THE EFFECT OF SEASONAL ADJUSTMENT FILTERS ON TESTS FOR A UNIT ROOT

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ABSTRACT

We consider the effect of seasonal adjustment filters in univariate dynamic models. While most of the literature has considered the effect of seasonal adjustments in the linear regression model, little is known in a dynamic context. We concentrate our analysis on the behavior of the least-squares estimator of the sum of the autoregressive coefficients in a univariate regression. We show the existence of a limiting upward bias with the X-11 filter when the process does not contain a unit root. We also quantify the extent of this bias for a range of models and filtering procedures. Such an asymptotic bias has interesting implications with respect to the power of tests for a unit root. In order to assess the importance of this effect we present an extensive simulation study of both the size and power of the usual Dickey-Fuller (1979) and Phillips-Perron (1988) statistics (both with and without an estimated trend). We show that, in many cases, there is a considerable reduction in power compared to the benchmark cases where the data is unfiltered. Finally some practical implications of our study are addressed with respect to tests for unit roots with seasonally adjusted data.

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1. INTRODUCTION

Much of the literature on seasonality has focused on univariate time series models emphasizing the design of adjustment filters. In comparison, relatively little has been said about the effects of seasonality and seasonal adjustment on inference and hypothesis testing. Sims (1974) and Wallis (1974) explored the nature of the asymptotic bias due to the seasonal noise in a linear regression model when adjusted and unadjusted data were used. The focal point of our paper is also inference and hypothesis testing, and how seasonal adjustment affects it. Unlike Sims and Wallis, our concern is not econometric inference and testing of (linear) relationships among a set of economic time series. Instead, we consider the simple univariate autoregressive model and study the effect of filtering on the behavior of the least-squares estimator of the sum of the autoregressive coefficients and on testing the hypothesis of a unit root. The motivation for exploring this particular hypothesis is the current general interest in such tests among applied econometricians because of its implications for economic theory and econometric inference.

Intensive research efforts have been devoted to the characterization of both the asymptotic and finite sample distribution of unit root tests and their power properties with unfiltered data. Yet, in many practical applications, the hypothesis of a unit root is tested not with raw data but with filtered series such as quarterly or monthly seasonally adjusted series using the X-11 procedure of the U.S. Bureau of the Census. In Ghysels (1990), it was pointed out that the use of seasonally adjusted data raises several practical problems[1]. It was also shown that the evidence supporting the unit root hypothesis in real GNP is less conclusive when seasonally unadjusted series are used. Although some Monte Carlo evidence was presented by Ghysels (1990), this simulation analysis was quite preliminary. In this paper we explore in more detail the effect of seasonal filtering from both analytical and simulation perspectives paying particular attention to the size and power of unit root tests as proposed by Dickey and Fuller (1979) and Phillips and Perron (1988).

Section 2 contains a preliminary discussion of seasonal adjustment procedures. We first review the commonly used X-11 filter and its associated linear approximation for both the quarterly and monthly frequencies. Specific attention is also given to the Henderson moving average filter as it is the subfilter which extracts the trend component of an already seasonally adjusted series. Details about the specific weights associated with these linear filters appear in an appendix. In Section 3 we outline the models considered in our study. They include nonseasonal ARMA(1,1) models as well as seasonal unobserved component models. We also present the test statistics considered.

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[1] As will be discussed further, there are at least three undesirable effects due to seasonal adjustment that were identified : (1) smoothing of series; (2) distant autocorrelations produced by symmetric filtering; and (3) time variation of the X-11 filter.

In Section 4 we consider the behavior of both the least squares estimator in a first-order autoregressive model and the least-squares estimator of the sum of the autoregressive coefficients in an AR(p) regression. These are of importance because they form the basis of the unit root tests considered. When the data generating process is stationary, we show the existence of a non-vanishing asymptotic bias in the estimates of the autoregressive parameters when filtered data are used, whether the original series contains a seasonal component or not. We quantify the extent of this bias for a variety of parametric specifications. The extent of the bias can be quite substantial especially when the underlying model contains a seasonal component and the length of the autoregression is less than the seasonal period. When the data generating process has a unit root, we show that the OLS estimator is consistent and that the asymptotic distributions of unit root tests remain invariant to filtering with X-11. The implications for the size and power of unit root tests are also discussed. Sims (1974) showed that the least-squares estimator is consistent in a linear regression model with no lagged dependent variable when each series is seasonally adjusted by the same filter. Our analysis shows that this result does not carry over to cases of regression models with lagged dependent variables. Wallis (1974) reports some results on the asymptotic bias of the least-squares estimator in the AR(1) model; our paper analyzes the inconsistency in much greater detail. Related work by Jaeger and Kunst (1990) shows the effect of seasonal adjustment on measures of persistence in aggregate output.

Section 5 contains a detailed examination of the finite sample behavior of the Dickey-Fuller and Phillips-Perron procedures applied to filtered data. Both exact sizes and powers are considered. We use a variety of data generating processes and filters at both the quarterly and monthly sampling frequency. The effects of the exact X-11 filter as well as its linear approximation are given special attention. The simulation results support and extend the qualitative properties suggested by the theoretical analysis of section 4. Finally, Section 6 offers some concluding comments.

2. FILTERING AND SEASONAL ADJUSTMENT

Seasonal adjustment filters are usually analyzed as linear filters. The U.S. Census X-11 and Statistics Canada X-11 ARIMA methods, which are widely applied in Canada, the U.S. and several other countries, use a set of moving averages to produce seasonally adjusted data. Likewise, the so-called "model-based" seasonal adjustment filters, which are designed on the principle of optimal linear signal extraction theory, are also linear filters. In practice, however, seasonal adjustment methods such as X-11 produce seasonally adjusted series which are not the output of a linear filter. Although the basic idea of moving averages is simple, the sets of filters used with X-11 changes through time and hence imply in a time-varying filtering of the original data[2]. Without completely ignoring the time variation associated with the X-11 filter, we focus most of our attention on its linear approximation. In this section we present the seasonal adjustment filters most commonly used and discuss their properties.

The Census X-11 program is considered first since it is the most widely applied adjustment procedures[3]. It was the product of several decades of research. Its development started in the early thirties by researchers at the N.B.E.R. [see, e.g., Macaulay (1931)] and it emerged as a fully operational procedure in the mid-sixties due to the work by Julius Shiskin and his collaborators at the U.S. Bureau of the Census [see Shiskin et al. (1967)]. The program consists of a set of moving average filters which are applied to the data sequentially. It was noted that the program does not amount to a linear data transformation in almost all practical circumstances. The X-11 program can be represented by a linear filter when the default option is considered. Several authors, including Wallis (1974), Laroque (1977), Dagum (1983), Burridge and Wallis (1984) and Ghysels (1984) have studied the quarterly and/or monthly linear X-11 filters. The exact structure of the linear approximation for the quarterly filter is presented in Laroque (1977). The actual filter weights of the linear approximation to the monthly filter, while used in several previous studies, are not readily available[4]. Hence we will pay slightly more attention to the structure of the monthly filter. Some of the details are deferred to appendix A.

The different steps of the X-11 monthly filter are as follows. A centered thirteen term MA is initially subtracted from the original series using the filter :

$$\begin{aligned} SM(L) &= 1 - (1/24)(1 + L)(1 + L \dots + L^{11})\, L^{-6} \\ &\approx - 0.042L^{6} - 0.083L^{5} - 0.083L^{4} - 0.083L^{3} - 0.083L^{2} - 0.083L + 0.917 \\ &\quad - 0.083L^{-1} - 0.083L^{-2} - 0.083L^{-3} - 0.083L^{-4} - 0.083L^{-5} - 0.042L^{-6} \qquad (2.1) \end{aligned}$$

The filtered series obtained is a first estimate of the seasonal plus noise part of the series[5]. A first estimate of the seasonal part is obtained by applying the filter :

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2 There are two main reasons why it is not a linear filter. First, the X-11 procedure switches between different (sub)filters depending on the presence of outliers. Secondly, the program uses different filters at each end of a data series since it requires an equal number of observations on each side of a data point.

3 Since only symmetric filters will be discussed, we do not cover the X-11 ARIMA variant [Dagum (1982)] of the Census X-11 seasonal adjustment procedure.

4 Several authors, including Dagum (1983) and Burridge and Wallis (1984), graph the transfer function of the linear monthly filter. Burridge and Wallis tabulated only some of the filter weights. In the appendix to this paper we provide tables with actual filter weights for the monthly and quarterly linear filters.

5 Filter weights were rounded off at the third decimal place.

$$M_1(L) = (1/9)(L^s + 1 + L^{-s})^2$$
$$\approx 0.111L^{2s} + 0.222L^s + 0.333 + 0.222L^{-s} + 0.111L^{-2s} \qquad (2.2)$$

with s = 12. In order to have the seasonal components sum to unity over one year the filter SM(L) is applied once more. The whole process yields a first estimate of the seasonal component. This estimate is subtracted from the original series after which a Henderson moving average is applied :

$$HM(L) = -0.019L^6 - 0.028L^5 + 0.066L^3 + 0.147L^2 + 0.214L + 0.240 + 0.214L^{-1}$$
$$+ 0.147L^{-2} + 0.066L^{-3} - 0.028L^{-5} - 0.019L^{-6} \qquad (2.3)$$

to obtain a second estimate of the trend cycle component[6]. The latter is subtracted from the original series to obtain a second estimate of the seasonal plus noise from which a final estimate of the seasonal component is obtained via the application of the filter :

$$M_2(L) = (1/15)(L^s + 1 + L^{-s})(L^{2s} + L^s + 1 + L^{-s} + L^{-2s})$$
$$\approx 0.067L^{3s} + 0.133L^{2s} + 0.200L^s + 0.200 + 0.200L^{-s} + 0.133L^{-2s} + 0.067L^{-3s} \qquad (2.4)$$

again with s = 12. Finally, in order to have the seasonal components sum to unity, the filter SM(L) is applied again. Consequently, adding up these steps and denoting $\upsilon^M_{X-11}(L)$ as the linear approximation, we have :

$$\upsilon^M_{X-11}(L) = 1 - SM(L)\{M_2(L)(1 - HM(L)[1 - SM(L)M_1(L)SM(L)])\}$$
$$= 1 - SM(L)M_2(L) + SM(L)M_2(L)HM(L)$$
$$- SM^3(L)M_1(L)M_2(L)HM(L). \qquad (2.5)$$

The coefficients of this filter are presented in Table A.1 in appendix A. Each coefficient is applied twice, once for the lag and once for the lead terms. The final two-sided symmetric monthly X-11 filter, appearing in (2.5) and the Henderson moving average filter in (2.3) will be used to study the effects of filtering on tests for unit roots. The Henderson MA filter is included, although it is only a subfilter, because it represents the trend estimate used in the program. The transfer functions of both filters are also reported for convenience in

6 The exact formula for calculating the weights of a Henderson moving average filter appears, for instance, in Macauley (1931, p. 54), Dagum (1985) or Gouriéroux and Monfort (1990, p. 102).

Appendix A, Figures A.1 and A.2. The transfer function of the final X-11 monthly filter has a large dip at the seasonal frequency and wiggles around one at all other frequencies. The Henderson MA filter is essentially a smoothing filter with a transfer function which equals one at low frequencies and drops off to zero at high frequencies.

The quarterly X-11 filter, analyzed by Laroque (1977), is constructed in a similar fashion. The final filter denoted by $\upsilon^{Q}_{X-11}(L)$ is composed of :

$$SQ(L) = 1 - (1/8)(1 + L)(1 + L + L^2 + L^3)L^{-2}$$

$$= - 0.125L^2 - 0.250L + 0.750 - 0.250L^{-1} - 0.125L^{-2}. \tag{2.6}$$

$$HQ(L) = - 0.073L^2 + 0.294L + 0.558 + 0.294L^{-1} - 0.073L^{-2} \tag{2.7}$$

along with $M_1(L)$ and $M_2(L)$, evaluated at s = 4. Again using (2.5) one can calculate the implied weights for the filter. They appear in Table A.2 in appendix A. The transfer functions of the quarterly final filter and its Henderson MA subfilter are reported in Figures A.3 and A.4 respectively. They exhibit features similar to the monthly filters.

Besides the linear X-11 filter and the Henderson MA filter we will also consider the X-11 filter, both monthly and quarterly, as it actually operates in practice. That is, instead of using the standard (default option) linear approximation, we consider the actual procedure as implemented in the SAS statistical package with its endogenous switching and use of different filters at each end of a data series[7]. Hence, the filter behaves in a nonlinear fashion like it does in almost all practical applications.

3. MODELS AND TEST STATISTICS

The quantification of the asymptotic biases and the Monte Carlo experiments to be presented center on data generating processes that are either described by a single component ARMA process or by standard unobserved component linear time series models. The first model considered is an ARMA(1,1) process :

$$y_t = \alpha y_{t-1} + e_t + \theta e_{t-1} \tag{3.1}$$

where e_t ~ i.i.d. N(0,1) without loss of generality. This nonseasonal ARMA(1,1) process is essentially a benchmark model, since it is widely used to study the power of unit root tests

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[7] It should be noted that the SAS version of the X-11 procedure corresponds to an older version of the X-11 Census program.

applied to unfiltered data processes. It will allow us to compare our results with previously reported power analyses such as those of Dickey (1984), Dickey, Bell and Miller (1986), Schwert (1989) and Perron (1988), among others. Considering the effect of filtering on the power of unit root tests in the context of an ARMA(1,1) model also extends earlier work by Ghysels (1990) who discussed pure autoregressive models. The size of the tests after filtering as well as the power against several alternatives will be analyzed. Studying the effect of seasonal adjustment filtering procedures on series that have no seasonal components also has its advantages. Indeed, in this context, the issue concerning whether the seasonal part has been removed adequately does not occur. Hence, it permits a more specific investigation of the properties of the filters and their effects on the correlation structure of the data.

Unobserved component models with both a seasonal and nonseasonal component form a second class of data generating processes. In this case, we have :

$$y_t = y_t^{ns} + y_t^{s} \tag{3.2a}$$

$$y_t^{ns} = \alpha_{ns} y_{t-1}^{ns} + \varepsilon_t^{ns} + \theta_{ns} \varepsilon_{t-1}^{ns} \tag{3.2b}$$

$$y_t^{s} = \alpha_s y_{t-sp}^{s} + \varepsilon_t^{s} + \theta_s \varepsilon_{t-sp}^{s} \tag{3.2c}$$

where ε_t^{ns} and ε_t^{s} are each i.i.d. N(0,1) and uncorrelated with each other, and sp denotes the seasonal period – 4 for quarterly data and 12 for monthly.

Two classes of unit root tests are studied. The first is the procedure of Dickey–Fuller (1979) and Said–Dickey (1984), which is obtained from the t–statistic for testing $\alpha = 1$ in the following regressions, estimated by OLS :

$$y_t = \hat{\mu} + \hat{\alpha} y_{t-1} + \sum_{j=1}^{k} \hat{c}_j \Delta y_{t-j} + \hat{e}_t ; \tag{3.3}$$

and

$$y_t = \tilde{\mu} + \tilde{\beta} t + \tilde{\alpha} y_{t-1} + \sum_{j=1}^{k} \tilde{c}_j \Delta y_{t-j} + \tilde{e}_t , \tag{3.4}$$

where k is a truncation lag parameter which specifies the number of lags of first–differences of the data to be included in the regression. We denote these statistics by $t_{\hat{\alpha}}$ ant $t_{\tilde{\alpha}}$, respectively. They will have the asymptotic distribution tabulated in Fuller (1976) if k increases at a controlled rate as T increases to infinity [see Said and Dickey (1984)]. The second class of tests is the one due to Phillips and Perron (1988), namely, the t–statistic and the normalized bias of the estimated first order autoregressive OLS parameter both applied with correction factors. Consider the following regressions estimated by ordinary least-squares :

$$y_t = \hat{\mu} + \hat{\alpha}\, y_{t-1} + \hat{e}_t \tag{3.5}$$

and

$$y_t = \tilde{\mu} + \tilde{\beta} t + \tilde{\alpha}\, y_{t-1} + \tilde{e}_t \tag{3.6}$$

Denote by $\hat{S}^2$ the residual sum of squares in a regression of y_{t-1} on a constant. The Phillips-Perron statistics, in the case of non-trending series, i.e. (3.5), are defined by :

$$Z(\hat{\alpha}) = T(\hat{\alpha} - 1) - T^2(\hat{\sigma}^2 - \hat{\sigma}_e^2)/2\hat{S}^2\ , \tag{3.7}$$

$$Z(t_{\hat{\alpha}}) = (\hat{\sigma}_e/\hat{\sigma})t_{\hat{\alpha}} - T(\hat{\sigma}^2 - \hat{\sigma}_e^2)/2\hat{\sigma}\hat{S}\ , \tag{3.8}$$

where $\hat{\sigma}_e^2 = T^{-1}\Sigma_1^T \hat{e}_t^2$ is the estimated variance of the residuals from (3.5), $t_{\hat{\alpha}}$ is the usual t–statistic for testing $\alpha = 1$ and

$$\hat{\sigma}^2 = T^{-1}\Sigma_1^T \hat{e}_t^2 + 2T^{-1} \sum_{\tau=1}^{k} w(\tau,k) \sum_{t=\tau+1}^{T} \hat{e}_t \hat{e}_{t-\tau}\ . \tag{3.9}$$

$w(\tau,k)$ is a lag window which, in the simulation experiments of Section 5, is specified as $w(\tau,k) = 1 - \tau/(k + 1)$ following Newey and West (1987). Here k is a truncation lag parameter which determines the number of autocovariances to be considered in constructing the estimate. The statistics for the case where a trend is fitted are defined analogously by using the residuals from (3.6) and defining the corresponding estimate in (3.9) by $\tilde{\sigma}^2$. Similar use is made, in (3.7) and (3.8), of $\tilde{S}^2$ defined as the residual sum of squares in a regression of y_{t-1} on a constant and a trend. The statistics in the case where a trend is fitted are denoted by $Z(\tilde{\alpha})$ and $Z(t_{\tilde{\alpha}})$.

4. ASYMPTOTIC BIASES

In this section, we analyze the limit of the least-squares estimators. We put the emphasis on the asymptotic bias, i.e. the difference between the expected value of the limiting random variable with and without filtering. We start by considering the following general time series model for a given variable of interest $\{y_t\}$:

$$y_t = \mu + \beta t + Z_t, \tag{4.1}$$

where $A(L)\ Z_t = B(L)\varepsilon_t$ and $\varepsilon_t \sim$ i.i.d. $(0, \sigma^2)$. B(L) is a q^{th} order polynomial in the lag operator L with roots outside the unit circle; i.e. B(L) is invertible. A(L) is a p^{th} order polynomial in L with at most one root on the unit circle and all other roots outside the unit circle. Hence we are allowing for the possibility that $\{y_t\}$ represents a difference stationary process. In that case (4.1) can be written as

$$y_t = \beta + y_{t-1} + v_t, \tag{4.2}$$

where $v_t = A^*(L)^{-1}\ B(L)\varepsilon_t$ and $A^*(L)$ is defined such that $(1 - L)A^*(L) = A(L)$. Note that, given our assumptions, $A^*(L)$ has all its roots outside the unit circle.

We denote by $\psi(L)$ a general linear two-sided filter. Given the structure of the filters described in section 2 we consider only polynomials $\psi(L)$ which are symmetric and whose coefficients sum to one, i.e. for which $\psi(L) = \psi(L^{-1})$ and $\psi(1) = 1$. Under these conditions, the filtered version of y_t, denoted $y_t^F = \psi(L)y_t$, is given by :

$$y_t^F = \mu + \beta t + \psi(L)Z_t. \tag{4.3}$$

Of particular interest is the fact that a symmetric filter with $\psi(1) = 1$ leaves unchanged the deterministic nonseasonal part of the series. The behavior of $\tilde{\alpha}$ in (3.4) and (3.6) is invariant to the true values of the parameters of the trend function, μ and β [see Dickey (1984)]. Though it is not numerically invariant to the specification of the initial condition y_0, the effect is trivial for usual values of this parameter [Dickey (1984)]. Hence we can, without loss of generality, specify $\mu = \beta = y_0 = 0$ in (4.1) and (4.2), and analyse the unfiltered and filtered processes :

$$y_t = Z_t \qquad ; \qquad y_0 = 0, \tag{4.4}$$

$$y_t^F = \psi(L)Z_t \qquad ; \qquad y_0^F = 0. \tag{4.5}$$

Similarly $\hat{\alpha}$ in (3.3) and (3.5) is invariant with respect to μ (though not with respect to β) and the effect of the initial condition is again negligible.

To ease the presentation of our analytical results, we first consider regressions with no estimated intercept and trend. We then show how the results remain identical in the more general cases. Hence we consider :

$$y_t = \alpha^* y_{t-1} + e_t^* , \tag{4.6}$$

and

$$y_t = \alpha^* y_{t-1} + \sum_{j=1}^{k} c_j^* \Delta y_{t-j} + e_t^* , \tag{4.7}$$

where the autoregressive parameter is specifically defined in the case of (4.6) as :

$$\alpha^* = \Sigma_1^T y_t y_{t-1} / \Sigma_1^T y_{t-1}^2 \tag{4.8}$$

with unfiltered data, and with filtered data as :

$$\alpha_F^* = \Sigma_1^T y_t^F y_{t-1}^F / \Sigma_1^T \left[y_{t-1}^F\right]^2 . \tag{4.9}$$

4.1 Limiting Results in the Unit Root Case

We first consider the limiting behavior of α^* and α_F^* when A(L) has a root on the unit circle. In this case both estimators converge to 1 as the sample size increases; hence, there is no asymptotic bias. To see this, note that from (4.2) with $\beta = 0$, we have

$$y_t^F = \psi(L)y_t = \psi(L)y_{t-1} + \psi(L)v_t$$

$$= y_{t-1}^F + w_t ,$$

where $w_t = \psi(L)A^*(L)^{-1}B(L)\varepsilon_t$. From (4.9), $(\alpha_F^* - 1) = T^{-1} \Sigma_1^T y_{t-1}^F w_t / T^{-1} \Sigma_1^T \left[y_{t-1}^F\right]^2$. Given that A*(L) and B(L) have roots outside the unit circle, w_t is a stationary and invertible ARMA process and $T^{-1} \Sigma_1^T y_{t-1}^F w_t \to 0$ (in probability) as $T \to \infty$. Hence $\alpha_F^* \to 1$ (in probability). α^* also converges to 1 using usual arguments [e.g., Phillips (1987)].

The asymptotic distributions of $T(\alpha_F^* - 1)$ and $T(\alpha^* - 1)$ are different because of the transformed correlation structure of the error term. First note, that both v_t and w_t satisfy the mixing conditions for the application of a functional central limit theorem to their partial sums [see, e.g., Phillips (1987) and Phillips and Perron (1988)]. Let $\sigma_v^2 = \lim_{T\to\infty} T^{-1} E[S_{T,v}^2]$ where $S_{T,v} = \Sigma_1^T v_t$; and similarly, $\sigma_w^2 = \lim_{T\to\infty} T^{-1} E[S_{T,w}^2]$, where $S_{T,w} = \Sigma_1^T w_t$. We note that $\sigma_v^2 = 2\pi h_v(0)$ and $\sigma_w^2 = 2\pi h_w(0)$ where $h_x(0)$ denotes the non-normalized spectral density function of x evaluated at frequency 0. We have $\sigma_v^2 = \sigma_w^2$ given that $\sigma_w^2 \equiv \sigma_\varepsilon^2[\psi(1)^2 B(1)^2 / A^*(1)^2] = \sigma_\varepsilon^2[B(1)^2 / A^*(1)^2] \equiv \sigma_v^2$ using the fact that $\psi(1) = 1$ since the weights of the filter sum to unity. The variances of the processes $\{v_t\}$ and $\{w_t\}$ are, however, different. To see this note that $S_w^2 \equiv Var(w_t) = Var(\psi(L)v_t) = \sum_{j=-m}^{m} \sum_{l=-m}^{m} \psi_j \psi_l Cov_v(j-l)$ (see, e.g., (4.10)), where $Cov_v(k)$ is the autocovariance function of $\{v_t\}$ at lag k. Hence, the limiting distribution of $T(\alpha^*-1)$ is $(1/2)\ (W(1)^2 - S_v^2/\sigma_v^2)/\int_0^1 W(r)^2 dr$ and the limiting distribution of $T(\alpha_F^*-1)$ is $(1/2)\ (W(1)^2 - S_w^2/\sigma_v^2)/\int_0^1 w(r)^2 dr$. The expressions for the limiting distributions change accordingly when an intercept and/or trend are estimated.

While the asymptotic distribution of $T(\alpha^*-1)$ and $T(\alpha_F^*-1)$ are different, the asymptotic distribution of the Dickey-Fuller and Phillips-Perron statistics are identical [and as tabulated in Fuller (1976)]. This is because the structure of the errors in each case is such that the corrections applied by each procedure effectively eliminate the dependency of the asymptotic distribution on nuisance parameters associated with the correlation in the errors.

The basic result that the asymptotic null distributions of the unit root test statistics are not affected by filtering the data does not imply, however, that the finite sample distributions are unaffected, and the simulation results of section 6 show that, indeed in some cases, the finite sample distributions can be substantially different after filtering.

4.2 Limiting Results in the Stationary Case

When the polynomial A(L) does not contain a unit root, things are rather different because there exists an asymptotic bias induced by filtering, and -- more importantly -- the bias is positive. Thus, in (4.6) and (4.7), α_F^* has a limit greater than α^*. Hence, one can expect unit root tests performed with filtered data to be less powerful against stationary alternatives.

Consider first the behavior of α^* in (4.6). We have $\operatorname{plim}_{T\to\infty} \alpha^* = \operatorname{plim}_{T\to\infty} T^{-1}\sum_1^T y_t y_{t-1} / \operatorname{plim}_{T\to\infty} T^{-1} \sum_1^T y_{t-1}^2 = \gamma_y(1) / \gamma_y(0)$ where $\gamma_y(j)$ is the autocovariance function of $\{y_t\}$ at lag j. With filtered data, we have $\operatorname{plim}_{T\to\infty} \alpha_F^* = \operatorname{plim}_{T\to\infty} T^{-1} \sum_1^T y_t^F y_{t-1}^F / \operatorname{plim}_{T\to\infty} T^{-1}(\sum_1^T y_{t-1}^F)^2$. For the more general case that will follow, consider the probability limit of $T^{-1} \sum_1^T y_t^F y_{t-s}^F$. Denoting the order of the symmetric MA polynomial $\psi(L)$ by m, we have :

$$\operatorname*{plim}_{T\to\infty} T^{-1} \Sigma_1^T y_t^F y_{t-s}^F = \operatorname*{plim}_{T\to\infty} T^{-1} \Sigma_1^T [\psi(L)y_t][\psi(L)y_{t-s}] ,$$

$$= \operatorname*{plim}_{T\to\infty} T^{-1} \Sigma_1^T \Sigma_{i=-m}^m \psi_i y_{t+i} \Sigma_{j=-m}^m \psi_j y_{t+j-s} ,$$

$$= \Sigma_{i=-m}^m \Sigma_{j=-m}^m \psi_i \psi_j \left[\operatorname*{plim}_{T\to\infty} T^{-1} \Sigma_1^T y_{t+i} y_{t+j-s}\right] ,$$

$$= \Sigma_{i=-m}^m \Sigma_{j=-m}^m \psi_i \psi_j \gamma_y(i - j + s) \equiv \gamma_y^F(s) . \tag{4.10}$$

Using (4.8) through (4.10), it is easy to deduce the following asymptotic bias of α_F^* which we denote as $b(\alpha_F^*, 0)$ for reasons that will become clear later :

$$b(\alpha_F^*, 0) = [\gamma_y^F(1) / \gamma_y^F(0)] - [\gamma_y(1) / \gamma_y(0)] . \tag{4.11}$$

With (4.10), it is easy to numerically compute this asymptotic bias provided we specify the covariance structure of the original series $\{y_t\}$, i.e. the prefiltered series. Before we do so we now consider the more general case of the AR(p) process (4.7) with $p = k + 1$, where now α^* represents the sum of the autoregressive coefficients. First write (4.7) as :

$$y_t = \theta_1 y_{t-1} + \theta_2 y_{t-2} + \dots + \theta_p y_{t-p} + e_t , \tag{4.12}$$

where, in accordance with (4.7), we have $\alpha^* = \sum_{j=1}^{p} \theta_j$ and $c_i^* = -\sum_{j=i}^{p-1} \theta_j$. Let $\theta' = [\theta_1, \theta_2, \dots, \theta_p]$. The least-squares estimator of the vector of parameters θ is given by $\hat{\theta} = A_n^{-1} V_n$ where $A_n = T^{-1} \sum_{t=p+1}^{T} X_t'X_t$, $V_n = T^{-1} \sum_{t=p+1}^{T} X_t'y_t$ where $X_t = (y_{t-1}, \dots, y_{t-p})$ is a p x 1 vector of lagged values of the data. For the filtered data, $\hat{\theta}_F$ is defined similarly with $\{y_{t-s}\}_{s=0}^p$ replaced by $\{y_{t-s}^F\}_{s=0}^p$. Let $e' = (1, 1, \dots, 1)$ be a 1 x p vector of ones. Then $\alpha_F^* = e'\hat{\theta}_F$ and $\alpha^* = e'\hat{\theta}$. Denote the asymptotic bias of α_F^* when k lags of first differences of the data are included in (4.7) by $b(\alpha_F^*, k)$ (note that $p = k + 1$). The following proposition characterizes this asymptotic bias.

Proposition 1 : Let $\{y_t\}$ be generated by (4.4). Let $\hat{\theta}$ be the OLS estimator of θ obtained from regression (4.12) and $\alpha^* = e'\hat{\theta}$. Similarly let $\{y_t^F\}$ be generated by $y_t^F = \psi(L)y_t$ and $\alpha_F^* = e'\hat{\theta}_F$

where $\hat{\theta}_F$ is the vector of least-squares estimators of θ in (4.12) when the filtered data, y_t^F, are used. Then

$$b(\alpha_F^*, k) \equiv \operatorname*{plim}_{T\to\infty} \alpha_F^* - \operatorname*{plim}_{T\to\infty} \alpha^*$$

$$= e'A^{-1}V - e'A_F^{-1}V_F , \tag{4.13}$$

where A is a p x p matrix with elements $a_{ij} = \gamma_y(i - j)$ (i, j = 1,...,p), V is a p x 1 vector with elements $v_i = \gamma_y(i)$ (i = 1,...,p) ; A_F and V_F are defined similarly with elements $a_{ij}^F = \gamma_y^F(i - j)$ and $v_i^F = \gamma_y^F(i)$, with $\gamma_y^F(s)$ defined by (4.10).

Proposition 1, whose proof is straightforward and omitted, provides a computable expression for the limiting bias of the least-squares estimator of α_F^* in (4.7) when k lags of first differences of the data are included (i.e. in a p^{th} order autoregression with p = k + 1). Given (4.10), all that is needed to compute this asymptotic bias is the covariance function $\gamma_y(s)$ of the original unfiltered data and the weights of the filter polynomial $\psi(L)$. Note that when k = 0, p = 1 and (4.13) reduces to (4.11).

Though the asymptotic bias as described in proposition 1 pertains to the estimators α_F^* when no constant and trend are included in the regression, the same quantitative result holds if a constant or a constant and a time trend are included in the regression. This is a consequence of the next proposition.

Proposition 2 : Denote by α^*, $\hat{\alpha}$ and $\tilde{\alpha}$, respectively, the OLS estimator of α in (4.6), (3.5) and (3.6). Similarly, denote by α_F^* , $\hat{\alpha}_F$ and $\tilde{\alpha}_F$ the OLS estimator in the same regressions using the filtered data $y_t^F \equiv \psi(L)y_t$. Consider the estimates obtained using the unfiltered data generated by (4.1) with the following specification : i) $\mu = \beta = 0$ for α^* and α_F^*; ii) $\beta = 0$ (μ unrestricted) for $\hat{\alpha}$ and $\hat{\alpha}_F$; and iii) μ and β unrestricted for $\tilde{\alpha}$ and $\tilde{\alpha}_F$. Then

$$\operatorname*{plim}_{T\to\infty} \alpha^* = \operatorname*{plim}_{T\to\infty} \hat{\alpha} = \operatorname*{plim}_{T\to\infty} \tilde{\alpha}$$

and

$$\operatorname*{plim}_{T\to\infty} \alpha_F^* = \operatorname*{plim}_{T\to\infty} \hat{\alpha}_F = \operatorname*{plim}_{T\to\infty} \tilde{\alpha}_F .$$

The result of Proposition 2 states that, when analyzing the limiting behavior of $\hat{\alpha}$, $\tilde{\alpha}$, $\hat{\alpha}_F$ and $\tilde{\alpha}_F$ we need only consider the limiting behavior of α^* and α_F^* defined by (4.6) in the first-order case and (4.7) in the p^{th} order case. In particular the asymptotic bias is given by

$$\text{bias} = \underset{T\to\infty}{\text{plim}}\ \hat{\alpha}_F - \underset{T\to\infty}{\text{plim}}\ \hat{\alpha} = \underset{T\to\infty}{\text{plim}}\ \tilde{\alpha}_F - \underset{T\to\infty}{\text{plim}}\ \tilde{\alpha} = \underset{T\to\infty}{\text{plim}}\ \alpha_F^* - \underset{T\to\infty}{\text{plim}}\ \alpha^*.$$

Note, however, that the asymptotic distributions will differ, but this causes no difficulties as we shall concentrate solely on the extent of the asymptotic bias. The proof of Proposition 2 is straightforward and details are omitted. The intuition is first that the filter $\psi(L)$ leaves unchanged the trend function of the series as defined by (4.1). Given that the least squares estimator of α in (3.6) is invariant to the values of μ and β, we may specify, without loss of generality $\mu = \beta = 0$ (similarly the estimator of α in (3.5) is invariant to the true value of μ; hence, we may set $\mu = 0$ again). The idea behind the proof of proposition 2 is simply that $\tilde{\alpha}_F$, $\hat{\alpha}_F$ and α_F^* have identical limiting values though not equal to the limit in the unfiltered case. Tedious algebra shows this to be the case.

4.3 The Covariance Structure of the Processes

In this section, we briefly describe the covariance function of the processes outlined in section 3. Consider first the nonseasonal ARMA(1,1) model given by (3.1). We have [e.g., Box and Jenkins (1970, p. 76)] :

$$\begin{aligned} \gamma_y(0) &= (1 + \alpha^2 + 2\alpha\theta) / (1 - \alpha^2)\,; \\ \gamma_y(1) &= (1 + \alpha\theta)(\alpha + \theta) / (1 - \alpha^2)\,; \qquad (4.15) \\ \gamma_y(j) &= \alpha\gamma_y(j-1)\,, \qquad j \geq 2\,. \end{aligned}$$

In the case of the seasonal unobserved component model, we have, given that ε_t^s and ε_t^{ns} are uncorrelated with each other, $\gamma_y(j) = \gamma_y^{ns}(j) + \gamma_y^s(j)$, i.e. the covariance, at lag j, of y is the sum of the covariances, at lag j, of the seasonal and nonseasonal components. The covariance structure of the nonseasonal component is given by (4.15) with α_{ns} and θ_{ns} as parameters instead of α and θ. The covariance structure of the seasonal component is given by [e.g., Box-Jenkins (1970, p. 333)] :

$$\gamma_y^s(0) = 1 + (\theta_s + \alpha_s)^2 / (1 - \alpha_s)^2$$

$$\gamma_y^s(1) = \dots = \gamma_y^s(s-1) = 0 \tag{4.16}$$

$$\gamma_y^s(s) = (\theta_s + \alpha_s)(1 + \alpha_s\theta_s) / (1 - \alpha_s^2)$$

$$\gamma_y^s(j) = \alpha_s\, \gamma_y^s(j-s) \qquad j \geq s + 1 ,$$

where s is the seasonal period (4 for quarterly data and 12 for monthly data). The autocovariance function of $\{y_t\}$ is simply the sum of the autocovariances in (4.15) and (4.16).

For the nonseasonal ARMA(1,1) process we present results for the following configuration of parameter values : α = -0.9, -0.5, 0.0, 0.1, 0.5, 0.8, 0.9 and 0.95 and θ = -0.8, -0.5, 0.0, 0.5 and 0.8 . For the seasonal unobserved component model we specify α_{ns} = 0.0, 0.5, 0.8 and 0.9; θ_{ns} = 0.0, 0.5, -0.5 and θ_s = 0.0, 0.5 and -0.5 . We present tabulated results only for the case where α_s = 0.85 for the sake of brevity.[8] In each case, we present results on the asymptotic bias of the least-squares estimator of the sum of the autoregressive coefficients in an autoregression of order k + 1 for k = 0, 4 and 8. For the seasonal unobserved component model, we also considered k = 12. The specifications are chosen to ensure adequate comparisons with both the finite sample results about the power of the test statistics presented in Section 5, as well as to provide some possible comparisons with earlier studies about the behavior of the statistics with unfiltered data.

In each of the cases described above, we consider the effect of applying four different linear and symmetric filters (whose coefficients sum to one). First, the linear approximation of the X-11 filter for both the quarterly and monthly versions whose weights are described in the appendix. We also consider the Henderson moving average filter, whose weights are described in (2.7) for the quarterly version and in (2.3) for the monthly version.

4.4 Description of the Results

Consider first the case of the nonseasonal ARMA(1,1) model presented in Tables 1 to 3 for k = 0, 4 and 8 respectively. The major feature that emerges from these tables is that the asymptotic bias is always positive (with one small exception). Hence, filtering the data causes an increase in the limiting value of the sum of the autoregressive coefficients over what the limit is with unfiltered data. Given that the Dickey-Fuller and Phillips-Perron statistics are

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8 We also considered asymptotic biases with α_s = 0.9 and 0.95. The pattern of the results is the same with the biases being larger throughout.

based on these estimators, we can expect a loss of power associated with these tests when the data are seasonally adjusted even if no seasonality is really present in the data.

In general, the asymptotic bias decreases as the AR parameter α increases with non-negative values of θ and α. For θ negative, non-monotonic behavior occurs when k = 0 and to some extent k = 4. With k = 8, the asymptotic bias decreases with α for any range of values (with very minor exceptions). For a given value of α the behavior of the asymptotic bias shows non-monotonicity in the MA parameter θ. Consider, for example, $\alpha = 0.0$ with the X-11 filter and k = 0. Here the asymptotic bias is greatest for values of θ around $\theta = 0.0$. When k = 4 or 8, the bias shows non-monotonic patterns that are different for the chosen values of θ and varies depending on the filter, i.e. the quarterly and monthly cases.

When k = 0, the asymptotic bias is in general small for the X-11 filter. It tends to increase as k increases especially for negative values of α. The bias is, however, quite large when the quarterly Henderson MA filter is applied. For example, when $\theta = 0.0$, the limit of $\hat{\alpha}^*_F$ varies between 0.903 and 0.99 as α varies between -0.9 and 0.9. It is, indeed, remarkable how the limit of $\hat{\alpha}^*_F$ is close to one for most configurations of parameters α and θ when applying the monthly Henderson MA filter. The behavior of the asymptotic bias is roughly similar when k is 4 or 8. We also experimented with k = 12 and obtained similar results.

The behavior of the asymptotic biases changes dramatically with the unobserved seasonal component model presented in Tables 4 through 7 for k = 0, 4, 8 and 12 respectively. Consider first the case where k = 0 (Table 4). The first thing to note is that the biases are positive (with one exception) and quite large. For example, consider the case k = 0, $\alpha_{ns} = \theta_{ns} = \theta_s = 0$ and $\alpha_s = 0.85$. The asymptotic bias induced by the quarterly X-11 filter is 0.405, by the monthly X-11 filter 0.193, and by the quarterly Henderson MA filter it is 0.916 . The relative ranking stays similar across experiments; in particular, the asymptotic bias is larger with the quarterly X-11 filter than with the monthly version. The opposite is true with the Henderson MA filters, the monthly version showing greatest bias overall. For many cases, it is remarkable how the limit of α^*_F is biased towards one (without exceeding one). For the monthly Henderson filter, the limit of α^*_F is again above 0.9 for the majority of cases irrespective of the limit of α^* using unfiltered data. For the X-11 filter the smallest asymptotic bias occurs when there is negative serial correlation in the moving average of the seasonal component, i.e. the smallest asymptotic bias occur in the least likely case in practice.

In the quarterly case, the picture is very different when considering an autoregression with k equal to 4 or 8 instead of k = 0. When k = 4, the asymptotic bias induced by the quarterly X-11 filter and the quarterly Henderson MA filter are reduced dramatically. Indeed, the biases become close to the level they are with the nonseasonal ARMA(1, 1) model, especially concerning the X-11 filter. When k increases from 4 to 8 the bias induced by the quarterly X-11 filter shows a further marginal decrease, while the bias due to the quarterly Henderson MA filter shows a marginal increase.

This decrease in bias is not observed for the monthly filters when k is increased from 0 to 4 or 8. Indeed, the asymptotic biases of both the X-11 and Henderson MA filters still increase as k increases up to 8 when α_N = 0.0 or 0.5 (sometimes with a slight decrease from k = 0 to k = 4 when α_{NS} = 0.5) and marginally decreases when α_{NS} = 0.8 or 0.9. While the limit of α_F^* induced by the monthly X-11 filter varies across parameter configurations it can reach as high a value as 0.675 when the limiting value of α^* with unfiltered data is 0.0 (e.g., $\alpha_{ns} = \theta_{ns} = 0$, $\alpha_s = 0.85$, $\theta_s = 0.5$ and k = 4; the corresponding figure with k = 8 is 0.778). With α_s = 0.95, the corresponding biases are 0.868 with k = 4 and 0.919 with k = 8. On the other hand, the limiting value of α_F^* induced by applying the Henderson MA filter appears to vary only slightly, irrespective of the limiting value of α^* with unfiltered data. With k = 4 this limiting value is around 0.96 and with k = 8 it is around 0.99 . Hence, as was the case with the nonseasonal ARMA model, the asymptotic biases induced by the various filters are, in almost all cases, positive. However, with a seasonal unobserved component model, the asymptotic biases can be very large. Table 7 presents results for the case where k = 12. It can be seen that the asymptotic biases with the monthly X-11 filter decrease substantially, as was the case with k = 4 in a quarterly seasonal model. Hence, our results clearly show that the bias in the sum of the autoregressive coefficients is substantial unless the order of the estimated autoregression is at least as great as the seasonal period when correlation at seasonal lags is present in the original data and the X-11 filter is applied.

We draw the following tentative implications of our results with respect to the effect of seasonal adjustment on tests for a unit root. First, given that the seasonal adjustment filters analyzed here cause an upward asymptotic bias in the estimator of the sum of the autoregressive coefficients when the model is stationary, tests for a unit root will have less power with filtered data than with unfiltered data. This fact is best illustrated when the underlying series does not contain a seasonal component and the filter is nevertheless applied.

More interestingly, the extent of the asymptotic bias depends upon the nature of the underlying seasonal component, the period of the seasonality and the length of the autoregression considered. The major feature that emerges from our results is that if the length of the autoregression estimated is less than the seasonal period, filtering the data induces a substantial upward bias in the sum of the autoregressive coefficients. For example, our results show a large bias when applying the quarterly filter and estimating an autoregression with k = 0 when seasonal correlation is present. However, the asymptotic bias decreases rapidly for all models considered when k is increased to 4. With the monthly filter our results showed an increasing bias as k increased from 0 to 4 and to 8 and a substantial decrease when k reached 12; i.e. at least as large as the seasonal period.

Consider first the implications for unit root tests of the Dickey-Fuller type based on the estimator of the sum of the autoregressive coefficients in an autoregression of order p, say. Our results indicate that one must consider an estimated autoregression of an order at least as great as the seasonal period in order to avoid a substantial bias. Even with such an order, however, there will still remain an upward bias. Hence, seasonal adjustment of the data does not permit a reduction in the order of the autoregression. This is basically due to the fact that even though seasonal adjustment reduces correlation in the data at seasonal frequencies, it induces a bias in the autocorrelation function at lags less than the seasonal period which does not vanish even asymptotically. Given that an autoregression of order at least as great as the seasonal period is needed with filtered data, it is reasonable to expect tests for a unit root to have higher power with unfiltered data than with filtered data. Indeed, not only does seasonal adjustment not permit a reduction in the length of the autoregression estimated (which would help increase power), but it also induces an upward bias in the statistic of interest.

Our results imply still more pronounced effects on the behavior of the Phillips-Perron statistics. The important element is that these statistics are based on an estimated first-order autoregression no matter what the correlation structure of the data. They apply a correction to the statistics based on this first-order autoregression which eliminates any dependency of the asymptotic distribution on the nature of the correlation present. Given our results, as long as there is a seasonal component present in the data, the Phillips-Perron statistics will be biased towards non-rejection if filtered data are used. This loss of power is due to the fact that seasonal filters induce a substantial upward bias in the first-order autocorrelation of the data when there is seasonal autocorrelation present. From an asymptotic perspective, it is therefore expected that the Phillips-Perron test statistics will lead to tests of the unit root hypothesis with greater power if unadjusted data are used. Of course the finite sample performance of the tests may be significantly affected, even with unfiltered data, if the finite sample properties of the estimator in a first-order autoregression or of the correction made to it are affected by the presence of seasonal correlation. These and other issues are the focus of the next section which presents an extensive simulation study of the finite sample behavior of both the Phillips-Perron and Dickey-Fuller statistics when seasonal issues are taken into account.

5. A SIMULATION STUDY OF THE FINITE SAMPLE BEHAVIOR

The finite sample properties of the Dickey-Fuller t-test and the Phillips-Perron $Z(t_{\hat{\alpha}})$ and $Z(\hat{\alpha})$ tests with filtered data series are examined by means of a detailed simulation experiment. The design of the Monte Carlo experiment is described first, followed by a discussion of the results.

The size and power of the unit root tests is first studied with data generated by the univariate (nonseasonal) ARMA(1,1) process described in equation (3.1). The design of this first experiment is similar to that of several studies, such as Dickey (1984), Dickey, Bell and Miller (1986) and Schwert (1989), which report finite sample simulation results on size and power with the same type of data generating process. In our experiment the time series are affected, however, by filtering. Three different types of filters will be considered, namely (1) the quarterly and monthly linear seasonal adjustment filters; (2) the quarterly and monthly Henderson MA filters and finally (3) the actual X–11 procedure. Comparisons with existing studies allow us to assess the effect of the filtering process on the finite sample distribution of each of the tests. Next we turn to seasonal unobserved component models as described in equations (3.2a) through (3.2c). Monthly and quarterly seasonal processes are considered. The size and power of the t–tests and the normalized bias test were simulated, using the same filters as in the first experiment. In each experiment we also report the size and power of the same unit root tests applied to the unfiltered time series, again in order to make the proper comparisons.

Simulations will be reported for two sample sizes, the first containing 100 observations and the second 200. The choice of both sample sizes was motivated by the fact that filters with a quarterly as well as a monthly frequency were used. Data sets ranging from 25 to 50 years of quarterly data are common. In addition, quite often one finds monthly series with between ten and twenty years of observations. Moreover, the sample sizes selected also correspond to those often appearing in previous studies. To obtain a sample of either 100 or 200 filtered time series with symmetric two–sided filters, it was necessary to generate extra data points before and after the actual sample. The unfiltered data sets were therefore simulated with 200 extra observations, half prior to the sample of interest and the other half following it. Hence a sample with 100 filtered observations starts with the 101st observation of the unfiltered data series after it is adjusted by the filtering process. A comparison with the corresponding unfiltered data also starts with the 101st observation. There is one exception to the aforementioned modus operandi. The simulations with the actual X–11 (SAS) procedure involves neither starting values nor extra tail observations. Lack of such extra observations makes the program switch filters at each end of the data sets it filters. The experiments were based on 10000 replications, except for the SAS X–11 experiments, where only 1000 replications were used.[9] In this latter case, we also restricted the experiment to the Dickey–Fuller unit root t–test. Due to space limitations not all the results can be reported. Moreover, there are similarities in the simulation findings which allows us to reduce the number of cases to report explicitly.

Table 8 reports the size and power of the three types of unit root tests, calculated from a regression equation without an estimated trend, for a nonseasonal ARMA(1,1) process filtered with the monthly linear X–11 filter. Table 9 reports size and power of the tests when an estimated trend is included in the regression equation. The parameter configurations for the ARMA(1,1) data generating process defined in equation (3.1) were as follows : α = 0.95, 0.9 or 0.8 and θ = 0.5, 0.0 or -0.5. One must be careful of course when reading this and subsequent tables. While the input process is a simple ARMA(1,1) it should be kept in mind that the tests are actually applied to a data set with a complex autocorrelation structure due to the filtering. The lag length for the DF tests considered are 4, 6 and 8. The number of lags in the correction term of the PP tests are 6, 8 and 12. Both are denoted k in the tables.

For the Dickey–Fuller t–test one finds the right size both at the five and ten percent nominal level when the series are filtered with the linear X–11 filter. This is true whether or not an estimated trend is included in the regression equation. Thus our simulation results show that the small sample properties of the OLS estimator with filtered series are the same in this respect as the asymptotic properties. In addition, our results also confirm similar findings about the size of DF t–tests reported in earlier Monte Carlo studies which considered unfiltered data series. Tables 8 and 9 also contain evidence on the size of the two Phillips–Perron unit root tests being studied. Here the results are quite different. Despite the fact that the asymptotic distribution of the OLS estimator is invariant to linear symmetric filters which sum to one for data generated by unit root processes, one observes a conservative test under the null hypothesis of a random walk input process or a unit root pre-filtered process with positively autocorrelated errors. In contrast, when the MA term θ is negative the Phillips-Perron tests are clearly too liberal. The results are similar to those reported by Schwert (1989), Perron (1988) and Phillips and Perron (1988), who studied the case of unfiltered ARMA process. We also considered the actual X-11 filter, although we do not report it in the table. Results with the actual X-11 procedure do not indicate the right size (i.e. they were too liberal) for the DF t–test in particular when the number of AR lags included in the regression increases. As noted in section 2, the X-11 procedure is characterized by endogenous filter-switching. The procedure is, therefore, no longer a time-invariant filtering

[9] The number of replications was reduced for the latter because it was prohibitively expensive to do otherwise. The structure of SAS programming, in particular the fact that SAS procedures cannot be nested forced us to store large collections of random samples into SAS data sets which were passed through the X-11 procedure. The output of PROC X11 was again retrieved into data sets before final analysis. The SAS programs were therefore CPU time-consuming and required a great deal of memory space.

process applied to the original data set. It is difficult, however, to formalize the effects of time-variation associated with this filter. We know, however, that the theoretical developments in section 4, specifically the invariance of the unit root limiting distribution with regard to linear filters, do not apply when the actual X-11 procedure is used. The simulation results seem to suggest, however, that longer AR expansions affect adversely the size of the DF t-tests in moderately sized samples as is the case with unfiltered data.

Let us turn now to the power of the unit root tests presented in Tables 8 and 9. The asymptotic bias calculations appearing in Tables 1 through 3 indicated that for univariate ARMA(1,1) models the asymptotic biases were usually minor. Hence we should expect, for small samples, results approximately similar to previous results with unfiltered data except for a slight reduction in power. The asymptotic bias being minor in many cases, it is no surprise to find that our results for $\alpha = 0.85$ are about the same as the empirical power reported in, for instance, Perron (1988) where tests were applied without filtering. In Table 2 of Perron (1988) it is reported that for k equal to 6 and a sample size of 200 the rejection frequency at the 5 % nominal level for the DF test is 0.94 for $\Theta = -0.5$. We do have the same result in Table 8. Likewise, when $\Theta = 0$. we find 0.88 for the DF test versus 0.86 in Perron (1988). The difference is probably due to the number of replications in the experiments, namely 10000 in ours rather than 1000 in Perron (1988). As α gets closer to one, the asymptotic bias should play a greater role. A comparison of the results in Tables 8 and 9 indicates that including an estimated trend in the regression model also leads to a substantial reduction in power, as expected. The power of the DF test when the actual X-11 filter is applied looks very similar to the results with the linear X-11 filter shown in Tables 8 and 9. A minor reduction in power can be detected in some cases, but it appears not to be uniform across the models. One must keep in mind, however, that filtering series with the actual X-11 filter resulted in a liberal DF test. Consequently, considering size-adjusted power would lead one to conclude that filtering with the actual X-11 filter reduces the power of the DF test more than the linear X-11 filter does.

We also calculated the power with the Henderson MA filters (monthly and quarterly). The Henderson MA filter destroys the power of all the tests. We do not explicitly report the results in tables because they are relatively easy to summarize. One striking example may illustrate how low the power of the DF test is with the Henderson MA filter. Consider the case of an AR(1) process with $\alpha = 0.85$. When the process is filtered with the Henderson MA filter the rejection frequency of the test in a sample of 100 is only between 20 and 30 percent at the five percent critical value. Such results are in fact not so surprising. The transfer function of the Henderson MA filters, quarterly and monthly, showed that the filter eliminates all the high frequencies. Unit root tests get their power in small samples from the implied high frequency behavior of low frequency models. When all the high frequency behavior has disappeared because the series was passed through a smoothing filter we should expect to have a reduction in power. Finally, we conducted experiments with the quarterly linear X-11, the quarterly Henderson MA filter and the actual X-11 procedure. While our study was not as extensive as in the monthly case, we observed the same pattern in the results.

We now turn to the small sample behavior of the tests when the unfiltered data sets are generated by seasonal unobserved component models. First, we discuss the Dickey-Fuller test. The size and power of the test, when no trend is included, appear in Table 10. The case of an estimated trend is not reported as it is similar. The quarterly unobserved component model is discussed first. The size of the test when it is applied to the unfiltered data sets is reported together with the results when the series are filtered. We report results for three different lag lengths of the AR polynomial expansion. The shortest lag length is 2 and hence less than the quarterly seasonal periodicity. The other two, namely 4 and 6, equal or exceed the seasonal period.

Three parameter configurations were selected for the MA parts of the unobserved components. They were selected on the basis of the asymptotic biases reported in Tables 4 through 7. A first case with MA components parameters $\theta_{ns} = 0.5$ and $\theta_s = -0.5$ represents one where the asymptotic bias is small (e.g., Table 4 with $\alpha_{ns} = 0.9$). A second case with $\theta_{ns} = 0.0$ and $\theta_s = 0.5$ has a large asymptotic bias (see also Table 4) and finally a third one has an intermediate asymptotic bias with parameters $\theta_{ns} = 0.0$ and $\theta_s = 0.0$.

A first thing to note is that the DF test applied to unfiltered data is top liberal when k = 2, except perhaps in the case where the asymptotic bias is low. When k equals 4 or 6, we do get approximately the right size. When the DGP is filtered with the linear and standard X-11 filters, we observe an empirical size in finite samples corresponding to the nominal size, with perhaps one exception, namely the high asymptotic bias case. Hence, when filtering induces a large asymptotic bias under the alternative it seems to also affect the null distribution of the test statistic in small samples. The size of the tests applied to data filtered with the Henderson MA filter is incorrect in particular with k = 4, where it tends to overreject. It may appear that the results with the Henderson MA filter obtained here are in conflict with the previous results we had with the same filter. That apparent conflict can be explained, however. While the Henderson MA filter eliminates the high frequencies it does not eliminate the seasonal frequency, neither in the monthly nor in the quarterly case. From figures A.2 and A.4 we note that the transfer function for both the quarterly and monthly Henderson MA filter are about 0.7 at the seasonal frequency. This means that the filter does not really erase the strong seasonal correlation in the unit root process. In contrast, the seasonal adjustment filters do eliminate the seasonal correlation, albeit imperfectly, which explains why they have the right size. It is also interesting to observe that here the time-varying actual X-11 filter yields results similar to the linear case.

The power of the DF tests also appears in Table 10. The reduction in power due to filtering the data with the linear X-11 filter depends on the magnitude of the asymptotic bias. With a low asymptotic bias for the OLS estimator the rejection rate only drops by one or two percent in many cases (see, e.g., Table 10, T = 200 and T = 100 with k = 2). The power reduction is larger when we turn to the intermediate and large asymptotic bias cases (see, e.g., T = 100 and k = 4 for the intermediate case). There are also a number of cases where the power increases with the application of the linear X-11 filter (see, for instance, T = 200 and k = 6 in the large asymptotic bias case). Such results are not surprising since in finite samples there are many determinants of the power function other than the asymptotic bias due to filtering, such as the number of AR lags included, the correlation structure of the errors, etc. We also simulated the size and power of a number of other cases characterized by large asymptotic biases (notably $\theta_{ns} = \theta_s = 0.5$; $\theta_{ns} = -0.5$ and $\theta_s = 0.0$; $\theta_{ns} = -0.5$ and $\theta_s = 0.5$) and found results similar to the ones reported in Table 10. We also observe in Table 10 that the actual X-11 program leads to less powerful tests compared to the linear version of the program as would be expected. It is also interesting to note that, in small samples, the power does not seem to increase when the expansion of the lag structure in the regression is increased from 2 to 4 and hence becomes greater than the seasonal lag, except for the Henderson filter. Consequently, the small sample evidence may appear to contradict the analytical large sample results in Tables 4 through 7 which suggested that the power should improve with AR expansions beyond the seasonal lag. This is not the case, however, since in small samples one faces a trade-off between more power due to the lower bias with AR expansions beyond the seasonal lag and less power due to a higher number of estimated parameters and loss of initial values in the auxiliary regression of the augmented Dickey-Fuller test.

The exact size of the Phillips-Perron tests is reported for the quarterly filters in Tables 11 and 12. We consider again three lag lengths, namely 6, 8 and 12, for the Newey-West correction applied to the regression errors. The size is not equal to the nominal one except when the MA parts of both the seasonal and nonseasonal are nonzero and equal to respectively -0.5 and 0.5. When we look at the power of the tests, also appearing in Tables 11 and 12, we find that in cases where both tests have the right size (namely when the MA parts are 0.5 and -0.5) the tests are most powerful when applied to the unfiltered series and much more powerful than the DF procedure. The tests applied to data transformed with the linear X-11 filter rank second and those with the Henderson MA filter are the least powerful. We also notice that the power reduction is more substantial in the large asymptotic bias case. This result is not surprising since both PP tests depend only on the estimated first AR coefficient. It confirms the asymptotic results discussed in the previous section. It is also interesting to note that the PP tests are much more powerful than the DF tests when the asymptotic bias of the OLS estimator is small.

The final results we report consider the finite sample size and power of unit root tests with data generated by monthly unobserved component models filtered by monthly seasonal adjustment filters. For this we turn to Table 13. Only the results for the DF are reported. The lag lengths chosen were 6 and 12, again to study the behavior of the finite sample distribution with a lag length smaller than and one equal to the seasonal period. Once again, we observe that the test has the right size with either of the two filters. With the unfiltered process, we note that the DF test has the right size except in the high asymptotic bias case when $k = 6$. Hence, like in Table 10 we notice that the filtering process may affect the null distribution in finite samples. We again face the power trade-off discussed earlier when we consider the lag length of the AR expansion. As in the quarterly case, we observe a reduction in power with the increase in lag length. A power reduction due to filtering is once again observed, sometimes very dramatically. The results in Table 13 show very convincingly that the asymptotic bias affects the power of DF tests. For example, when $T = 100$, $k = 6$ and the asymptotic bias is large, the power of the test drops from a 44 % rejection rate to 17 % (at the 5 % critical value) when the series is filtered. When $T = 200$, the power drops from 83 % to 52 %. When the asymptotic bias is low the power reduction is minor.

6. CONCLUSIONS

We have considered the effect of seasonal adjustment procedures on tests for a unit root. One result of interest is that the OLS estimator of the sum of the autoregressive coefficients in a univariate regression with a stationary series shows an upward bias when commonly used seasonal adjustment filters are applied. Sims (1974) showed that the OLS estimator remains consistent after filtering all the series with the same adjustment filter in a regression model with no lagged dependent variables. We show that this result does not go through in dynamic models. The limiting bias was quantified for a class of ARMA and seasonal unobserved component models and was found to be large in many cases. More interestingly, exact numerical calculations showed that the inconsistency of the OLS estimator is greatly reduced when one expands the AR polynomial in the regression beyond the seasonal lag. This result counters the common assumption that one can ignore seasonal lags in testing for a unit root once the series are adjusted. While the seasonal adjustment filter indeed eliminates seasonal correlation it induces a bias at the low frequencies, which makes unit root tests less powerful. Most of the asymptotic results seem also to hold in moderately sized samples. The results also provide additional justifications for using annual data when testing for a unit root (see Perron (1990) and Shiller and Perron (1985)).

Further extensions of our results call for some additional research projects. Namely, it is easy to see that the asymptotic bias shown here to exist in univariate dynamic models will hold for multivariate models as well. The extent of the bias in models such as VAR is of course difficult to assess at this stage. The impact of the asymptotic bias on impulse response functions would also be important to study. Empirical findings in Ghysels (1987) seem to suggest, however, that impulse response functions in VAR models estimated with adjusted data differ substantially from impulse response functions obtained from unadjusted series even if one concentrates on nonseasonal impulses. Our paper clearly suggests that there are important inference issues besides the methodological ones (discussed in Ghysels (1988)) associated with studying impulse response functions in seasonal dynamic models.

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APPENDIX A

Table A.1 : Filter Weights of the Linear Monthly X-11 Filter

Lags and Leads					
0	0.819	23	0.011	46	-0.003
1	0.019	24	-0.121	47	-0.005
2	0.018	25	0.013	48	-0.005
3	0.017	26	0.013	49	-0.003
4	0.016	27	0.013	50	-0.001
5	0.015	28	0.013	51	0.002
6	0.014	29	0.012	52	0.003
7	0.013	30	0.008	53	0.003
8	0.014	31	0.005	54	0.002
9	0.015	32	0.004	55	0.001
10	0.018	33	0.003	56	0.000
11	0.020	34	0.003	57	-0.001
12	-0.179	35	0.004	58	-0.001
13	0.021	36	-0.063	59	-0.001
14	0.020	37	0.005	60	-0.001
15	0.018	38	0.007	61	-0.001
16	0.016	39	0.008	62	-0.001
17	0.015	40	0.008	63	0.001
18	0.012	41	0.008	64	0.001
19	0.009	42	0.005	65	0.001
20	0.009	43	0.002	66	0.000
21	0.009	44	0.001	67	0.000
22	0.010	45	-0.001	68	0.000

Source : Ghysels (1984, Table A.3.3).

Table A.2 : Filter Weights of the Linear Quarterly X-11 Filter

Lags and Leads			
0	0.856	14	0.016
1	0.051	15	-0.005
2	0.041	16	-0.010
3	0.050	17	0.000
4	-0.140	18	0.008
5	0.055	19	-0.002
6	0.034	20	-0.003
7	0.029	21	0.000
8	-0.097	22	0.002
9	0.038	23	0.000
10	0.025	24	0.000
11	0.012	25	0.000
12	-0.053	26	0.000
13	0.021	27	0.000

Source : Laroque (1977, Table 1).

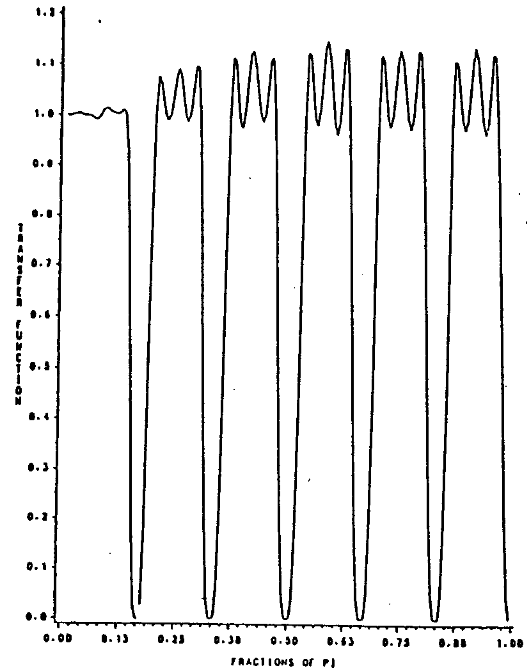

Figure A.1 : Transfer Function of the Linear Monthly X–11 Filter

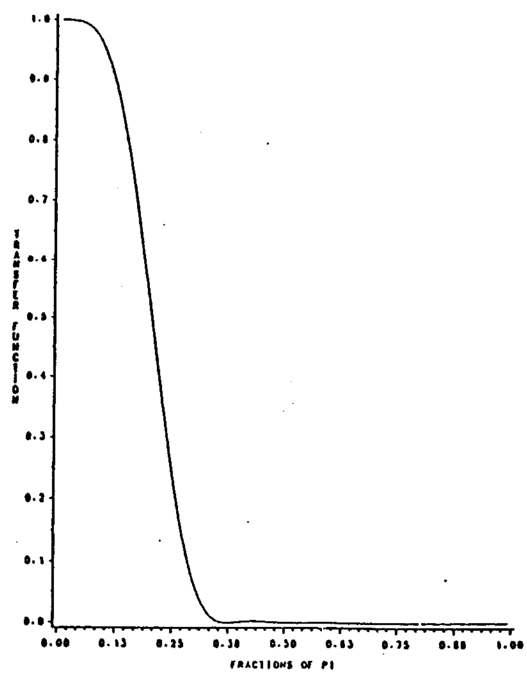

Figure A.2 : Transfer Function of the Monthly Henderson MA Filter

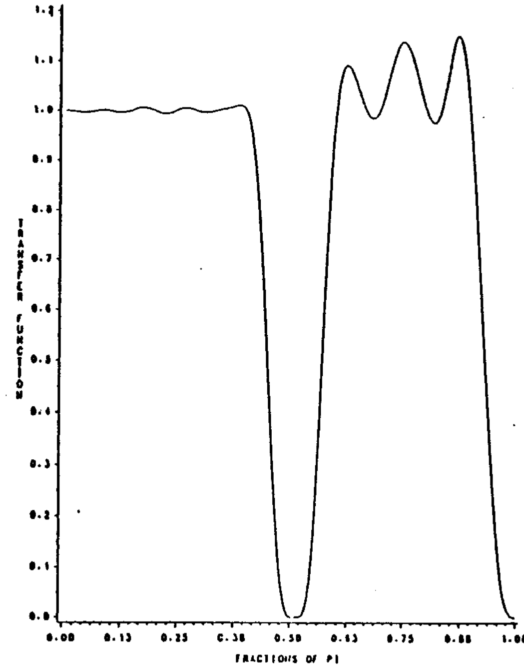

Figure A.3 : Transfer Function of the Linear Quarterly X-11 Filter

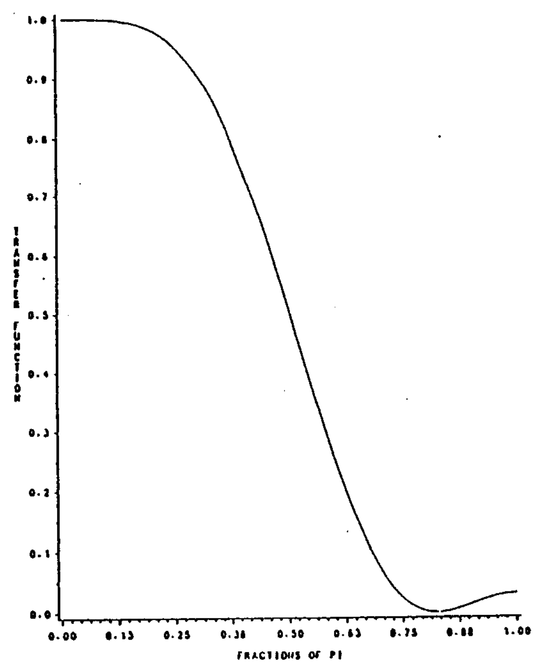

Figure A.4 : Transfer Function of the Quarterly Henderson MA Filter

Table 1 : $b(\alpha_F^*, 0)$; Asymptotic Bias of α_F^* with Filtered ARMA (1,1) Data; k = 0

	α	X-11 Filter Quarterly	X-11 Filter Monthly	Henderson Filter Quarterly	Henderson Filter Monthly	$\text{plim}_{T\to\infty} \alpha^*$
$\theta = -0.8$	−0.90	0.103	0.017	0.289	1.670	−0.949
	−0.50	0.038	0.00	0.707	1.558	−0.746
	0.00	0.006	0.001	0.679	1.318	−0.488
	0.10	0.008	0.002	0.667	1.270	−0.435
	0.50	0.039	0.006	0.620	1.080	−0.214
	0.80	0.071	0.027	0.576	0.915	0.000
	0.90	0.083	0.050	0.535	0.804	0.140
	0.95	0.089	0.067	0.467	0.666	0.300
$\theta = -0.5$	−0.90	0.113	0.018	0.372	1.783	−0.944
	−0.50	0.044	0.003	0.796	1.591	−0.714
	0.00	0.024	0.008	0.720	1.287	−0.400
	0.10	0.030	0.010	0.696	1.221	−0.330
	0.50	0.071	0.027	0.576	0.915	0.000
	0.80	0.082	0.058	0.389	0.555	0.400
	0.90	0.064	0.056	0.254	0.347	0.629
	0.95	0.042	0.040	0.150	0.200	0.788
$\theta = 0.0$						
	−0.90	0.184	0.033	0.777	1.803	−0.900
	−0.50	0.074	0.014	0.910	1.410	−0.500
	0.00	0.071	0.027	0.576	0.915	0.000
	0.10	0.076	0.029	0.512	0.817	0.100
	0.50	0.070	0.037	0.270	0.435	0.500
	0.80	0.033	0.031	0.104	0.166	0.800
	0.90	0.017	0.019	0.051	0.081	0.900
	0.95	0.009	0.010	0.025	0.040	0.950
$\theta = 0.5$	−0.90	0.340	0.084	1.053	1.542	−0.629
	−0.50	0.071	0.027	0.576	0.915	0.000
	0.00	0.070	0.028	0.271	0.519	0.400
	0.10	0.068	0.028	0.229	0.454	0.467
	0.50	0.042	0.026	0.101	0.223	0.714
	0.80	0.015	0.018	0.034	0.079	0.888
	0.90	0.008	0.010	0.016	0.038	0.944
	0.95	0.004	0.005	0.008	0.018	0.972
$\theta = 0.8$	−0.90	0.161	0.056	0.701	1.055	−0.140
	−0.50	0.043	0.026	0.388	0.701	0.214
	0.00	0.064	0.026	0.196	0.432	0.488
	0.10	0.062	0.026	0.168	0.382	0.540
	0.50	0.036	0.024	0.075	0.192	0.746
	0.80	0.013	0.016	0.026	0.069	0.899
	0.90	0.006	0.009	0.012	0.033	0.949
	0.95	0.003	0.005	0.006	0.016	0.975

Table 2 : $b(\alpha_F^*, 4)$; Asymptotic Bias of α_F^* with Filtered ARMA (1,1) Data; k = 4

	α	X-11 Filter Quarterly	X-11 Filter Monthly	Henderson Filter Quarterly	Henderson Filter Monthly	$\text{plim}_{T\to\infty} \alpha^*$
$\theta = -0.8$	-0.90	0.875	0.209	2.514	4.950	-4.317
	-0.50	0.483	0.082	1.941	4.062	-3.351
	0.00	0.190	0.072	1.463	2.938	-2.131
	0.10	0.166	0.073	1.366	2.709	-1.884
	0.50	0.141	0.086	0.937	1.761	-0.867
	0.80	0.112	0.115	0.536	0.951	0.000
	0.90	0.081	0.097	0.345	0.593	0.381
	0.95	0.054	0.067	0.210	0.352	0.634
$\theta = -0.5$	-0.90	0.576	0.303	1.859	3.463	-2.633
	-0.50	0.293	0.188	1.410	2.756	-1.890
	0.00	0.134	0.151	0.999	1.863	-0.954
	0.10	0.127	0.144	0.912	1.683	-0.765
	0.50	0.112	0.115	0.536	0.951	0.000
	0.80	0.061	0.067	0.225	0.389	0.590
	0.90	0.034	0.036	0.114	0.196	0.793
	0.95	0.018	0.018	0.058	0.098	0.896
$\theta = 0.0$	-0.90	0.199	0.228	0.941	1.806	-0.900
	-0.50	0.108	0.155	0.741	1.427	-0.500
	0.00	0.112	0.115	0.536	0.951	0.000
	0.10	0.115	0.107	0.488	0.856	0.100
	0.50	0.095	0.073	0.280	0.475	0.500
	0.80	0.043	0.032	0.114	0.191	0.800
	0.90	0.023	0.015	0.057	0.093	0.900
	0.95	0.012	0.007	0.029	0.048	0.950
$\theta = 0.5$	-0.90	0.145	0.164	0.574	1.206	-0.281
	-0.50	0.112	0.115	0.536	0.951	0.000
	0.00	0.123	0.084	0.387	0.631	0.338
	0.10	0.119	0.078	0.350	0.567	0.405
	0.50	0.075	0.051	0.195	0.314	0.671
	0.80	0.030	0.021	0.078	0.126	0.869
	0.90	0.015	0.009	0.039	0.061	0.934
	0.95	0.007	0.004	0.019	0.028	0.967
$\theta = 0.8$	-0.90	0.152	0.139	0.527	1.027	-0.084
	-0.50	0.091	0.096	0.478	0.771	0.190
	0.00	0.095	0.070	0.311	0.506	0.468
	0.10	0.090	0.064	0.278	0.455	0.521
	0.50	0.051	0.042	0.151	0.252	0.735
	0.80	0.019	0.016	0.060	0.100	0.894
	0.90	0.009	0.007	0.030	0.054	0.947
	0.95	0.005	0.003	0.015	0.020	0.974

Table 3 : $b(\alpha_F^*, 8)$; Asymptotic Bias of α_F^* with Filtered ARMA (1,1) Data; k = 8

	α	X-11 Filter Quarterly	X-11 Filter Monthly	Henderson Filter Quarterly	Henderson Filter Monthly	plim α^* $T\to\infty$
$\Theta = -0.8$	−0.90	1.508	0.764	4.317	7.054	−6.497
	−0.50	0.917	0.558	3.357	5.659	−5.004
	0.00	0.425	0.261	2.206	3.899	−3.127
	0.10	0.375	0.254	1.994	3.545	−2.749
	0.50	0.264	0.300	1.169	2.106	−1.217
	0.80	0.152	0.181	0.533	0.955	0.000
	0.90	0.092	0.098	0.294	0.522	0.455
	0.95	0.052	0.052	0.159	0.279	0.709
$\Theta = -0.5$	−0.90	0.684	0.657	2.261	3.602	−2.789
	−0.50	0.399	0.492	1.721	2.849	−1.993
	0.00	0.213	0.290	1.092	1.903	−0.997
	0.10	0.199	0.269	0.977	1.714	−0.798
	0.50	0.152	0.181	0.533	0.954	0.000
	0.80	0.075	0.060	0.212	0.382	0.599
	0.90	0.040	0.025	0.106	0.190	0.800
	0.95	0.021	0.011	0.053	0.096	0.900
$\Theta = 0.0$	−0.90	0.238	0.383	1.131	1.813	−0.900
	−0.50	0.160	0.279	0.845	1.432	−0.500
	0.00	0.152	0.181	0.533	0.954	0.000
	0.10	0.151	0.166	0.477	0.859	0.100
	0.50	0.113	0.087	0.260	0.476	0.500
	0.80	0.051	0.021	0.103	0.188	0.800
	0.90	0.026	0.008	0.051	0.099	0.900
	0.95	0.013	0.004	0.026	0.005	0.950
$\Theta = 0.5$	−0.90	0.174	0.258	0.729	1.210	−0.268
	−0.50	0.152	0.181	0.533	0.954	0.000
	0.00	0.150	0.129	0.342	0.636	0.334
	0.10	0.143	0.118	0.307	0.572	0.400
	0.50	0.087	0.054	0.170	0.318	0.667
	0.80	0.035	0.011	0.068	0.134	0.867
	0.90	0.017	0.004	0.034	0.082	0.933
	0.95	0.009	0.002	0.017	−0.007	0.967
$\Theta = 0.8$	−0.90	0.175	0.208	0.580	1.017	−0.066
	−0.50	0.134	0.137	0.431	0.786	0.177
	0.00	0.122	0.108	0.295	0.520	0.455
	0.10	0.114	0.099	0.267	0.466	0.510
	0.50	0.063	0.042	0.151	0.254	0.728
	0.80	0.024	0.007	0.061	0.097	0.891
	0.90	0.012	0.003	0.030	0.027	0.946
	0.95	0.006	0.001	0.016	0.035	0.973

Table 4 : $b(\alpha_F^*, 0)$; Asymptotic Bias of α_F^* with Filtered Data from an Unobserved Component Model; $k = 0$, $\alpha_S = 0.85$.

			Quarterly			Monthly		
	θ_{NS}	θ_S	X-11 Filter	Henderson Filter	plim α^* $T\to\infty$	X-11 Filter	Henderson Filter	plim α^* $T\to\infty$
$\alpha_{NS} = 0.0$	0.00	0.00	0.405	0.511	0.000	0.193	0.916	0.000
	0.00	0.50	0.562	0.497	0.000	0.313	0.917	0.000
	0.00	–0.50	0.174	0.547	0.000	0.069	0.916	0.000
	0.50	0.00	0.448	0.449	0.103	0.300	0.815	0.103
	0.50	0.50	0.592	0.466	0.057	0.410	0.861	0.057
	0.50	–0.50	0.205	0.421	0.186	0.123	0.732	0.186
	–0.50	0.00	0.293	0.570	–0.103	0.042	1.017	–0.103
	–0.50	0.50	0.469	0.530	–0.057	0.162	0.972	–0.057
	–0.50	–0.50	0.104	0.647	–0.186	–0.002	1.096	–0.186
$\alpha_{NS} = 0.5$	0.00	0.00	0.460	0.458	0.135	0.334	0.790	0.135
	0.00	0.50	0.601	0.472	0.075	0.442	0.847	0.075
	0.00	–0.50	0.216	0.429	0.240	0.144	0.689	0.240
	0.50	0.00	0.425	0.385	0.281	0.358	0.650	0.281
	0.50	0.50	0.574	0.435	0.168	0.480	0.758	0.168
	0.50	–0.50	0.190	0.300	0.442	0.148	0.492	0.442
	–0.50	0.00	0.405	0.511	0.000	0.193	0.916	0.000
	–0.50	0.50	0.562	0.497	0.000	0.313	0.917	0.000
	–0.50	–0.50	0.174	0.547	0.000	0.069	0.916	0.000
$\alpha_{NS} = 0.8$	0.00	0.00	0.416	0.385	0.348	0.377	0.605	0.348
	0.00	0.50	0.569	0.439	0.215	0.508	0.730	0.215
	0.00	–0.50	0.181	0.294	0.527	0.161	0.433	0.527
	0.50	0.00	0.306	0.272	0.544	0.296	0.417	0.544
	0.50	0.50	0.469	0.363	0.381	0.448	0.574	0.381
	0.50	–0.50	0.118	0.168	0.708	0.112	0.256	0.708
	–0.50	0.00	0.452	0.487	0.103	0.318	0.831	0.103
	–0.50	0.50	0.594	0.487	0.057	0.423	0.871	0.057
	–0.50	–0.50	0.214	0.485	0.186	0.146	0.757	0.186
$\alpha_{NS} = 0.9$	0.00	0.00	0.324	0.296	0.534	0.314	0.438	0.534
	0.00	0.50	0.488	0.382	0.369	0.468	0.596	0.369
	0.00	–0.50	0.127	0.192	0.707	0.122	0.271	0.707
	0.50	0.00	0.202	0.179	0.716	0.201	0.262	0.716
	0.50	0.50	0.347	0.274	0.566	0.343	0.408	0.566
	0.50	–0.50	0.070	0.097	0.838	0.071	0.143	0.838
	–0.50	0.00	0.449	0.458	0.213	0.375	0.741	0.213
	–0.50	0.50	0.591	0.476	0.123	0.483	0.820	0.123
	–0.50	–0.50	0.211	0.415	0.353	0.176	0.611	0.353

Table 5 : $b(\alpha_F^*, 4)$; Asymptotic Bias of α_F^* with Filtered Data from an Unobserved Component Model; $k = 4$, $\alpha_s = 0.85$.

	θ_{NS}	θ_S	Quarterly X-11 Filter	Quarterly Henderson Filter	Quarterly $\text{plim}_{T\to\infty} \alpha^*$	Monthly X-11 Filter	Monthly Henderson Filter	Monthly $\text{plim}_{T\to\infty} \alpha^*$
$\alpha_{NS} = 0.0$	0.00	0.00	0.081	0.116	0.665	0.527	0.962	0.000
	0.00	0.50	0.047	0.057	0.809	0.675	0.966	0.000
	0.00	−0.50	0.130	0.319	0.297	0.259	0.956	0.000
	0.50	0.00	0.128	0.133	0.599	0.462	0.870	0.094
	0.50	0.50	0.075	0.066	0.753	0.616	0.912	0.054
	0.50	−0.50	0.176	0.315	0.325	0.223	0.801	0.162
	−0.50	0.00	0.062	0.132	0.686	0.584	1.076	−0.116
	−0.50	0.50	0.039	0.066	0.829	0.720	1.025	−0.060
	−0.50	−0.50	0.104	0.380	0.211	0.291	1.185	−0.239
$\alpha_{NS} = 0.5$	0.00	0.00	0.109	0.108	0.636	0.421	0.766	0.202
	0.00	0.50	0.067	0.056	0.764	0.576	0.844	0.124
	0.00	−0.50	0.141	0.249	0.435	0.199	0.659	0.308
	0.50	0.00	0.153	0.128	0.592	0.334	0.629	0.343
	0.50	0.50	0.105	0.071	0.697	0.478	0.732	0.238
	0.50	−0.50	0.159	0.233	0.501	0.156	0.513	0.461
	−0.50	0.00	0.081	0.116	0.665	0.527	0.962	0.000
	−0.50	0.50	0.047	0.057	0.809	0.675	0.966	0.000
	−0.50	−0.50	0.130	0.319	0.297	0.259	0.956	0.000
$\alpha_{NS} = 0.80$	0.00	0.00	0.080	0.069	0.744	0.244	0.429	0.553
	0.00	0.50	0.057	0.041	0.796	0.384	0.553	0.426
	0.00	−0.50	0.084	0.131	0.698	0.105	0.318	0.666
	0.50	0.00	0.098	0.075	0.752	0.167	0.311	0.675
	0.50	0.50	0.079	0.048	0.776	0.265	0.405	0.577
	0.50	−0.50	0.085	0.116	0.754	0.075	0.234	0.754
	−0.50	0.00	0.071	0.086	0.715	0.423	0.714	0.259
	−0.50	0.50	0.045	0.047	0.816	0.583	0.811	0.161
	−0.50	−0.50	0.098	0.203	0.535	0.196	0.584	0.390
$\alpha_{NS} = 0.90$	0.00	0.00	0.049	0.041	0.844	0.138	0.240	0.750
	0.00	0.50	0.038	0.027	0.860	0.239	0.339	0.649
	0.00	−0.50	0.048	0.071	0.835	0.056	0.167	0.824
	0.50	0.00	0.056	0.043	0.859	0.088	0.165	0.827
	0.50	0.50	0.049	0.029	0.862	0.147	0.225	0.765
	0.50	−0.50	0.046	0.062	0.869	0.038	0.123	0.872
	−0.50	0.00	0.052	0.060	0.791	0.309	0.490	0.493
	−0.50	0.50	0.036	0.037	0.847	0.472	0.631	0.349
	−0.50	−0.50	0.064	0.126	0.712	0.130	0.349	0.636

Table 6 : $b(\alpha_F^*, 8)$; Asymptotic Bias of α_F^* with Filtered Data from an Unobserved Component Model; $k = 8$, $\alpha_s = 0.85$.

	Θ_{NS}	Θ_S	Quarterly X-11 Filter	Quarterly Henderson Filter	Quarterly plim α^* $T\to\infty$	Monthly X-11 Filter	Monthly Henderson Filter	Monthly plim α^* $T\to\infty$
$\alpha_{NS} = 0.0$	0.00	0.00	0.066	0.136	0.739	0.656	0.984	0.000
	0.00	0.50	0.047	0.093	0.827	0.778	0.990	0.000
	0.00	−0.50	0.123	0.296	0.424	0.377	0.968	0.000
	0.50	0.00	0.086	0.149	0.708	0.562	0.886	0.094
	0.50	0.50	0.054	0.095	0.815	0.708	0.933	0.054
	0.50	−0.50	0.154	0.289	0.429	0.302	0.808	0.162
	−0.50	0.00	0.051	0.126	0.758	0.763	1.103	−0.116
	−0.50	0.50	0.038	0.088	0.833	0.842	1.052	−0.060
	−0.50	−0.50	0.100	0.318	0.399	0.485	1.204	−0.239
$\alpha_{NS} = 0.5$	0.00	0.00	0.086	0.151	0.709	0.478	0.778	0.203
	0.00	0.50	0.058	0.103	0.808	0.638	0.860	0.126
	0.00	−0.50	0.133	0.255	0.495	0.239	0.665	0.308
	0.50	0.00	0.102	0.159	0.690	0.364	0.637	0.343
	0.50	0.50	0.067	0.106	0.789	0.518	0.743	0.239
	0.50	−0.50	0.142	0.231	0.548	0.174	0.517	0.461
	−0.50	0.00	0.066	0.136	0.739	0.656	0.984	0.000
	−0.50	0.50	0.047	0.093	0.827	0.778	0.990	0.000
	−0.50	−0.50	0.123	0.296	0.424	0.377	0.968	0.000
$\alpha_{NS} = 0.8$	0.00	0.00	0.070	0.118	0.770	0.229	0.419	0.566
	0.00	0.50	0.055	0.095	0.819	0.361	0.533	0.453
	0.00	−0.50	0.082	0.144	0.713	0.095	0.317	0.668
	0.50	0.00	0.067	0.103	0.797	0.157	0.312	0.677
	0.50	0.50	0.053	0.082	0.831	0.250	0.398	0.588
	0.50	−0.50	0.076	0.118	0.770	0.064	0.236	0.754
	−0.50	0.00	0.069	0.132	0.744	0.428	0.689	0.294
	−0.50	0.50	0.052	0.099	0.819	0.598	0.796	0.191
	−0.50	−0.50	0.097	0.210	0.581	0.198	0.559	0.419
$\alpha_{NS} = 0.9$	0.00	0.00	0.044	0.073	0.856	0.118	0.225	0.764
	0.00	0.50	0.039	0.067	0.870	0.199	0.306	0.685
	0.00	−0.50	0.047	0.079	0.842	0.045	0.165	0.826
	0.50	0.00	0.039	0.059	0.884	0.079	0.162	0.830
	0.50	0.50	0.033	0.051	0.894	0.130	0.215	0.777
	0.50	−0.50	0.042	0.063	0.878	0.031	0.123	0.872
	−0.50	0.00	0.054	0.102	0.801	0.259	0.431	0.557
	−0.50	0.50	0.046	0.087	0.839	0.416	0.566	0.424
	−0.50	−0.50	0.062	0.130	0.740	0.104	0.312	0.675

Table 7 : $b(\alpha^*_F, 12)$; Asymptotic Bias of α^*_F with Filtered Data from an Unobserved Component Model;$k = 12$, $\alpha_S = 0.85$.

	Θ_{NS}	Θ_S	Quarterly X-11 Filter	Quarterly Henderson Filter	Quarterly plim α^* $T\to\infty$	Monthly X-11 Filter	Monthly Henderson Filter	Monthly plim α^* $T\to\infty$
$\alpha_{NS} = 0.0$	0.00	0.00	0.068	0.143	0.759	0.042	0.317	0.665
	0.00	0.50	0.050	0.102	0.829	0.031	0.180	0.809
	0.00	–0.50	0.126	0.300	0.489	0.056	0.667	0.297
	0.50	0.00	0.080	0.154	0.741	0.078	0.380	0.599
	0.50	0.50	0.055	0.106	0.823	0.046	0.234	0.753
	0.50	–0.50	0.149	0.304	0.486	0.106	0.641	0.325
	–0.50	0.00	0.059	0.139	0.762	0.051	0.299	0.686
	–0.50	0.50	0.047	0.105	0.824	0.038	0.162	0.829
	–0.50	–0.50	0.108	0.306	0.478	0.057	0.749	0.212
$\alpha_{NS} = 0.5$	0.00	0.00	0.081	0.157	0.738	0.059	0.350	0.629
	0.00	0.50	0.057	0.109	0.819	0.036	0.230	0.756
	0.00	–0.50	0.135	0.276	0.532	0.079	0.537	0.434
	0.50	0.00	0.092	0.167	0.721	0.111	0.389	0.589
	0.50	0.50	0.066	0.120	0.800	0.071	0.294	0.689
	0.50	–0.50	0.138	0.254	0.575	0.119	0.475	0.501
	–0.50	0.00	0.068	0.143	0.759	0.042	0.317	0.665
	–0.50	0.50	0.050	0.102	0.829	0.031	0.180	0.809
	–0.50	–0.50	0.126	0.300	0.489	0.056	0.667	0.297
$\alpha_{NS} = 0.8$	0.00	0.00	0.070	0.132	0.779	0.064	0.253	0.732
	0.00	0.50	0.058	0.107	0.822	0.045	0.213	0.773
	0.00	–0.50	0.086	0.165	0.720	0.068	0.288	0.698
	0.50	0.00	0.065	0.116	0.805	0.095	0.242	0.746
	0.50	0.50	0.057	0.103	0.828	0.077	0.226	0.761
	0.50	–0.50	0.075	0.133	0.778	0.082	0.236	0.753
	–0.50	0.00	0.072	0.145	0.757	0.023	0.275	0.706
	–0.50	0.50	0.053	0.104	0.826	0.018	0.186	0.800
	–0.50	–0.50	0.106	0.233	0.599	0.035	0.424	0.552
$\alpha_{NS} = 0.9$	0.00	0.00	0.046	0.085	0.857	0.047	0.152	0.839
	0.00	0.50	0.043	0.079	0.867	0.036	0.143	0.848
	0.00	–0.50	0.049	0.091	0.845	0.044	0.157	0.836
	0.50	0.00	0.038	0.067	0.887	0.061	0.139	0.856
	0.50	0.50	0.037	0.067	0.888	0.054	0.138	0.854
	0.50	–0.50	0.041	0.071	0.881	0.049	0.125	0.869
	–0.50	0.00	0.058	0.117	0.803	0.017	0.189	0.797
	–0.50	0.50	0.048	0.095	0.841	0.012	0.150	0.838
	–0.50	–0.50	0.069	0.148	0.745	0.025	0.245	0.741

Table 8 : Size and Power of Unit Root Tests with Monthly Linear X-11 Filter

Model : $y_t = \alpha y_{t-1} + \varepsilon_t + \theta\varepsilon_{t-1}$

Regression Model without Trend

	T = 100 10 %	T = 100 5 %	T = 200 10 %	T = 200 5 %	T = 100 10 %	T = 100 5 %	T = 200 10 %	T = 200 5 %	T = 100 10 %	T = 100 5 %	T = 200 10 %	T = 200 5 %
	$\theta = -0.5$				$\theta = 0.0$				$\theta = 0.5$			
DF t-test					$\alpha = 1.00$							
k = 4	0.09	0.05	0.10	0.05	0.11	0.06	0.10	0.05	0.09	0.05	0.10	0.05
6	0.09	0.05	0.10	0.05	0.09	0.05	0.08	0.06	0.10	0.05	0.11	0.06
8	0.09	0.05	0.10	0.05	0.12	0.06	0.11	0.04	0.11	0.06	0.12	0.06
					$\alpha = 0.95$							
k = 4	0.18	0.10	0.41	0.25	0.17	0.09	0.39	0.23	0.17	0.09	0.38	0.23
6	0.17	0.10	0.39	0.23	0.19	0.10	0.42	0.26	0.19	0.11	0.43	0.27
8	0.18	0.10	0.40	0.25	0.21	0.12	0.45	0.28	0.22	0.13	0.47	0.30
					$\alpha = 0.85$							
k = 4	0.73	0.56	1.00	0.99	0.60	0.41	0.99	0.95	0.55	0.37	0.98	0.92
6	0.57	0.38	0.98	0.94	0.51	0.34	0.96	0.88	0.49	0.32	0.95	0.87
8	0.47	0.30	0.95	0.87	0.48	0.31	0.95	0.86	0.49	0.33	0.95	0.87
PP $Z(t_{\hat{\alpha}})$					$\alpha = 1.00$							
k = 6	0.49	0.40	0.42	0.33	0.10	0.05	0.07	0.03	0.07	0.03	0.08	0.03
8	0.52	0.44	0.44	0.36	0.06	0.03	0.07	0.03	0.07	0.03	0.08	0.04
12	0.56	0.48	0.49	0.40	0.05	0.03	0.07	0.03	0.06	0.03	0.08	0.04
					$\alpha = 0.95$							
k = 6	0.88	0.82	0.99	0.97	0.20	0.11	0.44	0.27	0.11	0.04	0.30	0.15
8	0.90	0.85	0.99	0.97	0.21	0.11	0.47	0.28	0.10	0.04	0.32	0.15
12	0.93	0.88	1.00	0.99	0.21	0.10	0.49	0.29	0.08	0.03	0.30	0.13
					$\alpha = 0.85$							
k = 6	1.00	1.00	1.00	1.00	0.92	0.81	1.00	1.00	0.63	0.37	1.00	0.98
8	1.00	1.00	1.00	1.00	0.93	0.81	1.00	1.00	0.60	0.32	1.00	0.98
12	1.00	1.00	1.00	1.00	0.93	0.79	1.00	1.00	0.47	0.20	1.00	0.96
PP $Z(\hat{\alpha})$					$\alpha = 1.00$							
k = 6	0.49	0.40	0.42	0.34	0.08	0.03	0.05	0.02	0.04	0.01	0.06	0.02
8	0.52	0.44	0.45	0.37	0.03	0.01	0.04	0.02	0.04	0.01	0.06	0.02
12	0.57	0.49	0.50	0.42	0.02	0.00	0.04	0.01	0.03	0.01	0.06	0.02
					$\alpha = 0.95$							
k = 6	0.90	0.83	0.99	0.97	0.23	0.12	0.55	0.34	0.13	0.04	0.41	0.21
8	0.92	0.86	0.99	0.98	0.24	0.12	0.58	0.37	0.12	0.04	0.43	0.22
12	0.95	0.90	1.00	0.99	0.24	0.11	0.60	0.39	0.09	0.02	0.43	0.20
					$\alpha = 0.85$							
k = 6	1.00	1.00	1.00	1.00	0.96	0.87	1.00	1.00	0.77	0.50	1.00	1.00
8	1.00	1.00	1.00	1.00	0.97	0.88	1.00	1.00	0.74	0.44	1.00	0.99
12	1.00	1.00	1.00	1.00	0.97	0.86	1.00	1.00	0.63	0.26	1.00	0.99

Notes : DF t-test is the Dickey-Fuller (1979) t-test with k the AR expansion; PP $Z(t_{\hat{\alpha}})$ is the Phillips-Perron t-test with a correction factor using a Newey-West covariance estimator with k number of lags. PP $Z(\hat{\alpha})$ is the Phillips-Perron normalized bias test using the same correction factor.

Table 9 : Size and Power of Unit Root Tests with Monthly Linear X-11 Filter

Model : $y_t = \alpha y_{t-1} + \varepsilon_t + \theta\varepsilon_{t-1}$

Regression Model with Trend

	T = 100		T = 200		T = 100		T = 200		T = 100		T = 200	
	10 %	5 %	10 %	5 %	10 %	5 %	10 %	5 %	10 %	5 %	10 %	5 %
	$\theta = -0.5$				$\theta = 0.0$				$\theta = 0.5$			
DF t-test					$\alpha = 1.00$							
k = 4	0.08	0.04	0.09	0.04	0.11	0.06	0.03	0.03	0.08	0.04	0.09	0.04
6	0.08	0.04	0.09	0.05	0.09	0.04	0.05	0.02	0.11	0.06	0.11	0.05
8	0.09	0.05	0.10	0.05	0.13	0.07	0.09	0.04	0.13	0.07	0.12	0.06
					$\alpha = 0.95$							
k = 4	0.12	0.06	0.25	0.14	0.11	0.06	0.23	0.13	0.11	0.06	0.23	0.12
6	0.12	0.06	0.24	0.13	0.14	0.07	0.27	0.15	0.14	0.07	0.28	0.16
8	0.12	0.06	0.25	0.14	0.15	0.08	0.39	0.24	0.17	0.10	0.32	0.20
					$\alpha = 0.85$							
k = 4	0.50	0.33	0.98	0.92	0.37	0.22	0.91	0.79	0.33	0.19	0.87	0.73
6	0.40	0.28	0.90	0.77	0.35	0.20	0.84	0.68	0.32	0.20	0.82	0.67
8	0.27	0.16	0.82	0.65	0.30	0.18	0.82	0.66	0.32	0.20	0.83	0.69
PP $Z(t_{\hat{\alpha}})$					$\alpha = 1.00$							
k = 6	0.75	0.67	0.67	0.59	0.03	0.01	0.01	0.00	0.04	0.02	0.05	0.02
8	0.78	0.71	0.71	0.63	0.03	0.01	0.01	0.00	0.04	0.02	0.05	0.02
12	0.82	0.76	0.76	0.69	0.02	0.00	0.00	0.00	0.03	0.01	0.05	0.02
					$\alpha = 0.95$							
k = 6	0.90	0.85	0.98	0.96	0.14	0.07	0.28	0.15	0.05	0.02	0.13	0.05
8	0.93	0.88	0.98	0.97	0.14	0.07	0.30	0.16	0.04	0.01	0.13	0.05
12	0.95	0.91	0.99	0.98	0.12	0.06	0.35	0.21	0.03	0.01	0.11	0.03
					$\alpha = 0.85$							
k = 6	1.00	1.00	1.00	1.00	0.77	0.58	1.00	0.99	0.28	0.10	0.95	0.82
8	1.00	1.00	1.00	1.00	0.77	0.58	1.00	0.99	0.21	0.07	0.94	0.79
12	1.00	1.00	1.00	1.00	0.72	0.52	1.00	0.99	0.12	0.04	0.91	0.68
PP $Z(\hat{\alpha})$					$\alpha = 1.00$							
k = 6	0.72	0.63	0.66	0.57	0.03	0.01	0.03	0.01	0.01	0.00	0.04	0.01
8	0.76	0.68	0.70	0.61	0.03	0.01	0.02	0.01	0.01	0.00	0.04	0.01
12	0.81	0.74	0.76	0.69	0.03	0.01	0.01	0.00	0.00	0.00	0.03	0.01
					$\alpha = 0.95$							
k = 6	0.89	0.83	0.98	0.95	0.11	0.05	0.28	0.14	0.03	0.00	0.13	0.05
8	0.92	0.86	0.98	0.97	0.11	0.04	0.30	0.15	0.01	0.00	0.14	0.05
12	0.94	0.90	0.99	0.98	0.09	0.03	0.37	0.20	0.01	0.00	0.11	0.03
					$\alpha = 0.85$							
k = 6	1.00	1.00	1.00	1.00	0.75	0.53	1.00	0.99	0.25	0.06	0.97	0.86
8	1.00	1.00	1.00	1.00	0.74	0.52	1.00	0.99	0.16	0.03	0.96	0.83
12	1.00	1.00	1.00	1.00	0.67	0.44	1.00	1.00	0.07	0.01	0.94	0.73

Table 10 : Size and Power of Dickey–Fuller Unit Root Tests with Quarterly Filters
Quarterly Unobserved Component Model : $y_t^{ns} = \alpha_{ns} y_{t-1}^{ns} + \varepsilon_t^{ns} + \Theta_{ns}\varepsilon_{t-1}^{ns}$
$y_t^{s} = \alpha_{s} y_{t-4}^{s} + \varepsilon_t^{s} + \Theta_{s}\varepsilon_{t-4}^{s}$
Regression Model without Trend

		Linear X-11		Standard X-11		Henderson		Unfiltered	
k	T	10 %	5 %	10 %	5 %	10 %	5 %	10 %	5 %
		Low asymptotic bias case							
		Size : $\alpha_{ns} = 1.0/\alpha_s = 0.9/\Theta_{ns} = 0.5/\Theta_s = -0.5$							
2	100	0.12	0.06	0.14	0.08	0.06	0.03	0.13	0.06
4	100	0.11	0.05	0.11	0.05	0.22	0.11	0.18	0.09
6	100	0.12	0.05	0.12	0.05	0.15	0.08	0.10	0.05
2	200	0.13	0.06	0.13	0.07	0.06	0.03	0.12	0.06
4	200	0.12	0.05	0.12	0.05	0.25	0.13	0.18	0.09
6	200	0.13	0.06	0.13	0.06	0.18	0.09	0.11	0.04
		Power : $\alpha_{ns} = \alpha_s = 0.9/\Theta_{ns} = 0.5/\Theta_s = -0.5$							
2	100	0.51	0.29	0.45	0.23	0.13	0.04	0.53	0.32
4	100	0.38	0.20	0.30	0.16	0.64	0.47	0.61	0.41
6	100	0.39	0.20	0.44	0.29	0.50	0.32	0.36	0.19
2	200	0.94	0.81	0.65	0.57	0.47	0.22	0.95	0.82
4	200	0.84	0.65	0.70	0.51	0.98	0.91	0.96	0.86
6	200	0.83	0.64	0.83	0.67	0.91	0.79	0.79	0.58
		Intermediate asymptotic bias case							
		Size : $\alpha_{ns} = 1.0/\alpha_s = 0.9/\Theta_{ns} = \Theta_s = 0.0$							
2	100	0.13	0.06	0.12	0.07	0.06	0.03	0.35	0.24
4	100	0.12	0.06	0.11	0.08	0.32	0.22	0.14	0.07
6	100	0.13	0.06	0.14	0.08	0.17	0.09	0.13	0.05
2	200	0.14	0.07	0.16	0.09	0.06	0.03	0.38	0.25
4	200	0.14	0.06	0.10	0.05	0.39	0.26	0.14	0.07
6	200	0.14	0.06	0.13	0.06	0.20	0.09	0.12	0.06
		Power : $\alpha_{ns} = \alpha_s = 0.9/\Theta_{ns} = \Theta_s = 0.0$							
2	100	0.50	0.30	0.45	0.22	0.13	0.05	0.88	0.77
4	100	0.43	0.24	0.35	0.20	0.80	0.67	0.49	0.30
6	100	0.38	0.21	0.36	0.24	0.48	0.29	0.38	0.22
2	200	0.93	0.78	0.55	0.43	0.38	0.18	1.00	0.99
4	200	0.87	0.68	0.69	0.51	1.00	0.98	0.88	0.71
6	200	0.80	0.58	0.74	0.57	0.88	0.71	0.79	0.58

Table 10 (continued)

k	T	Linear X-11 10 %	Linear X-11 5 %	Standard X-11 10 %	Standard X-11 5 %	Henderson 10 %	Henderson 5 %	Unfiltered 10 %	Unfiltered 5 %
				Large asymptotic bias case Size : $\alpha_{ns} = 1.0/\alpha_s = 0.9/\Theta_{ns} = 0.0/\Theta_s = 0.5$					
2	100	0.15	0.08	0.19	0.11	0.07	0.03	0.60	0.49
4	100	0.13	0.06	0.14	0.08	0.37	0.24	0.12	0.05
6	100	0.14	0.07	0.11	0.06	0.22	0.12	0.12	0.06
2	200	0.16	0.08	0.20	0.14	0.07	0.04	0.55	0.43
4	200	0.14	0.06	0.12	0.06	0.34	0.22	0.12	0.06
6	200	0.15	0.07	0.11	0.05	0.20	0.11	0.12	0.05
				Power : $\alpha_{ns} = \alpha_s = 0.9/\Theta_{ns} = 0.0/\Theta_s = 0.5$					
2	100	0.52	0.33	0.45	0.39	0.17	0.08	0.95	0.90
4	100	0.38	0.21	0.30	0.81	0.77	0.63	0.37	0.21
6	100	0.36	0.20	0.32	0.19	0.50	0.33	0.32	0.17
2	200	0.91	0.78	0.73	0.58	0.43	0.25	1.00	1.00
4	200	0.78	0.57	0.63	0.48	0.98	0.94	0.75	0.54
6	200	0.75	0.54	0.58	0.34	0.86	0.71	0.68	0.47

Table 11 : Size and Power of Phillips-Perron $Z(t_{\hat{\alpha}})$ Unit Root Tests with Quarterly Filters

Quarterly Unobserved Component Model : $y^{ns}_t = \alpha_{ns} y^{ns}_{t-1} + \varepsilon^{ns}_t + \theta_{ns}\varepsilon^{ns}_{t-1}$

$y^s_t = \alpha_s y^s_{t-4} + \varepsilon^s_t + \theta_s \varepsilon^s_{t-4}$

Regression Model without trend

		Linear X-11		Henderson		Unfiltered	
k	T	10 %	5 %	10 %	5 %	10 %	5 %
		Low asymptotic bias case					
		Size : $\alpha_{ns} = 1.0/\alpha_s = 0.9/\theta_{ns} = 0.5/\theta_s = -0.5$					
6	100	0.16	0.09	0.12	0.05	0.12	0.05
8	100	0.15	0.09	0.11	0.05	0.11	0.05
12	100	0.16	0.08	0.10	0.05	0.10	0.05
6	200	0.16	0.08	0.14	0.06	0.14	0.06
8	200	0.16	0.08	0.14	0.06	0.14	0.06
12	200	0.16	0.09	0.14	0.06	0.14	0.06
		Power : $\alpha_{ns} = \alpha_s = 0.9/\theta_{ns} = 0.5/\theta_s = -0.5$					
6	100	0.66	0.44	0.50	0.24	0.91	0.81
8	100	0.66	0.43	0.47	0.21	0.92	0.84
12	100	0.65	0.41	0.38	0.15	0.93	0.86
6	200	0.98	0.90	0.93	0.79	1.00	0.99
8	200	0.98	0.91	0.93	0.77	1.00	0.99
12	200	0.98	0.92	0.92	0.73	1.00	1.00
		Intermediate asymptotic bias case					
		Size : $\alpha_{ns} = 1.0/\alpha_s = 0.9/\theta_{ns} = \theta_s = 0.0$					
6	100	0.41	0.29	0.42	0.31	0.42	0.31
8	100	0.41	0.30	0.43	0.33	0.43	0.33
12	100	0.45	0.34	0.47	0.36	0.47	0.36
6	200	0.35	0.24	0.40	0.29	0.40	0.29
8	200	0.37	0.26	0.41	0.30	0.41	0.30
12	200	0.41	0.29	0.39	0.29	0.38	0.28
		Power : $\alpha_{ns} = \alpha_s = 0.9/\theta_{ns} = \theta_s = 0.0$					
6	100	0.93	0.84	0.92	0.83	1.00	1.00
8	100	0.94	0.87	0.92	0.84	1.00	1.00
12	100	0.95	0.90	0.93	0.86	1.00	1.00
6	200	1.00	0.99	1.00	0.99	1.00	1.00
8	200	1.00	1.00	1.00	0.99	1.00	1.00
12	200	1.00	1.00	1.00	1.00	1.00	1.00

Table 11 (continued)

k	T	Linear X-11 10 %	Linear X-11 5 %	Henderson 10 %	Henderson 5 %	Unfiltered 10 %	Unfiltered 5 %
		Large asymptotic bias case Size : $\alpha_{ns} = 1.0/\alpha_s = 0.9/\Theta_{ns} = 0.0/\Theta_s = 0.5$					
6	100	0.51	0.41	0.62	0.53	0.95	0.93
8	100	0.53	0.44	0.65	0.56	0.96	0.93
12	100	0.58	0.48	0.69	0.60	0.97	0.95
6	200	0.47	0.36	0.61	0.50	0.94	0.92
8	200	0.49	0.38	0.63	0.53	0.95	0.92
12	200	0.54	0.44	0.66	0.58	0.97	0.94
		Power : $\alpha_{ns} = \alpha_s = 0.9/\Theta_{ns} = 0.0/\Theta_s = 0.5$					
6	100	0.95	0.90	0.97	0.95	1.00	1.00
8	100	0.97	0.93	0.98	0.96	1.00	1.00
12	100	0.98	0.95	0.98	0.97	1.00	1.00
6	200	1.00	0.99	1.00	1.00	1.00	1.00
8	200	1.00	1.00	1.00	1.00	1.00	1.00
12	200	1.00	1.00	1.00	1.00	1.00	1.00

Table 12 : Size and Power of Phillips–Perron $Z(\hat{\alpha})$ Unit Root Tests with Quarterly Filters
Quarterly Unobserved Component Model : $y_t^{ns} = \alpha_{ns} y_{t-1}^{ns} + \varepsilon_t^{ns} + \Theta_{ns}\varepsilon_{t-1}^{ns}$

$$y_t^s = \alpha_s y_{t-4}^s + \varepsilon_t^s + \Theta_s \varepsilon_{t-4}^s$$

Regression Model without Trend

		Linear X-11		Henderson		Unfiltered	
k	T	10 %	5 %	10 %	5 %	10 %	5 %
		Low asymptotic bias case Size : $\alpha_{ns} = 1.0/\alpha_s = 0.9/\Theta_{ns} = 0.5/\Theta_s = -0.5$					
6	100	0.13	0.07	0.08	0.04	0.08	0.04
8	100	0.13	0.07	0.08	0.04	0.08	0.04
12	100	0.13	0.08	0.08	0.02	0.08	0.02
6	200	0.14	0.08	0.11	0.05	0.11	0.05
8	200	0.15	0.08	0.11	0.05	0.11	0.05
12	200	0.15	0.09	0.10	0.04	0.10	0.04
		Power : $\alpha_{ns} = \alpha_s = 0.9/\Theta_{ns} = 0.5/\Theta_s = -0.5$					
6	100	0.71	0.50	0.55	0.32	0.92	0.85
8	100	0.72	0.50	0.52	0.28	0.94	0.87
12	100	0.71	0.48	0.43	0.19	0.95	0.90
6	200	0.99	0.94	0.96	0.88	1.00	1.00
8	200	0.99	0.95	0.96	0.87	1.00	1.00
12	200	0.99	0.96	0.95	0.85	1.00	1.00
		Intermediate asymptotic bias case Size : $\alpha_{ns} = 1.0/\alpha_s = 0.9/\Theta_{ns} = \Theta_s = 0.0$					
6	100	0.38	0.28	0.40	0.31	0.40	0.31
8	100	0.40	0.31	0.41	0.33	0.41	0.33
12	100	0.43	0.35	0.44	0.36	0.44	0.36
6	200	0.33	0.25	0.38	0.29	0.38	0.29
8	200	0.36	0.27	0.39	0.30	0.39	0.30
12	200	0.45	0.32	0.49	0.40	0.51	0.39
		Power : $\alpha_{ns} = \alpha_s = 0.9/\Theta_{ns} = \Theta_s = 0.0$					
6	100	0.94	0.87	0.92	0.86	1.00	1.00
8	100	0.95	0.90	0.93	0.87	1.00	1.00
12	100	0.96	0.92	0.94	0.89	1.00	1.00
6	200	1.00	1.00	1.00	0.99	1.00	1.00
8	200	1.00	1.00	1.00	0.99	1.00	1.00
12	200	1.00	1.00	1.00	1.00	1.00	1.00

Table 12 (continued)

k	T	Linear X-11 10 %	Linear X-11 5 %	Henderson 10 %	Henderson 5 %	Unfiltered 10 %	Unfiltered 5 %
		Large asymptotic bias case Size : $\alpha_{ns} = 1.0/\alpha_s = 0.9/\Theta_{ns} = 0.0/\Theta_s = 0.5$					
6	100	0.49	0.41	0.61	0.53	0.95	0.93
8	100	0.53	0.44	0.64	0.56	0.96	0.94
12	100	0.58	0.50	0.68	0.61	0.97	0.96
6	200	0.47	0.37	0.60	0.52	0.94	0.92
8	200	0.48	0.40	0.62	0.54	0.95	0.93
12	200	0.54	0.46	0.66	0.59	0.97	0.95
		Power : $\alpha_{ns} = \alpha_s = 0.9/\Theta_{ns} = 0.0/\Theta_s = 0.5$					
6	100	0.96	0.92	0.98	0.95	1.00	1.00
8	100	0.97	0.94	0.98	0.97	1.00	1.00
12	100	0.98	0.96	0.99	0.97	1.00	1.00
6	200	1.00	1.00	1.00	0.99	1.00	1.00
8	200	1.00	1.00	1.00	0.99	1.00	1.00
12	200	1.00	1.00	1.00	1.00	1.00	1.00

Table 13 : Size and Power of Dickey-Fuller Unit Root Tests with Monthly Filters
Monthly Unobserved Component Model : $y_t^{ns} = \alpha_{ns} y_{t-1}^{ns} + \varepsilon_t^{ns} + \Theta_{ns}\varepsilon_{t-1}^{ns}$

$$y_t^s = \alpha_s y_{t-12}^s + \varepsilon_t^s + \Theta_s \varepsilon_{t-12}^s$$

Regression Model without Trend

		Linear X-11		Henderson		Unfiltered	
k	T	10 %	5 %	10 %	5 %	10 %	5 %
		Low asymptotic bias case Size : $\alpha_{ns} = 1.0/\alpha_s = 0.9/\Theta_{ns} = 0.5/\Theta_s = -0.5$					
6	100	0.11	0.05	0.12	0.05	0.10	0.05
12	100	0.10	0.05	0.10	0.06	0.11	0.06
6	200	0.12	0.05	0.12	0.05	0.10	0.05
12	200	0.11	0.06	0.11	0.06	0.10	0.05
		Power : $\alpha_{ns} = \alpha_s = 0.9/\Theta_{ns} = 0.5/\Theta_s = -0.5$					
6	100	0.33	0.18	0.29	0.16	0.36	0.19
12	100	0.23	0.11	0.19	0.09	0.27	0.13
6	200	0.77	0.57	0.74	0.52	0.82	0.62
12	200	0.59	0.41	0.69	0.41	0.75	0.59
		Intermediate asymptotic bias case Size : $\alpha_{ns} = 1.0/\alpha_s = 0.9/\Theta_{ns} = \Theta_s = 0.0$					
6	100	0.10	0.05	0.09	0.04	0.14	0.07
12	100	0.10	0.04	0.08	0.03	0.12	0.07
6	200	0.11	0.05	0.09	0.04	0.14	0.07
12	200	0.10	0.05	0.09	0.04	0.11	0.06
		Power : $\alpha_{ns} = \alpha_s = 0.9/\Theta_{ns} = \Theta_s = 0.0$					
6	100	0.33	0.18	0.21	0.10	0.49	0.32
12	100	0.20	0.09	0.24	0.14	0.40	0.29
6	200	0.76	0.55	0.56	0.33	0.88	0.75
12	200	0.58	0.35	0.61	0.45	0.81	0.78

Table 13 (continued)

k	T	Linear X-11 10 %	Linear X-11 5 %	Henderson 10 %	Henderson 5 %	Unfiltered 10 %	Unfiltered 5 %
		Large asymptotic bias case Size : $\alpha_{ns} = 1.0/\alpha_s = 0.9/\Theta_{ns} = 0.0/\Theta_s = 0.5$					
6	100	0.10	0.04	0.07	0.04	0.20	0.11
12	100	0.11	0.05	0.11	0.05	0.10	0.05
6	200	0.11	0.05	0.07	0.03	0.21	0.12
12	200	0.11	0.05	0.11	0.05	0.10	0.04
		Power : $\alpha_{ns} = \alpha_s = 0.9/\Theta_{ns} = 0.0/\Theta_s = 0.0$					
6	100	0.32	0.17	0.13	0.06	0.60	0.44
12	100	0.24	0.12	0.22	0.11	0.19	0.08
6	200	0.74	0.52	0.37	0.19	0.92	0.83
12	200	0.61	0.39	0.55	0.33	0.51	0.30