature of the "jump" in De(t) are key to understanding the findings in table 1.

Consider the effects of a shorter expected commitment. From panel B of figure IV it can be seen that, for a given "jump" in De(t), the loss of competitiveness $[e_2(t)-e_1(t)]$ will be greater the shorter the expected commitment. As a result, the combination of the appreciating exchange rate and expansionary monetary policy will tend to lead to a more acute stagflation. Under these conditions the government will have a greater incentive to renege prior to the date expected by the public. Although this problem could be alleviated by a smaller anticipated "jump" in De(t), this is not consistent with the expected new policy. Hence, by anticipating that the public expects less commitment to the original policy, the government is induced to renege prematurely. The size of h[®] reported in table I reflects the acuteness of this problem.

This analysis also suggests why the value of θ has such an important influence on the set of incentive compatible policies. As noted above, the incentive to renege prematurely will be reduced if the expected "jump" in De(t) accompanying the switch is small. This will depend on the form of the government's cost function. In particular, if deviations of the real

exchange rate from its targeted level contribute heavily to costs, a less dramatic reduction in monetary growth at $t = h^{\theta}$ will be preferred, implying a smaller prior appreciation of e(t). Hence relatively small anticipated "jumps' in De(t) will be consistent with the policy switch when θ is large. This allows shorter expected commitments to be incentive compatible, permitting governments with poorer reputations to follow partially credible policies.

V:Summary and Conclusions

In the preceding sections I have presented and applied a technique capable of identifying the "optimal credible policy" available to a government with a pre-specified reputation. Since the analysis does not rely on the features of a specific structural model, it has many potential applications within the class of linear rational expectations/perfect foresight models. The results in one application were presented in the previous section. While these are interesting in themselves, they also convey a far more general message: The potential time inconsistency of an optimal policy need not result in the complete loss of its credibility. Although the government may not be able to adopt a credible policy given its reputation, this will depend on both the structure of the economy and the form of the proposed policy.

In section IV, I showed that in cases where a partially credible policy could be adopted, the economy experienced a period of rising inflation, falling output and an appreciating real exchange rate. Moreover, these trends could only be reversed by adopting a new policy. Although this represents a departure from the original policy, and is therefore time inconsistent, the switch does not come unexpectedly. Rather it is viewed as an inevitable consequence of the optimal policy consistent with the government's reputation.

The analysis also suggests that an anti-inflationary monetary policy designed with specific regard to the behavior of the real exchange rate is more likely to enjoy some credibility. If the consequences for competitiveness are ignored, the public foresees a situation in which the appreciation of the real exchange rate is sufficient to place the ultimate policy goals in jeopardy, and hence induce the government to renege. Under these circumstances, the policy has no credibility.

Finally some general comments about the structure of the model are in order. First, it should be emphasized that the structure of the model relies heavily on the assumed presence of perfect information. When the government designs its policy it assumes that the public knows the complete structure of the model, including the specification of the government's costs. Consequently, the announcement of an incentive compatible policy enjoys the same degree of credibility as the government anticipated in its design. Relaxing these assumptions would complicate the model considerably. For example, suppose that only incomplete knowledge of the government's cost function is available to the public. Under these circumstances, the government may anticipate taking advantage of the public's misperceptions when formulating its policy. Moreover, although the ultimate loss of credibility is reflected in e(0) under complete information, this signal about the public's reaction to a time inconsistent policy will become blurred once there is uncertainty about the cost facing the government. Consequently, the government could easily misinterpret e(0) to mean that credibility could be re-established after a switch in policy. It may then decide to renege when, ex post, the original policy would have been more profitable.

Another crucial aspect of the model concerns the actual and anticipated loss of credibility induced by the government reneging. Although this may be viewed as an extreme response, alternatives suffer from other problems. In this model, unlike Backus and Driffill (1984) for example, the public is assumed to behave atomistically, in the sense that each individual treats the government's behavior and the actions of everyone else as given. Although, as Barro (1985) points out, the assumption of a monolithic private sector may be applicable in some situations (i.e. where a monopoly trade union negotiates economy-wide wages), in general such coalitions coordinating the behavior of the public do not exist. This implies that sophisticated threats may themselves suffer from a credibility problem. The approach taken in this paper avoids this by employing a device that is consistent with an atomistic private sector: From an individual's viewpoint. a complete loss of credibility places them in a safe position since unexpected "jumps" in the exchange rate are no longer possible. Once this state is achieved, it seems unlikely that individuals will unilaterally re-expose themselves to the risks of believing government announcements. Therefore, in the absence of any collaboration, a loss of public confidence will be extremely hard to reverse. In this sense, the actual and anticipated Loss of credibility is far less extreme than it may appear at first sight.

The complete loss of credibility may be criticized on other grounds. Suppose that the economy is subjected to a series of exogenous shocks. (Here the assumption of perfect foresight would be replaced with rational expectations.) Although it may seem reasonable for the government to anticipate loosing all credibility if a switch in policy is engineered purely to delude the public, it may not be if the public can be convinced that the switch was desirable given the unanticipated change in circumstances. On the other hand, the public will be expected to treat these "explanations" with some skepticism because of the obvious incentive to misreport. Since the choice of a reasonable conjecture in this situation is unclear a priori, an examination of the effects of exogenous shocks on policy credibility remains an important issue.

Some further remarks about the role of the government's reputation are also in order. Unlike Backus and Driffill (1984,1985) and Barro (1985), in this model the government is unable to enhance its reputation through the choice of policy. In principle, however, one could consider whether the government would prefer to adopt a sub-optimal policy initially, in order to improve (lengthen) the period of commitment expected by the public. This requires a specification for the mechanism through which the expected commitment is revised. One candidate is the Baysian mechanism used by Backus and Driffill (1984,1985) and Barro (1985). Unfortunately this in turn needs the absence of complete information, which substantially complicates the model. It is also important to remember that the analysis refers only to a specific policy episode. Although the model could be extended to examine a sequence of such episodes, the government's reputation at the beginning of each could no longer be treated as exogenous. Instead, the government's past performance would affected its current reputation.

In the light of these remarks, the specific results reported here should be treated with some caution. Although the analysis provides some important insights into the role played by credibility in the conduct of macroeconomic policy, it is far from a definitive account. While I suspect that the qualitative findings will remain unaffected when incomplete information is introduced, this remains as issue for ongoing research.

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- $^{\prime}$ V(0) may also be interpreted as the expected cost over a poisson distributed finite horizon, where δ is the certainty equivalent discount factor [see Merton (1971)].
- 10 In the presence of discounting, the co-state variables $\lambda_z(s)$ where $\partial V(s)/\partial z \equiv \lambda_s(s)$ [for state variable z], must be distinguished from their current value, $\mu_z(s) \equiv \exp[\delta s]\lambda_z(s)$.
- $^{
 m II}$ This "solution" to the problem was originally suggested by Buiter (1983). The characterization is due to Miller and Salman (1984).
- 12 Although the assumed complete loss of credibility may, at first sight, appear questionable, alternatives suffer from greater problems. These are discussed below.
- 13 I shall establish this as an equilibrium strategy in subsection B below.
- ¹⁴Note that the exchange rate is effectively being treated as an exogenous variable. Even if the government were to re-optimize during $[h^{\circ}, \infty)$ there would be no change in the non-predetermined variable because $\mu_{e}(h^{\circ}) = 0$ for $t \geq h^{\circ}$. e(t) is therefore effectively exogenous with respect to changes in policy. For a further discussion see Buiter (1984).
- Although the dynamics of the economy during $[h^{\bullet}, \infty)$ are governed by (2.1), Pontryagin's principle can still be applied provided the system (1.5) is completely controllable [see Murata (1977)].
- 16 Note that the public's beliefs are treated as given. More will be said about this below.

Footnotes

In taking this approach cooperative behavior between the government and the public is implicitly ruled out. Aside from the problem of how such a coalition could be enforced, the non-cooperative framework allows the benefits of misguiding the privates sector's beliefs to be examined. These will be present even when the government is benevolent due to the influence of externalities on private sector decision-making. See Calvo (1978) and Cohen and Michel (1986) for a discussion.

²Although each proposes a different solution procedure, they all can be interpreted in terms of a Nash solution to the dynmamic game. Here the reaction function of the private sector is treated as given so that the state variables are only influenced by the direct effects of the policy instruments. An extensive discussion of these policies is contained in Blackburn (1986).

 3 The concept of incentive compatibility is originally due to Hurwicz (1972).

 4 I shall retain the use of the expectations operator to emphasize the role played by the private sector's expecations in the analysis. The implications of perfect foresight will be discussed at length below.

 $^{5}\mathrm{By}$ assuming that the price level is predetermined, the model incorporates the notion of short-run price stickiness.

 6 As e(t) is non-predetermined, De(t) must be interpreted as the right-hand side time derivative.

 7 Buiter and Miller (1981) show that this requires $\alpha \alpha_{2} < \alpha_{1}$.

 8 Since the behavior of (1.1) - (1.4) can be deduced from the dynamics of (1.5) [i.e. the model is comp[letely observable], C(t) can contain state variables from both sets of equations.

¹/Figure I does not consider the possibility that the cost curves intersect more than once to the left of h. Under these circumstances the technique cannot be applied because the government will find if profitable to renege before the end of the expected period of commitment despite the fact that $(\tau; h^{\bullet}) > 0$ for τ close to h. Consequently it is necessary to rule out such cost structures before employing the technique described in the text. This is done in the appendix.

¹⁸Since $\mu(\tau^*)$ is the discountd value at t=0 of the costate at τ^* , and $\partial V(s)/\partial e(s) \equiv \lambda_e(s)$, its current value is $\lambda_e(\tau^*) \equiv \exp\{-\delta \tau^*\}\mu_e(\tau^*)$.

 19 The complexity of the model precludes the use of analytic techniques.

 20 After a number of experiments it was found that the dynamics reported in Figure IIa are not particularly sensitive to the choice of θ . The reason for this particular value will become apparent below.

 21 Algebraically this is given by,

$$y(t) = \beta_1 (1 - \beta_1 \sigma)^{-1} [Dm(t) - r(t)] + \beta_2 (1 - \beta_1 \sigma)^{-1} e(t)$$

 $^{22} \text{The real interest rate may be writtten as r}^{*} + \text{De}(t) - \text{Dp}(t)$ and so is represented by the height of \textbf{A}_{i} above the horizontal axis.

 23 This may be seen by comparing the slopes of the trajectories at A and A' in panel B of figure IV.

In order to apply the technique described in the text, only one intersection of the cost curves $V_1(\tau;h^{\circ})$ and $V_2(\tau;h^{\circ})$ can occur over the interval $[0,h^{\circ})$. This rules out cases where the cost have the form shown in figure VI. Here the analysis of the cost differential at τ^* using (2.11) would not allow the set of incentive compatible policies to be identified. Fortunately, it is easy to check for these "pathological cost structures" on a case by case basis.

Suppose that one of the policies reported in tale 1 is not incentive compatible. Specifically, let $V_1(t_1,h^s)=V_2(t_1,h^s)$ and $V_1(t_2,h^s)=V_2(t_2,h^s)$ for $t_1 \neq t_2$ where $t_1,t_2 \in h^s$. Under these circumstances it would be advantageous to renege at any date between t_1 and t_2 . Using (2.11) to evaluate the cost differential at t_1 and t_2 , this implies that either a) $\mu_{\bf e}(t_1)=0$, or b) ${\bf e}_2(t_1)={\bf e}_1(t_1)$ for ${\bf i}=1,2$. If it can be directly verified that $\mu_{\bf e}(t)\neq 0$ for ${\bf t}\in (0,h^s)$ (as it can in the model studied in section iV), I need only check condition b). This requires that a change in policy at t_1 and t_2 would not be accompanied by a "jump" in the exchange rate. However this is only possible if the saddle paths drawn in figures IIB and IIIB intersect twice within the interval $[0,h^s)$.

To rule out this possibility, note that in panel D of figure IV the inflation rate at $t=h^{\circ}$ is lower than at any point in the immediate past. Given the concurrent fall in competitiveness, this implies that the saddle-path S_3S_4 lies "below" S_4S_2 in panel A of Figure V. Consequently, the saddle-paths associated with the alternatives facing the government following a partially credible policy can only intersect once within the interval $[0,h^{\circ})$. Hence (2.11) can be legitimately used to identify incentive compatible polices.

