THE MULTIPLIER-ACCELERATOR MODEL IN THE LIGHT OF COINTEGRATION*

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Aggregate investment and consumption functions are among the most important relationships in macroeconomics. This paper reexamines the nature of these relationships using an econometric method that takes into account the possible nonstationarity of the time series observations. Two key empirical questions will be addressed. Is the acceleration principle valid, and what form should it take? What is the dynamic relationship between consumption and income, and is there an equilibrium relationship between these two variables in the long run? In section I these relationships will be formulated in a simple system of simultaneous equations. For estimating and testing these equations, a suitable econometric method will be presented in section II to deal with the possible nonstationarity of the variables.

The econometric procedure to be recommended consists of two steps. In the first step, one finds the number of unit roots in the model and the canonical variables (linear combinations of the original variables) which are stationary, i.e., correspond to the stable roots. In the second step, one gets rid of the unit roots and retains the stationary canonical variables, thus estimating a stationary model by standard econometric methods. In conception, this two-step procedure is similar to the approach of Box and Jenkins (1970) to nonstationary univariate time series. In the univariate case, the first step is to perform differencing. The resulting stationary series can then be analyzed. In the m-variate case, one should not simply difference all m variables, because each differencing imposes a unit root in the model, and there may be, and frequently are, fewer than m unit roots. If we know the number $d \leq m$ of unit roots, we can do the right amount of differencing, and the resulting system will be stationary.

Performing the right amount of differencing means getting rid of the right number of unit roots. In the second step the model consists of only stationary variables, including first differences of the original variables and those linear combinations of them that are stationary. The components of this two-step procedure are well known. We merely suggest a framework for putting them together into a procedure which is very easy to use for estimating linear simultaneous-equations models of nonstationary time series.

Section III addresses several econometric issues concerning error-correction models, simultaneous-equations models and cointegrated relations. The issues are both theoretical and empirical. Often, one dynamic econometric equation is formulated in an error correction form, with the first difference of the dependent variable affected by the first differences of current and lagged explanatory variables and by the error-correction term, i.e., the deviation in the last period of the dependent variable from its long-run equilibrium relationship with selected explanatory variables (also called a cointegrating relationship). Questions arise concerning the appropriateness of such a single-equation specification in the context of a simulatneous-equations system. If there are d unit roots in the model, there are d fewer long-run equilibrium cointegrating relations than the number of equations. It is therefore impossible for each equation to be associated with a unique long-run equilibrium cointegrating relation. One cannot specify each structural equation as dependent on an error correction term based on a cointegrating relation for that equation alone. Several cointegrating relations, or none, may affect one structural equation. A related issue is whether the omission of selected variables in the specification of each structural equation is empirically justified.

Using annual and quarterly data respectively, sections IV and V present empirical results from estimating a simple multiplier-accelerator model. Besides obtaining empirical investment and consumption functions, we also address the econometric issues concerning error-correction and simultaneous-equations models. Section VI concludes this paper.

I. MODEL FORMULATION

The model to be studied consists of three macroeconomic variables: aggregate real investment y_{1t}, aggregate real consumption expenditures y_{2t} and an income variable y_{3t} which equals $y_{1t}+y_{2t}$ plus government expenditures minus taxes net of transfers. Investment y_{1t} is a function of Δy_{3t} by the acceleration principle, and of lagged $y_{1,t-j}$ and $y_{3,t-j}$ (j>0). Consumption y_{2t} is a function of y_{3t} and of lagged $y_{2,t-j}$ and $y_{3,t-j}$ (j>0). Income y_{3t} is a function of lagged $y_{1,t-j}$, $y_{2,t-j}$ and $y_{3,t-j}$. Since the own lagged variable $y_{i,t-1}$ is an explanatory variable for each variable y_{it}, one can simply subtract it from both sides of each equation yielding Δy_{it} as the dependent variable. If the number of lagged variables in each equation is at most p+1, we can replace p+1 lagged variables $y_{i,t-1},\ldots,y_{i,t-p-1}$ by $y_{i,t-1}$, $\Delta y_{i,t-1}$, ..., $\Delta y_{i,t-p}$ without loss. Hence, assuming linearity the three structural equations in error correction form for the vector Δy_t of dependent variables are

$$(1)\qquad \begin{bmatrix} 1 & 0 & -\beta_{13} \\ 0 & 1 & -\beta_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta y_{1t} \\ \Delta y_{2t} \\ \Delta y_{3t} \end{bmatrix} = \begin{bmatrix} \phi_{11} & 0 & \phi_{13} \\ 0 & \phi_{22} & \phi_{23} \\ \phi_{31} & \phi_{32} & \phi_{33} \end{bmatrix} \begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \\ y_{3,t-1} \end{bmatrix}$$

$$+ \Gamma_1 \Delta y_{t-1} + \ldots + \Gamma_p \Delta y_{t-p} + \mu + u_t$$

By the acceleration principle, Δy_{3t} is the appropriate income variable explaining investment y_{1t} or Δy_{1t}. Therefore, when both Δy_{3t} and $y_{3,t-1}$ appear as explanatory variables, the coefficient ϕ_{13} of $y_{3,t-1}$ should be small relative to the coefficient β_{13} of Δy_{3t}. On the other hand, in the consumption function, y_{3t} is the appropriate income variable explaining y_{2t} or Δy_{2t}; the coefficient ϕ_{23} of $y_{3,t-1}$ should not be small relative to the coefficient β_{23} of Δy_{3t}, as these income variables enter as $\beta_{23}y_{3t}$ plus $(\phi_{23}-\beta_{23})y_{3,t-1}$. The small ratio ϕ_{13}/β_{13} for the investment equation as compared with the ratio ϕ_{23}/β_{23} for the consumption equation is an important hypothesis to be examined.

If there is a long-run equilibrium relation between consumption and income generated by the consumption function alone, it would be given by $\phi_{22}y_{2,t-1} + \phi_{23}y_{3,t-1} + \mu_2$, which is the second row of model (1) when we set all Δy_{t-j} $(j = 0,\ldots,p)$ and u_t equal to zero. By the error-correction interpretation, ϕ_{22} should be negative; when the deviation from equilibrium $y_{2,t-1} + (\phi_{23}y_{3,t-1} + \mu_2)/\phi_{22}$ is positive, y_{2t} in the following period will tend to decrease. ϕ_{23} should be positive, with $-\phi_{23}/\phi_{22}$ measuring the long-run marginal propensity to consume. For a recent application of the error-correction formulation see Hendry and Ericsson (1989). On the other hand, by the acceleration principle, while capital stock may have a long-run equilibrium relation with income, investment need not have such a long-run relationship. We have pointed out that ϕ_{13} is expected to be small. One should not expect the coefficients ϕ_{11} and ϕ_{13} to be respectively negative and positive and be significant. Thus the signs and significance of ϕ_{22} and ϕ_{23}, as compared with ϕ_{11} and ϕ_{13}, are important hypotheses of the model to be tested.

Third, based on error-corrections applied to individual equations separately and on possible additional identification restrictions on simultaneous structural equations, ϕ_{12} and ϕ_{21} may be expected to be zero. Further $y_{2,t-1}$ should be absent from the investment function of (1) and $y_{1,t-1}$ should be absent from the consumption function. These hypotheses will be tested in sections IV and V.

II. ECONOMETRIC METHOD

Consider a linear model for a vector y_t of m time series to be written as

(2) $$y_t = Ay_{t-1} + A_1\Delta y_{t-1} + \ldots + A_p\Delta y_{t-p} + b + \varepsilon_t$$

where ε_t is serially uncorrelated. If all the roots of the matrix A are smaller than one in absolute value, the model is stationary and standard econometric methods apply. The model is nonstationary if some roots of A equal unity. Let

(3) $$A = PJQ = [P_1 \quad P_2] \begin{bmatrix} I_d & 0 \\ 0 & J_2 \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = P_1Q_1 + P_2J_2Q_2$$

where the d columns of P_1 are the right characteristic vectors of A corresponding to the unit roots, the r = m-d columns of P_2 correspond to the stationary roots on the diagonal of J_2; $Q = P^{-1}$, its first d rows Q_1 are the left characteristic vectors of A corresponding to the d unit roots and the remaining rows Q_2 correspond to the r stationary roots. Premultiplying (2) by Q gives

(4) $$Qy_t = JQy_{t-1} + QA_1\Delta y_{t-1} + \ldots + QA_p\Delta y_{t-p} + Qb + Q\varepsilon_t$$

Defining the canonical variables $z_t = Qy_t$, $z_{1t} = Q_1y_t$ and $z_{2t} = Q_2y_t$, we rewrite (4) in two parts as

(5) $$Q_1y_t = z_{1t} = z_{1,t-1} + Q_1A_1P\Delta z_{t-1} + \ldots + Q_1A_pP\Delta z_{t-p} + Q_1b + Q_1\varepsilon_t$$

(6) $$Q_2y_t = z_{2t} = J_2z_{2,t-1} + Q_2A_1P\Delta z_{t-1} + \ldots + Q_2A_pP\Delta z_{t-p} + Q_2b + Q_2\varepsilon_t$$

By (5) and (6) the first d canonical variables z_{1t} have unit roots while the remaining m-d = r canonical variables $z_{2t} = Q_2y_t$ are stationary, the r rows of Q_2 being the cointegrating vectors.

Alternatively, subtract y_{t-1} from both sides of (2) and write

(7) $$\Delta y_t = \Pi y_{t-1} + A_1 \Delta y_{t-1} + \dots + A_p \Delta y_{t-p} + b + \varepsilon_t$$

where

(8) $$\Pi \equiv A - I = PJQ - I = P(J-I)Q$$

$$= [P_1 \quad P_2] \begin{bmatrix} 0 & 0 \\ 0 & J_2 - I_r \end{bmatrix} = P_2(J_2-I)Q_2 \equiv P_2 D_2 Q_2$$

Π has d zero roots and r=m-d nonzero roots equal to the nonzero roots of A minus one, and has the same characteristic vectors as A. Denoting the m×r matrix P_2D_2 by α, we can replace Πy_{t-1} in (7) by $\alpha Q_2 y_t$ or αz_{2t}. If Q_2 were known, the parameters α, $A_1, \dots, A_p$ can be estimated by least squares to yield estimates with the usual distributional properties since all variables z_{2t} and Δy_{t-k} are stationary.

To estimate the cointegrating vectors $Q_2 \equiv \beta'$ we apply the method of maximum likelihood in Johansen (1988, 1989) assuming normal ε_t. Let r_{0t} be the residual of the regression of Δy_t on $\Delta y_{t-1}, \dots, \Delta y_{t-p}$ and 1, and r_{kt} be the residual of the regression of y_{t-1} on the same variables. Π in (7) is the matrix of coefficients of the regression of the residuals r_{0t} on the residuals r_{kt}. Let $S_{ij} = T^{-1} \sum_{t=1}^{T} r_{it} r'_{jt}$ (i,j = 0,k). If the rank of Π is m, its least squares estimate $\hat{\Pi}$ is $S_{0k}S_{kk}^{-1}$. If the rank of Π is r<m, $\Pi=\alpha\beta'$ with α being m×r. Given β, the L.S. estimate of α is

(9) $$\hat{\alpha}(\beta) = S_{0k}\beta(\beta' S_{kk}\beta)^{-1}$$

and the covariance matrix of regression residuals $r_{0t} - \hat{\alpha}\beta' r_{kt}$ is estimated by

(10) $$\hat{\Lambda}(\beta) = S_{00} - \hat{\alpha}\beta' S_{kk}\beta\hat{\alpha}' = S_{00} - S_{0k}\beta(\beta' S_{kk}\beta)^{-1}\beta' S_{k0}$$

Analogous to limited-information maximum likelihood estimation of linear simultaneous equations with normal residuals, β is found by minimizing the determinant of (9) which equals

$$(11) \qquad |S_{00} - S_{0k}\beta(\beta' S_{kk}\beta)^{-1}\beta' S_{k0}| = \begin{vmatrix} \beta' S_{kk}\beta & \beta' S_{k0} \\ S_{0k}\beta & S_{00} \end{vmatrix} / |\beta' S_{kk}\beta|$$

$$= |S_{00}| \; |\beta' S_{kk}\beta - \beta' S_{k0}S_{00}^{-1}S_{0k}\beta| \; / \; |\beta' S_{kk}\beta|$$

β is estimated by the r characteristic vectors corresponding to the r largest roots $\lambda_1 > \lambda_2 \ldots > \lambda_r$ of

$$(12) \qquad |\lambda S_{kk} - S_{k0}S_{00}^{-1}S_{0k}| = 0$$

with the normalization $\beta' S_{kk}\beta = I_r$.

The problem of determining the number r of cointegrating vectors, or the number m-r=d of zero roots in Π, can be solved by a likelihood ratio test. The null hypothesis $\Pi=\alpha\beta'$, α being m×r, is tested by using the likelihood ratio L (the alternative hypothesis being r=m)

$$(13) \qquad -2 \ln L = - T \sum_{i=r+1}^{m} \ln(1-\hat{\lambda}_i)$$

The null hypothesis (of m-r zero roots) would be rejected if the m-r smallest roots $\hat{\lambda}_{r+1},\ldots,\hat{\lambda}_m$ of (12) are large enough to make (13) greater than the critical value tabulated in Tables T.I to T.III on pp. 80-82 of Johansen (1989). Table T.III is for using model (2) without the intercept. Tables T.I and T.II are for using the above model including the intercept b. There are two subcases when b≠0. The first case is for $Q_1 b \neq 0$ in (5), when the variables z_{1t} incorporate a time trend (see Table T.I). The second case is for $Q_1 b=0$ (Table T.II). Since the distribution of (13) has a thicker tail in the second case, Table T.II is used if one wishes to avoid undercounting zero roots in Π or unit roots in A. The null hypothesis of r nonzero roots in Π against the alternative of r+1 nonzero roots is tested by using the likelihood ratio statistic

$$- 2 \ln L = -T \ln(1-\hat{\lambda}_{r+1}). \tag{14}$$

The critical values for (14) are also tabulated in the above 3 tables.

Once the number r of cointegrating vectors is determined, we know the appropriate degree of differencing by imposing m-r zero roots in (7). Johansen estimates β by the r characteristic vectors corresponding to the r largest roots of (12). This estimator $\hat{\beta}$ is super-consistent, converging at rate T. $\hat{\beta}$ can be treated as given when estimating the remaining coefficients α, $A_1,\ldots,A_p$ of (7). A computer program written by S. Johansen, K. Juselius and H. Hansen using RATS for computing the test statistics for the rank of Π has been used to estimate r and β for the empirical results of sections IV and V. An alternative method to Johansen's for estimating r and β can be found in Ahn and Reinsel (1900).

Turning to the estimation and testing of linear simultaneous equations with some unit roots. Let the model be

$$B\Delta y_t = \Phi y_{t-1} + \Gamma_1 \Delta y_{t-1} + \ldots + \Gamma_p \Delta y_{t-p} + \mu + u_t \tag{15}$$

where B is nonsingular with diagonal elements set equal to one and other selected elements set equal to zero. Equation (1) is an example of (15). Identification restrictions may also be imposed on $\Phi, \Gamma_1,\ldots,\Gamma_p$ and $\Sigma = \mathrm{cov}(u_t)$. Premultiplying (15) by B^{-1} gives the reduced-form (7). To find out the number r of cointegrating vectors in the structural equations (15), one can apply the same analysis as for the reduced-form (7). Since the matrix Φ in (15) equals a nonsingular matrix B times Π in (7), the rank of Φ is the same as the rank r of Π. Writing $\Phi = B\Pi = B\alpha\beta' \equiv \alpha^{*}\beta'$, we find that the cointegrating vectors of the structural equations (15) are the same as those of (7).

To estimate the i^{th} structural equation or the i^{th} row of (15) one can apply a limited information method such as instrumental variables (IV). The IV estimates are consistent. The problem is that when unit roots exist the distribution of the estimates of the i^{th} row of Φ is non-standard, since the variables y_{t-1} are nonstationary. To resolve this problem, I recommend using a two-step procedure. In the first step, one finds r and $\beta=\hat{\beta}$ as suggested above. Once r<m and β are determined , we replace Φy_{t-1} in (15) by $\alpha^{*}\beta' y_{t-1} = \alpha^{*} z_{2,t-1}$ in the second step. Since z_{2t} and Δy_t are stationary, all parameters of the model can be estimated by standard methods for simultaneous equations. In particular the distribution of the estimator $\hat{\alpha}^{*}_{i}$ of the i^{th} row of α^{*} has a covariance matrix, denoted by V_i, which can be consistently estimated. If we estimate Φ by $\hat{\Phi} = \hat{\alpha}^{*}\beta'$, the covariance matrix of its i^{th} row $\hat{\phi}_i = \hat{\alpha}^{*}_i\beta'$ is simply $\beta V_i \beta'$ which can be used for inference concerning $\hat{\phi}_i$. Treating $\beta=\hat{\beta}$ as given does not affect the asymptotic distribution in the second step because $\hat{\beta}$ converges at rate T, as in the two-step procedure of Engle and Granger (1987). This two-step procedure is much more convenient to use than applying the results of Sims, Stock and Watson (1990) in one step. Once the nonstandard distribution theory is applied in the first step, inference becomes standard in the second step. Imposing a unit root is equivalent to differencing in scalar time series. For a vector time series, imposing d unit roots or d zero roots in equation (15) is a generalization of differencing for univariate time series. Retaining the canonical variables $z_{2t} = \beta' y_t$ corresponding to the remaining stationary roots enables one to deal with a stationary, appropriately differenced, model for standard analysis.

III. ISSUES ON ERROR CORRECTIONS AND SIMULTANEOUS EQUATIONS

When there are unit roots in the model, the number r of cointegrating vectors (rows of β') is smaller than the number m of structural equations. It is therefore impossible to associate each structural equation with an error-correction mechanism generated by an equilibrium relationship attributed to that equation alone. There simply are not enough equilibrium relationships to be assigned to all structural equations. In general, all of the r canonical variables $z_{2,t-1} = \beta' y_{t-1}$ corresponding to the r cointegrating vectors β' may affect the dependent variable of the ith structural equation, with the effects measured by the coefficients $\alpha^*_{i\cdot}$. It is possible for a structural equation to have an error-correction term attributable only to an equilibrium relationship among its own variables. An example is the consumption function specified by model (1). This cannot apply to all equations. Hence one has to be careful in specifying any economic relation in terms of an error correction due to that equation only. A related issue is the estimation of a cointegrating vector by single-equation methods as proposed by Engle and Granger (1987) and by Stock (1987). These methods assume that one and only one cointegrating relation or stationary canonical variable is influencing the dependent variable in question.

When unit roots exist, one cannot associate one structural equation with one cointegrating vector. A structural equation may have no cointegrating vector associated with it. An example is the investment equation specified in model (1). A structural equation may have several cointegrating vectors associated with it, as it may be affected by several stationary canonical variables. The algebra of equation (15) makes clear the distinction between structural equations and stationary relations. The latter are obtained by setting to zero all Δy_{t-j} $(j = 0,\ldots,p)$ and u_t, and dropping the time subscript of y_{t-1}, yielding

$$\Phi y + \mu = 0 \tag{16}$$

When there are unit roots, the rank r of Φ is smaller than m and (16) does not have sufficient linearly independent equations to determine y.

If we write Φy as $\alpha\beta' y = \alpha z_2$, the choice of α and β' requires a normalization.

Since $\alpha\beta' = \alpha W^{-1}W\beta'$ for any nonsingular r×r matrix W, αW^{-1} and $W\beta'$ will serve our purpose as well as α and β'. In order to associate one and only one cointegrating vector with the i^{th} structural equation, one can normalize by letting the i^{th} row α_i of α be a vector of zeros except for the i^{th} element α_{ii}. The error correction term for the i^{th} equation of (16) will then be $\alpha_{ii}\beta_i'y$ or $\alpha_{ii}z_2$, where β_i' is the i^{th} row of β'. β_i' is interpreted as the cointegrating vector associated with the i^{th} equation and α_{ii} is the negative adjustment coefficient. This error-correction specification may require that certain elements of β_i' be zero as the corresponding variables are assumed to be absent from the i^{th} structural equation.

The above discussion suggests that assigning an error-correction mechanism to a structural equation when there are unit roots in the model imposes strong restrictions on the structural coefficients. Whether these restrictions are justified can be tested by using the distribution of the estimators in the second step of the two-step procedure recommended in section II.

IV. EMPIRICAL FINDINGS FROM ANNUAL OBSERVATIONS

The multiplier-accelerator model studied in this paper is a simplified version of an already very aggregative annual model of Chow (1967). The two main simplifications consist of eliminating the liquidity preference equation, justified by the small effect of the rate of interest on investment found in the previous model, and aggregating the two components of investment, including new construction as a separate component, into one investment variable. A third difference is that, in the previous model price deflation was performed by introducing the GNP deflator as a separate explanatory variable in each linear equation, whereas we now divide each nominal variable by the GNP deflator. The data are standard national income data for the United States, recorded quarterly from 1947.1 in the City Bank data file, citibase.rat. The consumption series is GC, personal comsumption expenditures. The investment series is CPI, gross private domestic investment. The income series is GC+CPI-(GGFNET+GGSNET), the last two series being respectively surplus or deficit of the Federal and State-Local government receipts and expenditures. All three variables are divided by GD, the implicit price deflator of GNP.

I was led to reexamine this model because I consider the accelerator to be one of the most basic and empirically valid laws of economics, having found in previous studies strong supporting evidence by examining data for the demand for automobiles in the United States (1957, 1960), for aggregate U.S. investment (1967, 1968 and 1969), and for aggregate investment in China (1985a, p. 236, and 1985b). There is no need to learn dynamic programming and rational expectations econometrics to understand that if capital stock K_t is dependent on income then the change in capital stock or investment will be dependent on the change in income. In the references cited, every time I regressed investment on both current income and lagged income, I found the coefficient of lagged income to be negative and approximately equal in absolute value (or slightly smaller if gross investment $K_t-(1-\delta)K_{t-1}$ is explained) to the coefficient of current income, but not so for consumpion. A second interesting empirical question is whether there exists a long-run equilibrium relation between consumption and income. In the model of Chow (1967) a unit root was imposed in the consumption function by differencing. With the new tools for differencing multivariate models at our disposal, it is of interest to reexamine the former model.

Denoting investment, consumption and income by y_{1t}, y_{2t} and y_{3t} respectively, we can write the investment and consumption functions of the former model as

(17) $$\Delta y_{1t} = \beta_{13}\Delta y_{3t} + \phi_{11}y_{1,t-1} + \phi_{13}y_{3,t-1} + \mu_1 + u_{1t}$$

(18) $$\Delta y_{2t} = \beta_{23}\Delta y_{3t} + \gamma_{22}\Delta y_{2,t-1} + \mu_2 + u_{2t}$$

The coefficient of $y_{3,t-1}$ in (17) is expected to equal δ times the coefficient of Δy_{3t}, δ being the rate of depreciation. (18) is derived by assuming y_{2t} to be a function of y_{3t} and $y_{2,t-1}$ and taking first difference. A unit root is thus imposed in the model. Furthermore, by the absence of $y_{2,t-1}$ and $y_{3,t-1}$ in (18), i.e., by assuming ϕ_{22} and ϕ_{23} of model (1) to be zero, any cointegrating relation between consumption and income, if it exists, is not allowed to influence Δy_{2t}.

The model of equations (17)-(18) has been estimated by first estimating a more general model as specified by equation (1) with more lagged Δy_1 and Δy_3 in the investment equation and with $y_{2,t-1}$, $y_{3,-1}$ and more lagged Δy_2 and Δy_3 in the consumption equation. Using annual data from 1947 to 1989, we first estimate an equation explaining Δy_{3t} by 3 lagged y_1, y_2 and y_3, using variables $y_{1,t-1}$, $y_{2,t-1}$, $y_{3,t-1}$ and two lag Δy_i (i=1,2,3) in the form of the third equation of model (1). Using the above explanatory variables as instruments, we estimate the investment and consumption equations of model (1) by the method of instrumental variables. The results are given in the first three equations of Table 1 where the three variables are labeled I, C and Y respectively. The sample period is 1950-1989, excluding the three initial observations. The number in parentheses below each coefficient is the "t" statistic in absolute value. We first interpret the statistical results by ignoring the possible existence of unit roots, leaving the issue of nonstationarity in a discussion that follows.

The investment equation (17) is in perfect agreement with the data. The coefficient of Δy_3 is .771 and significant. The coefficient of lagged income is practically zero as the theory predicts. The acceleration principle is strongly confirmed. We have included two insignificant lagged variables $\Delta y_{1,-1}$ and $\Delta y_{3,-1}$ which will be dropped in the next estimation. These variables were absent in the specification of equation (17). Since the coefficients of both $y_{1,-1}$ and $y_{3,-1}$ are insignificant, there is no long-run equilibrium relation between investment and income. The consumption function (18) is partly correct and partly incorrect. The correct part is the very significant

Table 1

The Multiplier-Accelerator Model: Annual Data 1947-1989

No Unit Roots

Dependent variable	Coefficients of ΔY	I_{-1}	C_{-1}	Y_{-1}	ΔI_{-1}	ΔC_{-1}	ΔY_{-1}	ΔI_{-2}	ΔC_{-2}	ΔY_{-2}	R^2/s
1. ΔI	.771 (2.2)	-.243 (.82)		.020 (.33)	-.024 (.11)		.119 (.40)				.668/.028 DW=1.810
2. ΔC	.573 (7.3)		-.325 (2.0)	.256 (2.1)		.213 (1.2)	-.061 (.48)				.821/.014 DW=2.066
3. ΔY		-.790 (2.9)	.239 (.55)	-.005 (.015)	-.040 (.14)	-.130 (.28)	.708 (2.0)	.413 (1.8)	-.650 (1.9)	-.075 (.25)	.685/.029 DW=2.178
4. ΔI	.916 (4.3)	-.130 (.65)		-.002 (.04)							.651/.028 DW=1.901
5. ΔC	.561 (7.7)		-.275 (2.3)	.218 (2.3)		.144 (1.45)					.823/.014 DW=2.007
6. ΔY		-.660 (3.1)	.083 (.24)	.072 (.25)	-.247 (1.0)	-.038 (.11)	.629 (2.0)				.610/.030 DW=1.724
7. ΔI	.897 (3.2)	-.156 (.66)		.006 (.12)							.641/.028 DW=1.944
8. ΔC	.618 (7.4)		-.285 (2.2)	.224 (2.3)		.119 (1.1)					.805/.014 DW=1.942

coefficient .573 for the change in income, in relation to the also significant coefficient .256 of lagged income. This shows the main difference between the consumption and investment equations according to the acceleration principle, as the coefficient of lagged income in the investment equation is zero. The incorrect part is that both $y_{2,-1}$ and $y_{3,-1}$ are significant, indicating the presence of a long-run relation between consumption and income, with a point estimate of .256/.325 or .788 for the marginal propensity to consume. This aspect of the equation was overlooked in the specification of equation (18). After dropping $\Delta y_{1,-1}$ and $\Delta y_{3,-1}$ from the investment equation and $\Delta y_{3,-1}$ from the consumption equation, we obtained equations 4 and 5 of Table 1. The results confirm the above conclusions concerning these two equations more strongly, with smaller standard errors for the key coefficients. As the lagged variables in equations 4 and 5 are so few, I dropped the last three instrumental variables. The regression of Δy_3 on this shorter list of instruments is given by equation 6, while the investment and consumption functions are given by equations 7 and 8 respectively, all with observations from 1949 to 1989. The results are almost the same.

To deal with possible nonstationarity, the first step is to find the number of zero roots of model (1) and the characteristic vectors β corresponding to the nonzero roots of (12). Using lags of order two, as in euqation 6 of Table 1, we find the roots λ_i to be .4479, .3838, and .1522, yielding $-T\ln(1-\lambda_i)$ of 24.352, 19.850 and 6.769 respectively. The value 6.769 is smaller than the 5% critical value 8.083 from Table T.II of Johansen (1989, p. 81), leading to accepting the hypothesis of one zero root. The critical value at the 10% level of significance is 6.691. At the 10% level, the hypothesis of the smallest root being zero can barely be rejected. One would then conclude that the model is stationary and hence the standard errors of Table 1 are valid. If we accept one zero root, we next ask whether there are two zero roots by comparing the sum 6.769+19.850 or 26.619 with the 5% critical value 17.844 and the 1% critical value 21.962 of Table T.II. The hypothesis of two zero roots is rejected at levels much below

17%. One may conclude that model (1) has one zero root or the rank of Φ is 2. The two characteristic vectors associated with the nonzero roots are

$$\beta' = \begin{bmatrix} -44.661 & -30.893 & 32.777 \\ -4.566 & -7.135 & 8.083 \end{bmatrix}$$

Using these two cointegrating vectors to form two canonical variables $\beta' y_t \equiv (z_{1t}, z_{2t})'$, we reestimate equations 7 and 8 of Table 1. The results are given in Table 2. Equation 1 of Table 2 expresses the investment equation in terms of the canonical variables $z_{1,-1}$ and $z_{2,-1}$, with coefficients $\hat{\alpha} = (\hat{\alpha}_1 \ \ \hat{\alpha}_2)$ being .0056 and -.015. In equation 2, the coefficients of I_{-1}, C_{-1} and Y_{-1} are estimated by $\hat{\phi} = \hat{\alpha}\beta'$. The estimate of the covariance matrix of $\hat{\phi}$ is $\beta(\text{Cov}\ \hat{\alpha})\beta'$, yielding the "t" ratios of the coefficients to their standard errors as given in parentheses. Equation 2 of Table 2 is very similar to the corresponding equation 7 of Table 1. None of the coefficients of I_{-1}, C_{-1} and Y_{-1} is significant in both cases, though in Table 1 we imposed a zero coefficient for C_{-1}. The coefficient of ΔY is very similar. The acceleration principle is very strongly confirmed. By imposing a unit root in the model, we did not change the results in this case.

The consumption function is given by equations 3 and 4 of Table 2. Equation 4 is somewhat different from equation 8 of Table 1. The difference comes mainly from the presence of the coefficient -.313 for I_{-1}, with a "t" ratio of -1.97, whereas in Table 1, this coefficient was assumed to be zero by the specification of the consumption function. Allowing for the presence of I_{-1} and imposing one unit root in Table 2, one finds the effect of ΔY on consumption reduced and the effect of ΔC_{-1} increased while the cointegrating vector relating C_{-1} and Y_{-1} are approximately the same. In order to isolate the effect of imposing a unit root, I have added I_{-1} as an explanatory variable in equation 8 of Table 1 and presented the result in equation 5 of Table 2. Equation 5 is not

Table 2

The Multiplier-Accelerator Model: Annual Data 1947-1989

One Unit Root; * for No Unit Root

Dependent variable	Coefficients of ΔY	$z_{1,-1}$	$z_{2,-1}$	I_{-1}	C_{-1}	Y_{-1}	ΔC_{-1}	R^2/s
1. ΔI	.874 (2.9)	.0056 (.92)	-.015 (3.2)					.644/.028 DW=1.937
2. ΔI	.874 (2.9)			-.182 (.65)	-.066 (.33)	.051 (.23)		.644/.028 DW=1.937
3. ΔC	.263 (1.4)	.0066 (1.8)	.0040 (1.7)				.295 (2.6)	.856/.012 DW=2.645
4. ΔC	.263 (1.4)			-.313 (1.97)	-.232 (2.10)	.252 (2.14)	.295 (2.6)	.856/.012 DW=2.645
5.* ΔC	.402 (2.14)			-.202 (1.25)	-.377 (2.93)	.336 (2.78)	.216 (1.86)	.872/.012 DW=2.374

very different from equation 4. The main difference is the smaller "t" ratio for the coefficient of I_{-1}, making it insignificant. The coefficient of ΔY is .401 with a standard error of .188; it is .263 with a standard error of .182 for equation 4. The standard errors of the coefficients of I_{-1}, C_{-1}, and Y_{-1} are respectively .162, .129 and .121 in equation 5; they are respectively .159, .111 and .118 in equation 4. The differences between the coefficients of equations 4 and 5 are not large as compared with the standard errors. The similarity of the standard errors in the two equations again illustrates a small effect of imposing a unit root on statistical inference as in the case of the investment equation 2.

The major findings of this section can be summarized. First, the acceleration principle as formulated in equation (17) is strongly confirmed in the presence or absence of unit roots; this principle does not apply to the consumption function. Second, there exists an equilibrium relation between consumption and income which affects short-run consumption behavior, and there is no long-run equilibrium relation between investment and income affecting investment behavior. Third, coefficient estimates and their standard errors obtained by ignoring the presence of unit roots happened to be close enough to those obtained by imposing one unit root that no major changes in inference occurred concerning our investment and consumption equations. Note that both sets of standard errors are similar because both are derived under the assumption of a stationary model. Fourth, for those who like to keep unit roots, there appears to be one unit root in the multiplier-accelerator model that we have formulated. The hypothesis of one unit root, i.e., one zero root in the model (1) explaining Δy, cannot be rejected at the 9% level, but the hypothesis of two unit roots, i.e., two zero roots in model (1), can be rejected at levels much below 1%. Fifth, our investment equation illustrates the usefulness and validity of the simultaneous-equations approach to econometric modelling by excluding variables from structural equations, an approach questioned by Liu (1960) and Sims (1980). The coefficients of C_{-1} and Y_{-1} turned out to be zero as the simultaneous-equations model (1) specifies. On the other hand, the possible effect of I_{-1} in the consumption function as illustrated by equations 4 and 5 of Table 2 suggests caution to simultaneous-equations model builders in imposing zero restrictions on selected coefficients. Sixth, our estimation of the Φ matrix in model (1) serves as a warning to those who formulate and estimate error-correction mechanisms equation by equation in the explanation of a set of interrelated economic variables. It is here suggested that the number of unit roots and the subsequent cointegrating vectors be estimated by using a simultaneous-equations model. Otherwise, the possible presence of I_{-1} in the consumption function can never be discerned.

V. EMPIRICAL FINDINGS FROM QUARTERLY OBSERVATIONS

Using quarterly data from 1947.1 to 1989.4 we first estimate the investment and consumption functions of model (1) by the method of instrumental variables without regard to unit roots. The instrumental variables used include all three lagged variables up to order 3, or lagged differences up to order 2, since three additional lagged variables are all insignificant in explaining ΔY, and including them increases the standard error (adjusted for the degrees of freedom) of the regression of ΔY slightly. Equations 1, 2 and 3 of Table 3 are estimates of the investment function; equations 4 and 5 are estimates of the consumption function. The investment functions again confirm the acceleration principle strongly. The coefficient of ΔY is very significant while the coefficient of Y_{-1} is practically zero. This is true after we drop ΔI_{-1} and ΔY_{-1} to obtain equation 2. The coefficients of I_{-1}, C_{-1} and Y_{-1} are all virtually zero in equation 3. A zero coefficient for C_{-1} confirms the validity of the specification of our investment equation in a simultaneous system of structural equations. The zero coefficient for Y_{-1} supports the absence of a cointegrating relation between Y and I affecting investment behavior. The consumption functions also support the accelerations principle since the coefficients of Y_{-1} are not very small as compared with the coeffi-

Table 3

The Multiplier-Accelerator Model: Quarterly Data 1947.1-1989.4

No Unit Roots

Dependent Variable	ΔY	Coefficients of I_{-1}	C_{-1}	Y_{-1}	ΔI_{-1}	ΔC_{-1}	ΔY_{-1}	ΔC_{-2}	R^2/s
1. ΔI	1.014 (5.5)	-.022 (.43)		-.003 (.31)	.088 (.93)		.135 (1.5)		.282/.0186 DW=1.936
2. ΔI	1.087 (5.8)	.013 (.03)		-.007 (.71)					.179/.0197 DW=1.936
3. ΔI	1.014 (5.3)	-.023 (.42)	-.001 (.01)	-.003 (.04)	.088 (.93)		.135 (1.5)		.281/.0186 DW=1.936
4. ΔC	.599 (3.8)		-.157 (3.2)	.125 (3.3)		-.374 (2.9)	.115 (2.1)	.071 (.81)	.113/.0124 DW=2.014
5. ΔC	.398 (2.4)	-.064 (2.1)	-.169 (4.0)	.146 (4.4)		-.273 (2.3)	.088 (1.8)	.104 (1.4)	.372/.0105 DW=2.050

cients of ΔY, and are both very significant. The significant coefficients of C_{-1} and Y_{-1} suggest the existence of a long-run equilibrium relation between C and Y affectiong consumption behavior. From equation 4, the long-run marginal propensity to consume is estimated to be .125/.157 or .80. The significant coefficient of I_{-1} in equation 5 suggests the possible effect of lagged investment on consumption, casting doubt on the single-equation approach to the formulation of an error-correction mechanism for consumption. If ΔY_{-2} is added to equation 5, the coefficient has "t" ratio of .52 while the coefficients of ΔY, Y_{-1} and I_{-1} and their corresponding "t" ratios are hardly affected. The results from using quarterly observations are in agreement with the results from annual observations presented in Table 1.

To allow for unit roots, we first estimate the number of zero roots of model (1). Using data from 1947.1 to 1989.4 and three lags for the three variables, we find the roots λ_i of (12) to be .34722, .27801 and .18651 with corresponding $-T\ln(1-\lambda_i)$ of 37.960, 28.991 and 18.372. The hypothesis of one zero root is rejected at levels much lower than 1%, for which the critical value from Table T.II of Johansen (1989, p. 81) is only 11.576, much smaller than 18.372. The model is judged to be stationary! The reader might accept the analysis of Table 3 assuming no unit roots. To continue our discussion note that if we employ data from 1951.1 to 1989.4, with the first observation beginning at 1951.4, we find the roots of λ_i of (12) to be .33094, .14148 and .04225, with $-T\ln(1-\lambda_i)$ for the smallest root being 6.604, smaller than the 10% critical value of 6.691 and leading to accepting one zero root. The sum of $-T\ln(1-\lambda_i)$ for the two smallest roots is 29.944, much larger than the 1% critical value of 21.962, leading to rejecting strongly the existence of two zero roots. The rank of Φ is two, with cointegrating vectors equal to

$$\beta' = \begin{bmatrix} 10.993 & 12.246 & -12.571 \\ 32.405 & 19.143 & -20.576 \end{bmatrix}$$

Estimates of the investment and consumption functions are given respectively by equations 1 and 2 of Table 4. We omit presenting estimates of the coefficients of the two canonical variables $z_{-1} = \beta' y_{-1}$ and report only the estimates of the coefficients of the original lagged variables y_{-1}. The results are the same as reported in Table 3 ignoring unit roots. To make the results more comparable, we use the same sample period 1951.4-1989.4 assuming stationarity and report the results in equations 3 and 4 of Table 4. The assumption of stationarity has no effect on the investment equation and increases slightly the absolute values of the coefficients of ΔY, C_{-1} and Y_{-1} in the consumption equation but gives very similar standard errors for both equations. Overall, statistical inference is little affected by assuming the existence of one unit root. The major conclusions from Table 3 and from using annual observations as reported in Tables 1 and 2 remain valid.

It might be of interest to investigate whether our conclusions are valid for two separate samples using data 1951.1-1969.4 and 1969.2-1989.4. The first sample has an estimate of 5.795 for the smallest $-T\ln(1-\lambda_i)$, leading to accepting one zero root at the 10% level. It has an estimate of 26.021 for the sum of the two smallest $-T\ln(1-\lambda_i)$, leading to the rejection of two zero roots at much lower than the 1% level. The second sample has corresponding estimates of 6.730 and 26.935 respectively, leading to accepting one zero root at the 9.5% level and rejecting two zero roots strongly. Assuming two stationary canonical variables, we present estimates of the investment and consumption functions for the first period in equations 5 and 6 of Table 4, and for the second period in equations 7 and 8. The conclusions from both periods are the same as for the entire sample, except for two qualifications. First, in equation 5 for the first

Table 4

The Multiplier-Accelerator Model: Quarterly Data 1951.1-1989.4

One Unit Root; * for No Unit Root

Dependent Variable	ΔY	I_{-1}	C_{-1}	Y_{-1}	ΔI_{-1}	ΔC_{-1}	ΔY_{-1}	ΔC_{-2}	R^2/s
1. ΔI	.820	-.032	.045	-.035	.089		.145		.415/.0167
	(5.7)	(.63)	(1.14)	(.88)	(.99)		(1.65)		DW=2.002
2. ΔC	.256	-.071	-.091	.089		-.173	.108	.063	.436/.0097
	(1.97)	(2.46)	(4.20)	(4.01)		(1.45)	(2.2)	(.85)	DW=2.046
3.* ΔI	.887	-.022	.001	-.004	.074		.140		.383/.0172
	(5.2)	(.42)	(.017)	(.070)	(.79)		(1.54)		DW=2.003
4.* ΔC	.345	-.062	-.139	.123		-.203	.101	.055	.421/.0099
	(2.35)	(2.06)	(3.39)	(3.67)		(1.65)	(2.0)	(.73)	DW=2.019
5. ΔI	.427	-.275	.012	.048	.199		.064		.426/.011
[-69.4]	(1.55)	(3.03)	(.855)	(1.87)	(1.76)		(.045)		DW=2.048
6. ΔC	.173	.025	-.0181	.0159		-.246	.242	.038	.417/.0063
[-69.4]	(.98)	(.46)	(2.08)	(.97)		(2.1)	(3.1)	(.37)	DW=2.125
7. ΔI	.851	-.0018	.047	-.045	.031		.222		.439/.021
[70.1-]	(5.2)	(.025)	(.84)	(.79)	(.23)		(1.7)		DW=2.032
8. ΔC	.273	-.081	-.116	.114		-.207	.097	.015	.429/.012
[70.1-]	(1.83)	(1.97)	(3.9)	(3.9)		(1.3)	(.13)	(.13)	DW=2.023
9.* ΔI	.399	-.275	.039	.027	.195		.025		.422/.011
[-69.4]	(1.3)	(2.99)	(.35)	(.29)	(1.7)		(.16)		DW=2.040
10.* ΔC	.293	.028	-.121	.095		-.242	.173	.067	.474/.0060
[-69.4]	(1.6)	(.55)	(1.97)	(1.93)		(2.2)	(2.05)	(.66)	DW=2.023
11.* ΔI	.875	.002	.018	-.024	.022		.219		.427/.012
[70.1-]	(4.9)	(.025)	(.18)	(.29)	(.16)		(1.7)		DW=2.030
12.* ΔC	.310	-.078	-.171	.156		-.204	.081		.431/.012
[70.1-]	(2.0)	(1.9)	(3.0)	(3.3)		(1.2)	(1.1)		DW=2.011

period, the coefficient of I_{-1} explaining ΔI is significant. However, one can still accept the null hypothesis of no cointegrating relation between I and Y because the 95% confidence interval for the ratio of the coefficient of Y_{-1} to the coefficient of I_{-1} is -.301 to .018, including the value zero. Second, in equation 6 for ΔC in the first period, the coefficient for C_{-1} and Y_{-1} are smaller than in equation 2 for the full sample and in equation 8 for the second half of the sample. All other conclusions remain valid. Equations 9* to 12* are estimated by assuming stationarity. They are very similar to the corresponding equations 5 to 8, except for the somewhat larger (in absolute value) point estimates of the coefficients of C_{-1} and Y_{-1} in equations 10 and 12, as compared with the corresponding equations 6 and 8. Note the larger standard deviations s of the residuals of the equations for the later half of the sample as one would expect. The "t" ratios presented are based on the standard errors of the coefficients without adjusting for heteroskedasticity because they are perhaps more interesting to examine for the purpose of comparing the estmation methods with and without the presence of unit roots.

We have replaced the three variables by their logarithms and repeated the analysis of Table 4 (except for omitting the $\Delta \ln C_{-2}$ variable in the consumption equations). For the entire period 1951.4-1989.4, the smallest $-T\ln(1-\lambda_3)$ is 12.497, leading to rejection of one zero root at levels below 1% (11.576 being the 1% critical value). One can conclude that the data are stationary. For the subperiods 1951.4-1969.4 and 1970.1-1989.4, $-T\ln(1-\lambda_3)$ are 6.264 and 10.846 respectively, leading to accepting one zero root at the 10% level for the first subperiod and rejecting one zero root at about the 2% level for the second subperiod. If we assume one unit root for comparison with the results of Table 4, we obtain the non-starred equations of Table 5. For Table 5 all conclusions stated in the last paragraph remain valid, except for the more homogeneous standard errors of the regressions for the two subperiods as expected. Using GNP in 1982 dollars for the entire period, $-T\ln(1-\lambda_3)$ becomes 3.223, accepting one zero root, but two zero

Table 5

The Multiplier-Accelerator Model: Quarterly Data in Logs 1951.1-1989.4
One Unit Root; * for No Unit Root

Dependent Variable	$\Delta \ln Y$	$\ln I_{-1}$	$\ln C_{-1}$	$\ln Y_{-1}$	$\Delta \ln I_{-1}$	$\Delta \ln C_{-1}$	$\Delta \ln Y_{-1}$	R^2/s
			Coefficient of					
1. ΔlnI	4.37	-.079	-.104	.178	.146		.495	.402/.0435
	(4.16)	(1.37)	(.37)	(.61)	(1.7)		(1.1)	DW=2.002
2. ΔlnC	.430	-.0046	-.113	.117		-.179	.168	.365/.0058
	(2.24)	(.57)	(3.02)	(3.02)		(1.64)	(2.95)	DW=1.982
3.* ΔlnI	4.41	-.078	-.125	.197	.146		.486	.398/.0438
	(4.03)	(1.34)	(.38)	(.60)	(1.7)		(1.05)	DW=2.003
4.* ΔlnC	.509	-.0027	.144	.143		-.201	.159	.347/.0059
	(2.50)	(.32)	(3.26)	(3.3)		(1.79)	(2.73)	DW=1.953
5. ΔlnI [-69.4]	3.34	-.211	.111	.125	.193		.171	.462/.0420
	(1.86)	(2.13)	(.34)	(.37)	(1.79)		(.21)	DW=2.032
6. ΔlnC [-69.4]	.179	.0082	-.072	.068		-.234	.260	.363/.0055
	(.68)	(.60)	(1.69)	(1.53)		(1.86)	(2.58)	DW=2.056
7. ΔlnI [70.1-]	4.58	-.0080	.0032	-.0005	.074		1.15	.404/.0445
	(4.86)	(.11)	(.012)	(.002)	(.57)		(1.73)	DW=2.003
8. ΔlnC [70.1-]	.455	-.015	-.116	.138		-.220	.161	.409/.0060
	(2.59)	(1.54)	(3.31)	(3.22)		(1.45)	(1.87)	DW=2.014
9.* ΔlnI [-69.4]	3.64	-.210	-.079	.306	.199		.0064	.456/.0425
	(1.88)	(2.10)	(.15)	(.58)	(1.80)		(.007)	DW=2.042
10.* ΔlnC [-69.4]	.271	.009	-.126	.119		-.242	.220	.423/.0053
	(1.01)	(.67)	(1.88)	(1.80)		(2.00)	(2.10)	DW=1.995
11.* ΔlnI [70.1-]	4.64	-.0054	-.060	.058	.070		1.14	.397/.0451
	(4.56)	(.70)	(.13)	(.12)	(.52)		(1.69)	DW=2.003
12.* ΔlnC [70.1-]	.507	-.014	0.189	.206		0.213	.132	.406/.0060
	(2.81)	(1.36)	(.308)	(.3.23)		(1.40)	(1.48)	DW=2.006
GNP82 for Y								
13. ΔlnI	6.03	-.088	-.156	.248	.059		-.690	.575/.0367
	(4.51)	(1.48)	(.90)	(1.30)	(.69)		(1.0)	DW=2.115
14. ΔlnC	.477	-.0052	-.0037	.0091		-.244	.152	.437/.0054
	(2.19)	(.60)	(.14)	(.32)		(2.63)	(1.7)	DW=2.024
15.* ΔlnI	6.24	-.087	-.207	.304	.055		-.756	.553/.0377
	(4.36)	(1.43)	(1.03)	(1.37)	(.63)		(1.1)	DW=2.115
16.* ΔlnC	.508	-.0051	-.010	.016		-.248	.143	.434/.0054
	(2.22)	(.58)	(.35)	(.50)		(2.65)	(1.55)	DW=2.021

roots are strongly rejected. The results are given in equations 13 to 16 of Table 5, supporting all previous conclusions except for the lack of an equilibrium relation between consumption and GNP.

VI. CONCLUSIONS

In this paper I set out to reexamine the aggregate investment and consumption functions which were estimated a quarter of a century ago in Chow (1967) using annual data from 1929 to 1963. The investment function stood up extremely well against new data of twenty-six years. The acceleration principle has received very strong additional support. The consumption function has been partly reconfirmed concerning the short-run effect of income, but I have now found the existence of a long-run equilibrium relation between consumption and income which was ruled out by the previous formulation in first differences of the two variables. I happened to be correct in imposing a unit root in the multiplier-accelerator model, but I was incorrect in placing it entirely in the consumption function. I did not know how to impose one unit root in a model of three equations, to be shared by them.

The major empirical findings of this paper are the six points stated in the last paragraph of section IV after the model was estimated by annual observations. The six points regarding the acceleration principle, the long-run income-consumption relation, the similarity of standard errors in the three-equation models assuming stationarity and nonstationarity with the presence of one unit root, the possible existence of one but not two unit roots in the model the validity of the simultaneous-equations approach to econometric modelling and the caution required in single-equation formulations of the error-correction mechanism are all valid for models estimated by quarterly data, and separately for two subsamples each of twenty years. Any theory of aggregate consumption has to explain the short-run and long-run effects of income as formulated and estimated here.

Concerning the two-step procedure here recommended to estimate and test linear simultaneous equations with unit roots, three comments shoud be made. First, it is possible to find the standard errors of the estimated coefficients in the second step, having imposed the rank of Φ in (15) to achieve stationarity, to be similar to the standard errors of the estimates based on stationarity in the first place because both are derived under the assumption of a stationary model. Second, if an econometrician does not take the first step and tries to derive the asymptotic distribution of the estimates of Φ assuming the true rank of Φ to be less than full, she will obtain a different set of standard errors based on nonstandard distributions involving integrals of Wiener processes. Third, if one insists on the existence of some unit roots as the null hypothesis, there is a choice between a one-step procedure and the two-step procedure. By imposing the rank of Φ in (15), the two-step procedure can produce different point estimates for the coefficients from those obtained without the rank restriction, as illustrated by the coefficients of C_{-1} and Y_{-1} in the consumption functions of Table 4 and by the coefficients of $\ln C_{-1}$ and $\ln Y_{-1}$ in the consumption functions of Table 5, but the asymptotic distribution of these estimates is normal and standard. The two-step procedure is much easier to use. It is subject to possible pretest bias as the Box and Jenkins (1970) differencing procedure and many other currently practiced econometric procedures are.

REFERENCES

Ahn, S. K. and Reinsel, G. C., "Estimation for Partially Nonstationary Multivariate Autoregressive Models," *Journal of the American Statistical Association*, 85, September 1990, 813-823.

Box, G. and Jenkins, J. M., *Time-Series Analysis Forecasting and Control*, San Francisco: Holden-Day, 1970.

Chow, G. C., *Demand for Automobiles in the United States: A Study in Consumer Durables* Amsterdam: North-Holland Publishing Company, 1957.

________, "Statistical Demand Functions for Automobiles and Their Use for Forecasting," in A. C. Harberger, *The Demand for Durable Goods*, Chicago: University of Chicago Press, 1960, 147-178.

________, "Multiplier, Accelerator, and Liquidity Preference in the Determination of National Income in the United States," *The Review of Economics and Statistics*, February 1967, XLIV, 1, 1-15.

________, "The Acceleration Principle and the Nature of Business Cycles," *Quarterly Journal of Economics*, August 1968, LXXXII, 3, 403-418.

________, *The Chinese Economy*, New York: Harper & Row, Pulblishers, Inc., 1985.

________, "A Model of Chinese National Income Determination," *Journal of Political Economy*, August 1985, 93, 782-792.

________ and Levitan, R. E., "Nature of Business Cycles Implicit in a Linear Economic Model," *Quarterly Journal of Economics*, August 1969, LXXXIII, 3, 504-517.

Engle, R. F. and Granger, C. W. J., "Cointegration and Error Correction: Representation, Estimation, and Testing," *Econometrica,* 1987, 55, 251-176.

Hendry, D. F. and Ericsson, N. R., "An Econometric Analysis of UK Money Demand in *Monetary Trends in the United States and the United Kingdom* by Milton Friedman and Anna J. Schwartz," Washington, D.C.: Board of Governors of the Federal Reserve System, International Finance Discussion Paper No. 355, July 1989.

Johansen, S., "Statistical Analysis of Cointegration Vectors," *Journal of Economic Dynamics and Control*, June/September 1988, 12, 231-254.

_________, *Likelihood Based Inference in Cointegration Theory and Applications*, Bagni di Lucca, Italy: Centro Interuniversitario di Econometrica, 1989.

Liu, T. C., "Underidentification, Structural Estimation and Forecasting," *Econometrica*, 1960, 28, 855-865.

Sims, C. A., "Macroeconomics and Reality," *Econometrica*, January 1980, 48, 1-48.

Sims, C. A., Stock, J. and Watson, M.,"Inference in Linear Time Series Models with Some Unit Roots," *Econometrica*, January 1990, 58, 113-144.

Stock, J. H., "Asymptotic Properties of Least Squares Estimators of Cointegrating Vectors," *Econometrica*, 1987, 55, 1035-1056.