FURTHER EVIDENCE ON BREAKING TREND FUNCTIONS IN MACROECONOMIC VARIABLES

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ABSTRACT

The aim of this study is twofold. First to reexamine the findings of Perron (1989) regarding the claim that most macroeconomic time series are best construed as stationary fluctuations around a deterministic trend function if allowance is made for the possibility of a shift in the intercept of the trend function in 1929 (a crash) and a shift in slope in 1973 (a slowdown in growth). Unlike in my previous study, the date of a possible change is not fixed a priori but is considered as unknown. In this context, the statistic used to test for the null hypothesis of a unit root is the minimum of the t–statistics for testing the null hypothesis of a unit root over all possible break points, while maintaining the assumption of at most one change during the sample period. The critical values of this test statistic are derived using simulation methods. A detailed discussion about the choice of the truncation lag parameter in the autoregression and of its effect on the critical values is also included. Most of the rejections reported in Perron (1989) are confirmed using this approach. Secondly, this paper investigates an international data set of post–war quarterly real GNP (or GDP) series for the G–7 countries. The evidence favors, for most countries, the hypothesis of stationary fluctuations around a trend function with different slopes before and after some date near 1973 (the date varying across countries). A number of other series including real consumption are analyzed and strong evidence is again found against the unit root hypothesis.

Key words: Hypothesis testing, structural change, stochastic trends, deterministic trends, simulation experiment, unit root.

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1. INTRODUCTION

In a previous paper, Perron (1989), I argued that most macroeconomic time series are best represented as stationary fluctuations around a deterministic trend function if allowance is made for a possible change in its intercept in 1929 (a crash) and in its slope in 1973 (a slowdown in growth). The test statistics were constructed by adding dummy variables for different intercepts and slopes, extending the standard Dickey–Fuller procedure. The asymptotic distributional theory underlying the critical values obtained under the different models assumed that the dating of the break points were known a priori, or more precisely, that the dates chosen were uncorrelated with the data.

This postulate has been criticized, most notably by Christiano (1988) who argued that the choice of these dates had to be viewed, to a large extent, as being correlated with the data. This is an important problem because both the finite sample and asymptotic distribution of the statistics presented in such an analysis depend upon the extent of the correlation between the choice of the break points and the data. There is a sense, as I argued before, in which the choice of these dates can be regarded as independent of the data. First, the dates reported in my previous study were chosen ex–ante and not modified ex–post. Secondly, I view these dates as being related to exogenous events that then occurred and for which economic theory would suggest the effects that actually happened ; e.g. the stock market crash of 1929 with the ensuing dismantle of the economic organization and the exogenous sudden change in oil prices with the resulting alteration of international economic coordination and policies.

In the sense described above the choice of the dates can be viewed as uncorrelated with the data. There is, however, a validity to the argument that it is only ex–post (after looking at the data) that we can say that the changes that followed these exogenous events actually occurred as predicted by the theory. Furthermore, many other exogenous events did not have the major impact that some theories would have likely predicted. In this sense, the choice of the break points must be viewed as being correlated, at least to some extent, with the data. To what extent were the choices of 1929 and 1973 correlated with the data ? This is a difficult and practically impossible question to answer. At the very least, however, these choices were not perfectly correlated with the data as I did not systematically try various dates to maximize the chances that the unit root would be rejected ; nor did I systematically try to find where, according to some test criteria, are the most likely dates of change.

While I still believe that the assumption about the exogeneity of the choice of the break points is a good first approximation to the true extent of the correlation with the data, I think it is useful to investigate how robust the results are to different postulates. The aim of this paper is to take the extreme view that the choice of the break points is perfectly correlated with the data. This extreme case is instructive to study because if one can still reject the unit root hypothesis under such a scenario it must be the case that it would be rejected under a less stringent assumption.

I proceed as follows for the practical implementation. Again, as in my previous analysis, only one possible break point is allowed for any single series. This break point is chosen in such a way that the t–statistic for testing the null hypothesis of a unit root is smallest among all possible break points (given that the alternative hypothesis of interest is a stationary series). Hence, using such a procedure, the choice of the break point is indeed perfectly correlated with the data. To derive the critical values of this testing procedure, I resort to simulation experiments of the finite sample distribution using corresponding sample size for the series analyzed. I also investigate various issues regarding the choice of the truncation lag parameter in the estimated autoregressions and the effect on the critical values of using different criteria for choosing this lag length. I also consider choosing the break point that corresponds to a minimal t–statistic on the parameter of the change in the trend function. This allows the (presumably) innocuous a priori imposition of a one–sided change (i.e. a decrease in the intercept or the slope of the trend function). As will be seen, such a minor change allows substantial gains in power.

In a sense, I am investigating the following issue. Can we still reject the unit root hypothesis for most macroeconomic time series if the break point is chosen in such a way as to be perfectly correlated with the data ? The answer is, in general, yes. Hence the results given in my previous study can be viewed as robust to the worst possible scenario concerning the assumption about the extent of the correlation between the choice of the break points and the data.

The paper is organized as follows. Section 2 reviews the statistical models and statistics involved. Section 3 discusses the procedures investigated and presents the design of the simulation experiment and the critical values obtained under each of them. Section 4 gives the empirical results for the Nelson–Plosser (1982) data set. Section 5 analyzes an international data set of post–war quarterly real GNP series for the United States, Canada, Japan, the United Kingdom, West Germany, Italy and France. Section 6 presents additional evidence on other series such as consumption and on some series analyzed in Section 4 but obtained from alternative sources. Finally, Section 7 offers concluding comments.

2. THE MODELS AND STATISTICS

I briefly review, in this section, the models and statistical procedures involved in testing for a unit root allowing for the presence of a change in the trend function occurring at most at one date. The reader is referred to Perron (1989) for a more detailed discussion. Throughout this paper, the time at which the change in the trend function occurs is denoted by T_B. The first model is concerned with the case where only a change in the intercept of the trend function is allowed under both the null and alternative hypotheses. Furthermore this change is assumed to occur gradually and in a way that depends on the correlation structure of the noise function. This was termed the "innovational outlier model" and can be succinctly represented , under the null hypothesis of a unit root, by :

$$y_t = y_{t-1} + b + \psi(L)(e_t + \delta D(TB)_t) \qquad (t = 1,...,T) \tag{1.a}$$

where $D(TB)_t = 1$ if $t = T_B + 1$ and 0 otherwise. The sequence $\{e_t\}$ is i.i.d. $(0, \sigma^2)$ and $\psi(L)$ is a possibly infinite lag polynomial in L (with $\psi(0) = 1$) which defines the moving average representation of the noise function of the series. That is, denoting by z_t the noise function of the series, we have $A(L)z_t = B(L)e_t$ and $\psi(L) = A(L)^{-1}B(L)$. Throughout, I assume that the finite order polynomials A(L) and B(L) have all their roots outside the unit circle. The immediate impact of the change in the intercept is δ while the long run impact is $\psi(1)\delta$. Under the alternative hypothesis of stationary fluctuations, the model is represented by :

$$y_t = a + ct + \Phi(L)(e_t + \theta DU_t) \qquad (t = 1,...,T) \tag{1.b}$$

where $DU_t = 1$ if $t > T_B$ and 0 otherwise, and $\Phi(L) = C(L)^{-1}D(L)$ with C(L) and D(L) finite order polynomials in L having all their roots outside the unit circle. In the trend–stationary case, the immediate impact of the change in the intercept of the trend function is θ while the long run impact is $\theta\Phi(1)$. Model (1.a) can be tested against model (1.b) using the t–statistic for testing $\alpha = 1$ in the following regression estimated by ordinary least–squares (with Δ the difference operator such that $\Delta y_t = y_t - y_{t-1}$) :

$$\text{(1.c)} \qquad y_t = \mu + \theta DU_t + \beta t + \delta D(TB)_t + \alpha y_{t-1} + \Sigma_{i=1}^{k} c_i \Delta y_{t-i} + e_t .$$

Regression (1.c), like the others that will follow, is in the spirit of the Dickey–Fuller (1979) and Said–Dickey (1984) methodology whereby autoregressive–moving average processes are approximated by autoregressive processes of order k. The statistics of interest, such as the t–statistic for testing $\alpha = 1$, will have the appropriate asymptotic distributions provided k is a fixed increasing function of the sample size (the rate of increase being restricted not to be too rapid).

Under the second model, both a change in the intercept and a change in the slope of the trend function are allowed at time T_B. With a similar notation, the model under the null and alternative hypotheses can respectively be represented as :

$$\text{(2.a)} \qquad y_t = y_{t-1} + b + \psi(L)(e_t + \delta D(TB)_t + \eta DU_t) \qquad (t = 1,\ldots,T)$$

and

$$\text{(2.b)} \qquad y_t = a + ct + \Phi(L)(e_t + \nu DU_t + \gamma DT_t) \qquad (t = 1,\ldots,T)$$

where $DT_t = t$ if $t > T_B$ and 0 otherwise. Model (2.a) can be tested against Model (2.b) using the t–statistic for the null hypothesis that $\alpha = 1$ in the following regression estimated by OLS :

$$\text{(2.c)} \qquad y_t = \mu + \theta DU_t + \beta t + \gamma DT_t + \delta D(TB)_t + \alpha y_{t-1} + \Sigma_{i=1}^{k} c_i \Delta y_{t-i} + e_t .$$

Under the third model, a change in the slope of the trend function is allowed but both segments of the trend function are joined at the time of break. Here the change is presumed to occur rapidly and correspond to the "additive outlier model" in the terminology of Perron (1989). The model under the null and alternative hypotheses can be represented as :

$$(3.a) \qquad y_t = y_{t-1} + b + \gamma DU_t + \psi(L)e_t \qquad (t = 1,...,T)$$

and

$$(3.b) \qquad y_t = a + ct + \gamma DT_t^* + \Phi(L)e_t \qquad (t = 1,...T)$$

where $DT_t^* = t - T_B$ if $t > T_B$ and 0 otherwise. The null hypothesis given by Model (3.a) can be tested versus the alternative Model (3.b) using the following two–step procedure. First, the series is detrended using the following regression estimated by OLS :

$$(3.c.i) \qquad y_t = \mu + \beta t + \gamma DT_t^* + \tilde{y}_t \,.$$

The test is then performed using the t–statistic for $\alpha = 1$ in the next regression, again estimated by OLS :

$$(3.c.ii) \qquad \tilde{y}_t = \alpha\,\tilde{y}_{t-1} + \Sigma_{i=1}^{k} c_i \Delta\tilde{y}_{t-i} + e_t.$$

I denote by $t_{\hat{\alpha}}(i,T_B,k)$ $(i = 1,2,3)$, the t–statistic for testing $\alpha = 1$ under model i with a break date T_B and truncation lag parameter k (i.e. the t–statistic for $\alpha = 1$ in regressions (1.c), (2.c) and (3.c.ii) for i = 1,2, and 3, respectively). Given that no assumptions about the date of the change T_B are made, I study the behavior of the statistics $t^*_{\hat{\alpha}}(i) = \text{Min}_{T_B \in (k+2,\, T)}\, t_{\hat{\alpha}}(i,T_B,k)$ $(i = 1,2,3)$, where different specifications about the choice of k will be analyzed. This procedure is in the tradition of tests for structural change when the date of the change is assumed unknown. These tests are usually based on extremum estimators which consider the maximum of a sequence of random variables as the statistic of interest. Such is the case for instance with the CUSUM test of Brown, Durbin and Evans (1975).

For Models 1 and 3, the following procedure is also analyzed. Instead of choosing the break point T_B so that $t_{\hat{\alpha}}(1,T_B,k)$ or $t_{\hat{\alpha}}(3,T_B,k)$ is minimized, it is chosen to minimize either $t_{\hat{\theta}}$, the t–statistic on the parameter associated with the change in the intercept (Model 1) or $t_{\hat{\gamma}}$, the t–statistic on the change in slope (Model 3). I shall denote the t–statistic on α (for a null hypothesis that $\alpha = 1$) obtained from such a procedure by $t^*_{\alpha,\theta}(1)$ for Model 1 and by $t^*_{\alpha,\gamma}(3)$ for Model 3. More precisely, $t^*_{\alpha,\theta}(1) = t_{\hat{\alpha}}(1,T^*_B,k)$ where T^*_B is such that $t_{\hat{\theta}}(T^*_B) = \text{Min}_{T_B \in (k+2,T)} t_{\hat{\theta}}(T_B,k)$, where again different specifications about the choice of k will be analyzed. The statistic $t^*_{\alpha,\gamma}(3)$ is defined in an analogous fashion. This procedure is more akin to that used by Christiano (1988). The use of the t–statistic on the parameter associated with the change in the trend function gives the possibility of imposing the mild a priori restriction of a one–sided change. Hence by choosing T_B so that $t_{\hat{\theta}}$ in (1.c) or $t_{\hat{\gamma}}$ in (3.c.i) is minimized, I allow the date of the change in the trend function to be unknown but restrict the analysis to the cases of a "crash" or a slowdown in growth.

3. THE SIMULATION DESIGN AND THE CRITICAL VALUES OF THE STATISTICS

No attempts were made to derive the asymptotic distribution of the statistics $t^*_{\alpha}(i)$ under the null hypothesis of a unit root. Instead, I resort to simulation experiments to obtain critical values that will permit appropriate statistical inferences. Using standard invariance arguments the finite sample distribution of $t^*_{\alpha}(i)$ is invariant, under the null hypothesis, to the true value of the coefficients b and δ in (1.a) for i = 1 ; to b, δ and η in (2.a) for i = 2 ; and to b and γ in (3.a) for i = 3. The simulations are therefore based on replications of the following model generated to represent the null hypothesis :

(4) $$y_t = y_{t-1} + e_t \ ; \qquad y_0 = 0 \ ,$$

with $e_t \sim$ i.i.d. N(0,1). The distributions of the statistics $t^*_{\alpha}(i)$ are not invariant to the initial condition y_0 but its effect is usually considered negligible (e.g. Dickey (1984)) and hence it is fixed at 0 for simplicity. I specify the sequence of innovations $\{e_t\}$ to be i.i.d. N(0,1) even though the distribution of $t^*_{\alpha}(i)$ is not invariant to the correlation structure of the data represented by the polynomial $\psi(L)$ under the null hypothesis. The basic reason is that, as in the Said–Dickey (1984) procedure, the asymptotic distribution of $t^*_{\alpha}(i)$ is invariant to additional correlation in the data if the truncation lag parameter is a fixed function of the sample size (T) such that k increases at a controlled rate as T increases to infinity. Hence there is no loss in generality in specifying i.i.d. errors under this asymptotic interpretation. The implicit assumption is that, in finite samples, the introduction of additional lags of first differences of the data would mostly eliminate the dependency of the distribution of $t_\alpha^*(i)$ on $\psi(L)$. Furthermore, with i.i.d. errors, the simulated critical values are likely to be a better approximation to the asymptotic distribution of $t_\alpha^*(i)$. Nevertheless, to assess the sensitivity of the distribution of $t_\alpha^*(i)$ to the particular fixed value of k used, I provide, for each sample size considered, simulated critical values for different specifications on the truncation lags, namely k = 0, 2, 5, 8 and 11. Given the nature of the data sets analyzed in later sections, I present critical values for the following sample sizes for each model considered. For Model 1, T = 60, 80 and 100; for Model 2, T =70 and 100; and for Model 3, T = 100 and 150. The same comments and specifications apply to the statistics $t_{\alpha,\theta}^*(1)$ and $t_{\alpha,\gamma}^*(3)$.

In practice, the truncation lag parameter k in the estimated autoregressions is rarely chosen as a fixed function of the sample size T, i.e. in such a way that the choice of k is uncorrelated with the data. Rather, it is usually chosen by some test statistic on the significance of the lagged first differences of the data. Examples of procedures used to select a given value of k include the following. First, the order of the autoregressive model may be chosen according to the Akaike criterion. Other methods include choosing the value of k such that the t–statistic on the coefficient of the last included lagged first difference is significant (using the fact that its asymptotic distribution is normal, see e.g. Fuller (1976)) but the t–statistic on the coefficients of all additional lags (up to some prespecified maximum) are insignificant (e.g. Perron (1989)). Said and Dickey (1984) use yet a different method in their reported empirical application. It is based on testing whether additional lags are jointly significant using an "F–test" on the estimated coefficients.

Such data based procedures present theoretical difficulties. First, using such procedures, it is not evident that even the asymptotic distributions of the statistics $t_\alpha^*(i)$ are invariant to the correlation structure of the data. Such limiting distributions have never been derived and more analytical work is clearly needed to assess these issues. Presumably, the asymptotic distribution of the $t_\alpha^*(i)$ statistics with k chosen according to a significance criterion, such as those described above, is the same as in the fixed k case if the size of the test used converges to zero as the sample size increases. Under this asymptotic interpretation, the proposed statistical procedures, where k is chosen according to a significance criterion, are asymptotically valid and invariant to nuisance parameters. The finite sample distribution may, however, be badly approximated using such an asymptotic framework when, in practice, the size of the test on additional lags is fixed. A better approximation may be given by an asymptotic framework where the size of the test is held fixed as the sample size increases. Current research is underway to analyze these issues.

A recent investigation by Hall (1990) presents some results related to this issue. He shows that the standard Dickey–Fuller test will have the same asymptotic distribution as in the fixed k case if the truncation lag parameter is chosen according to a data based method in such a way that this choice of k is uncorrelated with the t–statistic of interest and if the method used asymptotically chooses the true order of the autoregression. Here none of the procedures considered satisfy these conditions. While Hall's results are silent about the effect induced by the correlation between the choice of k and the t–statistic, the simulation presented below show a substantial effect on the finite sample critical values. My presumption is that the asymptotic distributions of tests for a unit root with the truncation lag parameter chosen according to such data based methods (with a test of a given size) are different from the case where k is a fixed function of the sample size and may not be invariant to the true structure of the correlation present in the data (if the size of the test used to choose k is held fixed).

In order to obtain estimates of the divergence in the critical values compared to the fixed k case, I investigated three different procedures where the choice of k is correlated with the data. The first one is to choose the value of k so that the t–statistic for $\alpha = 1$ is smallest for a given value of T_B when k can take values between 0 and some upper bound kmax, say. The statistics are then defined as $\mathrm{Min}_{T_B \in (k+2,T)} \mathrm{Min}_{k \in (0,kmax)} t_{\hat{\alpha}}(i,T_B,k)$ and I shall refer to this procedure as the "min k" one given that k is chosen to minimize $t_{\hat{\alpha}}(i,T_B,k)$. This is probably the procedure which uses the least information possible and leads to tests with lowest power. Nevertheless, given the presumption of such low power, if a rejection of the null hypothesis of a unit root occurs it should be viewed as quite strong evidence against the unit root. Care, however, must be taken in interpreting the results using such a procedure. The critical values are obtained using (4) as the data generating process where i.i.d. errors are specified. As discussed previously, the asymptotic distribution of such a procedure may depend on the true structure of the correlation of the data. Given that this correlation is unknown for each series analyzed, it is difficult to assess the adequacy of the inference. The inference obtained using the critical values under this method should be viewed simply as a preliminary step in analyzing the robustness of the results to alternative specifications.

The other data based method considered uses a "F test" to assess the significance of the coefficients on the lagged first differences of the data in the estimated autoregressions. The exact procedure is as follows. First a maximum value of k, kmax, is specified. For a given value of T_B, the autoregression is estimated with kmax and (kmax – 1) lags. A 10% one–tailed F–test is used to assess whether the coefficient on the kmaxth lag is significant and if so, the value of k chosen is this maximum value. If not, the model is estimated with (kmax – 2) lags. The lag (kmax – 1) is deemed significant if either the F–test for (kmax – 2) versus (kmax – 1) lags or the F–test for (kmax – 2) versus kmax lags are judged significant based on the 10% critical values of the χ^2 distribution. The procedure is repeated by lowering k until a rejection of the null hypothesis that additional lags are insignificant occurs or the lower bound k = 0 is attained [1]. I shall call this procedure the "F–sig" where k is chosen to minimize $t_{\hat{\alpha}}(i,T_B,k)$ given that the value of k yields an insignificant F–test for additional lags of first differences in the autoregressions for each value of T_B.

The last procedure investigated is akin to the one used in Perron (1989). Here k is chosen to minimize $t_{\hat{\alpha}}(i,T_B,k)$ subject to the requirement that the t–statistic on the parameter of the last included lag in the autoregressions is significant and that the parameter on the last lag in higher order autoregressions is insignificant, up to some maximum order kmax [2]. Here the asymptotic N(0,1) distribution is used to carry inference with a 10% two–sided test.

The two procedures described above are intended to impose more restrictions on the choice of T_B and k. The critical values on the t–statistic for $\alpha = 1$ should therefore be smaller (in absolute value) than in the unrestricted case. Hence, one may hope for higher power.

Tables I, II and III present the simulated critical values obtained using the procedures described above for Models 1, 2 and 3 respectively. Each set of results was obtained using

2,000 replications of the t–statistic using the appropriate autoregression estimated from data generated by (4). The program was coded using the C language and N(0,1) random deviates were obtained from the routine RAN1 of Press et al. (1986). For the procedures where the choice of k is correlated with the data, the value of kmax was specified to be 5 for purely computational reasons [3].

Table I contains the critical values for Model 1 where only a change in the intercept of the trend function is allowed. Upon comparison with the results in Perron (1989), it is readily seen that the critical values are much lower when T_B is allowed to be data dependent than when it is considered fixed. For example, with T = 100 and k = 0, the 5% critical value is –4.93 when minimizing over T_B as opposed to –3.76 when the date of the break is considered fixed at mid–sample. The critical values are fairly stable as k changes provided that k is held fixed when minimizing over T_B. However, the critical values decrease substantially if the t–statistic is chosen by minimizing over both T_B and k. These are presented in the rows labelled k = k*min. For example, with T = 100 , the 5% critical value is –5.24. The critical values for the procedure where k is chosen according to recursive F–tests on the coefficients of the lagged first differences are presented in the rows labelled k = k(F–sig). The critical values obtained are somewhere in between the values given by the two procedures discussed earlier. For example, the 5% point with T = 100 is –5.09. Generally, the critical values increase as T increases. The critical values for the procedure where k is chosen according to a t–test on the last included lag in the autoregression are presented in the rows labelled k=k(t–sig). The resulting values are close to the values obtained using the recursive F–test procedure [4].

The critical values for Model 2 (allowing both a change in slope and intercept) and Model 3 (allowing only a change in slope) are presented in Tables II and III, respectively. The presentation of the results is as in Table I. The same general features hold when comparing different procedures and different sample sizes. While comparing the critical values for the three models, it is interesting to note that the highest critical values (in the left tail of the distribution) occur for Model 3. This is contrary to the fixed T_B case where the highest critical values correspond to Model 1.

Table IV presents the critical values of the statistic $t^*_{\alpha,\theta}(1)$ for Model 1 obtained with T_B chosen to minimize $t_{\hat{\theta}}$, the t–statistic on the parameter for the change in intercept of the trend function. The same procedures as before are used to select k : 1) fixed k ; 2) k chosen to minimize $t_{\hat{\theta}}$ for any given T_B, k*min ; 3) k chosen according to a recursive F–test on additional lags, k(F–sig) ; and 4) k chosen according to a t–test on the last included lag, k(t–sig). As can be seen from comparing the results in Tables I and IV, the critical values obtained when choosing T_B according to the values of $t_{\hat{\theta}}$ are substantially higher. This is simply due to the a priori imposition of a one–sided change in the intercept of the trend function.

Table V presents the corresponding critical values for the statistic $t^*_{\alpha,\gamma}(3)$, obtained with T_B chosen to minimize the value of $t_{\hat{\gamma}}$ in the first–stage regression (3.c.i). As stated earlier, this procedure does not impose any a priori restrictions on the date of the change but restricts the change to be a decrease in slope (i.e. a one–sided structural change). Much of the same comments made with respect to the statistic $t^*_{\alpha,\theta}(1)$ apply to $t^*_{\alpha,\gamma}(3)$, e.g. the procedures analyzed to choose k are the same and the resulting critical values are higher than with $t^*_{\alpha}(3)$. However, note that unlike $t^*_{\alpha,\theta}(1)$, here the selected value of T_B is independent of the method used to choose k. This is due to the fact that under Model 3 a two–step procedure is used where the trend function is estimated separately from the noise function.

The procedures described in this Section and the critical values obtained are used in the next sections to reassess my earlier findings and provide additional evidence using a wider range of macroeconomic time series.

4. EMPIRICAL RESULTS FOR THE NELSON–PLOSSER DATA SET

Table VI presents the empirical results for the Nelson–Plosser (1982) series for which, in Perron (1989), Model 1 was the specification of interest. The series, whose exact sources can be found in Nelson and Plosser (1982), have an horizon which starts at different dates for different series but which ends in 1970 for all series. The data set includes the following series : Real GNP, Nominal GNP, Real per capita GNP, Industrial Production, Employment, GNP Deflator, Consumer Price Index, Nominal Wage, Money Stock, Velocity and Interest Rate. In my previous study, I claimed a rejection of the unit root hypothesis for all these series except the Consumer Price Index, Velocity and Interest Rate. I also claimed, however, that the unit root hypothesis could be rejected for the Consumer Price Index and Velocity series when considering the post–1929 sample using the usual Dickey–Fuller (1979) test statistic.

The results presented in Table VI were obtained by choosing the break point T_B and the truncation lag parameter k in such a way that the t–statistic for testing that $\alpha = 1$ in regression (1.c) is minimized. For this set of results, kmax is specified to be 10 [3]. Hence, both the choice of T_B and k are correlated with the data. Results are presented for the three methods of choosing T_B and k. When all methods yield the same values only one regression is reported, otherwise separate regressions are listed. The statistics of most interest are the estimates of α and its t–statistic as well as four sets of p–values in the last 4 columns (reported to the nearest 1%). The first set of p–values is obtained using the distribution of the t–statistic in the fixed k scenario using the empirically selected value of k. These p–values are reported for reference purposes in each of the cases used to select the truncation lag parameter k. They are included because, as argued earlier, the asymptotic distributions of the statistics obtained under a data dependent method (using a test of significance on the included lags of the data in the regression) are likely to be the same as in the fixed k case under an asymptotic framework where the size of the test on the lags decreases to zero as the sample size increases.

The second set of p–values is obtained using the distribution of the t–statistic when k and T_B were chosen to minimize the t–statistic on α (k = k*min in Table I). The third set of p–values correspond to the critical values of the t–statistic when k and T_B are chosen according to a significance test on the coefficients of the lagged first differences of the data (k = k(F–sig) in Table I). The last set of p–values are those corresponding to the case where T_B and k are chosen so that the last lag is significant according to a t–test on its parameter and the last lag in higher order autoregressions has insignificant coefficients (k = k(t–sig) in Table I). The critical values used correspond to a sample of size 60, 80 or 100 whichever is closest to the actual sample size.

The empirical results show that the unit root hypothesis can be rejected at the 5% significance level or better, under either scenario about the choice of k, for Real GNP, Nominal GNP [6], Industrial Production and Nominal Wages. For the Employment series, the p–value under the fixed k scenario is .02 and when choosing k such as to minimize the t–statistic it is .07 ; finally when k is chosen according to a significance criterion, the p–value is .05 with the F–test and .09 with the t–test. Hence the null hypothesis of a unit root is also rejected for the employment series. The real per capita GNP and Money Stock series present a more ambiguous case. In the fixed k case the p–values are .06 and .07, respectively. When k is chosen according to a significance criterion with a F–test, the p–value for the Real per capita GNP series is .12, and .14 for the Money Stock series. These values are marginal for a rejection at the 10% level. Nevertheless, to analyze whether these results are due to low power or are specific to the data series used here, Section 6 presents results that were obtained with similar series drawn from alternative sources.

As in my previous study, the unit root hypothesis cannot be rejected for the Consumer Price Index, Velocity and Interest Rate series under any procedure. The choices of T_B and k obtained using the data dependent methods for choosing k are different but yield the same qualitative results.

The only series which offers a different picture from the fixed T_B case is the GNP Deflator. In the fixed k case the p–value is .12, with k unrestricted it is .30 and with k chosen according to either significance criteria it is .35. Hence, for this series, it appears that the rejection of the unit root hypothesis reported in Perron (1989) is not robust to the type of correlation between the choice of T_B and the data specified here. It must be kept in mind, however, that the type of correlation assumed here is an extreme one and it may well be the case that the rejection would not hold under a more plausible specification. Also of interest is the fact that the GNP Deflator series appears to behave in a manner similar to the Consumer Price Index : non–rejection of the unit root using the full sample but rejection using a standard Dickey–Fuller test on the post–1929 sample. Indeed from Table A.2 in Perron (1989), the t–statistic with $k = 7$ is –4.47 allowing rejection at less than a 1% significance level.

A comment is warranted about the choice of T_B selected according to these procedures. Except for the C.P.I, Velocity and Interest Rate series (for which the unit root is not rejected), the value of T_B is either 1929 or 1928. It is 1929 for the Nominal Wage and Money Stock series and 1928 for the others. While 1928 does not exactly correspond to the date specified in Perron (1989), the economic interpretation remains the same. The selection of 1928 is due to the presence of the dummy variable $D(TB)_t$ in regression (1.c), which takes value 1 at date $T_B + 1$ and 0 otherwise. This variable is needed to permit the change in the intercept of the trend function under both the null and alternative hypotheses. Hence, 1928 is often chosen because the dummy variable takes value 1 in 1929 and offers some additional fit to the 1929 crash over what the change in the intercept can do alone.

Table VII presents the results when T_B is chosen to minimize $t_{\hat{\theta}}$, the t–statistic on the parameter of the change in intercept, i.e. when imposing the one–sided restriction of a crash. The results are presented as in Table VI. The qualitative results are the same for most series. When a rejection of the unit root hypothesis occurred in Table VI, it does so again here and more strongly given that the tests have higher power. As was the case earlier, the unit root cannot be rejected for the GNP deflator series. Hence, for this particular series, my earlier conclusion is not robust to allowing the date of the change in the trend function to be unknown or perfectly correlated with the data. The results in Table VII offer, however, a different picture for three series. First, for the Employment series, the unit root hypothesis can be rejected at the 5% level (using any procedure) instead of 10% with the statistic $t_{\alpha}^*(1)$. More interestingly, the unit root hypothesis can now be rejected at the 10% level for the Real Per Capita GNP and Money Stock series. For example, the p–values under the F–sig procedure are .06 and .07, respectively. These results and the preceding ones show my (1989) conclusions to be robust to different specifications about the nature of the correlation between the data and the choice of the break point for all series except the GNP Deflator.

I now turn to the analysis of the Common Stock Price and Real Wage series of the Nelson–Plosser data set where Model 2 is specified, i.e allowing both a change in the intercept and the slope of the trend function. The procedures used and the presentation of the estimation results in Table VIII follow our previous analysis of Model 1 (in Table VI) except that kmax is now 5. Consider first the case where both T_B, the date of break, and k, the truncation lag parameter, are chosen to minimize the t–statistic. The date of break selected by the data for the Common Stock Price series is 1936 instead of 1929. Under the fixed k scenario, the p–value is .025, and with k unrestricted, it is .12. For the Real Wage series, the date T_B selected by the data is 1939; the p–value under the fixed k scenario is .04 and .16 when k is unrestricted.

Table VIII also presents the results obtained when T_B and k are chosen according to significance tests on the coefficients of the lagged first differences of the data. The results for the Real Wage series are basically identical but now k is equal to 3. Using the appropriate critical values from Table II (rows k(F–sig) and k(t–sig)), the p–values are .06 and .08, hence allowing rejection of the unit root hypothesis. The results are somewhat different for the Common Stock Price series. With the F–test and the t–test, the value of k selected is still 1 but the date of break is 1928, much more in accordance with results presented in Perron (1989). Moreover the value of the t–statistic for $\alpha = 1$ is very similar and yields p–values of .04 and .06, respectively, allowing a rejection of the unit root hypothesis.

5. RESULTS WITH AN INTERNATIONAL DATA SET FOR POSTWAR REAL GNP

This Section analyzes an international data set of postwar quarterly real GNP or GDP series. The type (GNP or GDP) and the sampling period of the series used were basically dictated by data availability, but also to some extent by a desire to obtain results that were comparable with previous studies. The countries analyzed and the type and sampling period of the series are the following : USA (GNP ; 1947:1–1986:3) ; Canada (GDP ; 1947:1–1989:1) ; Japan (GNP ; 1957:1–1988:4) ; France (GDP ; 1965:1–1988:3) ; Germany (GNP ; 1960:1–1986:2) ; Italy (GDP ; 1960:1–1985:1) ; and the United Kingdom (GDP ; 1957:1–1986:3). The USA series is the same as that used in Perron (1989) to allow proper comparison. The series for Canada was obtained from the Cansim data bank and the series for Japan and France from the IFS data tape. The remaining series (U.K., Germany and

Italy) are from Data Resources Inc. and are those used in Campbell and Mankiw (1989). All series are seasonally adjusted and at annual rates, except for the USA and the United Kingdom which are at quarterly rates. The plots of the logarithm of each series are presented in Figures 1 through 7. In these graphs the solid line is the estimated trend function allowing a one–time change in slope. The date of the change varies for each series and was selected according to the data dependent method where both T_B and k are chosen (without a significance test) to minimize the t–statistic on α using regression Model 3.

The first set of results concerning the unit root tests are presented in Table IX. Using the fixed k critical values (Table III), the unit root is rejected, at close to the 5% level, for all series except Italy. Using the critical values allowing for an unrestricted data dependent method for the choice of k, the results are not as sharp. For Japan, the unit root is strongly rejected (p–value of .02). Indeed, the case of Japan is particularly striking in view of both the estimated t–statistic and the visual inspection of the graph of the series (Figure 3). First, the slope of the trend function has changed from 2.43% before 1971:3 to 1.01% after 1971:3 (a 58% decrease in the rate of growth). But more surprisingly the actual series follows closely this breaking trend function with very little variability.

The results are not as clear for most of the other series but some interesting cases still emerge. Consider the case of the United Kingdom. Using the critical values with k unrestricted, the p–value is .09, allowing rejection of the unit root hypothesis at the 10% level. The graph of the series is presented in Figure 7. While the series behavior is not as striking as for Japan, the change in the slope of the trend function (with the break point evaluated at 1973:3) is still quite large. Indeed, the quarterly growth rate is .74% before 1973:3 and .34% after 1973:3, representing a 54% reduction. Using the F–test and t–test for selecting T_B and k, the p–values are .06 and .08 respectively, allowing rejection of the unit root at the 10% level.

The results for Canada, France and Germany are similar in terms of the t–statistics obtained. They range from –4.26 to –4.34 yielding p–values for the unit root hypothesis (using the critical values in the rows k(F–sig) and k(t–sig) in Table III) between .13 and .17. While the unit root cannot be statistically rejected at the 10% level, the results are not very much at odds with the hypothesis that the series are better construed as stationary fluctuations around a breaking trend function. Such is not the case, however, with the GDP series from Italy. Here the p–values are different enough to cast some doubts on the trend–stationary hypothesis and, hence, the unit root is not rejected for this country. For the USA, the results are somewhat ambiguous; with the F–test criterion the p–value is .13 but with the t–test criterion it is .51.

It is interesting to look at the estimated change in the slope of the trend function and the dating of the break implied by the estimation procedure. The results obtained for the percentage decrease in the rates of growth are as follows (besides those mentioned above) : USA, 37% ; Canada, 36% ; France 66% , Germany 56 % ; Italy 57%. These figures are indeed quite large and suggest, besides the unit root issue, that an important structural change has occurred. The dates of the break point are different for each country but they are all close to the year 1973, associated with the first oil price shock. They vary between 1971:3 (Japan) and 1976:3 (Canada). It is to be noted, however, that the statistical method used here is not directly geared at providing a consistent estimate of the date of change in the slope of the trend function. Hence, the break dates should be viewed as approximate.

As discussed in Section 3, a method for selecting T_B based on $t^*_{\alpha,\gamma}(3)$, the t–statistic associated with a minimized value of the parameter of the change in slope of the trend function, is likely to allow tests with greater power. The results obtained by applying such a procedure are presented in Table X. Indeed, such a procedure appears more powerful. Consider first the case where k is chosen to minimize the t–statistic for testing $\alpha = 1$. The p–values (corresponding to the critical values in Table V, rows k = k*min) are less than .05 for Japan and the United Kingdom and less than .10 for Canada and France. For the USA and Germany, the p–values are .12. For Italy, the evidence clearly again does not favor rejecting the unit root (with a p–value of .21). The results are still more conclusive using a significance test to select the order of the autoregression. Using a t–test on the last lag of the first differences of the data (t–sig), the p–values for the null hypothesis of a unit root are at most .11 for all countries except Italy. Using the recursive F–test procedure, the p–values are smaller than .10 for USA, Japan, France and the United Kingdom. They are .13 and .14 for Canada and Germany, respectively. The results in Table X show that a simple imposition of a one–sided downward change in the slope of the trend function (with the date of break being considered unknown) is enough to warrant rejection of the unit root hypothesis at close to the 10% level for all countries except Italy.

I view these results, especially given the small span of the data, as substantial evidence against the unit root. It is indeed somewhat remarkable to consistently obtain p–values in this range given the relatively low power of tests of the unit root hypothesis when using a data series over a short span (see Perron (1990)). Given that the statistical procedure used is one where an extreme assumption is made about the correlation of the choice of the break point and the data (yielding a procedure with low power compared to the case where T_B is assumed fixed), I view these results as consistent with the hypothesis that the series are best characterized as stationary fluctuations around a breaking trend function with a change in slope near 1973. In a sense, the p–values under the unrestricted k scenario are not, I believe, different enough to overturn the rejections obtained under either the fixed k case (still allowing T_B to be data dependent) or the fixed T_B case.

6. EMPIRICAL RESULTS FOR SOME ADDITIONAL SERIES

Some additional series from alternative sources are analyzed in this Section. First, for the Real per Capita GNP and the Money Supply series, I use data sources other than the Nelson–Plosser data set. As discussed in Section 4, rejection of the unit root hypothesis is borderline for these series when allowance is made for an unknown break point without restricting the analysis to a one–sided change (see Table VI). My aim is to see if this ambiguity is particular to the series in the Nelson–Plosser data set. To this effect I present in Table XI evidence related to the Friedman and Schwartz (1982) Real per Capita GNP series for the same period (1909–1970). The series is graphed in Figure 8. The results imply a maximum p–value for the unit root hypothesis of .03 under any selection method for k and T_B [7]. Hence, it is easy to reject the unit root hypothesis for this series.

Consider now an alternative source for the money supply variable. I used the annual M2 series supplied in Balke and Gordon (1986) from 1869 to 1973. A graph of the series is presented in Figure 9. The results in Table XI again show a strong rejection of the unit root hypothesis with a p–value of at most .05 under any procedure.

Following the work of Hall (1978), much interest has been given to the time series behavior of consumption. To this effect, I analyze a data set consisting of historical series extending from 1889 to 1973 for Nominal Consumption, Real Consumption, their per capita counterparts, the Consumption Price Index and also the Population series. This data set is a subset of that used in Grossman and Shiller (1981). The graph of these series are presented in Figures 10 through 15. The results concerning the unit root tests are also presented in Table XI. For the Nominal and Real Consumption series the unit root can be rejected with a p–value of less than .01 under any procedure. The series again exhibit a significant decline in their level in 1929. For the Nominal per Capita Consumption series, rejection is still possible with a p–value of at most .03 but the picture is different with the Real per Capita Consumption series. Here the unit root cannot be rejected even though the break point is again associated with the 1929 crash. The results for the Consumption Price Index and Population series also imply a non–rejection of the unit root.

Table XII presents the unit root tests obtained using the statistic $t^*_{\alpha,\theta}(1)$ where a one–sided structural change is considered. The qualitative results are the same except for the Real per Capita Consumption series where now the p–values are close to .10, casting some doubts on the unit root characterization for this series as well.

The results for the Population series are interesting because they may explain why it is more difficult to reject the unit root hypothesis for the Real per Capita GNP series than for the Real GNP series itself. The results for the Consumption Price Index parallel the results analyzed earlier for the CPI and the GNP Deflator. It therefore appears that the unit root hypothesis cannot be rejected for a broad class of price indices. Hence, either another break is needed to make the series stationary (witness the sharp rise in the consumption price index before 1920, followed by the sharp decline in the early thirties, see Figure 14), or the price series are actually integrated processes. The fact that the unit root can easily be rejected for all price indices using post 1930 data points to the plausibility of the former explanation.

7. CONCLUDING COMMENTS

This paper documents the robustness of the results presented in Perron (1989) which claimed that the unit root hypothesis could be rejected for most macroeconomic time series if allowance was made for the possibility of a one time shift in the intercept (in 1929) and the slope (in 1973) of the trend function. Unlike in my previous study, I analyze the case where the date of the break point is explicitly correlated with the data and provided a set of critical values to carry inference in such a case. This work is not intended as a substitute for the statistical procedure presented in my earlier paper but rather as a complement. Indeed, a case can often be made for using critical values that are based on the assumption of no correlation between the choice of the break point and the data. On the one hand, it may represent a close approximation to the actual extent of the correlation. On the other hand, each investigator may differ as to the amount of a priori information he or she is willing to incorporate into the analysis.

Another issue not yet investigated but worth of further research concerns the power of the tests. There appears to be a clear tradeoff between power and the amount of a priori information one is willing to incorporate with respect to the choice of the break point. The presumption is clearly that a procedure imposing no such a priori information, such as the ones presented in this paper, has relatively low power. In this respect, the widespread rejection of the unit root hypothesis, even when assuming a perfect correlation between the choice of the break point and the data, is quite remarkable. More work is indeed required to assess the power properties of these statistical procedures.

The results also show that the least convincing rejection occurs for the USA postwar real GNP (apart from Italy). This was the series analyzed in Christiano (1988). In the context of the international data set analyzed in Section 5, his critique appears less forceful. As shown, the USA had one of the smallest decline in the rate of growth of Real GNP after 1973. For the other G–7 countries, the fall was more severe and the results strongly support the fact that fluctuations ought to be viewed as stationary around a trend function with a changing slope near 1973. These results contrast with Campbell and Mankiw's (1989) conclusions that fluctuations have permanent effects on the level of real GNP. By comparing our results for the international data set, it appears that the magnitude of the long term effect of a shock is proportional to the extent of the change in the slope of the trend function.

FOOTNOTES

1 In the empirical applications, the lower bound was set at k = 1.

2 If no coefficients on the lags of the first differences of the data are significant, k = 1 is chosen for that value of T_B.

3 The computation time becomes exorbitant as kmax increases, e.g. in the order of 4 to 5 days on a 386 when kmax = 5.

4 The simulated critical values involving a test of significance on the lagged first differences of the data are for tests of size 10%. I chose this value on the principle that it is usually safer to include extra lags in terms of achieving the correct size in finite samples (at the expense of a loss in power). However, critical values with 5% tests were also computed and are available on request. They are not included since they are very similar to those with the 10% tests.

5 The choice of kmax is somewhat arbitrary. On the one hand, one would like a large value to have as unrestricted a procedure as possible. On the other hand a large value of kmax yields problems of multicollinearity in the data and also a substantial loss of power. I selected kmax as follows. First, I selected quite a large value (15 usually) and calculated the values of k and T_B for which the t–statistic for $\alpha = 1$ was smallest. I then selected kmax for the significance tests according to the value of k selected. If that value of k was smaller than 5, I chose kmax = 5; if it was above 5, I either selected kmax at 10 (Tables VI and VII) or 12 (Tables XI and XII). For some quarterly series I allowed kmax = 15 (e.g. USA in Table IX). This was done provided that the estimated autoregressions dis not show any sign of remaining correlation in the residuals as indicated by the Box–Pierce statistic. Most of the results are robust to alternative choices for kmax.

6 For the Nominal GNP series, a value of kmax was found binding in the sense that k = 10 was selected. Hence kmax was increased to 15 which again was found to be binding. I did not increase kmax further given the relatively few number of observations.

Nevertheless, the conclusion is robust to basically any value of the truncation lag parameter k chosen.

7 The rejection of the unit root for the Friedman and Schwartz series is robust to using the longer samples 1900–1973 and 1890–1973. It is not robust to using the whole sample 1869–1973. In the latter case, however, the unit root can be rejected using a standard Dickey–Fuller procedure without any allowance for a possible change in the trend function.

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TABLE I : Percentage Points of the Distribution of $t^*_\alpha(1)$; Model 1.

	1.0%	2.5%	5.0%	10.0%	50.0%	90.0%	95.0%	97.5%	99.0%
T = 60									
k = 0	–5.61	–5.25	–4.93	–4.59	–3.62	–2.85	–2.64	–2.43	–2.22
k = 2	–5.57	–5.18	–4.84	–4.54	–3.64	–2.82	–2.57	–2.31	–2.05
k = 5	–5.49	–5.11	–4.88	–4.55	–3.61	–2.67	–2.41	–2.13	–1.77
k = 8	–5.52	–5.15	–4.86	–4.50	–3.52	–2.52	–2.20	–1.85	–1.46
k = 11	–5.73	–5.20	–4.90	–4.51	–3.48	–2.37	–1.96	–1.60	–1.19
k = k*min	–6.06	–5.74	–5.40	–5.09	–4.11	–3.26	–3.02	–2.78	–2.51
k = k(F–sig)	–5.83	–5.49	–5.21	–4.91	–3.91	–3.00	–2.70	–2.41	–1.96
k = k(t–sig)	–5.92	–5.58	–5.23	–4.92	–3.91	–3.00	–2.74	–2.55	–2.25
T = 80									
k = 0	–5.38	–5.11	–4.92	–4.62	–3.68	–2.85	–2.65	–2.43	–2.14
k = 2	–5.47	–5.10	–4.85	–4.58	–3.64	–2.79	–2.57	–2.34	–2.05
k = 5	–5.38	–5.08	–4.78	–4.53	–3.64	–2.76	–2.48	–2.27	–1.71
k = 8	–5.40	–5.11	–4.84	–4.53	–3.59	–2.61	–2.32	–1.95	–1.53
k = 11	–5.43	–5.10	–4.83	–4.53	–3.57	–2.53	–2.20	–1.81	–1.39
k = k*min	–5.86	–5.50	–5.26	–5.00	–4.06	–3.19	–2.94	–2.77	–2.45
k = k(F–sig)	–5.77	–5.35	–5.15	–4.84	–3.87	–2.96	–2.70	–2.41	–2.12
k = k(t–sig)	–5.77	–5.31	–5.09	–4.84	–3.88	–2.95	–2.73	–2.55	–2.22
T = 100									
k = 0	–5.49	–5.15	–4.93	–4.60	–3.70	–2.95	–2.67	–2.46	–2.26
k = 2	–5.43	–5.12	–4.84	–4.56	–3.68	–2.87	–2.57	–2.33	–2.15
k = 5	–5.40	–5.05	–4.85	–4.55	–3.68	–2.84	–2.56	–2.28	–1.95
k = 8	–5.26	–5.00	–4.76	–4.49	–3.62	–2.75	–2.47	–2.15	–1.80
k = 11	–5.32	–4.97	–4.76	–4.48	–3.60	–2.68	–2.35	–2.02	–1.53
k = k*min	–5.86	–5.52	–5.24	–4.97	–4.04	–3.23	–2.96	–2.68	–2.50
k = k(F–sig)	–5.70	–5.35	–5.09	–4.82	–3.89	–3.00	–2.74	–2.46	–2.22
k = k(t–sig)	–5.70	–5.36	–5.10	–4.82	–3.87	–3.05	–2.75	–2.46	–2.22

TABLE II : Percentage Points of the Distribution of $t^*_\alpha(2)$; Model 2.

	1.0%	2.5%	5.0%	10.0%	50.0%	90.0%	95.0%	97.5%	99.0%
T = 70									
k = 0	−6.65	−5.80	−5.43	−4.99	−3.98	−3.13	−2.89	−2.73	−2.52
k = 2	−6.27	−5.67	−5.24	−4.87	−3.88	−3.06	−2.82	−2.63	−2.46
k = 5	−6.21	−5.58	−5.27	−4.90	−3.89	−3.00	−2.74	−2.54	−2.34
k = 8	−5.95	−5.58	−5.17	−4.80	−3.80	−2.91	−2.66	−2.45	−2.23
k = 11	−6.10	−5.52	−5.22	−4.82	−3.79	−2.89	−2.57	−2.36	−2.13
k = k*min	−7.50	−6.92	−6.35	−5.75	−4.53	−3.66	−3.43	−3.21	−2.91
k = k(F−sig)	−6.22	−5.81	−5.52	−5.22	−4.21	−3.28	−3.00	−2.76	−2.54
k = k(t−sig)	−6.32	−5.90	−5.59	−5.29	−4.24	−3.32	−3.08	−2.85	−2.67
T = 100									
k = 0	−6.77	−5.78	−5.41	−5.02	−4.01	−3.22	−3.01	−2.76	−2.59
k = 2	−6.30	−5.57	−5.29	−4.92	−3.95	−3.13	−2.89	−2.68	−2.48
k = 5	−6.00	−5.53	−5.22	−4.89	−3.93	−3.07	−2.83	−2.66	−2.38
k = 8	−5.83	−5.38	−5.13	−4.77	−3.84	−3.02	−2.81	−2.60	−2.35
k = 11	−6.13	−5.57	−5.16	−4.83	−3.83	−2.94	−2.70	−2.50	−2.34
k = k*min	−8.71	−7.28	−6.49	−5.74	−4.47	−3.62	−3.40	−3.20	−2.97
k = k(F−sig)	−6.07	−5.72	−5.48	−5.17	−4.17	−3.29	−3.05	−2.83	−2.58
k = k(t−sig)	−6.21	−5.86	−5.55	−5.25	−4.22	−3.35	−3.13	−2.85	−2.63

TABLE III : Percentage Points of the Distribution of $t^*_\alpha(3)$; Model 3.

	1.0%	2.5%	5.0%	10.0%	50.0%	90.0%	95.0%	97.5%	99.0%
T = 100									
k = 0	–5.15	–4.81	–4.49	–4.19	–3.20	–2.37	–2.15	–1.99	–1.84
k = 2	–5.04	–4.70	–4.43	–4.09	–3.12	–2.35	–2.17	–2.02	–1.89
k = 5	–4.88	–4.57	–4.33	–4.05	–3.11	–2.39	–2.19	–2.09	–1.91
k = 8	–4.78	–4.46	–4.13	–3.87	–3.04	–2.32	–2.17	–2.05	–1.91
k = k*min	–5.50	–5.15	–4.89	–4.57	–3.57	–2.77	–2.59	–2.41	–2.26
k = k(F–sig)	–5.41	–4.99	–4.74	–4.44	–3.36	–2.53	–2.34	–2.21	–2.08
k = k(t–sig)	–5.45	–5.11	–4.83	–4.48	–3.44	–2.60	–2.39	–2.22	–2.06
T = 150									
k = 0	–4.99	–4.62	–4.40	–4.12	–3.12	–2.32	–2.15	–1.97	–1.84
k = 2	–4.97	–4.66	–4.37	–4.06	–3.14	–2.35	–2.16	–2.01	–1.86
k = 5	–4.85	–4.57	–4.38	–4.06	–3.11	–2.35	–2.19	–2.04	–1.91
k = 8	–4.74	–4.48	–4.21	–3.99	–3.08	–2.34	–2.18	–2.01	–1.88
k = k*min	–5.34	–5.04	–4.76	–4.48	–3.50	–2.67	–2.47	–2.30	–2.17
k = k(F–sig)	–5.19	–4.85	–4.59	–4.31	–3.32	–2.47	–2.29	–2.11	–1.96
k = k(t–sig)	–5.28	–4.96	–4.65	–4.38	–3.33	–2.50	–2.30	–2.13	–1.93

TABLE IV : Percentage Points of the Distribution of $t^*_{\alpha,\theta}(1)$; Model 1 ; Choosing T_B by Minimizing $t_{\hat{\theta}}$

	1.0%	2.5%	5.0%	10.0%	50.0%	90.0%	95.0%	97.5%	99.0%
T = 60									
k = 0	–5.37	–4.96	–4.55	–4.24	–3.19	–1.98	–1.54	–1.23	–0.64
k = 2	–5.31	–4.88	–4.57	–4.26	–3.13	–1.85	–1.26	–0.74	0.03
k = 5	–5.23	–4.87	–4.51	–4.20	–3.07	–1.43	–0.62	–0.05	0.74
k = 8	–5.15	–4.80	–4.45	–4.19	–2.96	–0.88	–0.13	0.55	1.17
k = 11	–5.33	–4.88	–4.54	–4.20	–2.89	–0.44	0.25	0.95	1.56
k = k*min	–5.78	–5.37	–4.94	–4.52	–3.32	–1.50	–0.74	0.04	0.66
k = k(F–sig)	–5.58	–5.15	–4.88	–4.47	–3.33	–1.60	–0.84	–0.05	0.56
k = k(t–sig)	–5.70	–5.21	–4.92	–4.53	–3.32	–1.79	–1.14	–0.35	0.42
T = 80									
k = 0	–5.21	–4.91	–4.68	–4.34	–3.24	–2.11	–1.74	–1.28	–0.64
k = 2	–5.28	–4.85	–4.60	–4.27	–3.20	–1.98	–1.48	–0.81	–0.05
k = 5	–5.14	–4.84	–4.55	–4.21	–3.15	–1.73	–0.88	–0.26	0.69
k = 8	–5.25	–4.88	–4.57	–4.22	–3.06	–1.09	–0.36	0.27	0.95
k = 11	–5.17	–4.87	–4.51	–4.18	–3.00	–0.80	0.05	0.65	1.40
k = k*min	–5.63	–5.21	–4.92	–4.58	–3.34	–1.75	–0.99	–0.26	0.67
k = k(F–sig)	–5.50	–5.11	–4.85	–4.53	–3.33	–1.86	–1.06	–0.32	0.67
k = k(t–sig)	–5.59	–5.09	–4.83	–4.54	–3.33	–1.92	–1.19	–0.46	0.34
T = 100									
k = 0	–5.17	–4.90	–4.60	–4.30	–3.27	–2.16	–1.72	–1.33	–0.77
k = 2	–5.21	–4.85	–4.58	–4.28	–3.21	–1.98	–1.49	–0.94	–0.33
k = 5	–5.09	–4.85	–4.57	–4.27	–3.17	–1.81	–1.10	–0.58	0.15
k = 8	–5.05	–4.75	–4.50	–4.19	–3.11	–1.51	–0.62	0.02	0.88
k = 11	–5.06	–4.76	–4.47	–4.17	–3.08	–1.16	–0.26	0.45	0.99
k = k*min	–5.54	–5.13	–4.87	–4.56	–3.36	–1.86	–1.18	–0.55	0.12
k = k(F–sig)	–5.42	–5.03	–4.80	–4.47	–3.33	–1.92	–1.33	–0.77	0.02
k = k(t–sig)	–5.43	–5.05	–4.83	–4.50	–3.34	–2.02	–1.38	–0.84	–0.05

TABLE V : Percentage Points of the Distribution of $t^*_{\alpha,\gamma}(3)$; Model 3 ; Choosing T_B by Minimizing $t_{\hat{\gamma}}$.

	1.0%	2.5%	5.0%	10.0%	50.0%	90.0%	95.0%	97.5%	99.0%
T = 100									
k = 0	–4.96	–4.56	–4.20	–3.87	–2.73	–1.68	–1.37	–1.09	–0.67
k = 2	–4.75	–4.43	–4.08	–3.77	–2.68	–1.69	–1.35	–1.03	–0.69
k = 5	–4.57	–4.31	–4.00	–3.65	–2.59	–1.70	–1.38	–1.14	–0.82
k = 8	–4.37	–4.03	–3.72	–3.48	–2.51	–1.68	–1.46	–1.16	–0.92
k = k*min	–5.31	–4.93	–4.60	–4.22	–3.05	–2.05	–1.70	–1.42	–1.06
k = k(F–sig)	–5.02	–4.69	–4.40	–3.99	–2.76	–1.76	–1.46	–1.12	–0.79
k = k(t–sig)	–5.26	–4.82	–4.44	–4.07	–2.83	–1.76	–1.45	–1.12	–0.83
T = 150									
k = 0	–4.71	–4.37	–4.10	–3.75	–2.69	–1.66	–1.31	–1.03	–0.52
k = 2	–4.69	–4.35	–4.04	–3.76	–2.67	–1.66	–1.29	–0.94	–0.56
k = 5	–4.56	–4.35	–4.02	–3.69	–2.62	–1.64	–1.32	–1.00	–0.64
k = 8	–4.58	–4.21	–3.92	–3.57	–2.57	–1.67	–1.38	–1.04	–0.70
k = k*min	–5.12	–4.71	–4.46	–4.12	–2.98	–1.94	–1.56	–1.21	–0.83
k = k(F–sig)	–4.89	–4.54	–4.27	–3.93	–2.74	–1.70	–1.33	–1.01	–0.64
k = k(t–sig)	–5.00	–4.63	–4.36	–3.99	–2.78	–1.72	–1.40	–1.07	–0.49

TABLE VI : Empirical Results , Nelson – Plosser Data ; $t^*_\alpha(1)$, kmax = 10 .

Regression : $y_t = \mu + \theta DU_t + \beta t + \delta D(TB)_t + \alpha y_{t-1} + \Sigma_{i=1}^{k} c_i \Delta y_{t-i} + e_t$.

Series	Sample	T	T_B	k	$\hat{\mu}$	$\hat{\theta}$	$\hat{\beta}$	$\hat{\delta}$	$\hat{\alpha}$	$t_{\hat{\alpha}}$	p–value (k fixed)	p–value (min k)	p–value (F sig)	p–value (t sig)
Real GNP	1909–1970	62	1928	9	3.59 (5.96)	–.20 (–5.13)	.030 (6.10)	.217 (3.34)	.190	–5.93	<.01	.01	<.01	
			1928	8	3.27 (5.53)	–.19 (–4.79)	.027 (5.63)	.197 (2.90)	.268	–5.50	.01			.03
Nominal GNP [a]	1909–1970	62	1928	11	5.99 (8.14)	–.33 (–6.34)	.040 (8.40)	.399 (4.35)	.404	–8.16	<.01	<.01		<.01
			1928	15	5.05 (6.27)	–.28 (–5.94)	.034 (6.62)	.361 (4.70)	.497	–6.21	<.01		<.01	
Real per Capita GNP	1909–1970	62	1928	9	4.72 (4.81)	–.15 (–3.73)	.016 (4.93)	.158 (2.26)	.313	–4.81	.06	.17	.12	
			1928	7	3.56 (4.52)	–.12 (–3.90)	.012 (4.58)	.097 (1.62)	.484	–4.51	.10			.21
Industrial Production	1860–1970	111	1928	8	–.19 (–3.66)	–.33 (–5.18)	.035 (5.97)	.243 (2.34)	.272	–6.01	<.01	<.01	<.01	<.01
Employment	1890–1970	81	1928	8	4.17 (5.17)	–.06 (–3.42)	.007 (5.01)	.058 (1.67)	.586	–5.14	.02	.07	.05	
			1928	7	3.52 (4.94)	–.05 (–3.11)	.006 (4.76)	.043 (1.32)	.650	–4.91	.04			.09
GNP Deflator	1889–1970	82	1928	10	.80 (4.44)	–.11 (–3.07)	.009 (4.45)	.123 (1.88)	.712	–4.43	.12	.30		
			1928	5	.61 (4.17)	–.10 (–3.28)	.007 (4.18)	.065 (1.34)	.783	–4.14	.22		.35	.35

TABLE VI (cont'd) : Empirical Results , Nelson – Plosser Data ; $t^*_\alpha(1)$, kmax = 10 .

Regression : $y_t = \mu + \theta DU_t + \beta t + \delta D(TB)_t + \alpha y_{t-1} + \Sigma_{i=1}^{k} c_i \Delta y_{t-i} + e_t$.

Series	Sample	T	T_B	k	$\hat{\mu}$	$\hat{\theta}$	$\hat{\beta}$	$\hat{\delta}$	$\hat{\alpha}$	$t_{\hat{\alpha}}$	p–value (k fixed)	p–value (min k)	p–value (F sig)	p–value (t sig)
C.P.I.	1860–1970	111	1939	4	.18 (3.13)	.03 (2.10)	.0007 (3.17)	–.037 (–.95)	.942	–3.48	.62	.81		
			1939	5	.16 (2.90)	.03 (2.00)	.0005 (2.36)	–.034 (–.89)	.948	–3.09	.81		.88	.88
Wages	1900–1970	71	1929	7	2.22 (5.45)	–.19 (–4.32)	.020 (5.37)	.085 (1.36)	.619	–5.41	<.01	.03		.02
			1929	9	2.23 (4.98)	–.19 (–4.17)	.019 (4.90)	.090 (1.31)	.618	–4.61	.08		.16	
Money Stock	1889–1970	82	1929	7	.25 (5.31)	–.08 (–2.80)	.014 (4.54)	.036 (.75)	.783	–4.69	.07	.20	.14	
			1927	6	.20 (4.73)	–.06 (–2.50)	.011 (4.29)	.073 (1.48)	.831	–4.30	.17			.28
Velocity	1869–1970	102	1949	8	.23 (2.27)	.10 (2.95)	–.002 (–2.29)	–.022 (–.29)	.830	–2.81	.89		.94	
			1948	1	.25 (3.15)	.10 (3.09)	–.003 (–3.10)	–.114 (–1.63)	.830	–3.49	.62	.81		
			1946	1	.21 (3.00)	.09 (3.20)	–.002 (–3.07)	–.034 (–.48)	.855	–3.30	.75			.80
Interest Rate	1900–1970	71	1966	9	.50 (1.56)	.97 (3.97)	–.0009 (–.34)	–.726 (–2.06)	.876	–1.82	.98	>.99		
			1965	3	.27 (1.24)	.72 (3.86)	–.0005 (–.25)	–.254 (–.85)	.934	–1.35	>.99		>.99	
			1963	3	.30 (1.26)	.56 (3.44)	–.001 (–.51)	–.303 (–1.01)	.928	–1.35	>.99			>.99

[a] : For Nominal GNP, kmax = 15 (See footnote 4). In this, and subsequent tables, the values in parentheses are the t–statistics.

Table VII : Empirical Results ; Nelson–Plosser Data Set ; Model 1 .

$t^*_{\alpha,\theta}(1)$; Choosing T_B minimizing $t_{\hat{\theta}}$; kmax = 10.

Regression : $y_t = \mu + \theta DU_t + \beta t + \delta D(TB)_t + \alpha y_{t-1} + \Sigma_{i=1}^{k} c_i \Delta y_{t-i} + e_t$.

Series	T_B	k	$t_{\hat{\theta}}$	$\hat{\alpha}$	$t_{\hat{\alpha}}$	p–value (k fixed)	p–value (min k)	p–value (F sig)	p–value (t sig)
Real GNP	1928	9	–5.13	.190	–5.93	<.01	<.01	<.01	
	1928	8	–4.79	.267	–5.50	<.01			.02
Nominal GNP[a]	1929	11	–6.78	.231	–7.86	<.01	<.01		<.01
	1928	15	–5.94	.497	–6.21	<.01		<.01	
Real Per Capita GNP	1928	9	–3.73	.313	–4.81	.02	.06	.06	
	1928	7	–3.31	.484	–4.51	.04			.10
Industrial Production	1928	8	–5.18	.272	–6.01	<.01	<.01	<.01	<.01
Employment	1928	8	–3.42	.586	–5.14	.01	.03	.02	
	1928	7	–3.11	.650	–4.91	.02			.04
GNP Deflator	1919	10	–3.63	.802	–4.20	.09	.19		
	1919	5	–3.51	.886	–3.24	.46		.54	.54
C.P.I.	1919	4	–3.40	.983	–1.13	.95	.95		
	1919	5	–3.12	.982	–1.16	.95		.96	.96
Wages	1929	7	–4.32	.619	–5.41	<.01	.02		.01
	1929	9	–4.10	.635	–4.62	.04		.08	
Money Stock	1929	9	–2.91	.744	–4.36	.07	.15		
	1929	7	–2.80	.783	–4.69	.04		.07	
	1928	6	–2.63	.825	–4.28	.09			.15
Velocity	1880	6	–3.06	.923	–1.71	.91	.92		
	1880	5	–2.74	.928	–1.62	.92		.93	
	1880	1	–2.56	.892	–2.52	.81			.80
Interest Rate	1920	1	–4.22	1.074	1.84	>.99	>.99		>.99
	1920	7	–3.54	1.125	2.21	>.99		>.99	

[a] : For Nominal GNP, kmax = 15 (see footnote 4).

TABLE VIII : Empirical Results , Nelson – Plosser Data ; $t^*_\alpha(2)$; kmax = 5 .

Regression : $y_t = \mu + \theta DU_t + \beta t + \gamma DT_t + \delta D(TB)_t + \alpha y_{t-1} + \Sigma_{i=1}^{k} c_i \Delta y_{t-i} + e_t$.

Series	Sample	T	T_B	k	$\hat{\mu}$	$\hat{\theta}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\delta}$	$\hat{\alpha}$	$t_{\hat{\alpha}}$	p–value (k fixed)	p–value (min k)	p–value (F sig)	p–value (t sig)
Common Stock Prices	1871–1970	100	1936	1	.47 (5.05)	–1.75 (–4.65)	.0076 (4.85)	.0228 (4.70)	.102 (.65)	.632	–5.58	.025	.12		
			1928	1	.35 (4.48)	–1.06 (–4.97)	.0065 (4.43)	.0141 (4.61)	.545 (3.53)	.616	–5.50	.03		.04	.06
Real Wages	1900–1970	71	1939	4	2.05 (5.45)	–.15 (–2.48)	.0098 (5.23)	.0058 (3.71)	–.085 (–2.28)	.297	–5.42	.04	.16		
			1939	3	1.78 (5.43)	–.11 (–2.02)	.0086 (5.26)	.0047 (3.38)	–.073 (–2.03)	.290	–5.41	.04		.06	.08

TABLE IX : Empirical Results , $t_{\alpha}^{*}(3)$; Model 3 , International Data Set .

Regression : $y_t = \mu + \beta t + \gamma DT_t^* + \tilde{y}_t$; $\tilde{y}_t = \alpha \tilde{y}_{t-1} + \Sigma_{i=1}^{k} c_i \Delta \tilde{y}_{t-i} + e_t$.

Series	Sample	T	T_B	k	$\hat{\mu}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\alpha}$	$t_{\hat{\alpha}}$	p–value (k fixed)	p–value (min k)	p–value (F sig)	p–value (t sig)
USA ; GNP (kmax = 15)	47:1–86:3	159	73:3	11	6.98 (1162.3)	.0087 (98.9)	–.0032 (–11.90)	.841	–4.34	.04	.13		
			73:4	12	6.98 (1161.8)	.0087 (99.3)	–.0032 (–11.79)	.875	–3.30	.36		.51	
			74:2	10	6.98 (1158.0)	.0086 (99.9)	–.0033 (–11.51)	.861	–3.99	.10			.21
Canada ; GDP (kmax = 5)	47:1–89:1	169	76:3	3	11.17 (2506.8)	.0122 (205.3)	–.0044 (–21.17)	.818	–4.26	.06	.16	.11	.13
Japan ; GNP (kmax = 5)	57:1–88:4	128	71:3	4	10.66 (2251.2)	.0243 (211.9)	–.0142 (–76.11)	.650	–5.14	<.01	.02	.01	
			71:4	4	10.67 (2242.1)	.0241 (212.0)	–.0141 (–75.19)	.659	–5.11	<.01			.03
France[a]; GDP (kmax = 10)	65:1–88:3	95	74:2	3	9.66 (1394.5)	.0127 (106.7)	–.0080 (–46.93)	.679	–4.33	.03	.16	.12	.13
Germany ; GNP (kmax = 10)	60:1–86:2	106	73:4	7	6.45 (941.4)	.0104 (74.0)	–.0058 (–21.92)	.624	–4.33	.03	.16	.12	.13
Italy ; GDP (kmax = 10)	60:1–85:1	101	72:4	3	11.73 (1369.1)	.0125 (67.5)	–.0071 (–20.91)	.755	–3.90	.15	.32		
			72:3	3	11.72 (1336.1)	.0126 (65.6)	–.0071 (–20.60)	.757	–3.87	.16		.26	
			73:2	1	11.73 (1428.1)	.0123 (70.9)	–.0071 (–21.37)	.814	–3.31	.40			.57
U.K. ; GDP (kmax = 10)	57:1–86:3	118	73:3	7	3.47 (769.8)	.0074 (73.5)	–.0040 (–18.96)	.552	–4.62	.02	.09	.06	.08

[a] A dummy taking value 1 in 1968,2 was included for France to take account of the general strike in May 1968.

TABLE X : Empirical Results ; International Data Set ; Choosing T_B by minimizing $t_{\hat{\gamma}}$.

Regression : $y_t = \mu + \beta t + \gamma DT_t^* + \tilde{y}_t \; ; \; \tilde{y}_t = \alpha\tilde{y}_{t-1} + \Sigma_{i=1}^{k} c_i \Delta\tilde{y}_{t-i} + e_t$.

				Min $t_{\hat{\alpha}}$ unrestricted				With t–test on last lag				With F–test on additional lags			
	T_B	$\hat{\beta}$	$\hat{\gamma}$	k	$t_{\hat{\alpha}}$	p–value (fixed k)	p–value (min k)	k	$t_{\hat{\alpha}}$	p–value (fixed k)	p–value (t sig)	k	$t_{\hat{\alpha}}$	p–value (fixed k)	p–value (F sig)
U.S.A. (kmax = 5)	70:2	.0089 (88.1)	–.0028 (–12.19)	2	–4.03 (.851)	.05	.12	2	–4.03 (.851)	.05	.09	2	–4.03 (.851)	.05	.08
Canada (kmax = 5)	77:1	.0121 (209.2)	–.0045 (–21.24)	3	–4.25 (.817)	.03	.08	2	–4.25 (.817)	.03	.06	4	–3.77 (.829)	.09	.13
Japan (kmax = 5)	71:3	.0243 (211.8)	–.0142 (–76.11)	4	–5.14 (.650)	<.01	.02	3	–4.99 (.689)	<.01	.02	4	–5.14 (.650)	<.01	<.01
France (kmax = 10)	74:1	.0129 (106.7)	–.0081 (–47.76)	2	–4.26 (.705)	.04	.09	2	–4.26 (.705)	.04	.07	3	–4.16 (.685)	.04	.07
Germany (kmax = 10)	73:1	.0106 (72.7)	–.0058 (–22.55)	7	–4.14 (.630)	.02	.12	4	–4.00 (.705)	.05	.11	6	–3.82 (.681)	.06	.14
Italy (kmax = 10)	73:3	.0122 (72.3)	–.0072 (–21.51)	3	–3.77 (.761)	.10	.21	1	–3.26 (.817)	.27	.33	1	–3.26 (.817)	.27	.29
United Kingdom (kmax = 10)	73:2	.0074 (72.6)	–.0040 (–18.98)	7	–4.61 (.554)	<.01	.05	7	–4.61 (.554)	<.01	.04	7	–4.61 (.554)	<.01	.03

Note : t–statistics are in parentheses except for the columns labelled $t_{\hat{\alpha}}$ where the entries are the estimates of α.

TABLE XI : Empirical Results , Additional Series ; $t^*_\alpha(1)$, kmax = 12 .

Regression : $y_t = \mu + \theta DU_t + \beta t + \delta D(TB)_t + \alpha y_{t-1} + \Sigma_{i=1}^{k} c_i \Delta y_{t-i} + e_t$.

Series	Sample	T	T_B	k	$\hat{\mu}$	$\hat{\theta}$	$\hat{\beta}$	$\hat{\delta}$	$\hat{\alpha}$	$t_{\hat{\alpha}}$	p–value (k fixed)	p–value (min k)	p–value (F sig)	p–value (t sig)
Real per per Capita GNP	1909–1970	62	1928	7	3.24 (5.56)	–.13 (–4.17)	.014 (5.73)	.139 (2.34)	.372	–5.62	<.01	.03		
(Friedman–Schwartz)			1928	11	4.14 (5.40)	–.16 (–4.74)	.018 (5.53)	.136 (2.38)	.288	–5.42	.02		.03	.02
M2	1869–1973	105	1929	9	.03 (2.14)	–.09 (–3.43)	.018 (5.23)	.069 (1.35)	.724	–5.22	.01	.05	.04	
			1929	12	.01 (.91)	–.10 (–4.21)	.018 (4.87)	.068 (1.37)	.720	–4.69	.07			.14
Nominal Consumption	1889–1973	85	1928	11	–1.06 (–6.72)	–.22 (–6.00)	.020 (7.06)	.215 (3.52)	.579	–6.78	<.01	<.01		<.01
			1928	12	–.97 (–5.66)	–.21 (–5.65)	.018 (6.11)	.227 (3.78)	.614	–5.70	<.01		.01	
Real Consumption	1889–1973	85	1928	11	–.27 (–6.00)	–.11 (–5.96)	.014 (6.67)	.120 (3.68)	.202	–6.45	<.01	<.01	<.01	
			1929	11	–.30 (–5.81)	–.12 (–5.78)	.015 (6.32)	.043 (1.23)	.109	–6.19	<.01			<.01
Nominal Per Capita	1889–1973	85	1928	11	–2.79 (–6.21)	–.19 (–5.42)	.014 (6.48)	.183 (2.91)	.582	–6.20	<.01	<.01		
Consumption			1928	12	–2.59 (–5.28)	–.19 (–5.12)	.013 (5.71)	.20 (3.28)	.613	–5.25	.02		.03	.03

TABLE XI (cont'd) : Empirical Results , Additional Series ; $t^*_{\alpha}(1)$, kmax = 12 .

Regression : $y_t = \mu + \theta DU_t + \beta t + \delta D(TB)_t + \alpha y_{t-1} + \Sigma_{i=1}^{k} c_i \Delta y_{t-i} + e_t$.

Series	Sample	T	T_B	k	$\hat{\mu}$	$\hat{\theta}$	$\hat{\beta}$	$\hat{\delta}$	$\hat{\alpha}$	$t_{\hat{\alpha}}$	p–value (k fixed)	p–value (min k)	p–value (F sig)	p–value (t sig)
Real Per Capita Consumption	1889–1973	85	1928	11	–3.35 (–4.61)	–.07 (–3.93)	.002 (4.48)	.059 (1.73)	.254	–4.60	.08	.23		
			1928	12	–3.71 (–4.50)	–.08 (–4.14)	.003 (4.70)	.071 (2.08)	.174	–4.49	.11		.20	
			1928	10	–2.82 (–4.56)	–.06 (–3.69)	.002 (4.35)	.057 (1.66)	.372	–4.54	.10			.19
Consumption Price Index	1889–1973	85	1928	10	–.63 (–4.67)	–.11 (–3.53)	.009 (4.78)	.122 (2.10)	.709	–4.72	.06	.19		
			1928	8	–.63 (–4.64)	–.12 (–3.77)	.009 (4.71)	.062 (1.30)	.709	–4.71	.07		.13	
			1919	10	–.39 (–4.27)	–.07 (–3.86)	.005 (5.22)	.227 (4.66)	.810	–4.34	.14			.26
Population	1889–1973	85	1917	11	.279 (4.88)	.004 (3.35)	.001 (4.58)	–.016 (–7.02)	.933	–4.82	.05	.15	.10	
			1923	10	.224 (3.59)	–.003 (–1.94)	.001 (3.66)	.016 (3.38)	.948	–3.48	.55			.71

Table XII : Empirical Results ; Additional Series ; Model 1 .

$t^*_{\alpha,\theta}(1)$; Choosing T_B minimizing $t_{\hat{\theta}}$; kmax = 12.

Regression : $y_t = \mu + \theta DU_t + \beta t + \delta D(TB)_t + \alpha y_{t-1} + \Sigma_{i=1}^{k} c_i \Delta y_{t-i} + e_t$.

Series	T_B	k	$t_{\hat{\theta}}$	$\hat{\alpha}$	$t_{\hat{\alpha}}$	p–value (k fixed)	p–value (min k)	p–value (F sig)	p–value (t sig)
Real Per Capita GNP (F–S)	1928	11	–4.74	.288	–5.42	<.01	.02	.01	.02
M2	1929	12	–4.21	.720	–4.69	.03	.08	.07	.07
Nominal Consumption	1928	11	–6.00	.579	–6.78	<.01	<.01		<.01
	1928	12	–5.65	.614	–5.70	<.01		<.01	
Real Consumption	1928	11	–5.96	.202	–6.45	<.01	<.01	<.01	
	1929	11	–5.78	.109	–6.19	<.01			<.01
Nominal Per Capita Consumption	1928	11	–5.42	.582	–6.20	<.01	<.01		
	1928	12	–5.12	.613	–5.25	<.01		.02	.02
Real Per Capita Consumption	1928	12	–4.14	.174	–4.49	.05	.12	.11	
	1928	10	–3.69	.372	–4.54	.05			.10
Consumption Price Index	1919	10	–3.86	.810	–4.34	.07	.15	.14	.14
Population	1929	12	–2.48	.929	–3.47	.30	.45		
	1925	8	–2.36	.966	–2.39	.72		.81	.82

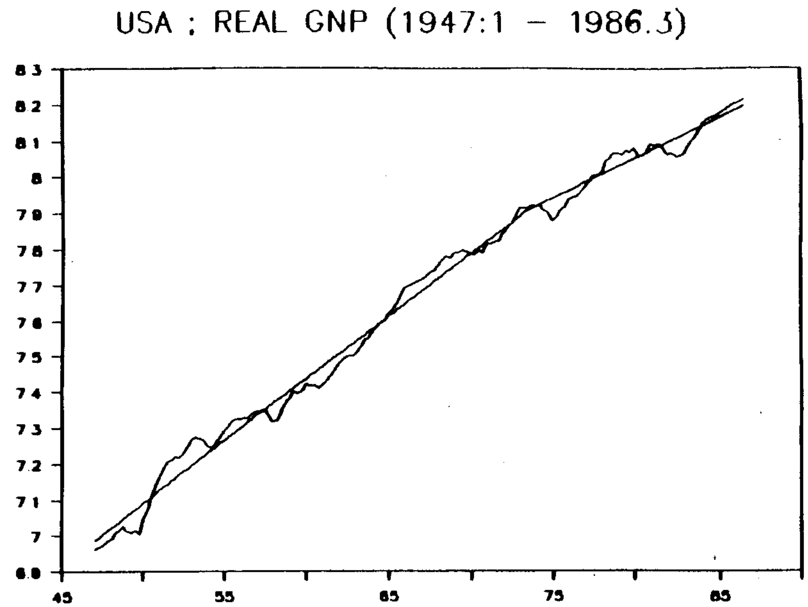


Figure 1

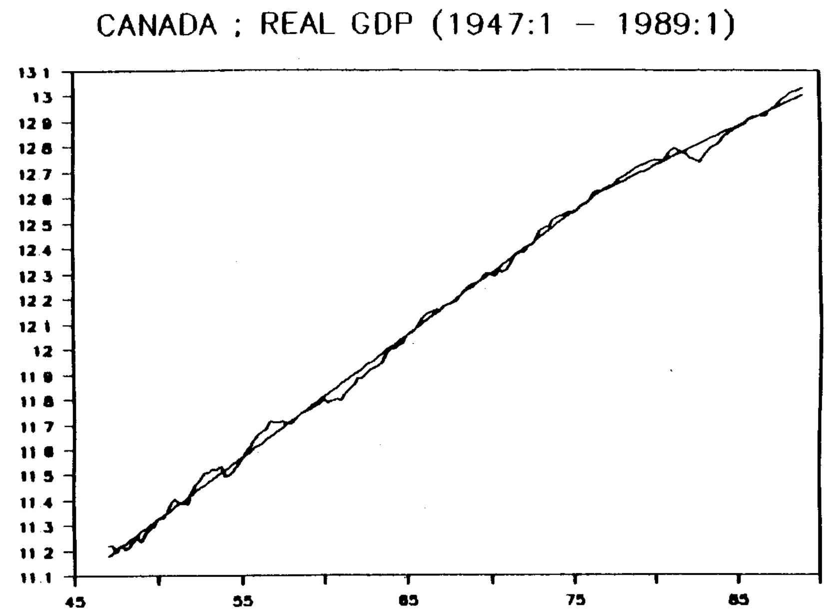
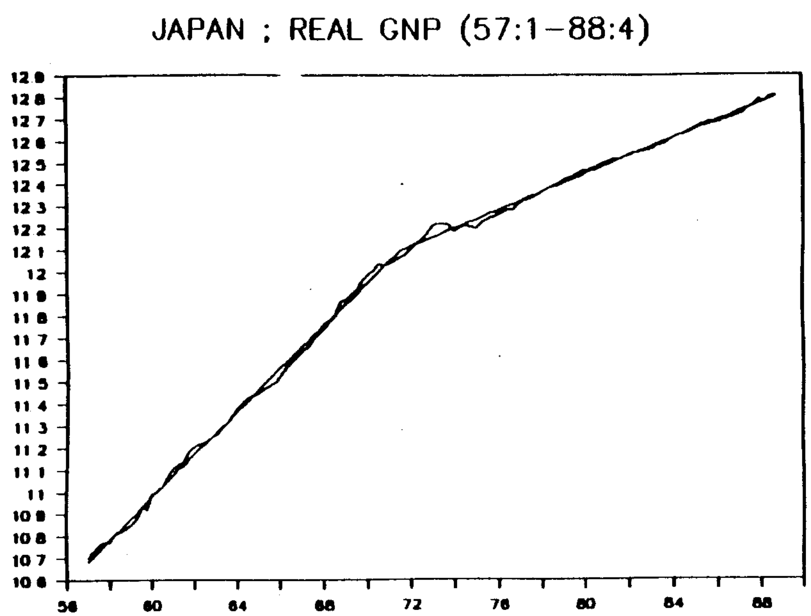


Figure 2



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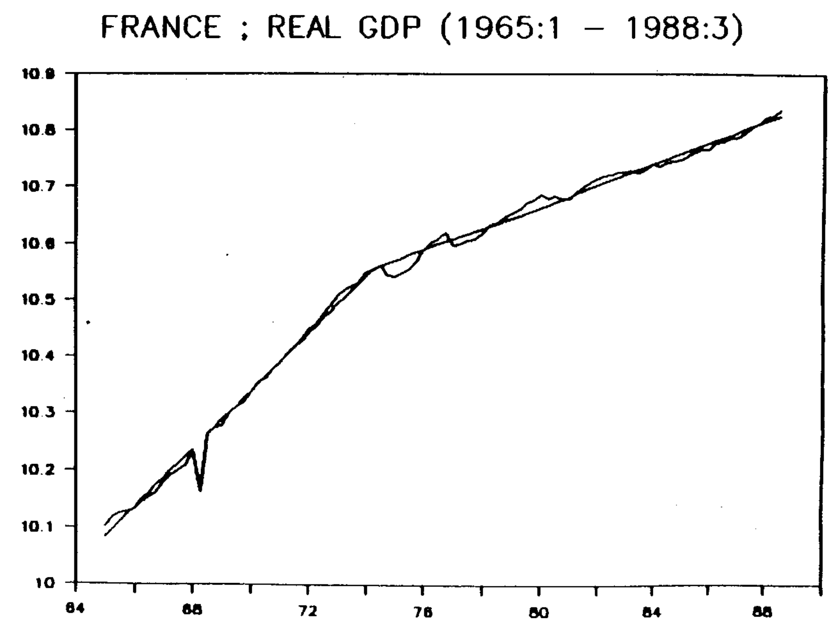

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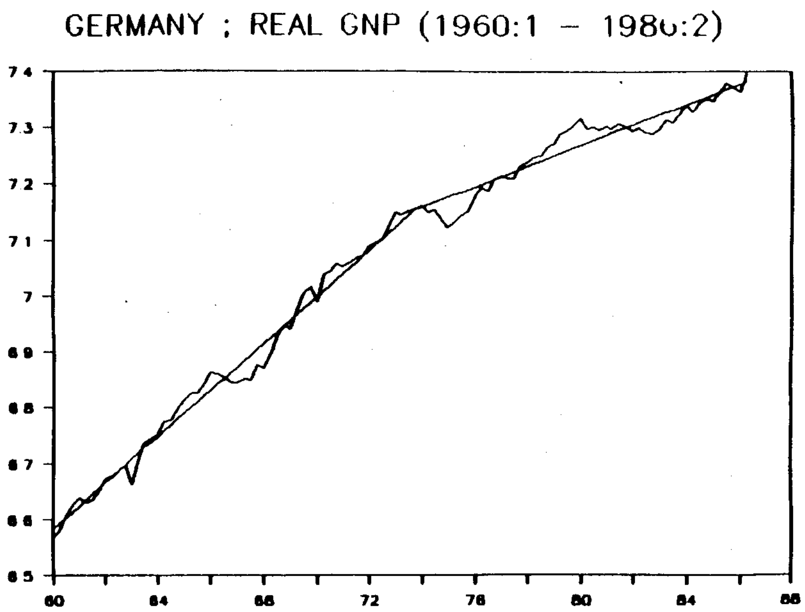

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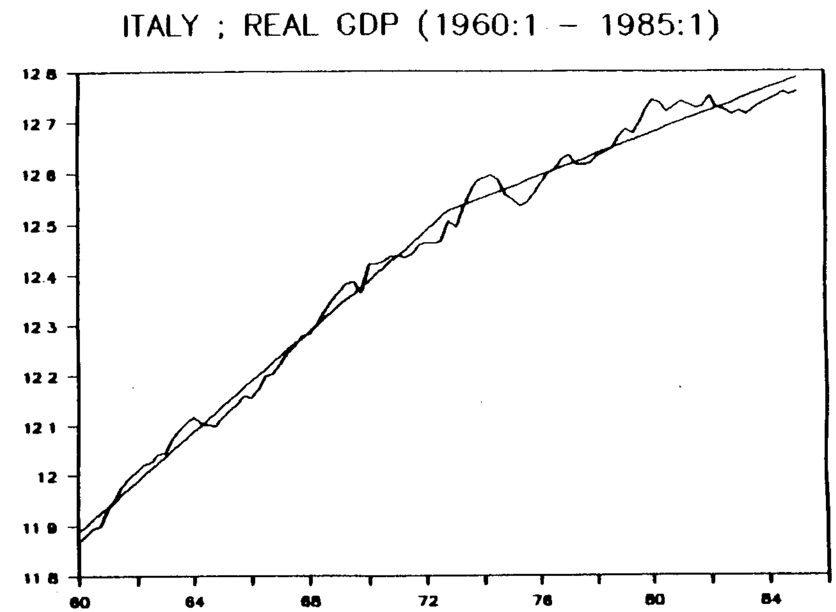

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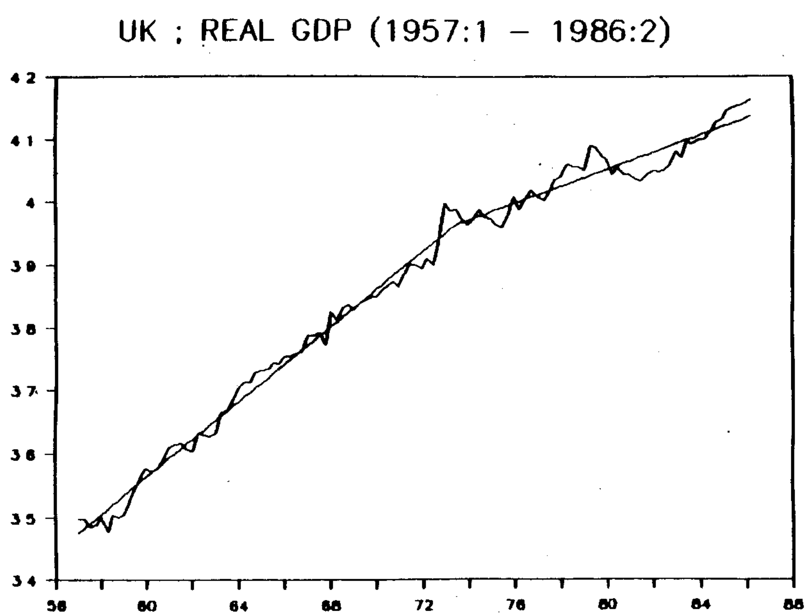

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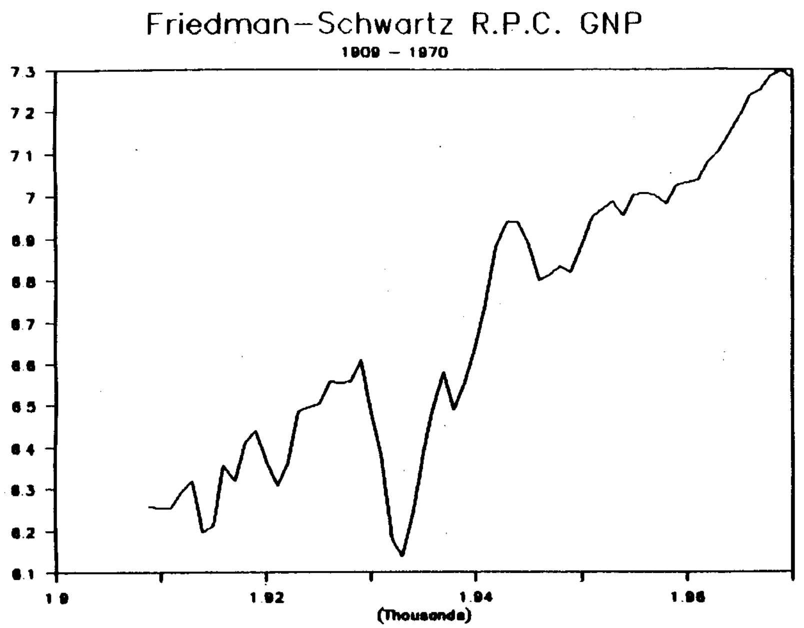

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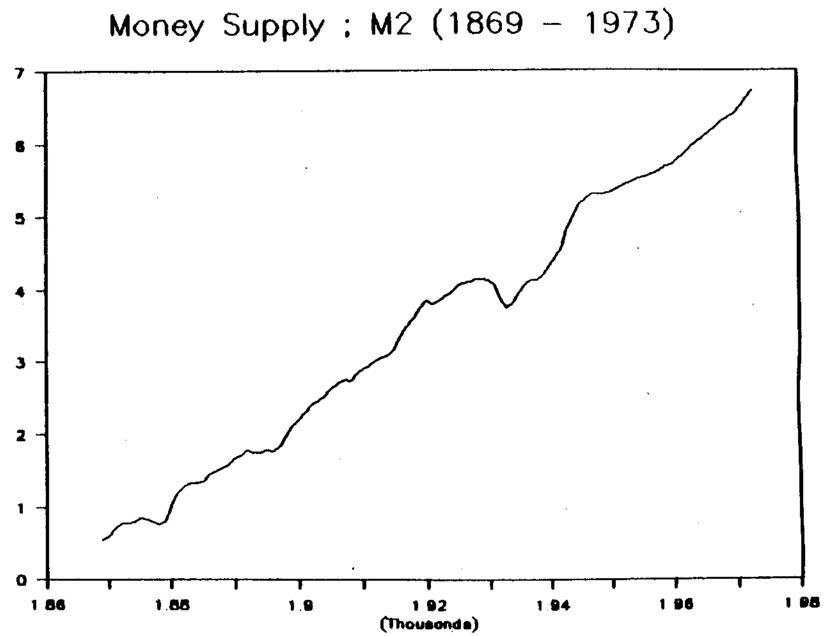

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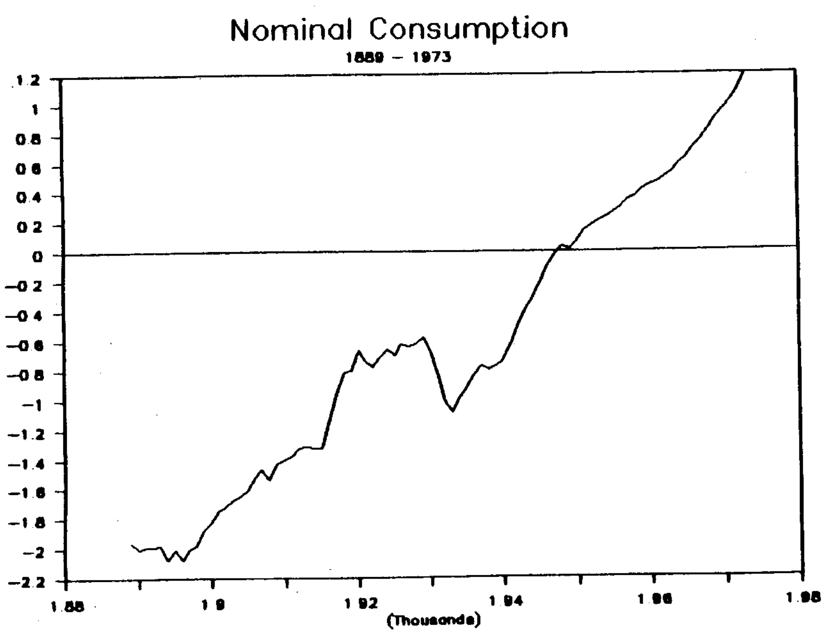


Figure 10

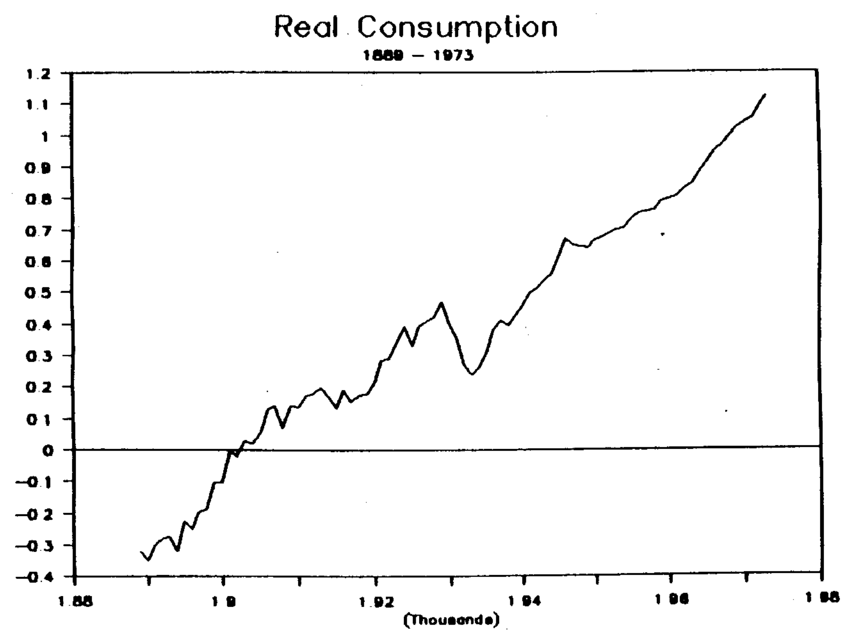
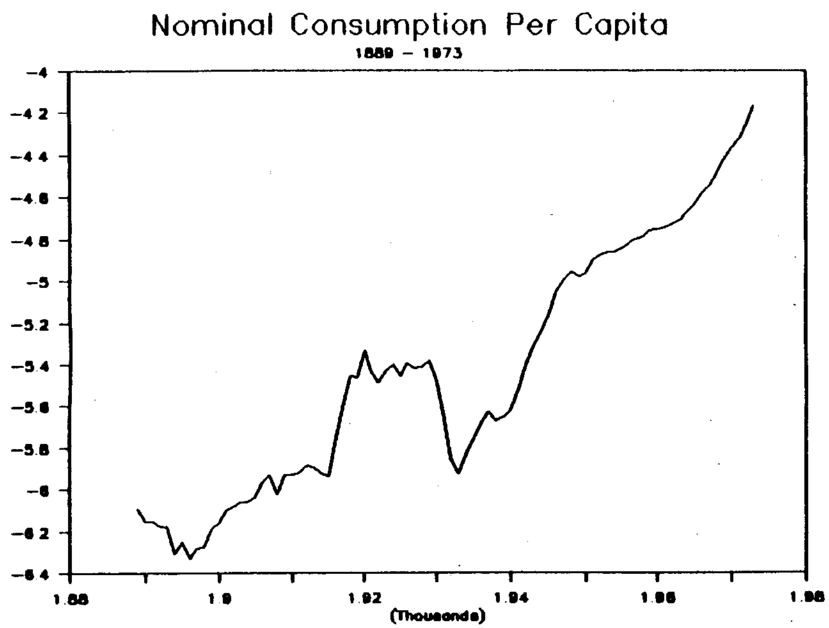


Figure 11



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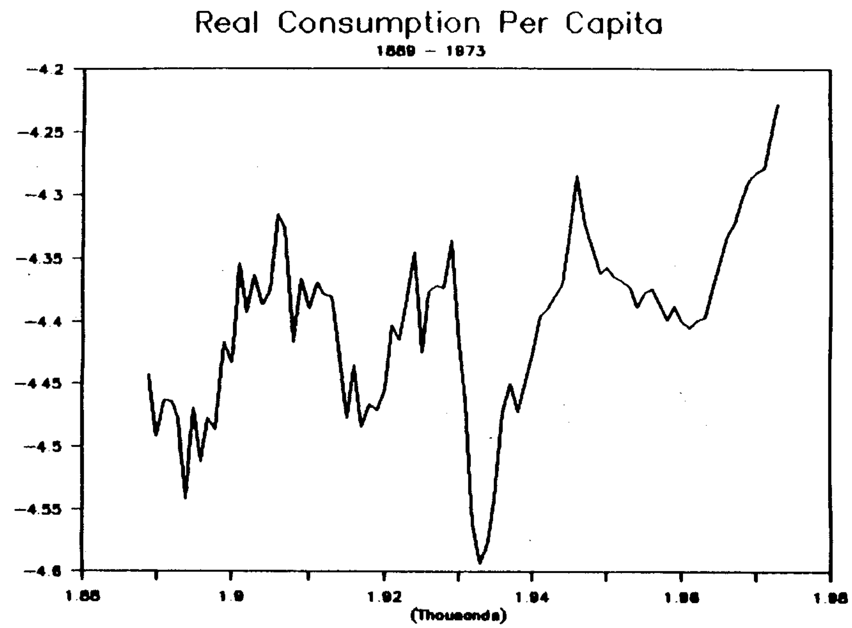

Figure 13

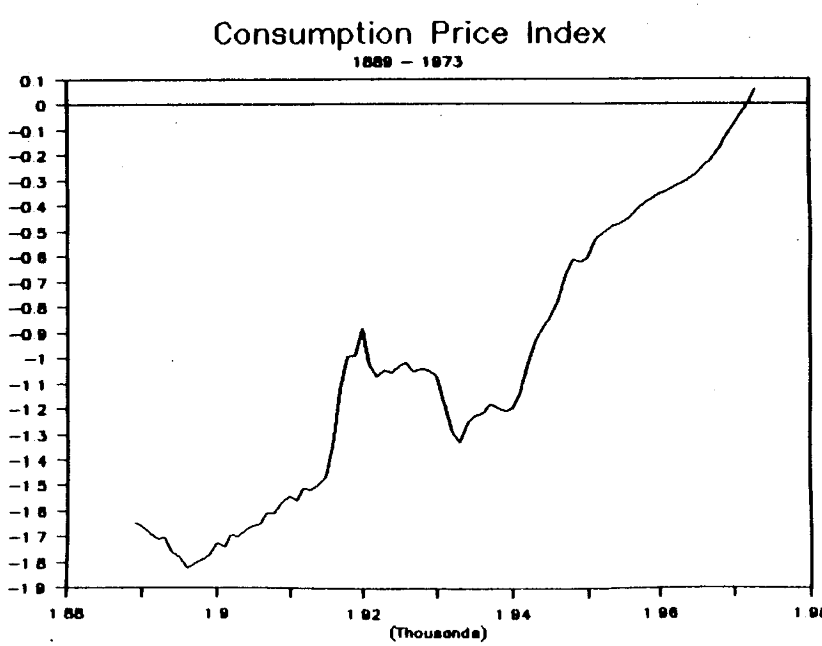

Figure 14

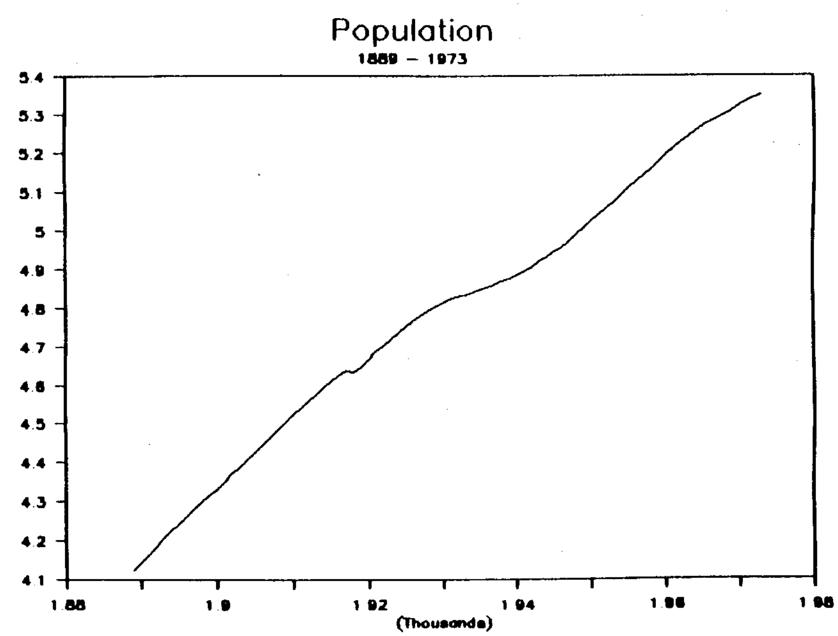

Figure 15