SOME SMALL SAMPLE PROPERTIES OF CERTAIN STRUCTURAL EQUATION ESTIMATORS

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1. Introduction

The last fifteen years have witnessed a proliferation of techniques by which the coefficients of a system of simultaneous linear structural equations can be estimated. This phenomenon can be explained with reference to at least three factors: (1) the development of macroeconomic theory and the improvement in measuring macroeconomic magnitudes resulted in new techniques for controlling the economy and thus necessitated more accurate predictions; (2) the advent of fast electronic computers made it possible to solve complex computational problems rapidly and frequently; (3) the recognition of the distinction between endogenous and exogenous variables compelled for theoretical reasons the abandonment of the direct least-squares approach to estimating the coefficients of structural equations containing several endogenous variables.

As is well known, the basic simultaneous equation model as proposed by the Cowles Commission is as follows. Variables fall into two categories: explanatory variables and variables to be explained. The former, denoted by $z_1,\ldots,z_k$, are either exogenous, that is to say, their values are given from outside the model in which they are embedded, they are nonstochastic, and their values are identical in repeated samples, or they are only predetermined and may then include not only exogenous variables but

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lagged values of variables to be explained. The latter, denoted by $y_1, \ldots, y_G$, are the endogenous variables which are jointly determined by the model.

We are now restricting ourselves to the case in which the explanatory variables are all exogenous. A structure consists of G, so-called structural, equations each of which explains a particular linear combination of y's in terms of a certain linear combination of z's and corresponds to a hypothesized causal mechanism operating in the economy. The T observations on the endogenous variables are assumed to be generated by the system

$$By_t + \Gamma z_t = u_t \qquad t = 1, \ldots, T$$

$|B| \neq 0$, where the elements of the vector u_t represent error terms and are customarily assumed to be jointly normally distributed with mean zero and covariance matrix Σ independent of t.

Numerous methods have been devised for estimating the structural coefficients, that is to say the elements of the $[B \quad \Gamma]$ matrix[2] Any list of them would have to include[2]

(a) Direct least squares

(b) Indirect least squares

(c) Full-information maximum likelihood

(d) Limited-information maximum likelihood

(e) Two-stage least squares or generalized classical linear estimation[3]

2. [1], [6], [7], [8], [9], [10], [13], [17], [19].

3. The former term being due to Theil, the latter to Basmann. Method (e) is a particular case of a wide class of estimators called Theil's k-class.

(f) Three-stage least squares

(g) Linearized maximum likelihood.

The above methods differ from each other in various respects. Among the most important of these is the extent to which a particular estimating technique makes use of the *a priori* restrictions on an econometric model which most often take the form of requirements that certain variables not appear in particular equations. With the exception of (a), the estimates derived by all these methods have some desirable asymptotic properties such as consistency. Some estimates are asymptotically normally distributed and full-information maximum likelihood estimates are efficient. It has been shown recently[4] that full information maximum likelihood is the correct generalization of the least squares principle, with other methods being conceptually defective in one or more respects.

None of these assertions says anything at all about the small sample behavior of the various estimators. It has been observed before [2] that it is generally assumed in the literature that the ranking of competing methods achieved in the light of their asymptotic properties is also applicable to small sample cases. Such a belief is obviously unjustified in the absence of knowledge about the small sample distributions of various estimators.[5] For example, knowledge that the full-information maximum

4. [4].

5. Consider two estimators $\hat{\theta}$ and $\hat{\omega}$ with small sample distributions $F_i(\hat{\theta})$, $G_i(\hat{\omega})$ for samples of size i ; let the sequence $F_i(\hat{\theta}) \rightarrow F(\hat{\theta})$ and the sequence $G_i(\hat{\omega}) \rightarrow G(\hat{\omega})$ as $i \rightarrow \infty$. Let α_i and β_i be some continuous functions of F_i and G_i respectively and let the sequences $\alpha_i \rightarrow \alpha$ and $\beta_i \rightarrow \beta$. Then, the assertion that, say, $\alpha < \beta$ allows us to conclude *only* that there exists *some* n such that for $i > n$, $\alpha_i < \beta_i$.

likelihood method yields (asymptotically) efficient estimates does not justify the assumption that the dispersion of such estimates from finite samples will be smaller than the dispersion of estimates obtained by some other method. A perhaps even more serious consequence of this is that the direct least squares method which can be shown to yield biased and inconsistent estimates may, in small samples, result in smaller bias than some alternative methods which are consistent. It is hardly necessary to point out that there is no way of intuiting the value of n which is large enough for asymptotic properties to hold in some approximate sense. It is therefore essential that further light be shed on the small sample properties of the various econometric estimators.

Section 2 of this paper discusses some earlier theoretical and experimental studies of the small sample properties of certain structural equation estimators and considers alternative measures of the goodness of estimates. Section 3 describes the nature of the sampling experiments conducted on Theil's k-class and reported in this paper. Section 4 presents results and some conclusions.

2. Previous Studies and the Goodness of Estimators

Three experimental studies of the small sample properties of certain structural estimators deserve special attention, namely those by H. M. Wagner [18], A. L. Nagar [10], R. Summers [16], and R. L. Basmann [3].[6] The numerous individual differences among these studies are somewhat obscured by the similarity of the general approach. The authors assume a system of simultaneous structural equations

6. A very good summary of various Monte Carlo approaches to the problem of uncovering the small sample properties of various econometric estimators can be found in Johnson, J. [7], pp. 275-95.

$$By + \Gamma z = u \tag{1}$$

and generate samples of the jointly determined variables y on the assumption of normally distributed error terms. From each of the samples the β and γ coefficients of one or more equations are estimated by various methods. Certain sample moments as well as some other statistics are calculated from the samples of estimates and statements are made on the basis of these statistics concerning the relative degrees of bias and efficiency resulting from the various estimating techniques.

Individual differences among these approaches are numerous. Wagner and Nagar employ the same three-equation model. Of these three equations one is an identity. Of the two predetermined variables one is truly exogenous and the other is a lagged value of an endogenous variable. Of the two genuine equations one is overidentified and one is just identified.[7] Wagner tests limited information maximum likelihood, direct least squares and instrumental variables by generating 100 samples of 20 observations for two alternative error covariance matrices. He finds, on the whole, that direct least squares gives more of a bias than the other two methods but has smaller sample variance. Nagar continues Wagner's investigation by applying to the Wagner model various estimating techniques all of which belong to Theil's k-class. Again the bias is found to be largest for direct least squares. It is smallest for two-stage least squares. Direct least squares again appears to yield minimum-variance estimates.

Summers [16] deals with a two equation model in which both equations are over-identified[8] and in which predetermined variables are genuinely

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7. Note that the identity may be used to eliminate one of the endogenous variables from the remaining equations. Some estimating techniques such as two-stage least squares are not invariant with respect to the manner in which this elimination is performed.

8. With the exception of the case in which the effects of misspecification are investigated in which overidentification of the first equation is assumed erroneously (but intentionally).

exogenous, that is to say, do not include lagged values of endogenous variables. Summers tests limited information maximum likelihood, two-stage least squares, direct least squares, indirect least squares, and full information maximum likelihood. Five sets of parameter constellations were tested. 50 samples of 20 observations each were generated[9] and two different sets of exogenous variables were tried out. Direct least squares does uniformly worst in terms of bias and tends to fare badly on the basis of the second moment about the true value. There is no clear contender for first place: only indirect least squares does not seem to qualify for the finals among the remaining four methods.

Basmann deals with a three equation model and estimates from samples of 16 observations the structural coefficients in the first, overidentified, equation. He finds that direct least squares gives a greater bias than either two stage least squares or limited information maximum likelihood. In terms of the second moment about the true value direct least squares does well, contrary to the findings of Summers. Both direct and two stage least squares appear better than limited information maximum likelihood by this criterion.

A significant difference between the Wagner-Nagar and the Summers approaches is that the former test the various estimating methods by comparing estimates of <u>structural coefficients</u> with the true values of these coefficients whereas the latter employs, in addition, the goodness of the conditional prediction of endogenous variables given a specification of values for the exogenous variables. A clearcut decision in favor of one of these techniques will ultimately rest on the objectives of the user. If

9. For one of the parameter constellations samples of 40 observations were used as well.

one is primarily interested in testing hypotheses about economic behavior one is likely to be interested in slopes and thus perhaps favor the first technique. If one is more interested in the predictive aspects of a model, the second technique is indicated. It is noteworthy, however, that the second technique may disguise some interesting facts. It is conceivable, for example, that the coefficients of one equation are estimated more accurately than those of another. Then, since conditional predictions take into account all coefficients in the system, the latter method may indicate an average bias which is not at all representative of the biases of the individual coefficients. There is no *a priori* reason to believe, however, that the ordering of estimating techniques will be different depending on which quantity we examine for bias. In the experiments to be described in subsequent sections the Wagner-Nagar approach was used, to the extent that attention is focused on the structural coefficients.

The Monte Carlo approach to the finite sample problem has been questioned in some instances by Basmann since he has shown that in certain simple cases the finite sample distributions of maximum likelihood estimators and two-stage least squares estimators of structural equation coefficients have no finite moments of any order.[10] Although Basmann provides proofs only for certain two-equation cases, his conjectured extension of these results to larger cases cannot be dismissed in view of the fact that estimators always appear as ratios of random variables. In the presence of the infinite moment problem extreme care must be exercised in the

10. Basmann conjectures that in general it will be found that no finite moments of order $\nu > k - g + 1$ exist where k is the number of exogenous variables missing and g the number of endogenous variables present in the equation in question. [2], p. 37.

interpretation of the results of Monte Carlo experiments.[11] If the first moment of the finite sample distribution of an estimator does not exist, it is meaningless to compare the mean estimate from a set of samples to the population value. If the variance does not exist, it is meaningless to consider the root mean square error.[12] These considerations then raise the question of what are statistically acceptable and rational measures or criteria of the goodness of an estimate.

In the absence of various statistical difficulties such as those indicated above it appears natural to consider first the mean error, the mean absolute error, and the mean square error. Let θ be the population value of a parameter and let θ_{1i} and θ_{2i} be two estimates of θ from the i^{th} sample. We shall then judge method 1 to be preferable to method 2 under the criteria named if

$$\left| \frac{\sum_i \theta_{1i}}{n} - \theta \right| < \left| \frac{\sum_i \theta_{2i}}{n} - \theta \right|$$

$$\sum_i \left| \theta_{1i} - \theta \right| < \sum_i \left| \theta_{2i} - \theta \right|$$

$$\Sigma (\theta_{1i} - \theta)^2 < \Sigma (\theta_{2i} - \theta)^2 .$$

Even if finite moments exist it is not obvious what criterion of goodness should be employed. It is relatively easy to argue that the first of the

11. There is some awareness of this problem in the literature. See [10], p. 33 and [16], p. 21.

12. It is to be noted that some of Nagar's procedures in forming expansions and omitting terms of higher order of smallness than T^{-1} become questionable in view of the infinite moment problem. Nagar, of course, is well aware of this. See [10], p. 33.

above measures is not a very good one in that it suffers from all the disadvantages of the arithmetic mean. But it may be a perfectly rational measure if what really matters is that on the average we be close to population values. Consider the following artificial and oversimplified example: it is desired to achieve a certain rate of growth in the economy by the judicious use of government expenditures. Their desired or recommended magnitude will be a function of the marginal propensity to consume. If we overestimate the marginal propensity to consume, our estimated national income multiplier will be greater than the true one; therefore government expenditures will be set at too low a level and the economy will fall below the desired growth rate. In the converse case, the economy will grow faster than desired. It may be rational to require from some long run point of view an estimating procedure that will result on the average in a growth rate close to the desired one.

Basmann believes[13] that if the estimates are at least approximately normally distributed and have finite variances then "...any rational measures of concentration we adopt will depend predominantly on their second moments...". But, in general, what is a rational measure of goodness can be defined only on the basis of a specification of the user's utility function over the errors. Assume, for example, that the finite sample distributions of estimators θ_1 and θ_2 are known exactly and that these distributions $f(\theta_1)$, $g(\theta_2)$ are members of some two-parameter family of distributions such as the normal distribution. Let the two parameters be the mean (μ_1 and μ_2 respectively) and the variance (σ_1^2 and σ_2^2 respectively). If we define σ_{1p}^2 and σ_{2p}^2 as the second moments about the

13. [2], p. 4.

population parameter θ and B_1 and B_2 as the biases of the two estimators such that $\theta = \mu_1 + B_1$ and $\theta = \mu_2 + B_2$ then it follows that

$$\sigma_{1p}^2 = \sigma_1^2 + B_1^2$$

$$\sigma_{2p}^2 = \sigma_2^2 + B_2^2 .$$

The first criterion suggests that θ_1 is to be preferred to θ_2 if $B_1^2 < B_2^2$; the third criterion suggests that θ_2 is to be preferred if $\sigma_{2p}^2 < \sigma_{1p}^2$. Obviously both of these can occur simultaneously. A rational choice requires that a utility function $U(\sigma_p^2, B^2)$ be specified. The set of possible estimators can be specified by the function $h(\sigma_p^2, B^2) = 0$. The choice among estimators is then accomplished by solving the following problem:

$$\begin{aligned} \text{Maximize} \quad & U(\sigma_p^2, B^2) \\ \text{subject to} \quad & h(\sigma_p^2, B^2) = 0 \\ & \sigma_p^2 \geqq 0 , \; B^2 \geqq 0 . \end{aligned}$$

Since we are dealing with utilities of estimates it would be desirable to find a reasonable axiomatization that would allow the construction of a utility index. The simplest, of course, would be a von Neumann-Morgenstern [11] axiomatization under which statisticians would turn out to be expected-bias-minimizers. This will hardly be adequate in view of the general requirement that the goodness of an estimator be dependent on its density or concentration in some neighborhood around the population value. At the same time it would be premature to select some concrete axiomatization which explicitly involves second (and possibly higher) moments.[14] Perhaps it is sufficient at this point to list some additional

14. See [12].

possible measures of the goodness of estimators:

(a) The median of the small sample distribution of the estimator minus the population value of the parameter estimated. An advantage of this measure is that it always exists (although it is not necessarily unique). Denoting the median by m, estimator 1 is to be preferred over estimator 2 if

$$|m_1 - \theta| < |m_2 - \theta| .$$

(b) The concentration c in some a priori neighborhood around the population value. Let this interval be $[\theta - k, \theta + k]$. Let the variable $y_1 = 1$ if an estimate θ_1 falls in the interval and let $y_1 = 0$ otherwise. The concentration c_1 associated with estimate 1 is then $c_1 = \frac{\sum_i y_{1i}}{n_1}$ and θ_1 is to be chosen as estimator if $c_1 > c_2$. c is a consistent estimator of the probability $P_c(\theta - k, \theta + k) = \int_{\theta-k}^{\theta+k} f(\theta_1)d\theta_1$ which always exists. The rationale for the measure is obvious: the greater the concentration around the population value the more often we can expect to be relatively close to it. This measure is also used by Basmann [3].

(c) The converse of the preceding measure is the decentralization d, an estimate of the probability $P_d(\theta - k, \theta + k) = 1 - \int_{\theta-k}^{\theta+k} f(\theta_1)d\theta_1$. We would consider method 1 better than method 2 if $d_1 < d_2$. The estimate of $P_d(\theta - k, \theta + k)$ is chosen as a criterion of goodness if frequent extreme errors are to be avoided. Of course, if k is taken to be the same as in (b) this measure is basically the same as the previous one. As a matter of fact, both concentration and decentralization criteria may be applied simultaneously and they may give conflicting evaluations of estimators if different values of k are used in constructing the criteria. In such a case a final evaluation cannot be made without a more concrete specification of the utility function, this time as a function of c and d .

(d) One may decide upon one of various measures of range such as the interquartile range. Comments similar to those under (a) and (b) apply.

(e) One may choose as one measure of goodness, as does Nagar [10], the sum of the ranks, summed over each estimator and obtained from an aggregated ranking of the absolute biases of the estimates. This method has the advantage of measuring, in essence, the mean absolute error without presupposing the existence of the first moment.

(f) Summers, in the previously discussed study [16], and, in a somewhat different context, Schupack [14] use the frequency of being closest to the true value of a parameter. Underlying this is the supposition that we wish to choose estimating technique i so as to maximize the probability $\Pr(|\theta_i - \theta| \leqq \min_{j \neq i} |\theta_j - \theta|)$.

There are numerous additional criteria possible. We might choose on the basis of that single observation which, for a given estimating technique i , makes $|\theta_i - \theta|$ the largest if large errors are to be avoided at all cost. Some of the criteria discussed above will be applied to experimental results in Section 4. It is to be remembered, however, that the rationality of these criteria is not a purely statistical question but depends on the costs (utilities or disutilities) associated with various types of error.

3. Objectives and Design of Experiments

Sampling experiments were carried out in order to investigate a number of questions with reference to Theil's k-class. Theil's k-class commends itself because of its simplicity, ease of computation, and the fact that it is a generalization of classical least squares. Chow has shown, however, that full information maximum likelihood is the correct generalization if the underlying model is one in which simultaneous linear combinations of endogenous variables are to be explained by the set of exogenous variables. Some questions one may raise about the k-class are as follows: (1) Can the tentative findings of other sampling experiments with respect to the relative ordering of estimating methods be confirmed by a different and larger model? (2) Can Theil's findings with respect to the relationship between the magnitude of estimates and the parameter k which is assumed in their derivation in the Girschick-Haavelmo model be confirmed by sampling experiments?[15] (3) Can one verify the conjecture that the magnitude of bias in the structural coefficients is a decreasing function of the sparseness of the coefficient matrix associated with the endogenous variables; that is to say, is it true that the more zeroes there are in the B matrix the less will be the degree of bias in the estimates? (4) Can one verify the conjecture that the magnitude of bias is a decreasing function of the sparseness of the covariance matrix of structural disturbances? This last conjecture has not been tested as yet. The former, namely (3), arises from the following heuristic considerations. Bias is caused by the fact that the y's are all stochastic variables which, in turn, is due to the fact that they are jointly determined, given a specification of exogenous variables and error terms. The sparser the B matrix the likelier it is that the causal chain connecting the variables in

15. [17], pp. 234-6.

the model contains no cycles. It seems plausible to argue that bias depends on how many causal feedbacks there are among the jointly determined variables.

Two models were employed, each of which contains 4 endogenous variables and thus 4 equations. Model 1 is

$$
\begin{aligned}
y_1 - .2y_2 + 2.0y_3 - y_4 - z_1 - .5z_2 + z_3 &= u_1 \\
-y_1 + y_2 + .5y_3 + .1y_4 - 2.0z_2 - z_5 &= u_2 \\
1.5y_1 - .5y_2 + y_3 + .2y_4 + .5z_3 + z_4 - 2.0z_5 &= u_3 \\
.4y_1 + y_2 - .5y_3 + y_4 - z_3 - .2z_5 - 3.0z_6 &= u_4
\end{aligned}
$$

Model 2 is the same as Model 1 with the exception that one more exogenous variable is introduced, z_7, the coefficients of which in the four equations are (0, .5, 0, 0) respectively. No predetermined variable is a lagged value of an endogenous variable. The values of the exogenous variables remain the same in repeated samples, although the experiments are performed on two separate sets of values of the exogenous variables. The correlation matrices of the exogenous variable data sets are

$$
\begin{bmatrix}
1.000 & .872 & .325 & .707 & -.726 & 0 & .152 \\
 & 1.000 & .440 & .847 & -.879 & 0 & .420 \\
 & & 1.000 & .429 & -.469 & 0 & -.049 \\
 & & & 1.000 & -.865 & 0 & .464 \\
 & & & & 1.000 & 0 & -.274 \\
 & & & & & 1.000 & 0 \\
 & & & & & & 1.000
\end{bmatrix}
$$

for Data Set 1 and

$$
\begin{bmatrix}
1.000 & -.423 & .335 & .589 & .068 & 0 & .193 \\
 & 1.000 & -.165 & -.539 & .439 & 0 & .065 \\
 & & 1.000 & .132 & -.291 & 0 & .556 \\
 & & & 1.000 & -.095 & 0 & -.030 \\
 & & & & 1.000 & 0 & .217 \\
 & & & & & 1.000 & 0 \\
 & & & & & & 1.000
\end{bmatrix}
$$

for Data Set 2.

In all cases the coefficients of the first equation are estimated. This equation is just identified in Model 1 and overidentified in Model 2. The generation of data consists of first generating 20 "observations" on the vector (u_1, u_2, u_3, u_4), where the u's are approximately normally distributed with zero mean and covariance matrix

$$\begin{bmatrix} 1.0 & .6 & .8 & -1.0 \\ .6 & 1.0 & 0 & -.2 \\ .8 & 0 & 2.0 & -.6 \\ -1.0 & -.2 & -.6 & 2.5 \end{bmatrix} .$$

Given the values of the vector z, corresponding y-values are then calculated.[16] Theil's k-class estimators are then obtained for the given sample for values of $k = -.4, -.3, \ldots, 0, \ldots, 1.0, \ldots, 1.9, 2.0$. This procedure is then repeated until a total of 100 samples have been generated. Conjecture (3) is tested by performing this procedure for four different B matrices of differing sparseness. The B matrices used are the one indicated earlier in the description of the model, namely

$$\begin{bmatrix} 1.0 & -.2 & 2.0 & -1.0 \\ -1.0 & 1.0 & .5 & .1 \\ 1.5 & -.5 & 1.0 & .2 \\ .4 & 1.0 & -.5 & 1.0 \end{bmatrix}$$

and

$$\begin{bmatrix} 1.0 & -.2 & 2.0 & -1.0 \\ -1.0 & 1.0 & 0 & .1 \\ 0 & -.5 & 1.0 & .2 \\ .4 & 0 & -.5 & 1.0 \end{bmatrix}$$

$$\begin{bmatrix} 1.0 & -.2 & 2.0 & -1.0 \\ 0 & 1.0 & 0 & .1 \\ 0 & -.5 & 1.0 & 0 \\ .4 & 0 & 0 & 1.0 \end{bmatrix}$$

$$\begin{bmatrix} 1.0 & -.2 & 2.0 & -1.0 \\ 0 & 1.0 & 0 & 0 \\ 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 1.0 \end{bmatrix}$$

16. Whenever matrix inversion was necessary, it was performed by a method designed to work well on ill-conditioned matrices. See [15]. The procedure was tested and its accuracy found to be good: if a_{ij} are elements of a matrix A and a^*_{ij} elements of A^{-1}, then $|\delta_{ij} - \Sigma a_{ik} a^*_{kj}| < 10^{-6}$ where δ_{ij} is the Kronecker delta. The method works well on the Hilbert matrix of size 7×7.

corresponding respectively to Run 1, Run 2, Run 3 and Run 4.

For the purpose of estimation the first equation is rewritten as

$$y_1 = \beta_{12}y_2 + \beta_{13}y_3 + \beta_{14}y_4 + \gamma_{11}z_1 + \gamma_{12}z_2 + \gamma_{13}z_3 + u_1 .$$

Mathematically the estimates are obtained in the following manner. Define Y_1, Y_2, Y_3, Y_4 the 20×1 column vectors containing the observations on the variables y_1, y_2, y_3, y_4. Denote by Y the 20×3 matrix $Y = [Y_2 \vdots Y_3 \vdots Y_4]$. Denote by Z the 20×6 (or 20×7 in Model 2) matrix containing the observations on the exogenous variables and by Z_* the submatrix of Z formed by its first three columns. We further define three 20×1 vectors V_i by $V_i = Y_i - Z(Z'Z)^{-1}Z'Y_i$ for $i = 2, 3, 4$ which makes the vectors V_i contain the residuals from the reduced form corresponding to y_2, y_3 and y_4. We define $V = [V_2 \vdots V_3 \vdots V_4]$. The estimates $(\hat{\beta}', \hat{\gamma}')$ are then obtained as follows:

$$\begin{bmatrix} \hat{\beta} \\ \hat{\gamma} \end{bmatrix} = \begin{bmatrix} Y'Y - kV'V & Y'Z_* \\ Z_*'Y & Z_*'Z_* \end{bmatrix}^{-1} \begin{bmatrix} Y' - kV' \\ Z_*' \end{bmatrix} Y_1$$

where the matrix, the inverse of which appears on the right hand side, will be referred to as the Theil-matrix.

Various safeguards were used to insure the proper operation of the computer program. The results of the first sample for $k = -.4$ and $k = 1.0$ were handcomputed on a desk calculator. These handcomputed results agree with those of the electronic computer except for rounding, accounting for discrepancies of the order of 10^{-6}. In addition the vector (β, γ) was estimated by setting the error terms u identically equal to zero in the generation of a sample. The results of this estimation are shown in Table 1.

	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
True value	.2	-2.0	1.0	1.0	.5	-1.0
Value estimated with k = -.4	.199545	-1.99893	.999115	.999622	.499531	- .999092

Table 1

Of course, in the absence of error terms the estimates should coincide exactly with the true values, since in this case the y's are not stochastic, all observations lie exactly on a hyperplane, and the k class (for any value of k) coincides with the least squares regression of y_1 on $y_2, y_3, y_4, z_1, z_2, z_3$. In fact, the residuals from estimating the reduced form corresponding to y_2, y_3 and y_4 are not zero, being on the order of 10^{-6} and 10^{-7}. This is undoubtedly due to rounding error in inverting (Z'Z). This error is then further propagated when the Theil-matrix is inverted. The figures in Table 1 are thus construed to support the judgment that the results obtained are correct except for rounding error.

4. Results and Conclusions

A number of statistics were collected during the course of the experiments. These are presented in the Appendix in Tables 1A-20D and Figures 1-16. The analysis of the results is based on various functions of the samples of estimates. These include (a) the median, (b) the concentration, that is the density in some neighborhood of the population parameter, (c) the decentralization, i.e., the density in the complement of some neighborhood about the population parameter, (d) the maximum deviation from the population parameter, (e) the arithmetic means and standard deviations of sample estimates. These latter are presented and discussed even though there is no assurance that the finite sample distributions of the various estimates possess finite moments of any order. It must be kept in mind, however, that the evidential character of these sample moments is weak and cannot therefore be decisive.

A. Analysis and Results Based on Sample Moments.

(a) Direct Least Squares Versus Two-Stage Least Squares. One may immediately observe that the sample standard deviations for two stage least squares are uniformly larger than for direct least squares by a factor ranging from less than five to over a hundred. This is in substantial agreement with previous results. [7].

Since the ordering of estimating techniques imposed by the average bias is essentially invariant with respect to which of the structural coefficients is being estimated and which of the Runs generated the data, aggregation over coefficients and Runs has been performed and the number of instances in which the average bias of direct least squares is smaller than that of two stage least squares has been counted. Since there are 6 coefficients and 4 Runs, there are 24 cases for each Model-Data Set combination. Direct least squares beats two-stage least squares as follows:

TABLE 2. MEANS. Relative Performance of k = 0 and k = 1.

Type	Number of Cases			Total Possible Cases
	k = 0 Beats	Ties	Loses to k = 1	
Model 1, Data Set 1	23	0	1	24
Model 1, Data Set 2	2	0	22	24
Model 2, Data Set 1	15	0	9	24
Model 2, Data Set 2	1	0	23	24

Two-stage least squares tends to result in smaller average biases when the model is overidentified than when it is just identified. What is much more striking, however, is the extent to which the relative goodness of the estimates depends on which set of exogenous variables is used. With the first set, direct least squares gives significantly better results than two-stage least squares: aggregating over Models 1 and 2 one can reject the null hypothesis that there is no difference between the two techniques on the .001 level of significance. On the other hand, two-stage least squares does overwhelmingly better than direct least squares with Data Set 2. In a sense one might expect (all) results to be worse in the case of Data Set 1 since the pairwise correlations between the exogenous variables are substantially higher than in the case of Data Set 2. In fact, aggregating over coefficients and runs, one finds that the average bias in the samples is smaller for Data Set 2 than for Data Set 1 in corresponding cases as indicated in Table 3. In point of fact, however, it

TABLE 3. MEANS. Relative Performance of Data Set 1 and Data Set 2.

Type	Number of Cases			Total Possible Cases
	Set 1 Beats	Ties	Loses to Set 2	
Model 1, k = 0	0	0	23	24
Model 1, k = 1	1	0	24	24
Model 2, k = 0	3	0	21	24
Model 2, k = 1	2	0	22	24

is irrelevant whether relatively strong multicollinearity among exogenous variables can explain the differential performance of direct and two-stage least squares.[17] It is sufficient to observe that in small sample situations two-stage least squares is not necessarily better than direct least squares. If the cause of this is the presence of multicollinearity, one has all the more reason to be concerned, since economic data frequently exhibit high multicollinearity. The asymptotic properties of two-stage least squares do not carry through to the small sample situation, which should not be considered surprising, particularly in view of the observation that the relative goodness of estimates appears to be highly sensitive to the configuration of the data. This then confirms Theil's tentative results in connection with the Girschick and Haavelmo model of the U.S. Food Market according to which the choice between two-stage and direct least squares cannot be made unambiguously.

(b) The Variation of Estimates with k. Theil has plotted the estimates obtained in the Girschick-Haavelmo model as a function of k varying from 0 to 1.5.[18] A similar procedure was undertaken in the present study since estimates were calculated for variations in k by steps of .1 from -.4 to 2.0. The results are graphed in Figures 1-16. Theil had observed a discontinuity in the graphs occurring roughly in the range $k = 1.1$ to $k = 1.5$. This was explained on the basis of the fact that the Theil matrix $\begin{bmatrix} Y'Y - kR'R & Y'Z_* \\ Z_*'Y & Z_*'Z_* \end{bmatrix}$ passes through singularity in that neighborhood. With two exceptions, the estimates in the Girschick-Haavelmo model

17. The difficulty that can be caused by multicollinearity is, of course, well known. See Theil, [17], p. 233.

18. [17], p. 235.

appear to be nearly constant for values of k between 0 and 1.

The following observations can be made in the present case. (i) For Data Set 1, that is the Data Set for which the average bias is smaller on the whole when $k = 0$, the estimates tend to be fairly constant for values of $k < .9$. Between $k = .9$ and $k = 1.6$ approximately, there are usually several wide swings in the values of the estimates. This is the range in which the Theil matrix passes through singularity. The fact that there are several apparent discontinuities is due to the fact that the figures represent arithmetic means: since the individual cases which are being averaged have their points of discontinuity at various locations in this interval, the graphs of means behave in the manner indicated.[19] For values of $k > 1.6$ the estimates have values which tend to suggest that they lie on an extrapolation of a line drawn through points representing the estimates for $k < .9$.[20] The change in the values of the estimates is then quite smooth except for the range in which the Theil matrix passes through the point of singularity. It is unfortunate that this range appears to include two-stage least squares. (ii) For Data Set 2 the estimates are generally worse for $k < 1.0$ than for $k = 1.0$ but they tend to improve monotonically as k increases towards 1.0. For $k > 1.0$ there are numerous wide swings in the estimates with no apparent tendency to continue the "trend" followed by the estimates obtained for $k < 1.0$.

(c) The Effect of Sparseness. The mean estimates for the six coefficients by direct and two-stage least squares have been arranged by Runs 1 through 4 in Tables 4 through 7 according to model and data set. It may

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19. I am indebted to Mr. John Cragg for this point.

20. There is some evidence for this in Theil's results as well.

be recalled that the sparseness of the B matrix increases with the run number: thus Run 1 involves the least sparse and Run 4 the sparsest B matrix. The basic hypothesis that the mean bias tends to be reduced by increasing sparseness can be examined and tested in several ways.

(i) If we rank the absolute values of the biases in each column of Tables 4-7, assigning the rank 1 to the largest and the rank 4 to the smallest absolute mean bias and calculate the rank correlation by Spearman's ρ between each of these rankings and the ranking of the four B matrices given by the run numbers (which thus associate higher rank with greater sparseness), we find that 45 out of the 48 rank correlation coefficients are positive and only 3 negative, strongly supporting the conjecture. Under the null hypothesis that there is no relation between absolute mean bias and sparseness we would expect approximately as many positive rank correlation coefficients as negative ones. Under this hypothesis the probability of the actual outcome is less than 10^{-4}. The null hypothesis is thus abandoned.

(ii) Inspection of Tables 4-7 reveals that for any given model-data set combination the variation in the bias from run to run tends to be similar for all six coefficients. This is tested by Friedman's two-way analysis of variance test. As before, the runs are ranked by each of the six coefficients. If these rankings were random, the sum of the ranks would be expected to be the same for each run. The null hypothesis that the rankings are random is rejected on at least the .05 level of significance in every case.

This test, of course, does not bear directly on the conjecture concerning the relationship between bias and sparseness. It merely indicates

TABLE 4. MEAN ESTIMATES
Model 1, Data Set 1

	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
True Values	.2	-2.0	1.0	1.0	.5	-1.0
k = 0						
Run 1	- .0951	-1.2004	.3612	.7158	.1154	- .3571
2	.2691	-1.2866	.3690	.7306	.1310	- .3634
3	.2876	-1.6823	.4463	.8326	.1896	- .4369
4	.3248	-2.0063	.5756	.9938	.2550	- .5784
k = 1						
Run 1	- .2904	- .9580	.1134	.6281	.0733	- .0907
2	.2618	-1.0398	.1322	.6344	.0746	- .1111
3	.6179	- .2299	-2.0395	- .0527	-1.0698	2.0905
4	.0344	-2.0254	1.9233	1.1283	.7837	-1.9622

TABLE 5. MEAN ESTIMATES
Model 2, Data Set 1

	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
True Values	.2	-2.0	1.0	1.0	.5	-1.0
k = 0						
Run 1	- .1050	-1.2128	.3636	.7265	.1297	- .3576
2	.2639	-1.2907	.3697	.7371	.1426	- .3627
3	.2835	-1.6805	.4450	.8340	.1962	- .4348
4	.3207	-2.0048	.5748	.9958	.2631	- .5763
k = 1						
Run 1	- .1569	-1.2379	.3530	.7242	.1925	- .3434
2	.2445	-1.2885	.3576	.7252	.1931	- .3477
3	.2702	-1.6391	.3840	.7981	.2114	- .3719
4	.3060	-1.9974	.5588	.9830	.3034	- .5590

TABLE 6. MEAN ESTIMATES
Model 1, Data Set 2

	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
True Values	.2	-2.0	1.0	1.0	.5	-1.0
k = 0						
Run 1	- .0837	-1.3333	.4444	.7673	.2130	- .4360
2	.2469	-1.4006	.4545	.7863	.2217	- .4454
3	.2609	-1.7324	.5294	.8849	.2683	- .5185
4	.2843	-2.0059	.6601	1.0235	.3385	- .6576
k = 1						
Run 1	.3084	-2.3610	1.2778	1.2275	.6813	-1.2783
2	.1790	-2.0919	1.0762	1.1066	.5696	-1.0741
3	.2417	-1.7853	.6387	.8642	.3079	- .6052
4	.2065	-2.0005	.9294	1.0725	.4913	- .9338

TABLE 7. MEAN ESTIMATES
Model 2, Data Set 2

	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
True Values	.2	-2.0	1.0	1.0	.5	-1.0
k = 0						
Run 1	- .0848	-1.3355	.4450	.7696	.2154	- .4374
2	.2461	-1.4021	.4552	.7885	.2239	- .4468
3	.2601	-1.7325	.5297	.8859	.2699	- .5194
4	.2834	-2.0057	.6607	1.0245	.3403	- .6586
k = 1						
Run 1	.0771	-1.7530	.7847	.9422	.4057	- .7803
2	.2109	-1.7783	.7940	.9482	.4112	- .7895
3	.2175	-1.8943	.8171	.9705	.4201	- .8166
4	.2254	-2.0030	.8787	1.0280	.4515	- .8766

a high degree of agreement between the rankings achieved by the six coefficients. In view of this high degree of agreement, the rankings by the six coefficients were averaged for each model - data set - k combination and a new ranking was established for the four runs on the basis of the average ranks. These are displayed in Table 8. In six out of the eight possible

TABLE 8. RANKING OF RUNS BY MEAN ABSOLUTE BIAS

	k = 0				k = 1			
	Data Set 1		Data Set 2		Data Set 1		Data Set 2	
	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2
Run 1	1	1	1	1	2	1	1.5	1
2	2	2	2	2	4	2	3	2
3	3	3	3	3	1	3	1.5	3
4	4	4	4	4	3	4	4	4

cases the average rankings of absolute mean bias are identical with the ranking of the B matrices based on sparseness. In no case do the average rankings reverse the ranking of the B matrices based on sparseness. It is noteworthy that the relationship between bias and sparseness is less clear-cut for $k = 1$ than for $k = 0$. This is by no means surprising and is clearly due to the extreme variability of the estimates when $k = 1$.

B. Analysis and Results Based on Measures of Concentration and Decentralization.

The previous analysis based on sample means must obviously be treated with extreme caution because the finite sample distribution of the estimates may possess no finite moments of any order. The previously examined questions must therefore be reanalyzed in the light of sample statistics for which corresponding population statistics are certain to exist.

Three measures, denoted by the letters M, C, D_1 and D_2 have been chosen. Their values for the various model - data set - k combinations are displayed in Tables 17A - 20D of the Appendix.

M is the sample median and its value is the arithmetic mean of the fiftieth and fifty-first items in an ordered list of the estimates of a coefficient. C is a measure of concentration defined as follows. Let θ be the true value of a coefficient and $\hat{\theta}$ an estimated value. Let n be the number of instances for a given model - data set - k - coefficient combination that $.8\theta \leqq \hat{\theta} \leqq 1.2\theta$, and let N be the total number of cases, that is, N = 100. Then C = n/N. C thus measures the concentration or density within 20 per cent of the population value. The figure 20 per cent was chosen arbitrarily, but on the generally defensible grounds that deviations from true values much in excess of 20 per cent could not be considered tolerable from a practical point of view, whereas most economists would probably consider errors which are less than 20 per cent more or less acceptable. D_1 is a measure of decentralization, defined here as the proportion of cases in which a particular estimated coefficient has the wrong sign. This, too, is an arbitrary criterion but one which makes good sense in an economic context in which we are primarily interested in qualitative answers, that is to say, whether two variables are positively or negatively related. D_1 represents a minimal criterion in the sense that the least requirement of an estimating process is that it give us estimates with the right sign. Finally, for the sake of completeness, data on D_2, the maximum absolute deviation from true value in each sample, are presented.

1. Analysis based on Median (M). Tables 9 and 10 are analogous to

TABLE 9. MEDIANS. Relative Performance of k = 0 and k = 1.

Type	Number of Cases			Total Possible
	k = 0 Beats	Ties	Loses to k = 1	
Model 1, Data Set 1	13	0	11	24
Model 1, Data Set 2	1	0	23	24
Model 2, Data Set 1	18	0	6	24
Model 2, Data Set 2	1	0	23	24

TABLE 10. MEDIANS. Relative Performance of Data Set 1 and Data Set 2.

Type	Number of Cases			Total Possible
	Set 1 Beats	Ties	Loses to Set 2	
Model 1, k = 0	1	0	23	24
Model 1, k = 1	1	0	23	24
Model 2, k = 0	1	0	23	24
Model 2, k = 1	2	0	22	24

Tables 2 and 3 but are based on a comparison of sample medians to true values. As before, aggregation over runs and individual coefficients has been performed. As before, we can overwhelmingly reject the null hypothesis that the two estimating methods are the same in performance. Again, two-stage least squares does overwhelmingly better than direct least squares with Data Set 2, but tends to perform relatively much worse with Data Set 1. The already conjectured sensitivity of any estimating method to the individual peculiarities of exogenous data, such as multicollinearity, is strongly confirmed by Table 10. On the whole the results are substantially the same as those displayed in Tables 2 and 3. If the absolute differences between the medians and corresponding true values are ranked in the usual fashion, assigning the rank 1 to the largest difference, and if the pairwise rank correlations by Spearman's ρ are obtained with the ranking of runs based on sparseness (Run 1 being ranked 1), we find that all 48 rank correlation coefficients are positive. Table 11,[21] which is analogous to Table 8, is based on the average ranking of runs averaged over the six coefficients. It confirms the previous findings.

TABLE 11. RANKING OF RUNS BY MEDIAN ABSOLUTE BIAS

	k = 0				k = 1			
	Data Set 1		Data Set 2		Data Set 1		Data Set 2	
Run	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2
1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4	4

2. Analysis based on Concentration (C). C measures the relative frequency of estimates within 20 per cent of the true values. Tables 12 and 13 display the relative performance of k = 0 versus k = 1 and the relative performance of the estimates for the two Data Sets and are based

TABLE 12. CONCENTRATION (C). Relative Performance of k = 0 and k = 1.

Type	Number of Cases: k = 0 Beats	Ties	Loses to k = 1	Total Possible
Model 1, Data Set 1	4	0	20	24
Model 1, Data Set 2	4	0	20	24
Model 2, Data Set 1	3	0	21	24
Model 2, Data Set 2	3	1	20	24

TABLE 13. CONCENTRATION (C). Relative Performance of Data Set 1 and Data Set 2.

Type	Number of Cases: Set 1 Beats	Ties	Loses to Set 2	Total Possible
Model 1, k = 0	1	1	22	24
Model 1, k = 1	1	0	23	24
Model 2, k = 0	1	0	23	24
Model 2, k = 1	1	1	22	24

21. See also Appendix, Tables 17A-17D.

on aggregation over runs and coefficients as before.[22] Direct least squares gives better estimates by this criterion in only 14 out of 96 possible cases, as opposed to 41 out of 96 cases if mean bias is the criterion and 33 out of 96 cases if median bias is the criterion. Although C is almost invariably higher for Data Set 2, irrespective of which model and what value of k are being used (Table 13), the relative performance of $k = 0$ versus $k = 1$ is not distinguished by which Data Set is being used.

3. Analysis based on Decentralization (D_1). D_1 is the relative frequency of cases in which an estimate of a coefficient has the wrong sign. Table 14 displays direct least squares as superior to two-stage least squares in terms of D_1. The superiority does not seem to depend on which Data Set is used. Table 15 results in a somewhat more ambiguous

TABLE 14. DECENTRALIZATION (D_1)
Relative Performance of $k = 0$ and $k = 1$

Type	Number of Cases: k = 0 Beats	Ties	Loses to k=1	Total Possible
Model 1, Data Set 1	21	2	1	24
Model 1, Data Set 2	23	0	1	24
Model 2, Data Set 1	21	2	1	24
Model 2, Data Set 2	21	2	1	24

TABLE 15. DECENTRALIZATION (D_1)
Relative Performance of Data Set 1 and Data Set 2

Type	Number of Cases: Set 1 Beats	Ties	Loses to Set 2	Total Possible
Model 1, k = 0	1	19	4	24
Model 1, k = 1	4	0	20	24
Model 2, k = 0	2	19	3	24
Model 2, k = 1	0	7	17	24

22. See also Appendix, Tables 18A-18D.

comparison between Data Sets 1 and 2 than the preceding ones but it is still clear that the estimators behave better with Data Set 2. Ties occur frequently with $k = 0$, indicating perhaps that direct least squares has a lesser sensitivity to peculiarities of exogenous data.

Values of D_1 by coefficient, model, data set, run and k are displayed in the Appendix.[23] If these are ranked in the usual fashion, with the lowest D_1 achieving rank 4 and the highest achieving rank 1 for any k-model - data set - coefficient combination, we find that all 31 of the relevant[24] rank correlation coefficients are positive. In addition, the mean values of D_1 are .166 for Run 1, .050 for Run 2, .044 for Run 3, and .029 for Run 4, confirming the previous findings.

4. Analysis based on Decentralization (D_2). D_2 measures the maximum absolute deviation in each sample from the corresponding true value. Tables 16 and 17 are obtained in the usual fashion by aggregation over runs

TABLE 16. DECENTRALIZATION (D_2). Relative Performance of $k = 0$ and $k = 1$

Type	Number of Cases			
	$k = 0$ Beats	Ties	Loses to $k = 1$	Total Possible
Model 1, Data Set 1	24	0	0	24
Model 1, Data Set 2	24	0	0	24
Model 2, Data Set 1	23	0	1	24
Model 2, Data Set 2	24	0	0	24

TABLE 17. DECENTRALIZATION (D_2). Relative Performance of Data Set 1 and Data Set 2

Type	Number of Cases			
	Set 1 Beats	Ties	Loses to Set 2	Total Possible
Model 1, $k = 0$	4	0	20	24
Model 1, $k = 1$	13	0	11	24
Model 2, $k = 0$	6	0	18	24
Model 2, $k = 1$	5	0	19	24

23. See Tables 19A-19D.
24. 17 of the 48 possible rankings are omitted since in these cases $D_1 = 0$ for all four runs.

and coefficients. It is not surprising that $k = 0$ consistently beats $k = 1$ in performance, that is, exhibits lower maximum absolute discrepancies. Data Set 2 again leads to better estimates (by the current criterion) than Data Set 1 in a majority of cases except for Model 1 and $k = 1$. The values of the maximum discrepancies, displayed in Tables 20A-20D of the Appendix generally confirm the conjecture for $k = 0$ that increasing sparseness improves estimates. This is less clearly so for $k = 1$ although even there only 7 of the possible 24 rank correlations are negative. The values of the maximum discrepancies are large and the rank correlations mostly negative with the just identified model.

C. Summary

We may summarize the findings of this study as follows:

1. The desirability of using direct least squares or two-stage least squares (in small sample situations) is judged relatively differently depending on the criterion employed.

2. The relative performance of estimators depends heavily on the peculiarities of exogenous variables. Specifically, estimators perform relatively poorly with relatively high multicollinearity among the exogenous variables.

3. The performance of estimators improves as the sparseness of the coefficient matrix associated with endogenous variables increases.

4. The distribution of two-stage least squares estimates tends to have higher density in the neighborhood of the population parameter than the distribution of direct least squares estimates, as is indicated by the behavior of C.

5. The distribution of two-stage least squares estimates tends to

have thicker tails than that of direct least squares, as is indicated by the behavior of D_1 and D_2.

6. The presence of multicollinearity tends to affect two-stage least squares relatively more unfavorably than direct least squares.

7. Two-stage least squares does not perform unambiguously better or worse in the overidentified case than in the just identified case, as indicated by M, C, D_1 and D_2.

8. The behavior of k-class estimates for various values of k is substantially as found by Theil in the Girschick-Haavelmo model.

9. Ultimately, the choice between direct least squares and two-stage least squares is not clear, even though the latter's asymptotic properties are superior to those of the former. A rational person may still prefer to use direct least squares.

10. It is finally conjectured that further investigation of larger systems estimated by a variety of methods will reveal similar ambiguities and difficulties in ordering methods.

APPENDIX

TABLE 1A. MODEL 1; DATA SET 1; RUN 1
ARITHMETIC MEANS

k	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
-.4	- .0855	-1.1884	.3588	.7100	.0993	- .3562
-.3	- .0878	-1.1913	.3594	.7122	.1032	- .3564
-.2	- .0902	-1.1943	.3600	.7134	.1072	- .3566
-.1	- .0926	-1.1973	.3606	.7146	.1113	- .3569
0	- .0951	-1.2004	.3612	.7158	.1154	- .3571
.1	- .0976	-1.2036	.3619	.7171	.1196	- .3573
.2	- .1002	-1.2068	.3625	.7184	.1239	- .3575
.3	- .1029	-1.2101	.3631	.7197	.1284	- .3577
.4	- .1057	-1.2134	.3637	.7210	.1329	- .3579
.5	- .1086	-1.2167	.3643	.7223	.1376	- .3579
.6	- .1118	-1.2201	.3647	.7237	.1424	- .3579
.7	- .1153	-1.2233	.3648	.7249	.1474	- .3575
.8	- .1196	-1.2260	.3642	.7259	.1526	- .3562
.9	- .1264	-1.2264	.3610	.7260	.1579	- .3521
1.0	- .2904	- .9580	.1134	.6281	.0733	- .0907
1.1	- .1606	-1.1171	.2756	.7210	.0927	- .2610
1.2	- .1026	-1.2748	.4037	.7480	.1779	- .3989
1.3	- .2264	- .9999	.1723	.6476	.0603	- .1602
1.4	- .0523	-1.3722	.4882	.7981	.2083	- .4814
1.5	- .1031	-1.3133	.4261	.7619	.2058	- .4176
1.6	- .1452	-1.2389	.3592	.7323	.1811	- .3484
1.7	- .1464	-1.2436	.3614	.7346	.1852	- .3500
1.8	- .1460	-1.2598	.3715	.7409	.1972	- .3596
1.9	- .1491	-1.2663	.3738	.7435	.2043	- .3614
2.0	- .1526	-1.2719	.3755	.7457	.2111	- .3624

TABLE 1B. MODEL 1; DATA SET 1; RUN 1
STANDARD DEVIATIONS

-.4	.0720	.1428	.1157	.0639	.0819	.1185
-.3	.0722	.1432	.1161	.0641	.0818	.1189
-.2	.0725	.1436	.1166	.0643	.0818	.1194
-.1	.0728	.1441	.1171	.0645	.0818	.1199
0	.0731	.1447	.1176	.0648	.0819	.1205
.1	.0736	.1454	.1184	.0651	.0821	.1212
.2	.0742	.1464	.1193	.0655	.0824	.1222
.3	.0751	.1477	.1205	.0660	.0829	.1234
.4	.0762	.1495	.1222	.0667	.0835	.1251
.5	.0778	.1523	.1247	.0676	.0844	.1277
.6	.0802	.1567	.1287	.0690	.0860	.1318
.7	.0844	.1647	.1358	.0714	.0886	.1390
.8	.0923	.1810	.1501	.0761	.0943	.1535
.9	.1125	.2250	.1880	.0890	.1104	.1918
1.0	.9952	2.1413	1.8065	.7909	.9106	1.8449
1.1	1.1197	3.3890	2.6427	1.1177	1.7866	2.6831
1.2	.9110	1.8131	1.5526	.5526	.7516	1.5906
1.3	.7001	1.6597	1.3615	.5945	.7606	1.3918
1.4	.7440	1.4867	1.2698	.5456	.6581	1.2948
1.5	.2488	.5401	.4518	.2076	.2391	.4609
1.6	.1305	.2788	.2282	.1118	.1429	.2331
1.7	.1082	.2490	.1971	.0980	.1401	.2014
1.8	.0968	.2027	.1608	.0832	.1197	.1644
1.9	.0946	.1938	.1531	.0808	.1177	.1565
2.0	.0938	.1891	.1486	.0798	.1178	.1519

TABLE 2A. MODEL 2; DATA SET 1; RUN 1
ARITHMETIC MEANS

k	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
-.4	-.0979	-1.2042	.3620	.7244	.1174	-.3570
-.3	-.0996	-1.2063	.3624	.7249	.1204	-.3571
-.2	-.1014	-1.2084	.3628	.7254	.1234	-.3573
-.1	-.1032	-1.2106	.3632	.7260	.1265	-.3574
0	-.1050	-1.2128	.3636	.7265	.1297	-.3576
.1	-.1069	-1.2150	.3640	.7271	.1329	-.3577
.2	-.1089	-1.2172	.3643	.7276	.1361	-.3577
.3	-.1110	-1.2194	.3646	.7281	.1394	-.3578
.4	-.1131	-1.2216	.3648	.7285	.1428	-.3577
.5	-.1155	-1.2237	.3649	.7290	.1463	-.3575
.6	-.1180	-1.2256	.3648	.7292	.1498	-.3570
.7	-.1211	-1.2271	.3642	.7293	.1534	-.3561
.8	-.1250	-1.2276	.3625	.7289	.1570	-.3540
.9	-.1316	-1.2250	.3576	.7270	.1602	-.3486
1.0	-.1569	-1.2379	.3530	.7242	.1925	-.3434
1.1	.0431	-1.5327	.6324	.8621	.2524	-.6268
1.2	.0051	-1.6299	.6756	.8489	.3671	-.6665
1.3	-.5189	-.2630	-.4231	.3985	-.3002	.4581
1.4	-.1524	-1.2687	.3744	.7528	.2038	-.3565
1.5	-.1775	-1.0289	.2152	.6763	.0398	-.2056
1.6	-.1064	-1.2812	.4047	.7500	.1837	-.3974
1.7	-.1268	-1.2483	.3740	.7372	.1749	-.3657
1.8	-.1422	-1.2913	.3923	.7590	.2144	-.3736
1.9	-.1192	-1.3079	.4142	.7602	.2124	-.4014
2.0	-.1555	-1.2316	.3494	.7307	.1831	-.3376

TABLE 2B. MODEL 2; DATA SET 1; RUN 1
STANDARD DEVIATIONS

-.4	.0712	.1448	.1184	.0628	.0809	.1214
-.3	.0713	.1450	.1186	.0629	.0808	.1216
-.2	.0714	.1452	.1188	.0630	.0807	.1218
-.1	.0716	.1455	.1191	.0632	.0807	.1221
0	.0718	.1459	.1195	.0633	.0808	.1225
.1	.0722	.1466	.1201	.0635	.0809	.1230
.2	.0726	.1475	.1209	.0638	.0812	.1238
.3	.0733	.1488	.1220	.0643	.0816	.1249
.4	.0743	.1509	.1238	.0649	.0823	.1267
.5	.0758	.1542	.1267	.0658	.0834	.1295
.6	.0783	.1598	.1314	.0674	.0854	.1341
.7	.0826	.1697	.1398	.0702	.0889	.1425
.8	.0909	.1892	.1563	.0758	.0961	.1589
.9	.1107	.2356	.1953	.0894	.1140	.1980
1.0	.4088	.9897	.8080	.3305	.4852	.8344
1.1	2.3224	5.0065	4.2137	1.9680	1.9549	4.1800
1.2	.8207	2.3957	1.8672	.6551	1.2662	1.8601
1.3	3.0272	7.5745	6.1232	2.6391	3.6355	6.3136
1.4	.7762	2.1729	1.7145	.7064	1.1131	1.7282
1.5	1.0711	1.9436	1.7127	.7242	.7650	1.7826
1.6	.2681	.5470	.4626	.2019	.2476	.4740
1.7	.3286	.7376	.6131	.2871	.3274	.6219
1.8	.6831	1.9693	1.5405	.8090	.9776	1.5176
1.9	.2619	.6309	.5133	.2489	.2910	.5106
2.0	.1900	.4414	.3607	.1706	.2076	.3604

TABLE 3A. MODEL 1; DATA SET 2; RUN 1
ARITHMETIC MEANS

k	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
-.4	-.0894	-1.3084	.4263	.7504	.1988	-.4178
-.3	-.0884	-1.3136	.4300	.7543	.2019	-.4215
-.2	-.0871	-1.3194	.4341	.7584	.2053	-.4257
-.1	-.0856	-1.3259	.4389	.7627	.2090	-.4305
0	-.0837	-1.3333	.4444	.7673	.2130	-.4360
.1	-.0814	-1.3417	.4507	.7724	.2175	-.4424
.2	-.0785	-1.3515	.4582	.7779	.2226	-.4499
.3	-.0747	-1.3631	.4672	.7840	.2284	-.4590
.4	-.0700	-1.3771	.4783	.7910	.2353	-.4702
.5	-.0637	-1.3947	.4923	.7992	.2437	-.4844
.6	-.0551	-1.4177	.5108	.8093	.2544	-.5032
.7	-.0427	-1.4494	.5367	.8226	.2688	-.5294
.8	-.0232	-1.4977	.5764	.8419	.2903	-.5696
.9	.0135	-1.5867	.6499	.8765	.3293	-.6442
1.0	.3084	-2.3610	1.2778	1.2275	.6813	-1.2783
1.1	-.1434	-1.2374	.3552	.8255	.1815	-.3530
1.2	-.0622	-1.4424	.5219	.8086	.2763	-.5067
1.3	1.2762	-3.1245	2.2492	-.0855	.6111	-2.2917
1.4	-.2161	-1.1444	.2573	.7537	.1735	-.2440
1.5	.2183	-2.0933	1.0749	.9932	.5367	-1.0548
1.6	-.0305	-1.5772	.6236	.8869	.3427	-.6056
1.7	-.5288	-.4805	-.3043	.6674	-.1164	.3333
1.8	-.4013	-.7113	-.1037	.6787	-.0125	.1143
1.9	2.8260	-8.0766	6.1008	3.8899	3.0315	-6.2077
2.0	-.1971	-1.1712	.2853	.7740	.1747	-.2726

TABLE 3B. MODEL 1; DATA SET 2; RUN 1
STANDARD DEVIATIONS

k	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
-.4	.0602	.1334	.1083	.1022	.0691	.1108
-.3	.0606	.1343	.1090	.1025	.0695	.1115
-.2	.0611	.1354	.1099	.1029	.0699	.1123
-.1	.0616	.1366	.1110	.1034	.0705	.1134
0	.0623	.1382	.1123	.1040	.0712	.1147
.1	.0632	.1402	.1140	.1047	.0721	.1164
.2	.0643	.1428	.1162	.1057	.0733	.1186
.3	.0658	.1462	.1190	.1069	.0747	.1214
.4	.0677	.1508	.1228	.1086	.0767	.1252
.5	.0704	.1571	.1281	.1109	.0795	.1305
.6	.0742	.1664	.1358	.1142	.0835	.1383
.7	.0802	.1809	.1479	.1196	.0898	.1505
.8	.0907	.2066	.1693	.1294	.1011	.1721
.9	.1149	.2661	.2184	.1530	.1271	.2219
1.0	1.4029	3.4784	2.8560	1.5856	1.5523	2.9022
1.1	1.7734	3.8117	3.2568	1.1536	1.5398	3.2747
1.2	1.0009	2.2090	1.8660	.6954	.9286	1.8943
1.3	20.9096	41.5872	36.4091	12.4900	15.7669	37.0844
1.4	1.6352	4.1156	3.3413	1.6113	1.9238	3.4233
1.5	3.5134	8.7966	7.2320	3.1414	3.9220	7.1997
1.6	1.9035	5.3093	4.2594	1.9631	2.4542	4.2023
1.7	2.8929	5.5613	4.8566	.5318	2.2306	5.0039
1.8	1.4969	2.9920	2.6200	.4994	1.1037	2.6111
1.9	28.9323	64.9009	54.8812	28.5670	26.6598	56.1345
2.0	.4037	1.1148	.8868	.5006	.5380	.8924

TABLE 4A. MODEL 2; DATA SET 2; RUN 1
ARITHMETIC MEANS

k	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
-.4	- .0900	- 1.3133	.4287	.7549	.2029	- .4211
-.3	- .0891	- 1.3180	.4321	.7583	.2056	- .4245
-.2	- .0879	- 1.3232	.4359	.7619	.2086	- .4283
-.1	- .0865	- 1.3290	.4402	.7656	.2119	- .4326
0	- .0848	- 1.3355	.4450	.7696	.2154	- .4374
.1	- .0827	- 1.3428	.4506	.7739	.2193	- .4430
.2	- .0801	- 1.3513	.4571	.7786	.2236	- .4496
.3	- .0769	- 1.3612	.4648	.7838	.2286	- .4573
.4	- .0728	- 1.3730	.4742	.7896	.2344	- .4667
.5	- .0676	- 1.3875	.4858	.7962	.2413	- .4784
.6	- .0607	- 1.4060	.5006	.8043	.2499	- .4934
.7	- .0510	- 1.4307	.5208	.8144	.2611	- .5137
.8	- .0366	- 1.4665	.5502	.8284	.2770	- .5435
.9	- .0116	- 1.5271	.6003	.8511	.3036	- .5941
1.0	.0771	- 1.7530	.7847	.9422	.4057	- .7803
1.1	.1166	- 1.7694	.8158	.8595	.3865	- .8065
1.2	- .0280	- 1.4954	.5730	.8360	.2889	- .5632
1.3	- .0503	- 1.4105	.5083	.8172	.2466	- .5077
1.4	- 2.2881	3.7646	- 3.8481	- .5815	- 1.8743	3.7723
1.5	- .0395	- 1.4173	.5187	.7600	.2460	.5118
1.6	- .2804	- .9065	.0801	.6258	.0411	- .0682
1.7	.8217	- 4.0976	2.6220	1.8393	1.5408	- 2.5597
1.8	35.9368	-87.4384	72.4727	32.1884	36.5590	-71.9636
1.9	- .1890	- 1.2436	.3360	.8701	.2144	- .3200
2.0	- .2511	- 1.0335	.1722	.7254	.1141	- .1605

TABLE 4B. MODEL 2; DATA SET 2; RUN 1
STANDARD DEVIATIONS

-.4	.0601	.1352	.1096	.1033	.0697	.1117
-.3	.0604	.1358	.1101	.1036	.0700	.1122
-.2	.0608	.1365	.1107	.1039	.0703	.1128
-.1	.0613	.1375	.1116	.1042	.0707	.1136
0	.0619	.1387	.1126	.1047	.0712	.1147
.1	.0627	.1404	.1141	.1052	.0719	.1161
.2	.0638	.1426	.1160	.1060	.0728	.1180
.3	.0652	.1456	.1186	.1069	.0740	.1206
.4	.0671	.1497	.1221	.1082	.0757	.1241
.5	.0698	.1555	.1271	.1100	.0780	.1290
.6	.0736	.1640	.1343	.1125	.0814	.1363
.7	.0795	.1773	.1456	.1165	.0868	.1476
.8	.0894	.2001	.1649	.1235	.0963	.1671
.9	.1104	.2494	.2062	.1397	.1171	.2090
1.0	.2563	.6795	.5427	.3583	.3357	.5558
1.1	1.6340	3.8020	3.1657	1.0919	1.6696	3.1975
1.2	1.0132	2.7411	2.2056	1.1871	1.2694	2.2040
1.3	1.0380	2.2317	1.9016	.6813	.9191	1.9244
1.4	19.1577	46.2630	38.4969	13.3833	19.5556	37.6958
1.5	1.9726	3.8401	3.3722	.6710	1.4673	3.4462
1.6	1.1698	2.5710	2.1919	.9114	1.0151	2.2010
1.7	11.7181	33.3718	26.6882	12.6359	15.5224	26.1806
1.8	358.000	855.786	715.819	312.521	361.163	710.864
1.9	1.1537	2.7395	2.2918	1.0864	1.1500	2.2900
2.0	.4487	1.0520	.8777	.4223	.4525	.8853

TABLE 5A. MODEL 1; DATA SET 1; RUN 2
ARITHMETIC MEANS

k	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
-.4	.2736	- 1.2829	.3681	.7292	.1192	- .3638
-.3	.2725	- 1.2838	.3683	.7295	.1221	- .3637
-.2	.2714	- 1.2847	.3686	.7299	.1250	- .3636
-.1	.2702	- 1.2856	.3688	.7302	.1280	- .3635
0	.2691	- 1.2866	.3690	.7306	.1310	- .3634
.1	.2679	- 1.2875	.3692	.7309	.1341	- .3633
.2	.2667	- 1.2885	.3693	.7313	.1372	- .3632
.3	.2655	- 1.2894	.3695	.7316	.1404	- .3630
.4	.2642	- 1.2904	.3696	.7320	.1437	- .3628
.5	.2629	- 1.2912	.3697	.7323	.1471	- .3625
.6	.2615	- 1.2920	.3696	.7325	.1506	- .3621
.7	.2601	- 1.2925	.3692	.7327	.1542	- .3613
.8	.2585	- 1.2923	.3681	.7325	.1579	- .3596
.9	.2566	- 1.2894	.3642	.7311	.1615	- .3550
1.0	.2618	- 1.0398	.1322	.6344	.0746	- .1111
1.1	.2218	- 1.5837	.6232	.8532	.3260	- .6152
1.2	.2503	- 1.8345	.8699	.9218	.3443	- .8854
1.3	.2781	- 1.0511	.1541	.6430	..0410	- .1423
1.4	.2661	- 1.5833	.6511	.8956	.2089	- .6495
1.5	.2487	- 1.3621	.4281	.7629	.2009	- .4202
1.6	.2512	- 1.2848	.3568	.7299	.1732	- .3466
1.7	.2498	- 1.2930	.3637	.7337	.1785	- .3532
1.8	.2474	- 1.3017	.3705	.7372	.1868	- .3597
1.9	.2457	- 1.3044	.3720	.7383	.1917	- .3608
2.0	.2440	- 1.3065	.3729	.7391	.1963	- .3613

TABLE 5B. MODEL 1; DATA SET 1; RUN 2
STANDARD DEVIATIONS

k	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
-.4	.0244	.1298	.1176	.0615	.0821	.1208
-.3	.0244	.1301	.1179	.0616	.0820	.1211
-.2	.0243	.1304	.1182	.0618	.0820	.1214
-.1	.0243	.1307	.1186	.0619	.0820	.1218
0	.0243	.1312	.1190	.0622	.0820	.1223
.1	.0244	.1318	.1196	.0624	.0821	.1229
.2	.0244	.1327	.1204	.0628	.0823	.1237
.3	.0245	.1339	.1215	.0632	.0827	.1249
.4	.0246	.1356	.1232	.0638	.0832	.1265
.5	.0248	.1383	.1256	.0647	.0840	.1290
.6	.0250	.1426	.1296	.0661	.0855	.1331
.7	.0254	.1505	.1368	.0685	.0881	.1404
.8	.0262	.1667	.1514	.0735	.0937	.1552
.9	.0284	.2096	.1900	.0870	.1102	.1941
1.0	.1473	1.9326	1.7397	.7645	.8820	1.7751
1.1	.2009	1.6564	1.4699	.6232	.9363	1.4925
1.2	.1313	5.7742	5.3228	1.9558	2.0096	5.5563
1.3	.1545	1.7841	1.5855	.6836	.8769	1.6261
1.4	.684	3.0528	2.9612	1.5086	.7759	3.0370
1.5	.0452	.5063	.4558	.2079	.2377	.4654
1.6	.0345	.2606	.2343	.1114	.1407	.2396
1.7	.0337	.2034	.1810	.0857	.1257	.1855
1.8	.0321	.1758	.1569	.0762	.1125	.1609
1.9	.0321	.1668	.1490	.0734	.1097	.1529
2.0	.0325	.1614	.1442	.0718	.1086	.1480

TABLE 6A. MODEL 2; DATA SET 1; RUN 2
ARITHMETIC MEANS

k	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
-.4	.2671	-1.2883	.3694	.7372	.1337	-.3630
-.3	.2663	-1.2889	.3695	.7372	.1359	-.3630
-.2	.2655	-1.2895	.3696	.7372	.1381	-.3629
-.1	.2647	-1.2901	.3697	.7371	.1403	-.3628
0	.2639	-1.2907	.3697	.7371	.1426	-.3627
.1	.2630	-1.2913	.3698	.7370	.1449	-.3625
.2	.2622	-1.2918	.3698	.7369	.1472	-.3623
.3	.2613	-1.2923	.3698	.7368	.1495	-.3621
.4	.2604	-1.2928	.3696	.7366	.1520	-.3618
.5	.2595	-1.2931	.3694	.7363	.1544	-.3613
.6	.2585	-1.2931	.3689	.7359	.1569	-.3605
.7	.2575	-1.2927	.3679	.7352	.1594	-.3593
.8	.2564	-1.2912	.3658	.7340	.1619	-.3570
.9	.2551	-1.2863	.3606	.7311	.1641	-.3514
1.0	.2445	-1.2885	.3576	.7252	.1931	-.3477
1.1	.4245	-1.8133	-2.4081	-.5341	-1.0632	2.4098
1.2	.2313	-1.5389	.5857	.8183	.2930	.5777
1.3	.3498	-.2555	-.5445	.3563	-.3805	.5814
1.4	.2603	-1.1503	.2373	.6994	.1051	-.2187
1.5	.2773	-.7519	-.1285	.5245	-.0434	.1528
1.6	.2550	-1.3378	.4086	.7546	.1783	-.4022
1.7	.2568	-1.2902	.3645	.7341	.1615	-.3573
1.8	.2509	-1.2476	.3211	.7189	.1607	-.3057
1.9	.2453	-1.3480	.4120	.7581	.2034	-.4000
2.0	.2496	-1.2765	.3481	.7285	.1742	-.3370

TABLE 6B. MODEL 2; DATA SET 1; RUN 2
STANDARD DEVIATIONS

k	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
-.4	.0223	.1317	.1196	.0614	.0808	.1229
-.3	.0223	.1317	.1196	.0614	.0817	.1230
-.2	.0222	.1318	.1198	.0614	.0806	.1231
-.1	.0222	.1321	.1200	.0615	.0805	.1232
0	.0222	.1324	.1203	.0616	.0805	.1235
.1	.0222	.1329	.1207	.0618	.0806	.1240
.2	.0222	.1337	.1215	.0620	.0807	.1247
.3	.0223	.1350	.1226	.0624	.0811	.1257
.4	.0224	.1369	.1243	.0630	.0816	.1275
.5	.0225	.1401	.1271	.0640	.0827	.1302
.6	.0228	.1454	.1319	.0656	.0845	.1349
.7	.0233	.1550	.1403	.0684	.0880	.1433
.8	.0242	.1737	.1569	.0741	.0952	.1598
.9	.0266	.2177	.1961	.0881	.1131	.1990
1.0	.0839	.8181	.7338	.2982	.4282	.7549
1.1	1.7412	26.6126	23.6696	11.4601	10.9487	23.4493
1.2	.1355	1.3514	1.1950	.4440	.7066	1.2002
1.3	.6559	7.3190	6.4052	2.7437	3.8100	6.6002
1.4	.1968	1.8587	1.6379	.6563	1.0148	1.6642
1.5	.2090	2.7325	2.5647	1.1692	.9519	2.6699
1.6	.0542	.5605	.5112	.2271	.2581	.5252
1.7	.0774	.7426	.6629	.3126	.3717	.6669
1.8	.1347	1.4394	1.2801	.6129	.7144	1.2969
1.9	.0561	.5835	.5182	.2453	.2852	.5167
2.0	.0416	.3936	.3507	.1628	.1981	.3511

TABLE 7A. MODEL 1; DATA SET 2; RUN 2
ARITHMETIC MEANS

k	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
-.4	.2499	- 1.3799	.4369	.7725	.2086	- .4275
-.3	.2492	- 1.3842	.4405	.7756	.2115	- .4311
-.2	.2485	- 1.3890	.4445	.7789	.2146	- .4353
-.1	.2477	- 1.3944	.4492	.7825	.2179	- .4400
0	.2469	- 1.4006	.4545	.7863	.2217	- .4454
.1	.2460	- 1.4077	.4607	.7905	.2258	- .4517
.2	.2449	- 1.4162	.4681	.7952	.2306	- .4592
.3	.2438	- 1.4262	.4770	.8004	.2361	- .4683
.4	.2425	- 1.4386	.4880	.8065	.2427	- .4794
.5	.2409	- 1.4542	.5019	.8138	.2507	- .4936
.6	.2390	- 1.4747	.5202	.8228	.2610	- .5122
.7	.2364	- 1.5032	.5459	.8349	.2751	- .5384
.8	.2327	- 1.5469	.5854	.8527	.2961	- .5785
.9	.2259	- 1.6275	.6585	.8850	.3344	- .6528
1.0	.1790	- 2.0919	1.0762	1.1066	.5696	- 1.0741
1.1	.3082	- .6227	- .2583	.5880	- .1373	.2608
1.2	.2427	- 1.3954	.4446	.7726	.2310	- .4305
1.3	- .1290	- 5.5911	4.1923	2.8303	2.2843	- 4.3399
1.4	.3019	- .5390	- .3471	.5465	- .1419	.3648
1.5	- .1088	- 5.4592	4.0981	2.1056	2.2644	- 4.1634
1.6	.2639	- 1.1574	.2305	.7079	.1128	- .2183
1.7	.2717	- .9100	.0006	.6871	.0220	.0180
1.8	.2984	- .4697	- .4168	.6231	- .1508	.4270
1.9	.2495	- 1.0822	.1448	.7668	.1276	- .1320
2.0	.2519	- 1.1863	.2495	.7561	.1508	- .2368

TABLE 7B. MODEL 1; DATA SET 2; RUN 2
STANDARD DEVIATIONS

-.4	.0182	.1207	.1086	.0988	.0695	.1112
-.3	.0182	.1214	.1092	.0991	.0698	.1118
-.2	.0183	.1223	.1100	.0993	.0702	.1126
-.1	.0185	.1233	.1110	.0997	.0707	.1136
0	.0186	.1247	.1123	.1002	.0713	.1148
.1	.0188	.1264	.1139	.1008	.0722	.1164
.2	.0190	.1286	.1159	.1016	.0732	.1185
.3	.0193	.1315	.1187	.1026	.0746	.1212
.4	.0197	.1355	.1224	.1040	.0765	.1249
.5	.0202	.1411	.1275	.1060	.0791	.1301
.6	.0209	.1494	.1351	.1090	.0829	.1377
.7	.0220	.1624	.1471	.1139	.0890	.1498
.8	.0241	.1856	.1683	.1230	.1000	.1713
.9	.0287	.2394	.2172	.1453	.1257	.2209
1.0	.2293	2.5117	2.2744	1.1735	1.2182	2.3113
1.1	.4682	6.1889	5.7046	1.8775	2.7802	5.6974
1.2	.1259	1.6662	1.5283	.5700	.7447	1.5542
1.3	4.2198	53.6464	48.6023	21.3035	24.7852	50.2057
1.4	.5486	6.7620	6.1927	2.0536	3.1462	6.2246
1.5	3.1630	31.8387	28.4073	12.1303	16.7311	28.9876
1.6	.1488	1.6128	1.4841	.5486	.7554	1.4879
1.7	.1017	1.9616	1.8028	.3832	.8093	1.8472
1.8	.2720	4.9429	4.6345	.8829	1.9511	4.6204
1.9	.2750	4.6968	4.4015	1.8353	1.8997	4.4809
2.0	.0793	.7986	.7218	.3601	.3965	.7317

TABLE 8A. MODEL 2; DATA SET 2; RUN 2
ARITHMETIC MEANS

k	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
-.4	.2487	- 1.3836	.4393	.7765	.2124	- .4307
-.3	.2481	- 1.3875	.4426	.7792	.2149	- .4340
-.2	.2475	- 1.3918	.4463	.7821	.2177	- .4377
-.1	.2468	- 1.3966	.4504	.7852	.2206	- .4420
0	.2461	- 1.4021	.4552	.7885	.2239	- .4468
.1	.2453	- 1.4084	.4607	.7920	.2275	- .4523
.2	.2445	- 1.4157	.4671	.7959	.2316	- .4588
.3	.2435	- 1.4243	.4747	.8003	.2362	- .4666
.4	.2424	- 1.4347	.4840	.8053	.2417	- .4759
.5	.2411	- 1.4476	.4955	.8111	.2484	- .4876
.6	.2396	- 1.4641	.5103	.8183	.2566	- .5026
.7	.2376	- 1.4863	.5303	.8274	.2676	- .5229
.8	.2348	- 1.5188	.5597	.8402	.2832	- .5527
.9	.2302	- 1.5743	.6101	.8616	.3096	- .6037
1.0	.2109	- 1.7783	.7940	.9482	.4112	- .7895
1.1	.2562	- 1.4091	.4713	.7348	.1943	- .4554
1.2	.3483	- .6202	- .2346	.3973	- .2241	.2410
1.3	.2552	- 1.3753	.4309	.7757	.1998	- .4311
1.4	.2888	- .5416	- .3471	.5654	- .1154	.3660
1.5	.2424	- 1.6833	.7195	.7720	.3224	- .7200
1.6	- .0654	- 4.6798	3.5465	2.1228	1.7320	- 3.4820
1.7	.2081	- 1.7707	.7813	.9315	.4224	- .7727
1.8	.2373	- 1.5060	.5505	.8050	.2738	- .5393
1.9	.2577	- 1.0010	.0743	.7601	.0830	- .0588
2.0	.2638	- 1.0491	.1223	.7100	.0871	- .1108

TABLE 8B. MODEL 2; DATA SET 2; RUN 2
STANDARD DEVIATION

k	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
-.4	.0182	.1220	.1097	.0998	.0701	.1120
-.3	.0182	.1224	.1101	.1000	.0703	.1124
-.2	.0183	.1230	.1107	.1002	.0705	.1129
-.1	.0184	.1237	.1114	.1004	.0709	.1137
0	.0185	.1248	.1124	.1008	.0713	.1146
.1	.0186	.1262	.1138	.1012	.0719	.1160
.2	.0188	.1281	.1156	.1018	.0727	.1177
.3	.0190	.1307	.1180	.1026	.0738	.1202
.4	.0193	.1343	.1214	.1037	.0754	.1236
.5	.0197	.1395	.1263	.1052	.0776	.1284
.6	.0203	.1473	.1335	.1075	.0808	.1356
.7	.0211	.1594	.1446	.1111	.0861	.1468
.8	.0226	.1804	.1639	.1176	.0953	.1662
.9	.0260	.2262	.2054	.1330	.1161	.2084
1.0	.0731	.6284	.5558	.3581	.3472	.5695
1.1	.3874	4.4232	3.9541	1.5995	2.2374	4.0627
1.2	1.0497	8.0723	7.1260	3.9092	4.6042	7.0914
1.3	.1780	2.5912	2.3690	.7286	1.1498	2.3961
1.4	.6011	8.6873	7.9056	2.5342	3.8906	8.0085
1.5	.2738	6.5446	6.0884	.9088	2.5748	6.2419
1.6	3.7857	42.3165	39.6763	17.0927	19.2140	39.2190
1.7	.4416	5.3032	4.8305	2.0039	2.4914	4.8553
1.8	.1626	2.4195	2.2549	.6391	1.0184	2.2512
1.9	.1613	2.5232	2.3622	.6936	1.0554	2.4005
2.0	.0990	1.1119	1.0232	.4596	.5202	1.0309

TABLE 9A. MODEL 1; DATA SET 1; RUN 3
ARITHMETIC MEANS

k	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
-.4	.2897	-1.6831	.4469	.8336	.1855	- .4378
-.3	.2892	-1.6829	.4468	.8334	.1865	- .4376
-.2	.2887	-1.6827	.4466	.8331	.1875	- .4374
-.1	.2882	-1.6825	.4465	.8328	.1885	- .4371
0	.2876	-1.6823	.4463	.8326	.1896	- .4369
.1	.2871	-1.6821	.4461	.8323	.1906	- .4366
.2	.2865	-1.6818	.4459	.8319	.1917	- .4363
.3	.2859	-1.6815	.4456	.8316	.1928	- .4359
.4	.2853	-1.6812	.4453	.8312	.1940	- .4355
.5	.2847	-1.6808	.4449	.8308	.1951	- .4350
.6	.2840	-1.6803	.4443	.8303	.1963	- .4342
.7	.2833	-1.6795	.4433	.8296	.1976	- .4331
.8	.2824	-1.6782	.4414	.8285	.1988	- .4310
.9	.2814	-1.6749	.4364	.8261	.1996	- .4255
1.0	.6179	- .2299	-2.0395	- .0527	-1.0698	2.0905
1.1	.2648	-1.7181	.5148	.8529	.2488	- .5035
1.2	.2507	-1.7960	.6423	.8661	.3177	- .6365
1.3	.3226	-1.5147	.1604	.7272	.0501	- .1423
1.4	.2422	-1.8096	.6743	.9075	.3331	- .6673
1.5	.2811	-1.6908	.4626	.8381	.2067	- .4538
1.6	.2841	-1.6681	.4243	.8231	.1913	- .4139
1.7	.2779	-1.6842	.4532	.8323	.2108	- .4435
1.8	.2792	-1.6794	.4447	.8295	.2061	- .4345
1.9	.2786	-1.6791	.4444	.8291	.2073	- .4341
2.0	.2778	-1.6789	.4444	.8288	.2087	- .4339

TABLE 9B. MODEL 1; DATA SET 1; RUN 3
STANDARD DEVIATIONS

k	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
-.4	.0360	.0725	.1239	.0557	.0869	.1285
-.3	.0360	.0726	.1240	.0558	.0869	.1287
-.2	.0360	.0727	.1243	.0558	.0869	.1289
-.1	.0361	.0729	.1245	.0559	.0870	.1292
0	.0361	.0731	.1249	.0560	.0871	.1295
.1	.0362	.0734	.1254	.0562	.0873	.1301
.2	.0363	.0739	.1262	.0564	.0876	.1308
.3	.0364	.0746	.1273	.0566	.0879	.1319
.4	.0366	.0756	.1289	.0570	.0885	.1335
.5	.0369	.0771	.1315	.0576	.0895	.1361
.6	.0373	.0796	.1357	.0586	.0911	.1403
.7	.0380	.0841	.1433	.0602	.0940	.1480
.8	.0395	.0931	.1587	.0636	.1002	.1634
.9	.0436	.1167	.1992	.0727	.1180	.2041
1.0	2.0253	9.1959	15.5774	5.7671	7.8269	15.9357
1.1	.1584	.5792	1.0001	.2900	.5415	1.0243
1.2	.1889	.6599	1.1162	.2878	.6423	1.1421
1.3	.2615	1.4451	2.4293	.9262	1.0703	2.5104
1.4	.3443	.7703	1.3530	.5137	.9225	1.3624
1.5	.0588	.1695	.2910	.1001	.1739	.2977
1.6	.0537	.1382	.2391	.0872	.1538	.2470
1.7	.0526	.1222	.2131	.0781	.1450	.2194
1.8	.0443	.0936	.1603	.0635	.1122	.1656
1.9	.0437	.0885	.1514	.0612	.1083	.1566
2.0	.0435	.0855	.1460	.0600	.1064	.1512

TABLE 10A. MODEL 2; DATA SET 1; RUN 3
ARITHMETIC MEANS

k	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
-.4	.2850	-1.6811	.4455	.8352	.1932	-.4355
-.3	.2846	-1.6810	.4454	.8349	.1939	-.4354
-.2	.2842	-1.6808	.4453	.8346	.1947	-.4352
-.1	.2838	-1.6807	.4451	.8343	.1954	-.4350
0	.2835	-1.6805	.4450	.8340	.1962	-.4348
.1	.2831	-1.6803	.4448	.8336	.1970	-.4346
.2	.2827	-1.6800	.4445	.8332	.1979	-.4343
.3	.2822	-1.6798	.4442	.8327	.1987	-.4340
.4	.2818	-1.6794	.4439	.8322	.1996	-.4336
.5	.2813	-1.6790	.4433	.8316	.2004	-.4330
.6	.2808	-1.6784	.4425	.8309	.2013	-.4321
.7	.2803	-1.6774	.4412	.8299	.2022	-.4308
.8	.2797	-1.6758	.4388	.8282	.2031	-.4283
.9	.2788	-1.6719	.4330	.8250	.2037	-.4224
1.0	.2702	-1.6391	.3840	.7981	.2114	-.3719
1.1	.3455	-2.5155	1.7404	1.1721	.4317	-1.7985
1.2	.3171	-1.6018	.2915	.7747	.0904	-.2661
1.3	.3092	-1.6397	.3673	.8220	.1266	-.3595
1.4	.2265	-1.8792	.7939	.9046	.3984	-.7856
1.5	-.0654	-2.4365	1.8286	1.2056	1.2421	-1.8384
1.6	.3094	-1.6371	.3630	.8172	.1257	-.3573
1.7	.2801	-1.6639	.4173	.8275	.1958	-.4034
1.8	.2710	-1.7155	.5062	.8561	.2359	-.4967
1.9	.2837	-1.6582	.4076	.8194	.1862	-.3947
2.0	.3013	-1.5707	.2601	.7693	.1183	-.2504

TABLE 10B. MODEL 2; DATA SET 1; RUN 3
STANDARD DEVIATIONS

-.4	.0329	.0727	.1251	.0570	.0853	.1297
-.3	.0329	.0727	.1250	.0570	.0851	.1296
-.2	.0328	.0727	.1250	.0569	.0850	.1296
-.1	.0328	.0728	.1251	.0569	.0849	.1297
0	.0328	.0729	.1253	.0569	.0849	.1299
.1	.0328	.0731	.1257	.0570	.0849	.1302
.2	.0328	.0735	.1263	.0570	.0851	.1308
.3	.0329	.0742	.1274	.0572	.0854	.1318
.4	.0331	.0752	.1291	.0575	.0860	.1335
.5	.0333	.0769	.1319	.0580	.0870	.1362
.6	.0337	.0797	.1366	.0589	.0889	.1409
.7	.0345	.0847	.1452	.0606	.0924	.1494
.8	.0362	.0947	.1623	.0641	.0998	.1664
.9	.0406	.1192	.2043	.0735	.1192	.2086
1.0	.1044	.4367	.7454	.2548	.3708	.7670
1.1	.9023	7.3515	11.2684	2.8066	1.9169	11.9290
1.2	.5558	1.6342	2.9686	.9357	1.8531	3.1166
1.3	.4486	.9657	1.7623	.4099	1.3471	1.7753
1.4	.4102	1.7039	2.9435	.6649	1.6002	3.0116
1.5	2.4919	4.5572	8.4659	2.2031	7.0568	8.5540
1.6	.2014	.4894	.8724	.2492	.5955	.8612
1.7	.1477	.5590	.9699	.3418	.5002	.9937
1.8	.0792	.2777	.4792	.1741	.2620	.4891
1.9	.1024	.3890	.6725	.2448	.3640	.6929
2.0	.2203	1.0246	1.7395	.5912	.8356	1.7239

TABLE 11A. MODEL 1; DATA SET 2; RUN 3
ARITHMETIC MEANS

k	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
-.4	.2640	- 1.7225	.5120	.8781	.2579	- .5005
-.3	.2633	- 1.7245	.5155	.8796	.2601	- .5042
-.2	.2626	- 1.7268	.5195	.8812	.2625	- .5083
-.1	.2618	- 1.7294	.5241	.8829	.2652	- .5130
0	.2609	- 1.7324	.5294	.8849	.2683	- .5185
.1	.2599	- 1.7359	.5356	.8871	.2718	- .5248
.2	.2587	- 1.7400	.5429	.8896	.2758	- .5323
.3	.2574	- 1.7450	.5517	.8925	.2806	- .5414
.4	.2558	- 1.7511	.5625	.8959	.2864	- .5525
.5	.2537	- 1.7589	.5762	.9002	.2937	- .5665
.6	.2512	- 1.7691	.5942	.9057	.3030	- .5849
.7	.2476	- 1.7832	.6192	.9132	.3159	- .6105
.8	.2423	- 1.8047	.6570	.9246	.3354	- .6493
.9	.2326	- 1.8433	.7253	.9456	.3705	- .7192
1.0	.2417	- 1.7853	.6387	.8642	.3079	- .6052
1.1	.3522	- 1.3349	- .1682	.7980	- .0756	.1807
1.2	.2882	- 1.7267	.5154	.7739	.2098	- .5023
1.3	.6325	- .4531	- 1.7559	.1343	- .9870	1.8144
1.4	.1647	- 2.0119	1.0353	1.1076	.5809	- 1.0460
1.5	.2702	- 1.6399	.3694	.8593	.2134	- .3532
1.6	.3784	- 1.1558	- .4829	.5968	- .1884	.5127
1.7	.3415	- 1.4968	.0956	.7333	.0148	- .0800
1.8	2.0519	- 3.6396	- 9.4663	- 3.6198	- 5.1219	9.5157
1.9	.3244	- 1.3312	- .1710	.8727	- .0172	.1857
2.0	.2749	- 1.6455	.3769	.8887	.2059	- .3655

TABLE 11B. MODEL 1; DATA SET 2; RUN 3
STANDARD DEVIATION

k	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
-.4	.0273	.0626	.1097	.0911	.0717	.1131
-.3	.0273	.0628	.1100	.0912	.0719	.1134
-.2	.0274	.0631	.1105	.0913	.0721	.1139
-.1	.0275	.0634	.1111	.0915	.0724	.1145
0	.0277	.0639	.1120	.0917	.0728	.1153
.1	.0278	.0645	.1131	.0920	.0733	.1165
.2	.0281	.0654	.1147	.0924	.0740	.1180
.3	.0284	.0665	.1168	.0931	.0750	.1202
.4	.0288	.0682	.1199	.0939	.0764	.1232
.5	.0294	.0706	.1242	.0952	.0785	.1276
.6	.0304	.0743	.1309	.0973	.0818	.1343
.7	.0319	.0803	.1417	.1007	.0872	.1453
.8	.0348	.0915	.1618	.1074	.0975	.1657
.9	.0423	.1182	.2097	.1250	.1234	.2146
1.0	.6061	2.2684	3.9257	2.0635	2.2535	4.2004
1.1	.6417	2.9760	5.1918	1.2291	2.4970	5.2359
1.2	.6949	2.3681	4.2070	1.6586	2.3185	4.3277
1.3	3.6566	12.1495	21.7374	7.0720	12.2122	22.2076
1.4	1.0162	4.5648	8.0952	2.0069	3.8463	8.3487
1.5	.2651	1.0813	1.8909	.4992	.9849	1.9342
1.6	.9152	4.2459	7.5049	2.3402	3.4122	7.6176
1.7	.6716	1.9082	3.5286	1.0768	2.0521	3.5379
1.8	17.3488	51.0407	95.2247	43.9985	51.7299	95.4990
1.9	.3480	2.5801	4.4865	.3993	1.6965	4.4863
2.0	.1623	.7724	1.3570	.4778	.6284	1.3955

TABLE 12A. MODEL 2; DATA SET 2; RUN 3
ARITHMETIC MEANS

k	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
-.4	.2628	- 1.7235	.5138	.8801	.2607	- .5030
-.3	.2622	- 1.7253	.5171	.8814	.2627	- .5064
-.2	.2616	- 1.7274	.5208	.8827	.2649	- .5102
-.1	.2609	- 1.7298	.5249	.8842	.2673	- .5145
0	.2601	- 1.7325	.5297	.8859	.2699	- .5194
.1	.2592	- 1.7356	.5352	.8877	.2730	- .5250
.2	.2582	- 1.7393	.5417	.8897	.2765	- .5317
.3	.2571	- 1.7436	.5494	.8921	.2806	- .5395
.4	.2557	- 1.7489	.5586	.8949	.2855	- .5490
.5	.2541	- 1.7555	.5702	.8983	.2915	- .5609
.6	.2520	- 1.7639	.5850	.9026	.2992	- .5760
.7	.2492	- 1.7752	.6051	.9083	.3095	- .5965
.8	.2451	- 1.7918	.6343	.9166	.3244	- .6263
.9	.2381	- 1.8197	.6836	.9310	.3497	- .6767
1.0	.2175	- 1.8943	.8171	.9705	.4201	- .8166
1.1	.1986	- 2.0538	1.0828	.9335	.5320	- 1.0808
1.2	.3609	- 1.3126	- .2200	.7438	- .0990	.2289
1.3	.2419	- 1.8043	.6556	.9256	.3363	- .6487
1.4	.1773	- 2.2446	1.4142	1.1688	.6349	- 1.4405
1.5	.3531	- 1.3227	- .1897	.7137	- .0844	.2179
1.6	.3155	- 1.5131	.1449	.7693	.0664	- .1241
1.7	.2723	- 1.6453	.3798	.8635	.2105	- .3674
1.8	.2626	- 1.5993	.3103	.9010	.2068	- .2890
1.9	.2880	- 1.5483	.2174	.8466	.1324	- .2020
2.0	.1928	- 2.0918	1.1697	.9787	.5505	- 1.1806

TABLE 12B. MODEL 2; DATA SET 2; RUN 3
STANDARD DEVIATIONS

-.4	.0273	.0628	.1105	.0919	.0723	.1136
-.3	.0273	.0629	.1107	.0919	.0723	.1138
-.2	.0274	.0631	.1109	.0920	.0724	.1140
-.1	.0275	.0633	.1114	.0921	.0725	.1144
0	.0275	.0637	.1120	.0923	.0727	.1150
.1	.0277	.0642	.1130	.0925	.0731	.1160
.2	.0278	.0650	.1143	.0928	.0736	.1173
.3	.0281	.0661	.1163	.0933	.0744	.1192
.4	.0284	.0677	.1192	.0939	.0755	.1221
.5	.0289	.0701	.1234	.0949	.0773	.1263
.6	.0296	.0738	.1299	.0964	.0802	.1328
.7	.0309	.0798	.1405	.0989	.0850	.1435
.8	.0332	.0904	.1594	.1038	.0941	.1626
.9	.0390	.1142	.2015	.1161	.1158	.2055
1.0	.0908	.2811	.4967	.2547	.2950	.5157
1.1	.2863	1.3742	2.3879	.5255	1.1321	2.4051
1.2	1.1810	4.6518	8.2901	2.1953	4.2459	8.3550
1.3	.2222	.9152	1.6158	.4817	.7984	1.6422
1.4	1.3601	6.3680	11.2218	3.3970	5.2042	11.6028
1.5	.4397	1.9374	3.3669	.8959	1.6974	3.4787
1.6	.2474	.9330	1.6536	.5073	.8684	1.6762
1.7	.2912	1.2497	2.2366	.5514	1.0437	2.2440
1.8	.3490	1.5044	2.6843	.8067	1.2157	2.7253
1.9	.3676	2.2163	3.8238	.4869	1.6590	3.8467
2.0	.8523	4.9575	8.7584	1.0029	3.7054	8.9908

TABLE 13A. MODEL 1; DATA SET 1; RUN 4
ARITHMETIC MEANS

k	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
-.4	.3270	-2.0071	.5761	.9950	.2498	- .5795
-.3	.3265	-2.0069	.5760	.9947	.2510	- .5793
-.2	.3259	-2.0067	.5759	.9944	.2523	- .5790
-.1	.3254	-2.0065	.5758	.9941	.2537	- .5787
0	.3248	-2.0063	.5756	.9938	.2550	- .5784
.1	.3242	-2.0061	.5754	.9935	.2563	- .5781
.2	.3236	-2.0059	.5752	.9931	.2577	- .5777
.3	.3230	-2.0057	.5750	.9928	.2591	- .5772
.4	.3224	-2.0055	.5747	.9924	.2605	- .5767
.5	.3218	-2.0052	.5742	.9920	.2620	- .5761
.6	.3212	-2.0050	.5736	.9915	.2635	- .5752
.7	.3206	-2.0047	.5726	.9910	.2650	- .5739
.8	.3200	-2.0044	.5706	.9904	.2665	- .5717
.9	.3198	-2.0039	.5657	.9896	.2676	- .5663
1.0	.0344	-2.0254	1.9233	1.1283	.7837	-1.9622
1.1	.2848	-2.0045	.7106	.9779	.3332	- .7063
1.2	.3470	-2.0099	.5210	.9829	.2121	- .5278
1.3	- .2688	-1.9770	2.3224	.5522	1.5179	-2.3240
1.4	.2982	-2.0041	.6561	.9981	.3060	- .6556
1.5	.3294	-2.0045	.5360	.9913	.2488	- .5387
1.6	.3307	-2.0053	.5376	.9918	.2445	- .5387
1.7	.3122	-2.0029	.5865	.9911	.2834	- .5877
1.8	.3139	-2.0029	.5789	.9904	.2800	- .5797
1.9	.3135	-2.0027	.5774	.9899	.2809	- .5780
2.0	.3129	-2.0025	.5768	.9894	.2823	- .5771

TABLE 13B. MODEL 1; DATA SET 1; RUN 4
STANDARD DEVIATIONS

k	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
-.4	.0440	.0137	.1226	.0473	.0959	.1258
-.3	.0441	.0136	.1228	.0473	.0961	.1259
-.2	.0442	.0136	.1229	.0473	.0962	.1260
-.1	.0443	.0136	.1232	.0473	.0965	.1263
0	.0445	.0136	.1235	.0473	.0968	.1266
.1	.0447	.0137	.1241	.0474	.0971	.1271
.2	.0450	.0137	.1248	.0474	.0976	.1277
.3	.0453	.0137	.1258	.0475	.0983	.1287
.4	.0458	.0138	.1274	.0475	.0992	.1303
.5	.0465	.0138	.1300	.0476	.1004	.1327
.6	.0476	.0139	.1342	.0478	.1023	.1368
.7	.0494	.0141	.1418	.0481	.1055	.1442
.8	.0529	.0143	.1575	.0487	.1119	.1596
.9	.0624	.0149	.1998	.0502	.1292	.2017
1.0	2.2099	.3777	12.3432	1.3129	3.8236	12.8027
1.1	.4702	.0729	1.4906	.1705	.9460	1.4542
1.2	.2211	.0292	.6980	.1193	.4285	.6857
1.3	8.2926	.7565	23.3868	4.2871	17.5467	23.3051
1.4	.1961	.0234	.7326	.0784	.3732	.7157
1.5	.1033	.0195	.3487	.0577	.2095	.3457
1.6	.1244	.0201	.3986	.0599	.2572	.4035
1.7	.0775	.0168	.2339	.0509	.1629	.2360
1.8	.0593	.0157	.1750	.0497	.1267	.1776
1.9	.0559	.0156	.1596	.0492	.1205	.1625
2.0	.0545	.0157	.1516	.0490	.1182	.1546

TABLE 14A. MODEL 2; DATA SET 1; RUN 4
ARITHMETIC MEANS

k	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
-.4	.3223	- 2.0053	.5750	.9972	.2591	- .5769
-.3	.3219	- 2.0052	.5750	.9969	.2600	- .5768
-.2	.3215	- 2.0051	.5749	.9965	.2611	- .5766
-.1	.3211	- 2.0049	.5749	.9961	.2621	- .5765
0	.3207	- 2.0048	.5748	.9958	.2631	- .5763
.1	.3202	- 2.0046	.5747	.9954	.2642	- .5761
.2	.3198	- 2.0045	.5746	.9949	.2653	- .5759
.3	.3193	- 2.0043	.5744	.9945	.2665	- .5756
.4	.3189	- 2.0041	.5742	.9940	.2677	- .5752
.5	.3184	- 2.0039	.5738	.9935	.2689	- .5748
.6	.3179	- 2.0037	.5733	.9928	.2702	- .5742
.7	.3173	- 2.0035	.5725	.9921	.2717	- .5732
.8	.3168	- 2.0031	.5708	.9911	.2734	- .5714
.9	.3162	- 2.0024	.5669	.9893	.2758	- .5673
1.0	.3060	- 1.9974	.5588	.9830	.3034	- .5590
1.1	.2961	- 2.0061	.6711	1.0033	.3081	- .6721
1.2	.3052	- 2.0140	.6787	1.0211	.2753	- .6707
1.3	.0637	- 2.0187	1.4456	.8924	.7569	- 1.4129
1.4	.2936	- 2.0070	.6849	.9989	.3154	- .6903
1.5	.3848	- 2.0037	.2736	.9432	.1480	- .2670
1.6	.3227	- 2.0051	.5565	.9987	.2573	- .5542
1.7	.2488	- 1.9952	.7527	.9605	.4065	- .7316
1.8	.3173	- 2.0041	.5736	.9985	.2706	- .5751
1.9	.3109	- 2.0034	.5912	.9966	.2839	- .5919
2.0	.2594	- 2.0016	.7746	1.0104	.3856	- .7783

TABLE 14B. MODEL 2; DATA SET 1; RUN 4
STANDARD DEVIATIONS

-.4	.0418	.0126	.1242	.0489	.0931	.1273
-.3	.0418	.0125	.1242	.0489	.0930	.1272
-.2	.0418	.0125	.1241	.0489	.0930	.1271
-.1	.0418	.0125	.1242	.0489	.0930	.1272
0	.0419	.0125	..1243	.0489	.0931	.1273
.1	.0420	.0125	.1246	.0489	.0932	.1276
.2	.0422	.0125	.1252	.0489	.0935	.1281
.3	.0424	.0124	.1261	.0489	.0939	.1289
.4	.0428	.0125	.1276	.0490	.0946	.1304
.5	.0434	.0125	.1301	.0490	.0956	.1328
.6	.0444	.0125	.1344	.0492	.0974	.1370
.7	.0463	.0126	.1422	.0495	.1006	.1446
.8	.0499	.0127	.1578	.0500	.1071	.1600
.9	.0596	.0131	.1974	.0514	.1248	.1993
1.0	.2166	.0376	1.0694	.1281	.3920	1.0919
1.1	.2544	.0297	.9059	.1637	.5197	.9112
1.2	.8146	.0575	3.2136	.2774	1.6241	3.2471
1.3	2.7150	.3235	11.0438	.5641	5.0030	10.8465
1.4	.3552	.0391	1.0870	.1562	.7466	1.0983
1.5	.9115	.0704	4.2465	.5644	1.6805	4.3820
1.6	.2652	.0296	.9597	.1129	.5237	.9682
1.7	1.2130	.1576	3.3367	.4240	2.5091	3.1774
1.8	.1257	.0199	.3897	.0685	.2583	.3821
1.9	.0944	.0160	.3206	.0603	.1922	.3207
2.0	.5257	.0211	1.9149	.1707	1.0350	1.9521

TABLE 15A. MODEL 1; DATA SET 2; RUN 4
ARITHMETIC MEANS

k	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
-.4	.2893	- 2.0063	.6438	1.0211	.3284	- .6411
-.3	.2883	- 2.0062	.6472	1.0217	.3306	- .6445
-.2	.2871	- 2.0061	.6509	1.0223	.3329	- .6483
-.1	.2857	- 2.0060	.6552	1.0229	.3355	- .6526
0	.2843	- 2.0059	.6601	1.0235	.3385	- .6576
.1	.2826	- 2.0057	.6658	1.0241	.3418	- .6633
.2	.2807	- 2.0056	.6725	1.0246	.3455	- .6700
.3	.2784	- 2.0054	.6804	1.0252	.3499	- .6781
.4	.2757	- 2.0052	.6901	1.0257	.3552	- .6878
.5	.2724	- 2.0049	.7022	1.0262	.3616	- .7001
.6	.2682	- 2.0046	.7179	1.0266	.3697	- .7158
.7	.2626	- 2.0041	.7391	1.0271	.3806	- .7372
.8	.2543	- 2.0034	.7702	1.0277	.3964	- .7685
.9	.2405	- 2.0022	.8226	1.0289	.4229	- .8213
1.0	.2065	- 2.0005	.9294	1.0725	.4913	- .9338
1.1	- .0549	- 1.9877	2.0701	.8302	.9689	- 2.0469
1.2	.2057	- 2.0056	1.0418	.9660	.4836	- 1.0464
1.3	.3176	- 2.0107	.5625	.9480	.2758	- .5556
1.4	.5738	- 2.0252	- .7702	1.0962	- .1759	.7901
1.5	- .1415	- 2.0008	2.5127	.4008	1.1722	- 2.4989
1.6	.3516	- 2.0110	.3802	1.0294	.2119	- .3766
1.7	.3014	- 2.0022	.5235	1.0916	.3067	- .5158
1.8	.3142	- 2.0054	.5109	1.0377	.2836	- .5050
1.9	.2545	- 1.9928	.7173	1.0510	.3968	- .7065
2.0	.5566	- 2.0376	- .3864	1.0857	- .1855	.3793

TABLE 15B. MODEL 1; DATA SET 2; RUN 4
STANDARD DEVIATIONS

-.4	.0367	.0091	.1073	.1014	.0757	.1090
-.3	.0366	.0092	.1073	.1015	.0757	.1090
-.2	.0367	.0092	.1074	.1017	.0757	.1091
-.1	.0367	.0092	.1077	.1020	.0758	.1093
0	.0368	.0092	.1081	.1023	.0759	.1096
.1	.0369	.0092	.1086	.1026	.0761	.1101
.2	.0371	.0093	.1095	.1031	.0765	.1110
.3	.0374	.0093	.1109	.1038	.0770	.1123
.4	.0378	.0094	.1130	.1046	.0779	.1144
.5	.0386	.0094	.1163	.1058	.0794	.1176
.6	.0399	.0096	.1215	.1075	.0818	.1227
.7	.0421	.0097	.1305	.1100	.0861	.1317
.8	.0467	.0101	.1476	.1144	.0947	.1486
.9	.0577	.0111	.1864	.1239	.1159	.1872
1.0	.2794	.0286	1.2623	.5188	.5307	1.2610
1.1	2.4550	.2210	8.7886	1.0374	4.6538	8.5591
1.2	.6573	.0669	2.7455	.6083	1.2510	2.7372
1.3	.5757	.0499	2.4626	.3304	1.0783	2.5163
1.4	2.3164	.1214	11.9666	.9190	4.1138	12.1900
1.5	3.6327	.1359	15.2215	5.0705	7.1585	15.0424
1.6	.7451	.0397	3.0697	.6745	1.4493	3.0737
1.7	.5226	.0540	1.8246	.2703	1.0206	1.8351
1.8	.2685	.0377	.9646	.1865	.5200	.9615
1.9	.6966	.1481	2.3417	.1582	1.3339	2.2768
2.0	1.7398	.2116	6.9087	.6056	3.3191	6.8188

TABLE 16A. MODEL 2; DATA SET 2; RUN 4
ARITHMETIC MEANS

k	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
-.4	.2879	- 2.0061	.6455	1.0228	.3312	- .6432
-.3	.2869	- 2.0060	.6486	1.0233	.3332	- .6464
-.2	.2859	- 2.0059	.6522	1.0237	.3353	- .6500
-.1	.2847	- 2.0058	.6561	1.0241	.3377	- .6540
0	.2834	- 2.0057	.6607	1.0245	.3403	- .6586
.1	.2819	- 2.0056	.6659	1.0249	.3432	- .6638
.2	.2802	- 2.0055	.6719	1.0252	.3465	- .6699
.3	.2782	- 2.0053	.6791	1.0255	.3504	- .6771
.4	.2759	- 2.0051	.6876	1.0257	.3549	- .6858
.5	.2730	- 2.0049	.6982	1.0260	.3604	- .6964
.6	.2695	- 2.0046	.7116	1.0261	.3673	- .7099
.7	.2648	- 2.0043	.7294	1.0263	.3763	- .7277
.8	.2582	- 2.0037	.7545	1.0265	.3890	- .7530
.9	.2477	- 2.0028	.7947	1.0272	.4093	- .7934
1.0	.2254	- 2.003	.8787	1.0280	.4515	- .8766
1.1	.0468	- 1.9838	1.5148	1.0642	.7934	- 1.5082
1.2	- 2.2524	- 1.9857	11.8859	- 3.4674	5.2454	-11.7501
1.3	.1212	- 1.9800	1.1877	1.0723	.6669	- 1.1844
1.4	.2349	- 2.0028	.8660	1.0127	.4319	- .8642
1.5	.4600	- 2.0396	.1343	.7286	.0032	- .1425
1.6	.2263	- 2.0007	.8236	1.0276	.4559	- .8179
1.7	.3260	- 2.0033	.4198	1.0887	.2608	- .4089
1.8	.1713	- 1.9947	1.0933	1.0536	.5461	- 1.0859
1.9	.3987	- 2.0101	.1882	1.1165	.1149	- .1882
2.0	.2469	- 1.9938	.7800	1.0503	.4059	- .7702

TABLE 16B. MODEL 2; DATA SET 2; RUN 4
STANDARD DEVIATIONS

k	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
-.4	.0369	.0093	.1080	.1014	.0761	.1093
-.3	.0368	.0093	.1079	.1015	.0760	.1092
-.2	.0368	.0094	.1079	.1017	.0759	.1091
-.1	.0368	.0094	.1080	.1019	.0758	.1092
0	.0368	.0094	.1083	.1022	.0758	.1095
.1	.0368	.0094	.1089	.1025	.0759	.1099
.2	.0370	.0094	.1097	.1030	.0761	.1108
.3	.0372	.0094	.1111	.1036	.0766	.1121
.4	.0376	.0095	.1133	.1044	.0774	.1142
.5	.0384	.0095	.1168	.1054	.0788	.1176
.6	.0396	.0096	.1225	.1069	.0811	.1231
.7	.0419	.0098	.1320	.1092	.0854	.1325
.8	.0463	.0101	.1496	.1130	.0939	.1498
.9	.0565	.0109	.1866	.1205	.1133	.1865
1.0	.1004	.0148	.3366	.1497	.1994	.3392
1.1	1.3521	.1928	4.2714	1.2961	2.6442	4.2118
1.2	23.7078	.1127	105.737	44.1322	45.8346	104.376
1.3	2.1922	.2121	7.9477	.6318	4.3600	8.0504
1.4	.3908	.0414	1.5491	.3309	.7495	1.5426
1.5	4.0000	.4225	14.6042	2.3534	7.7791	14.3977
1.6	.7292	.0679	2.8678	.4689	1.4047	2.8753
1.7	.4433	.0410	1.6443	.2568	.8656	1.6678
1.8	1.7629	.1627	7.3672	.2623	3.2021	7.3102
1.9	.7725	.0626	2.9691	.5885	1.5173	2.9102
2.0	.5290	.1041	2.0708	.1674	.9730	2.0177

Mean Estimates

Figure 1

Mean Estimates

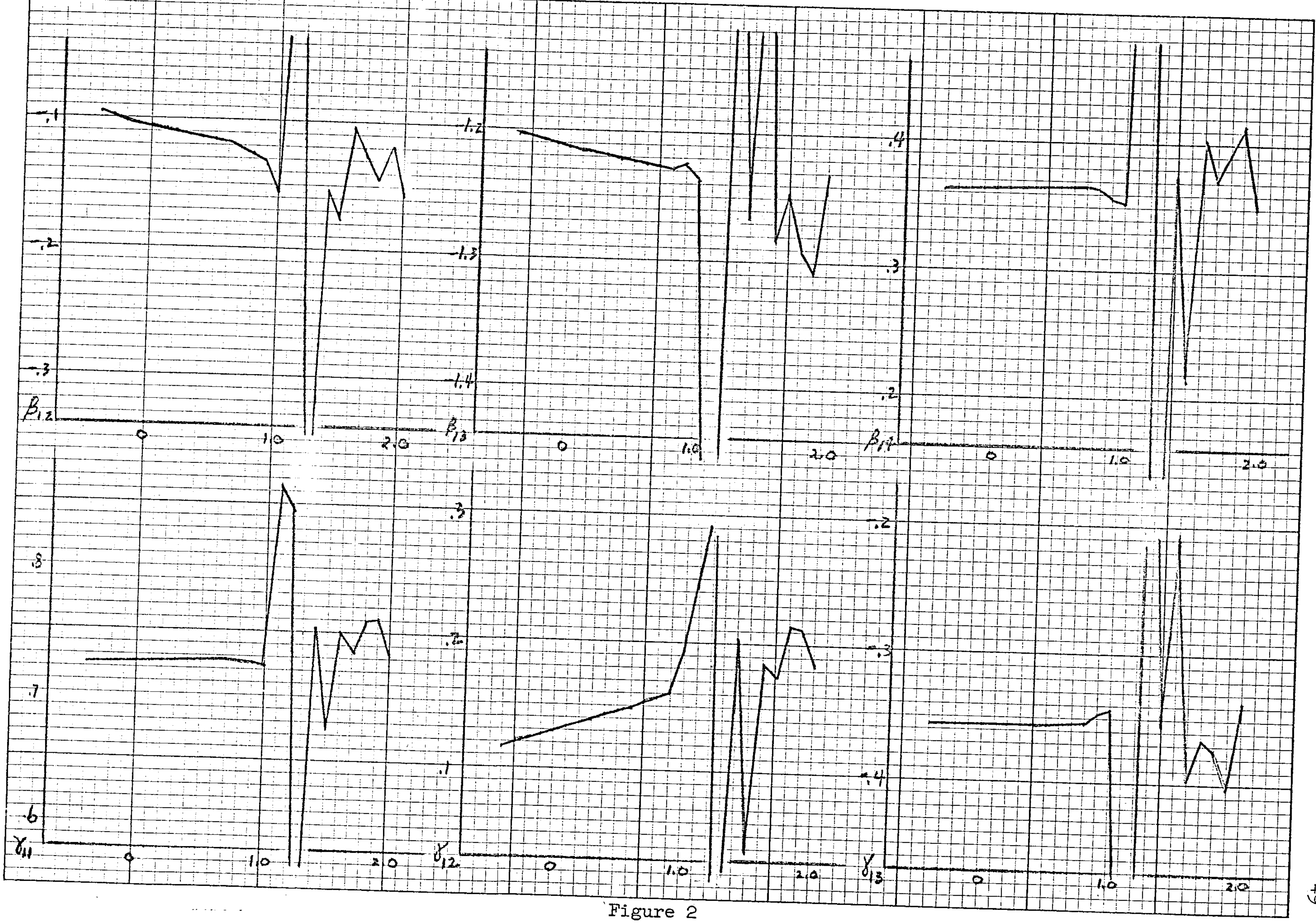

Figure 2

Mean Estimates

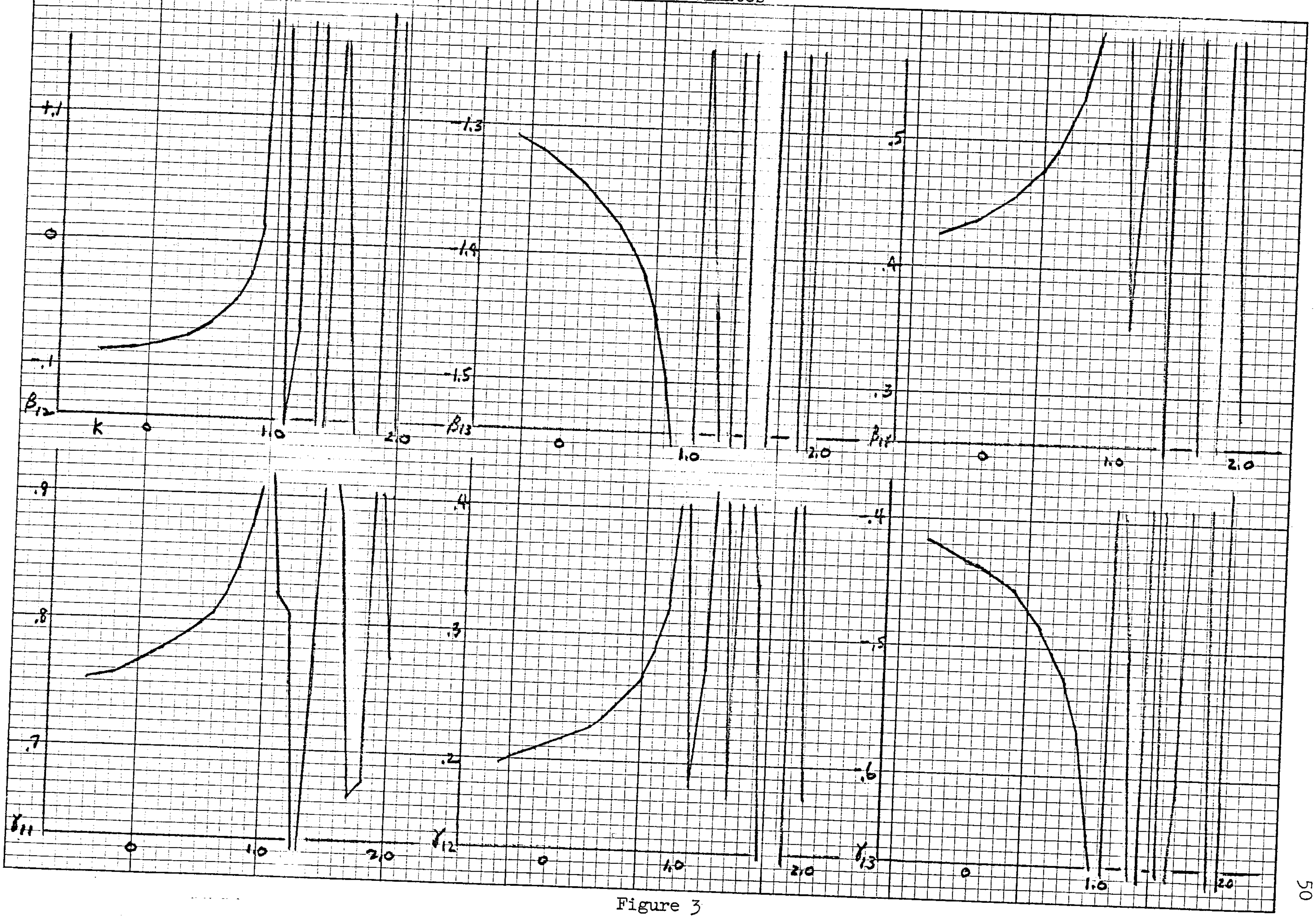

Figure 3

Mean Estimates

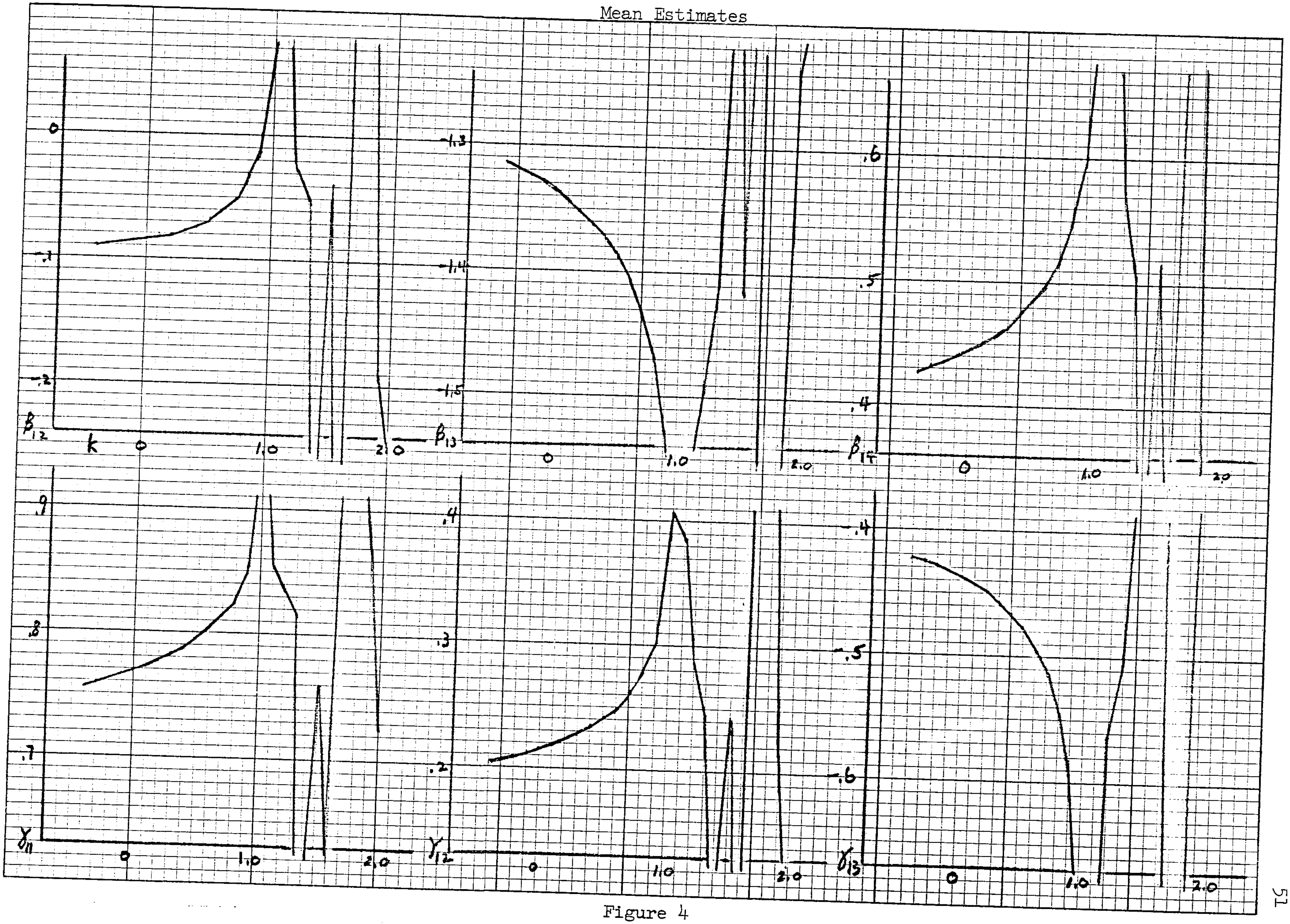

Figure 4

Mean Estimates

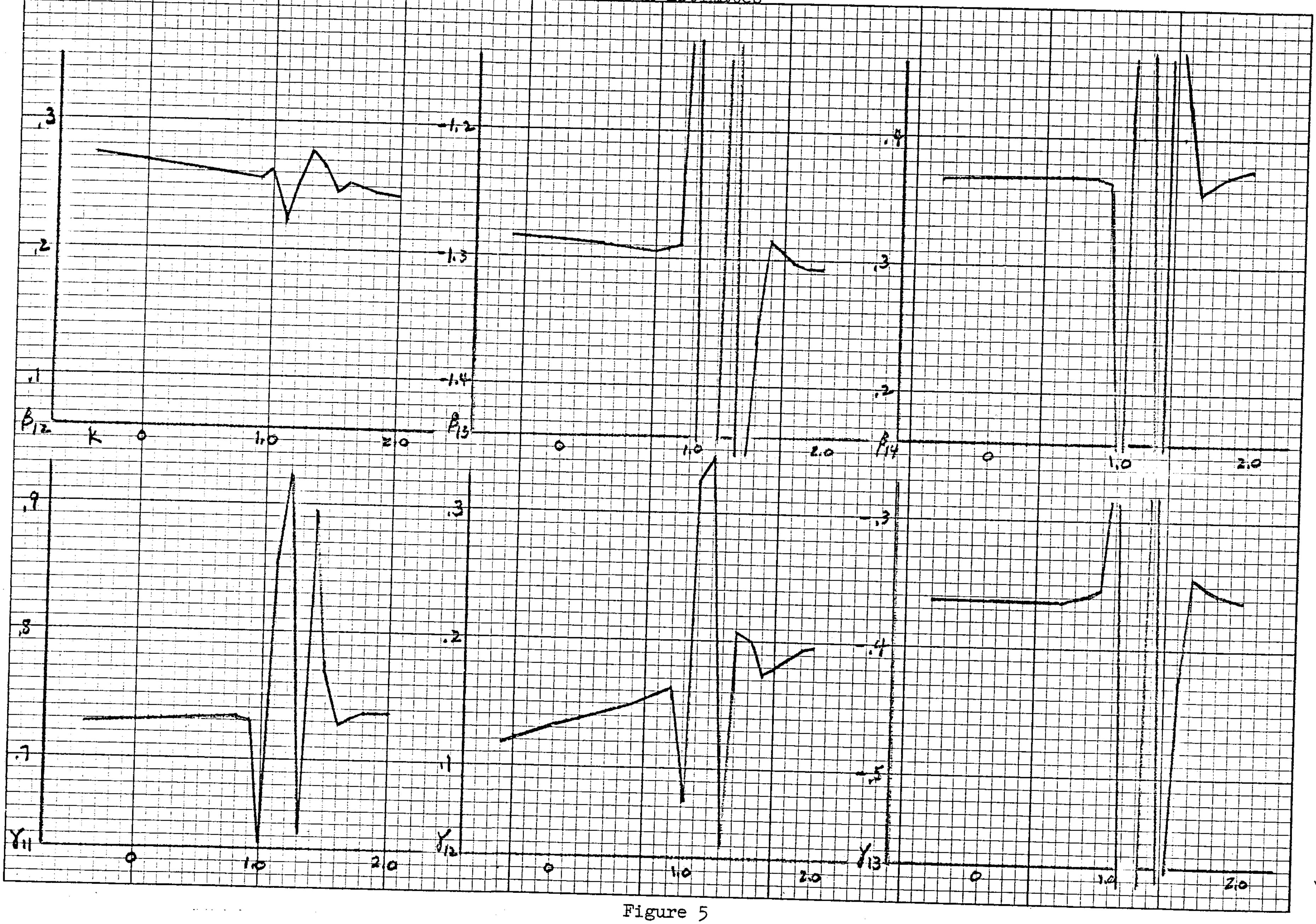

Figure 5

Mean Estimates

Figure 6

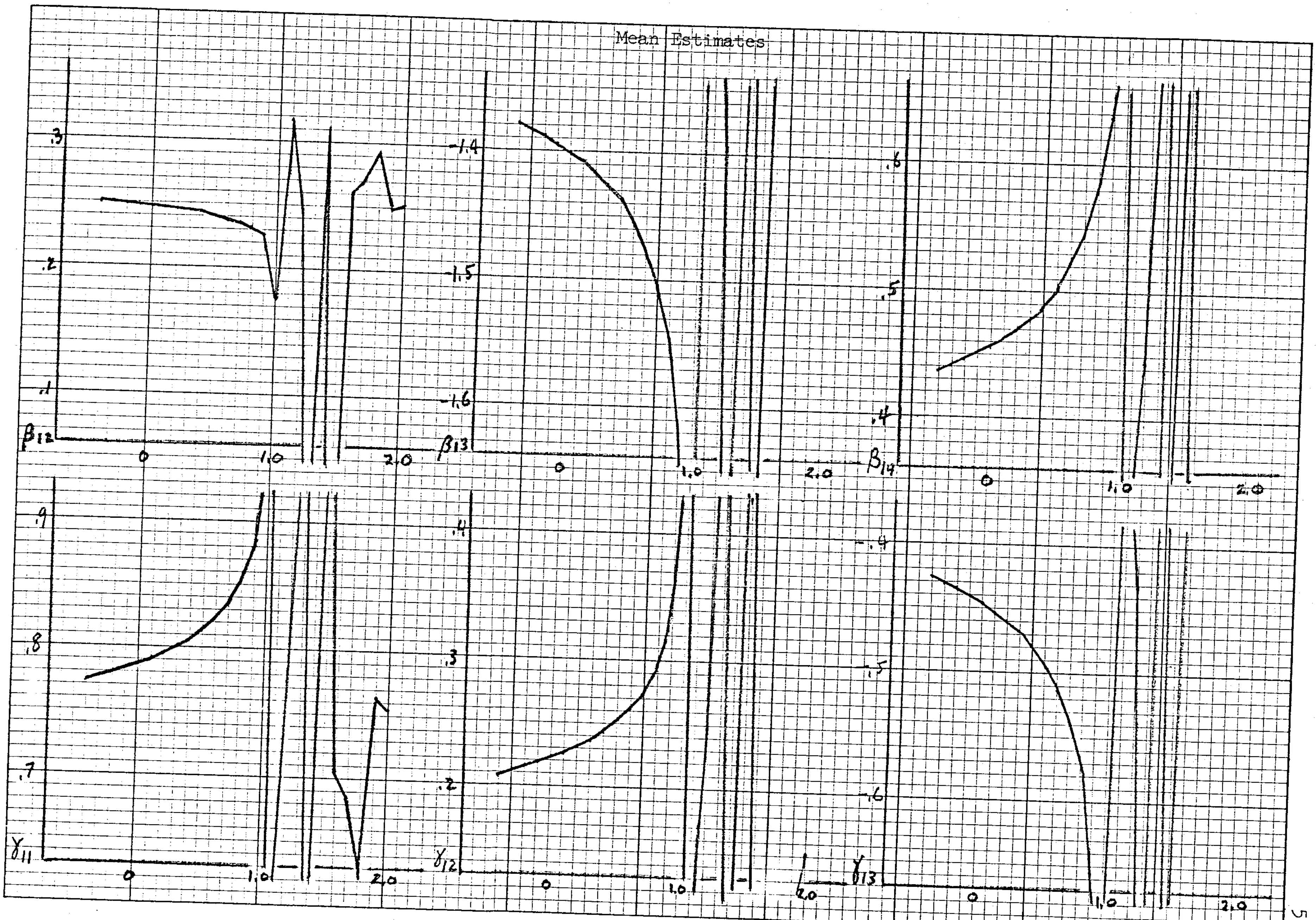
Mean Estimates
.3
.2
.1
β12
0
1.0
2.0
-1.4
-1.5
-1.6
β13
.6
.5
.4
β14
.9
.8
.7
γ11
.4
.3
.2
γ12
-.4
-.5
-.6
γ13

Figure 7

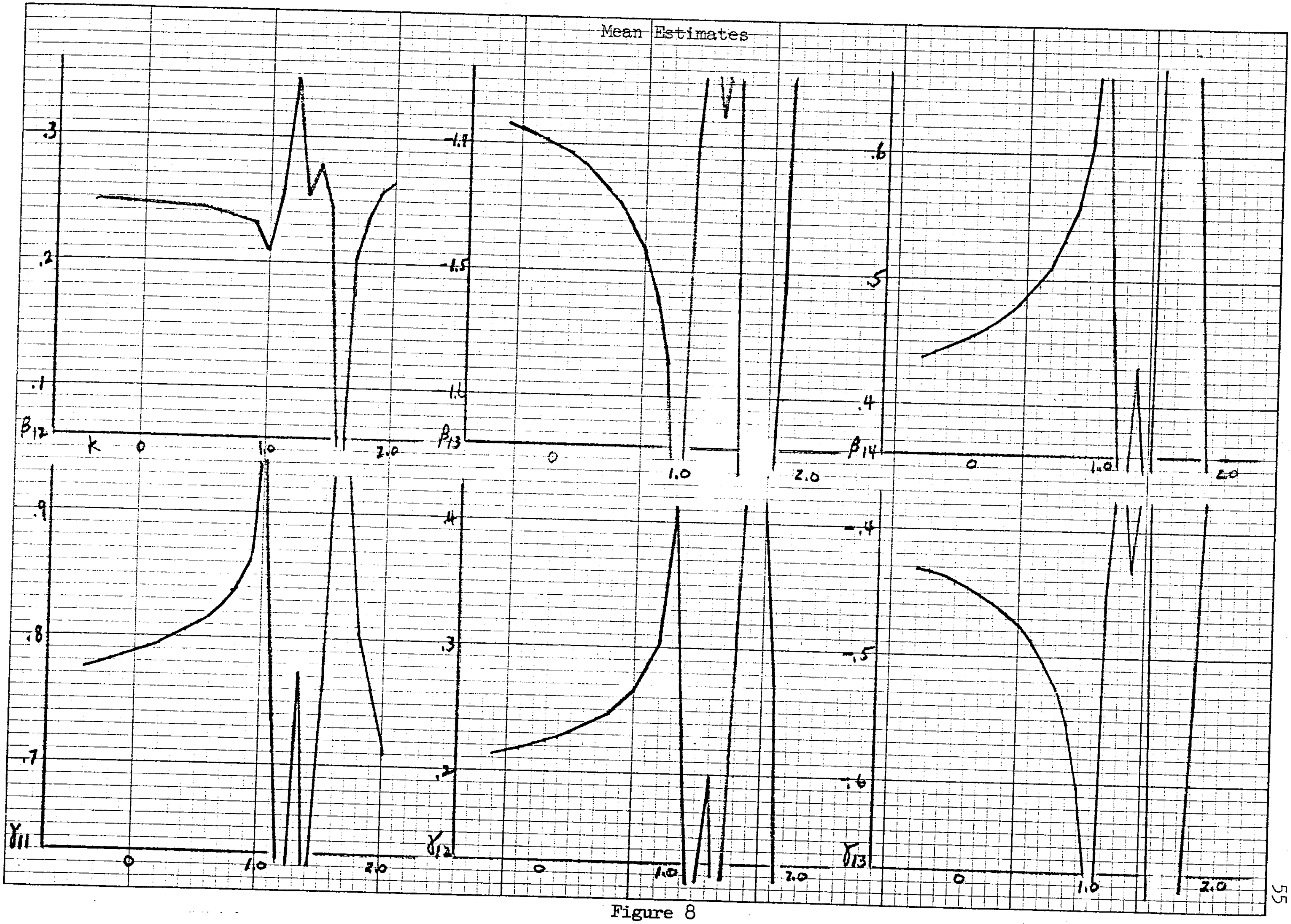
Mean Estimates
β_{12}
k
β_{13}
β_{14}
γ_{11}
γ_{12}
γ_{13}

Figure 8

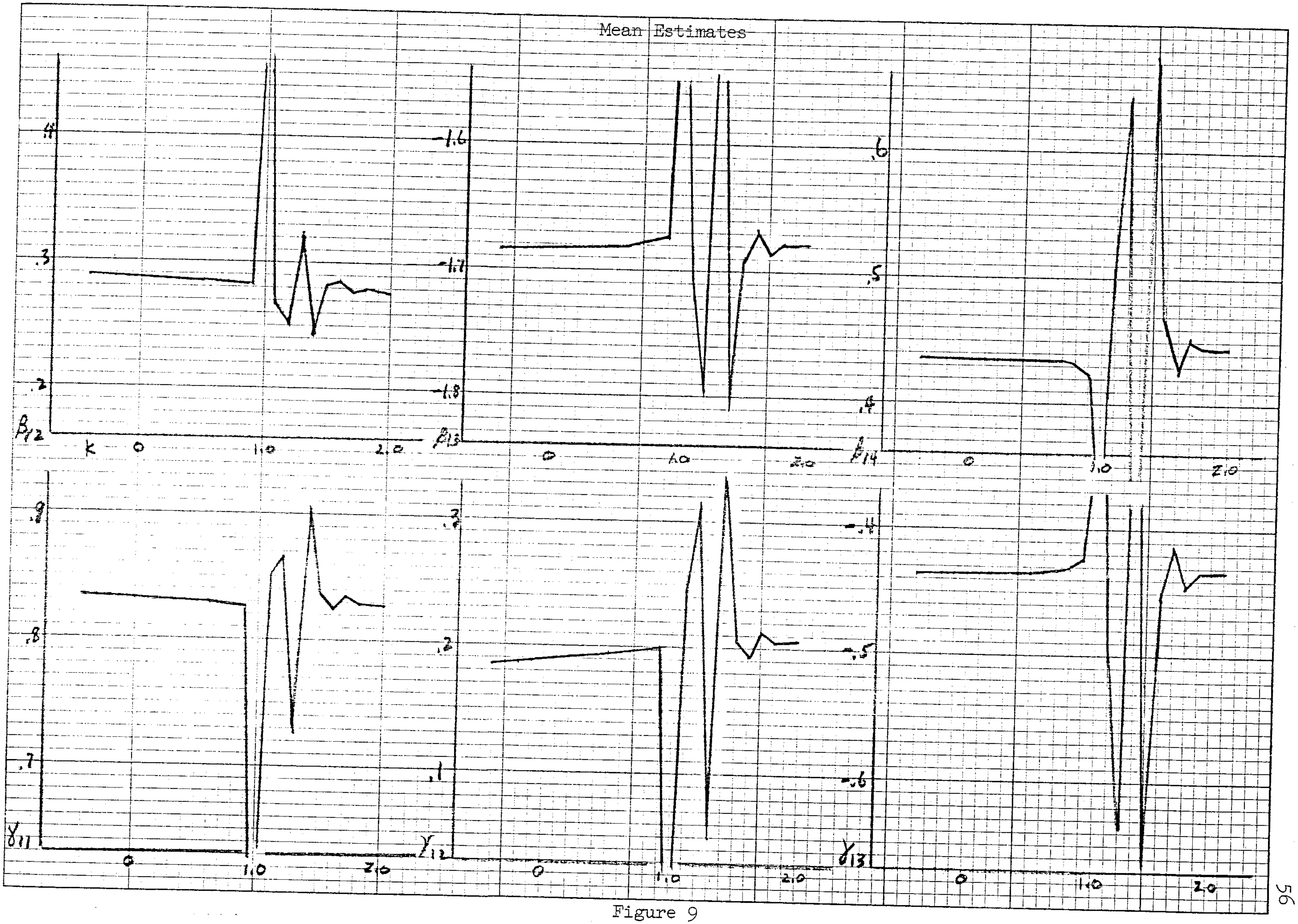
Mean Estimates
β_{12}
.4
.3
.2
k
0
1.0
2.0
β_{13}
-1.6
-1.7
-1.8
β_{14}
.6
.5
.4
γ_{11}
.9
.8
.7
γ_{12}
.3
.2
.1
γ_{13}
-.4
-.5
-.6

Figure 9

Mean Estimates

Figure 10

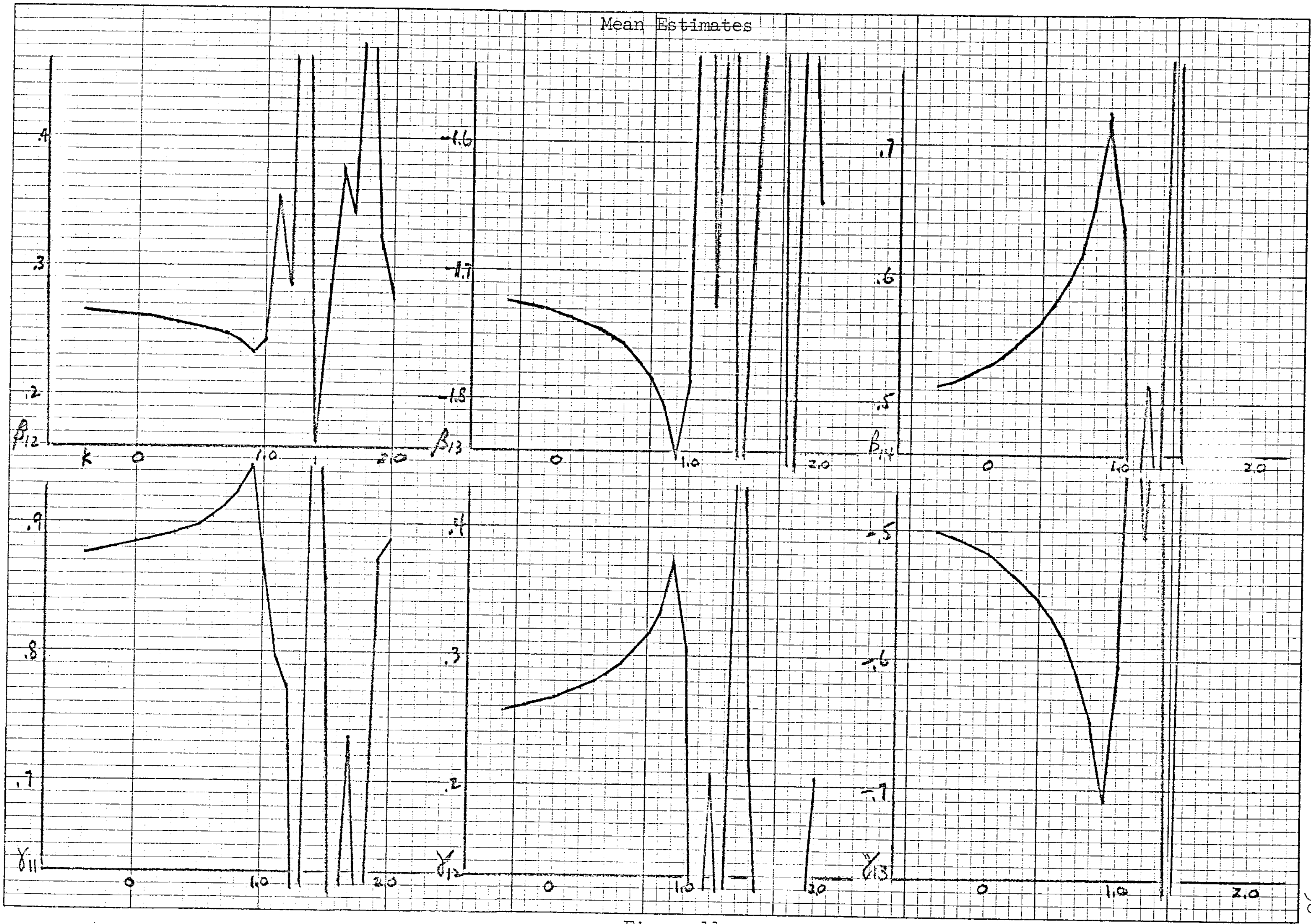
Mean Estimates
.4
.3
.2
β_{12}
k
0
1.0
2.0
-1.6
-1.7
-1.8
β_{13}
.7
.6
.5
β_{14}
.9
.8
.7
γ_{11}
.4
.3
.2
γ_{12}
-.5
-.6
-.7
γ_{13}

Figure 11

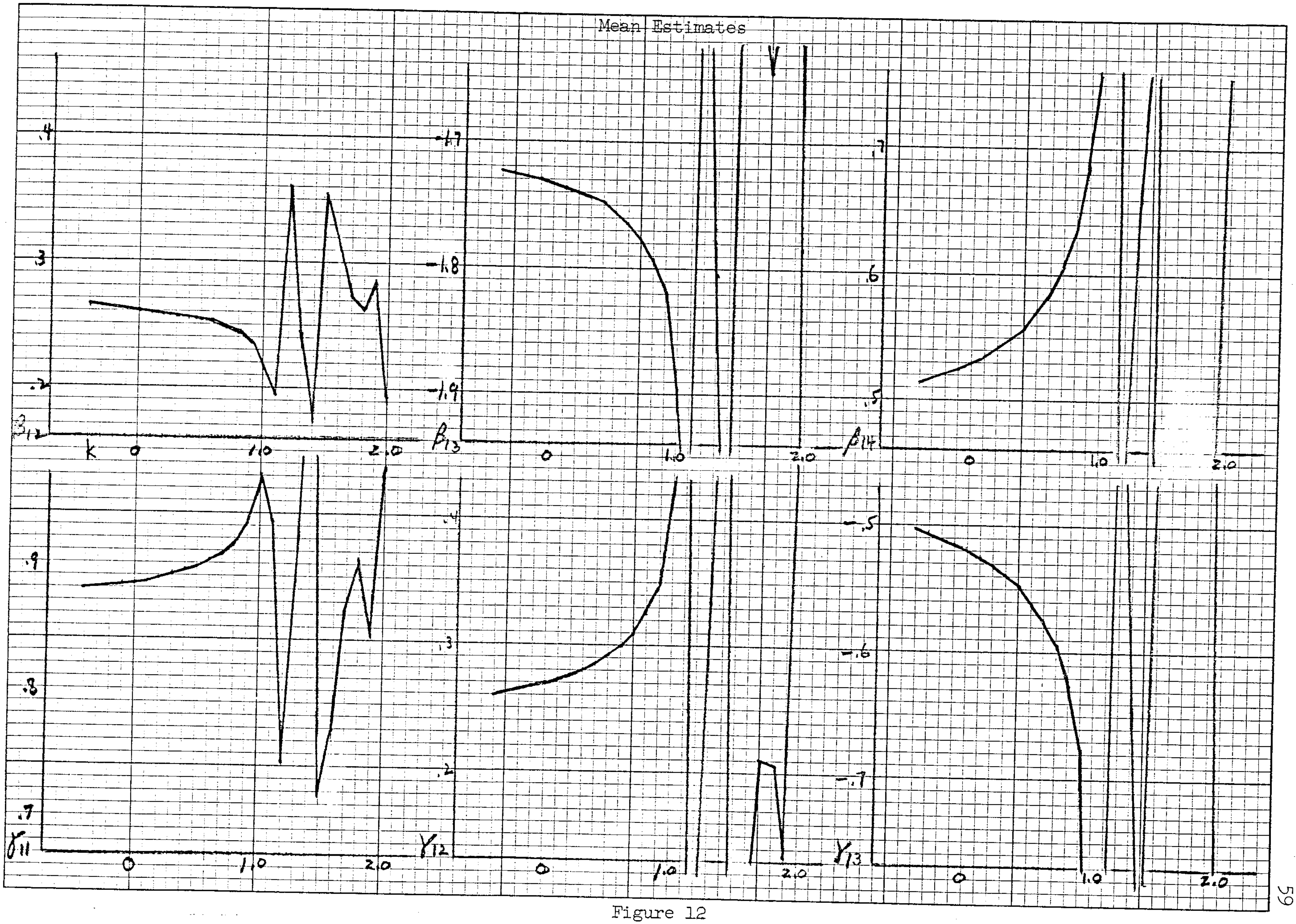
Mean Estimates
K
V
β_{12}
β_{13}
β_{14}
γ_{11}
γ_{12}
γ_{13}

Figure 12

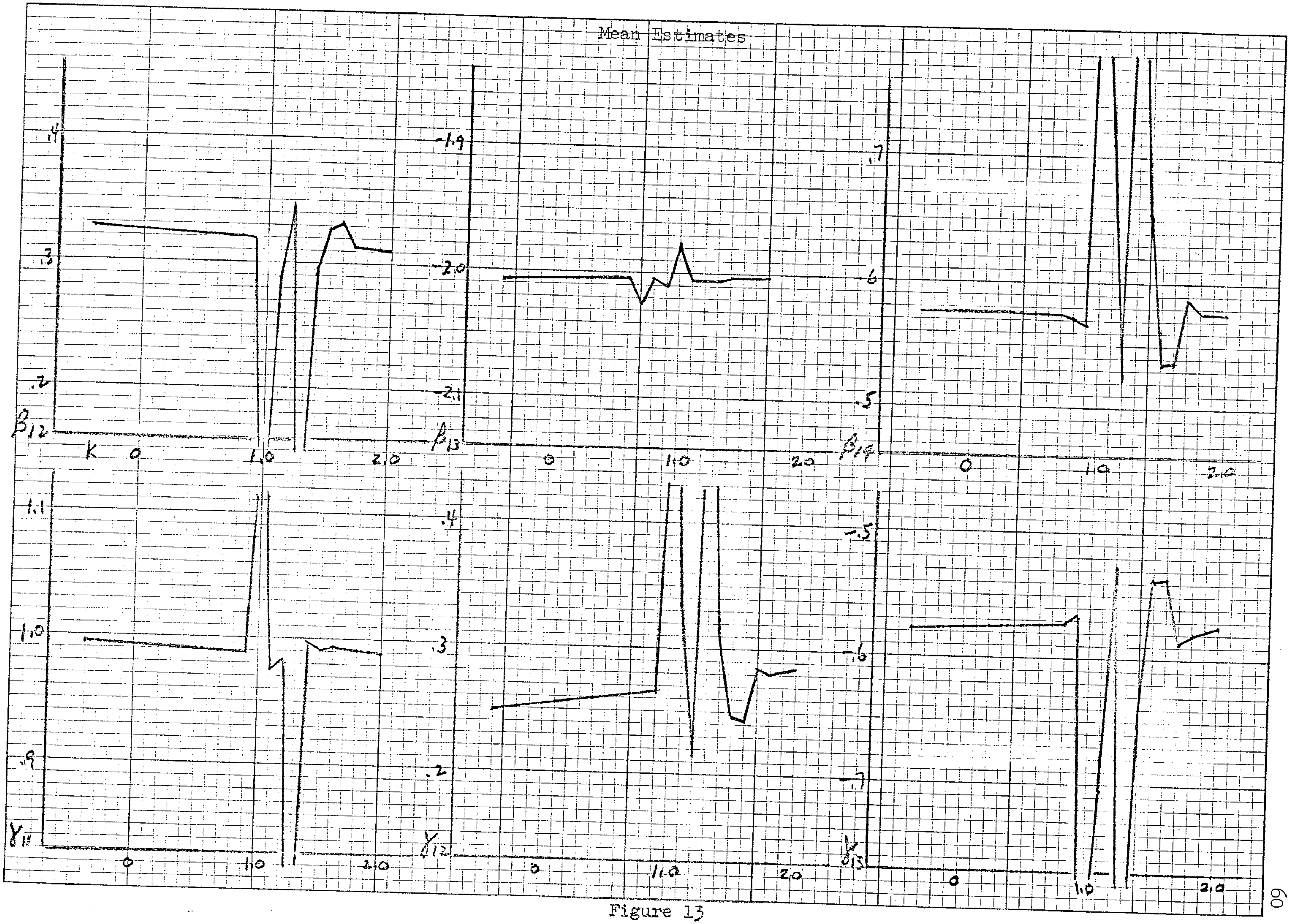
Mean Estimates
.4
.3
.2
β_{12}
K 0
1.0
2.0
-1.9
-2.0
-2.1
β_{13}
0
1.0
2.0
.7
.6
.5
β_{14}
0
1.0
2.0
1.1
1.0
.9
γ_{11}
0
1.0
2.0
.4
.3
.2
γ_{12}
0
1.0
2.0
-.5
-.6
-.7
γ_{13}
0
1.0
2.0

Figure 13


Mean Estimates
.4
.3
.2
β_{12}
K
0
1.0
2.0
-1.9
-2.0
-2.1
β_{13}
.7
.6
.5
β_{14}
1.1
1.0
.9
γ_{11}
.4
.3
.2
γ_{12}
-.5
-.6
-.7
γ_{13}

Figure 14

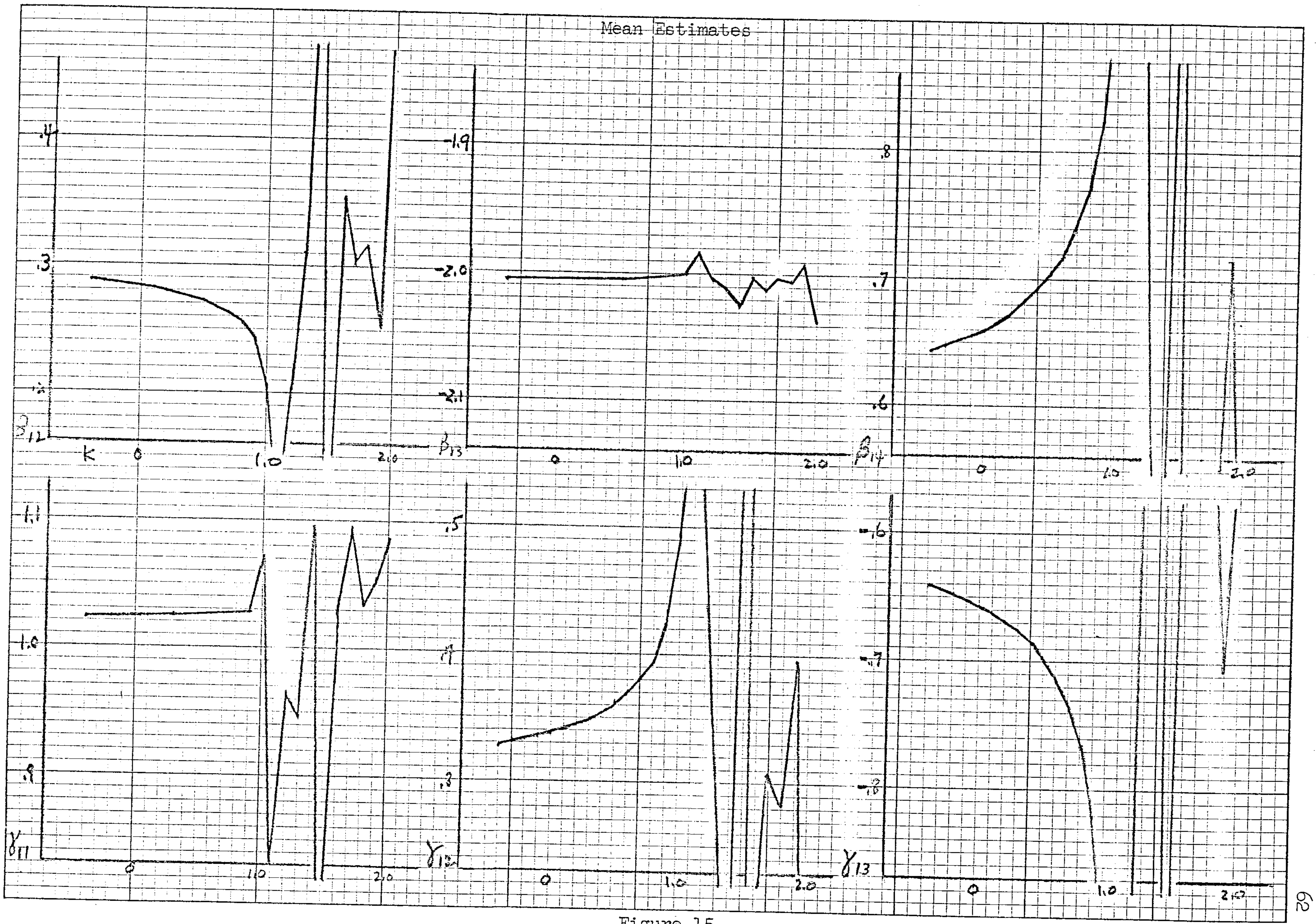
Mean Estimates
.4
.3
.2
β_{12}
K
0
1.0
2.0
-1.9
-2.0
-2.1
β_{13}
0
1.0
2.0
.8
.7
.6
β_{14}
0
1.0
2.0
1.1
1.0
.9
γ_{11}
0
1.0
2.0
.5
.4
.3
γ_{12}
0
1.0
2.0
-.6
-.7
-.8
γ_{13}
0
1.0
2.0

Figure 15

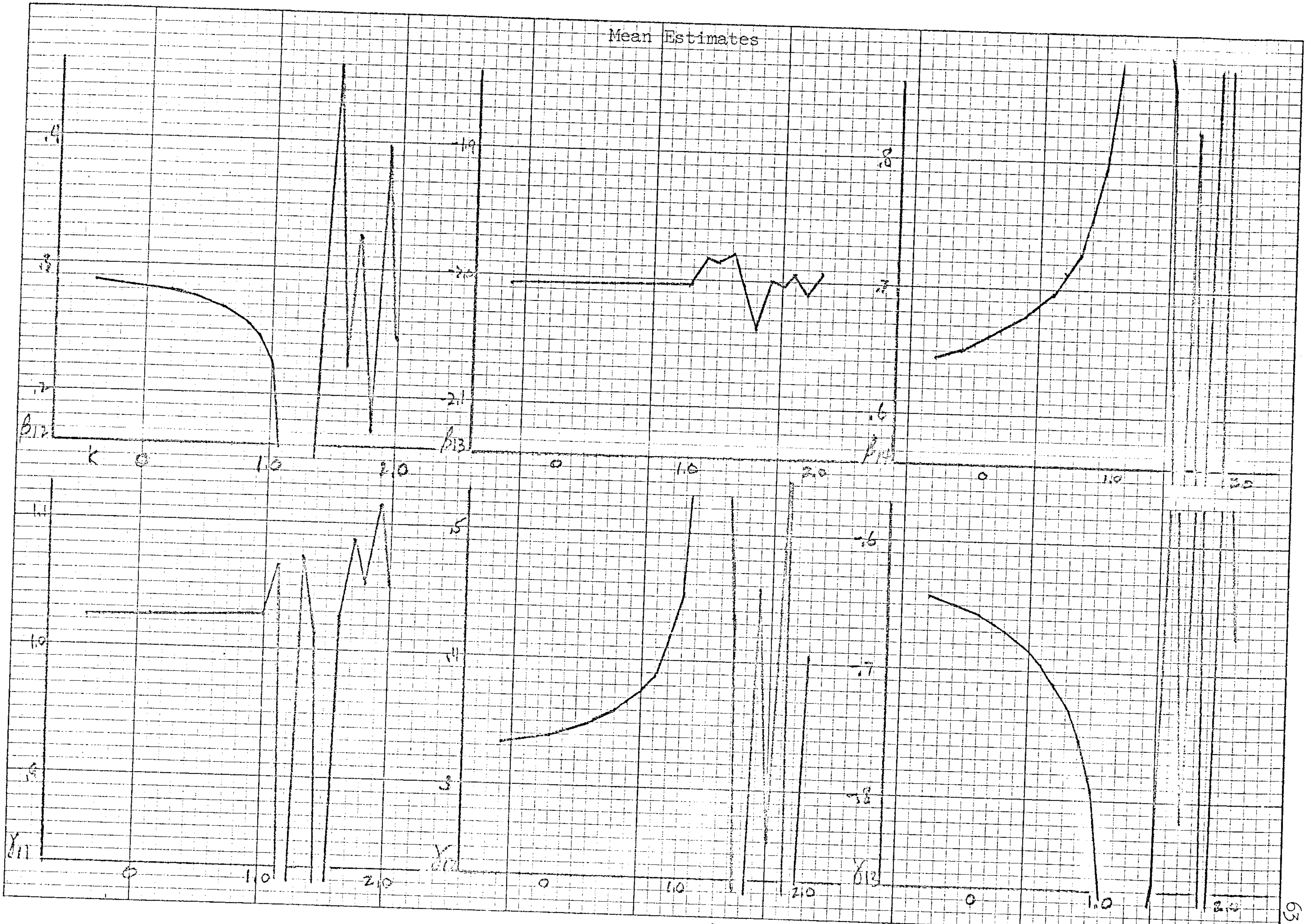
Mean Estimates
β_{12}
β_{13}
β_{14}
γ_{11}
γ_{12}
γ_{13}
K
0
1.0
2.0
.4
.3
.2
-1.9
-2.0
-2.1
.8
.7
.6
1.1
1.0
.9
.5
.4
.3
-.6
-.7
-.8

Figure 16

TABLE 17A. MEDIAN ESTIMATES. Model 1, Data Set 1

	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
True Values	.2	-2.0	1.0	1.0	.5	-1.0
k = 0						
Run 1	- .1026	-1.1808	.3428	.7159	.1157	- .3424
2	.2688	-1.2699	.3569	.7584	.1261	- .3554
3	.2831	-1.6755	.4307	.8334	.1890	- .4266
4	.3194	-2.0075	.5668	.9961	.2654	- .5605
k = 1						
Run 1	- .1474	-1.2040	.3368	.7184	.1540	- .3258
2	.2537	-1.2696	.3380	.7309	.1540	- .3370
3	.2799	-1.6402	.3835	.8276	.1756	- .3738
4	.3176	-2.0009	.5892	.9905	.2716	- .5838

TABLE 17B. MEDIAN ESTIMATES. Model 2, Data Set 1

	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
True Values	.2	-2.0	1.0	1.0	.5	-1.0
k = 0						
Run 1	- .1115	-1.1956	.3514	.7253	.1282	- .3438
2	.2640	-1.2771	.3587	.7324	.1402	- .3531
3	.2815	-1.6722	.4308	.8314	.1951	- .4232
4	.3207	-2.0065	.5612	1.0005	.2662	- .5604
k = 1						
Run 1	- .1441	-1.1773	.3227	.7145	.1534	- .3118
2	.2526	-1.2486	.3266	.7227	.1517	- .3112
3	.2800	-1.6442	.3875	.8272	.1831	- .3801
4	.3226	-2.0019	.5388	.9946	.2683	- .5275

TABLE 17C. MEDIAN ESTIMATES. Model 1, Data Set 2

	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
True Values	.2	-2.0	1.0	1.0	.5	-1.0
k = 0						
Run 1	- .0872	-1.3170	.4275	.7625	.2043	- .4160
2	.2480	-1.3805	.4381	.7822	.2133	- .4273
3	.2629	-1.7251	.5173	.8824	.2597	- .5004
4	.2896	-2.0064	.6504	1.0201	.3282	- .6494
k = 1						
Run 1	.1200	-1.8441	.8726	.9515	.4206	- .8667
2	.2105	-1.8588	.8768	.9515	.4238	- .8632
3	.2160	-1.9456	.9001	.9921	.4380	- .8883
4	.2100	-2.0007	.9283	1.0268	.4715	- .9253

TABLE 17D. MEDIAN ESTIMATES. Model 2, Data Set 2

	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
True Values	.2	-2.0	1.0	1.0	.5	-1.0
k = 0						
Run 1	- .0774	-1.3152	.4296	.7662	.2093	- .4216
2	.2477	-1.3824	.4394	.7873	.2164	- .4288
3	.2619	-1.7240	.5174	.8831	.2607	- .4986
4	.2876	-2.0063	.6522	1.0203	.3334	- .6495
k = 1						
Run 1	.0445	-1.7292	.7500	.8611	.3668	- .7498
2	.2227	-1.7403	.7490	.8667	.3735	- .7491
3	.2284	-1.8869	.7986	.9339	.3909	- .7903
4	.2329	-2.0023	.8773	1.0238	.4319	- .8726

TABLE 18A. CONCENTRATIONS. Model 1, Data Set 1

	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
True Values	.2	-2.0	1.0	1.0	.5	-1.0
k = 0						
Run 1	.00	.00	.00	.09	.00	.00
2	.09	.02	.00	.12	.00	.00
3	.08	.88	.00	.70	.01	.00
4	.02	1.00	.06	.99	.07	.06
k = 1						
Run 1	.02	.13	.07	.24	.10	.08
2	.27	.14	.07	.25	.12	.07
3	.13	.51	.08	.51	.12	.08
4	.11	.98	.12	.88	.12	.11

TABLE 18B. CONCENTRATIONS. Model 2, Data Set 1

	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
True Values	.2	-2.0	1.0	1.0	.5	-1.0
k = 0						
Run 1	.00	.01	.00	.12	.00	.00
2	.11	.03	.00	.15	.00	.00
3	.08	.87	.00	.72	.01	.00
4	.01	.99	.06	1.00	.07	.06
k = 1						
Run 1	.01	.16	.06	.28	.10	.06
2	.33	.15	.11	.30	.09	.05
3	.19	.64	.13	.60	.12	.14
4	.17	1.00	.21	.98	.19	.21

TABLE 18C. CONCENTRATIONS. Model 1, Data Set 2

	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
True Values	.2	-2.0	1.0	1.0	.5	-1.0
k = 0						
Run 1	.01	.03	.01	.30	.01	.01
2	.30	.05	.01	.40	.01	.01
3	.23	.98	.01	.83	.04	.01
4	.13	1.00	.08	.92	.22	.09
k = 1						
Run 1	.11	.51	.33	.49	.29	.30
2	.53	.56	.33	.53	.30	.31
3	.43	.81	.34	.67	.32	.32
4	.33	.99	.48	.77	.42	.47

TABLE 18D. CONCENTRATIONS. Model 2, Data Set 2

	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
True Values	.2	-2.0	1.0	1.0	.5	-1.0
k = 0						
Run 1	.01	.02	.01	.34	.01	.01
2	.34	.06	.01	.40	.01	.01
3	.25	.98	.01	.83	.04	.01
4	.15	1.00	.09	.93	.24	.09
k = 1						
Run 1	.09	.51	.31	.60	.27	.30
2	.59	.56	.33	.64	.28	.31
3	.44	.88	.37	.78	.33	.35
4	.36	1.00	.51	.84	.44	.49

TABLE 19A. DECENTRALIZATIONS, D_1. Model 1, Data Set 1

	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
True Values	.2	-2.0	1.0	1.0	.5	-1.0
k = 0						
Run 1	.86	.00	.00	.00	.10	.00
2	.00	.00	.00	.00	.06	.00
3	.00	.00	.00	.00	.01	.00
4	.00	.00	.00	.00	.01	.00
k = 1						
Run 1	.73	.15	.30	.10	.28	.30
2	.04	.13	.30	.09	.29	.30
3	.05	.09	.29	.08	.28	.29
4	.14	.00	.22	.00	.19	.19

TABLE 19B. DECENTRALIZATIONS, D_1. Model 2, Data Set 1

	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
True Values	.2	-2.0	1.0	1.0	.5	-1.0
k = 0						
Run 1	.90	.00	.00	.00	.06	.00
2	.00	.00	.00	.00	.04	.00
3	.00	.00	.00	.00	.02	.00
4	.00	.00	.00	.00	.00	.00
k = 1						
Run 1	.76	.04	.21	.01	.21	.24
2	.01	.02	.21	.01	.21	.24
3	.02	.01	.17	.01	.18	.20
4	.04	.00	.12	.00	.11	.11

TABLE 19C. DECENTRALIZATIONS, D_1. Model 1, Data Set 2

	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
True Values	.2	-2.0	1.0	1.0	.5	-1.0
k = 0						
Run 1	.92	.00	.00	.00	.00	.00
2	.00	.00	.00	.00	.00	.00
3	.00	.00	.00	.00	.00	.00
4	.00	.00	.00	.00	.00	.00
k = 1						
Run 1	.19	.05	.06	.04	.06	.06
2	.07	.04	.06	.04	.06	.06
3	.08	.01	.05	.03	.05	.05
4	.09	.01	.03	.01	.03	.03

TABLE 19D. DECENTRALIZATIONS, D_1. Model 2, Data Set 2

	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
True Values	.2	-2.0	1.0	1.0	.5	-1.0
k = 0						
Run 1	.93	.00	.00	.00	.00	.00
2	.00	.00	.00	.00	.00	.00
3	.00	.00	.01	.00	.00	.00
4	.00	.00	.00	.00	.00	.00
k = 1						
Run 1	.32	.01	.04	.01	.03	.04
2	.01	.01	.04	.01	.03	.04
3	.01	.01	.03	.01	.03	.03
4	.02	.00	.01	.00	.02	.02

TABLE 20A. DECENTRALIZATIONS, D_2. Model 1, Data Set 1

	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
True Values	.2	-2.0	1.0	1.0	.5	-1.0
k = 0						
Run 1	.4307	1.1152	.8914	.4375	.5430	.8960
2	.1320	.9888	.8791	.4144	.5364	.8847
3	.1971	.4525	.8030	.2866	.5229	.8144
4	.2522	.0370	.6990	8.9980	.5105	.6991
k = 1						
Run 1	5.3691	11.6320	9.8294	4.2087	4.9783	10.2095
2	.6767	10.4872	9.4426	4.0684	4.7820	9.8062
3	16.5812	84.8750	142.500	51.7500	69.3750	146.750
4	21.1219	3.6055	119.625	12.9687	36.0625	124.125

TABLE 20B. DECENTRALIZATIONS, D_2. Model 2, Data Set 1

	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
True Values	.2	-2.0	1.0	1.0	.5	-1.0
k = 0						
Run 1	.4392	1.1508	.9239	.4390	.5517	.9305
2	.1231	1.0224	.9112	.4163	.5383	.9186
3	.1832	.4702	.8362	.2852	.5034	.8494
4	.2375	.9955	.6991	.1491	.4864	.6926
k = 1						
Run 1	1.9221	6.3374	4.8960	1.8803	3.5717	5.1255
2	.5839	4.2084	3.7168	1.5702	1.3161	3.9012
3	.4301	3.0183	5.0313	1.9336	1.8468	5.2290
4	1.2046	.2658	6.6187	.8783	2.0206	6.8579

TABLE 20C. DECENTRALIZATIONS, D_2. Model 1, Data Set 2

	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
True Values	.2	-2.0	1.0	1.0	.5	-1.0
k = 0						
Run 1	.4393	1.0532	.8596	.4589	.3148	.8814
2	.0913	.9562	.8559	.4409	.4776	.8787
3	.1344	.4457	.7865	.3077	.4440	.8137
4	.1854	.0284	.6217	.2705	.3714	.6291
k = 1						
Run 1	8.6519	22.2939	18.2036	10.6636	10.0134	18.4546
2	1.1507	14.1727	12.9283	5.7277	6.5019	13.0192
3	5.5239	15.5918	33.9141	19.9561	20.2920	36.6523
4	1.8471	3.9895	3.1985	4.5707	1.7248	10.4780

TABLE 20D. DECENTRALIZATIONS, D_2. Model 2, Data Set 2

	β_{12}	β_{13}	β_{14}	γ_{11}	γ_{12}	γ_{13}
True Values	.2	-2.0	1.0	1.0	.5	-1.0
k = 0						
Run 1	.4396	1.0512	.8585	.4597	.4798	.8784
2	.0900	.9539	.8547	.4417	.4761	.8757
3	.1323	.4448	1.3535	.3062	.4436	.8116
4	.1860	.0278	.6241	.2758	.3695	.6295
k = 1						
Run 1	1.1254	3.8092	2.8319	2.2962	1.7484	2.8931
2	.5384	3.6194	3.0761	2.4159	6.4481	3.1454
3	.4781	3.8208	2.9198	1.5692	1.6781	3.1316
4	.3746	.0549	1.4138	.4584	.7800	1.4929

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