

PAYMENT OF DIVIDEND BY INSURANCE COMPANIES

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## 1. Introduction

1.1 An insurance contract gives the insured a right to claim certain amounts of money from the insurance company if some specified events should occur. If insurance shall serve its very purpose, the insured must be virtually certain that the company is able to meet its contractual obligations. This means that an insurance company must keep "special reserves" or "surplus funds" in addition to its technical reserve, which by definition is equal to the mathematical expectation of claim payments under the contracts in the company's portfolio.

An insurance company will usually acquire its surplus funds by adding a "safety loading" to the premiums, so that these funds can be expected to grow as time goes by. In practice, most insurance companies will pay out a dividend — to shareholders or policy holders as the case may be — when surplus funds in some sense become "larger than necessary." However theory has so far not been able to lay down any really satisfactory rules as to how large these funds should be before the company can consider paying dividends. In this paper, we shall try to bridge the gap between present theory and current insurance practice.

1.2 We shall begin by considering the following situation:

At the end of an underwriting period an insurance company finds itself with a surplus  $S$ , and considers the possibility of paying out an amount  $s$  as dividend. The amount not distributed as dividend,  $S - s$  will be kept by the company as a "special reserve" during the following underwriting period. The purpose of this reserve is to enable the company to meet contingencies in later periods, and to

safeguard future dividend payments.

The problem is to determine  $s$  when  $S$  is given. This clearly means that the company must balance the desirability of paying a high dividend today against the desirability of being able to pay dividends in the future.

1.3 The problem we have outlined is obviously of central importance in insurance, but it has seldom been explicitly formulated and discussed in actuarial literature.

The usual approach in modern actuarial theory is to consider  $S$  as given at some point of time, for instance when the company enters into business, and then calculate Lundberg's probability of ruin. If this probability turns out to be too high, either  $S$  must be increased, or the company has to make reinsurance arrangements so that the ruin probability is brought down to an acceptable level.

However Lundberg's probability of ruin is calculated under the assumption that the surplus earned by the company is retained in the special reserve during all future periods. Only under this assumption will the probability of ruin be smaller than one. This means, of course, that the special reserve must be allowed to increase to infinity. Any dividend policy which keeps the special reserve finite makes it virtually certain that the company will be ruined sometime in the future.

A theory of risk which cannot accommodate the fact that insurance companies sometimes pay dividends is obviously unrealistic. In the following we shall try to outline a more realistic theory. In doing so, we shall draw on mathematical techniques which have been developed fairly recently, and which so far do not seem to have been applied to problems in insurance.

## 2. A Simple Solution

2.1 In paragraph 1.2, we mentioned that the purpose of maintaining a "special reserve" is not just to avoid ruin, but also to safeguard future dividend payments. The latter purpose is clearly the more general, since dividend payments necessarily will stop if the company is ruined.

These considerations naturally lead us to make the tentative assumption that an insurance company seeks to maximize the expected value of dividend payments, discounted in a suitable manner. We shall formulate the problem as follows:

Let  $S$  be the surplus held by the company at the beginning of period 1, and let  $s_t$  be the dividend paid at the end of period  $t$ . The problem of the company is then to determine the sequence  $s_0, s_1 \dots s_t \dots$  which maximizes

$$E\left[\sum_{t=0}^{\infty} v^t s_t\right]$$

where  $v$  is a discount factor, which for the time being will be considered as constant over time.

This way of formulating the problem is more in the line of classical actuarial mathematics than the theory of Lundberg.

2.2 The expected value of future dividend payments will clearly depend on the initial surplus  $S$  which the company holds at the beginning of period 1. We shall therefore write  $V(S)$  for the expected discounted value of an optimal dividend sequence, and define this function by

$$(1) \quad V(S) = \max E\left[\sum_{t=0}^{\infty} v^t s_t\right]$$

We shall assume that this function exists, and that it has the following properties:

- (i)  $V(S) = 0$  for  $S < 0$ , i.e., if there is no initial surplus, the company is not allowed to operate, and no dividends can ever be paid
- (ii)  $V(S)$  is continuous everywhere, except possibly for  $S = 0$

2.3 We now consider an insurance company which in each period holds a portfolio of insurance contracts with claim distribution  $F(x)$ , i.e.,  $F(x)$  is the probability that the total amount of claims in a period shall not exceed  $x$ .

We shall assume that  $F(x) = 0$  for  $x < 0$ , and that the derivative  $f(x) = F'(x)$  exists, and is continuous for all non-negative  $x$ .

The net premium of this portfolio is by definition

$$P_1 = \int_0^{\infty} x f(x) dx$$

We shall assume that the gross premium collected by the company is

$$P = P_1 + P_2$$

where  $P_2$  is a positive safety loading.

2.4 We now assume that our company at the end of an arbitrary period has a surplus  $S$ . If the company pays out a dividend  $s$ , and then again underwrites the portfolio just described, its funds at the end of the following period will be

$$S - s + P - x$$

For an arbitrary value of  $s$ , we must have

$$V(S) \geq s + v \int_0^{\infty} V(S - s + P - x) f(x) dx$$

$V(S)$  is by definition the discounted expected value of an optimal sequence of dividend payments. The inequality merely states that an arbitrary payment  $s$  cannot increase  $V(S)$ . If, however,  $s$  is an optimal payment, the sign of equality must hold. Hence the function  $V(S)$ , if it exists, must satisfy the equation

$$(2) V(S) = \max_{0 \leq s \leq S} [s + v \int_0^{\infty} V(S - s + P - x) f(x) dx]$$

2.5 We now consider the function

$$(3) w(s) = s + v \int_0^{\infty} V(S - s + P - x) f(x) dx$$

which we shall write in the following form

$$w(s) = s + U(S - s + P)$$

Differentiating this function we obtain

$$w'(s) = 1 - U'(S - s + P)$$

If  $w(s)$  has a maximum for a value  $s'$  in the interval  $0 < s' < S$ ,  $s'$  must be a root of the equation

$$(4) U'(S - s + P) = 1$$

Hence, if  $s'$  is an optimal dividend payment when the surplus is  $S$ ,  $s' + \sigma$  will be an optimal payment when surplus is equal to  $S + \sigma$ .

It then follows that if  $w(s)$  has a maximum in the interval  $(0, S)$  the optimal dividend payment is determined by an equation of the form

$$(5) \quad s = S - Z \quad \text{if } S > Z$$

$$s = 0 \quad \text{if } S \leq Z$$

This means that the company will let surplus accumulate up to a limit  $Z$ , and distribute as dividend any surplus in excess of  $Z$ .

2.6 If  $w(s)$  has a maximum for  $s'$  in the interval  $0 < s' < S$ , we have

$$V(S) = s' + U(S - s' + P)$$

Differentiating this equation with respect to  $S$ , and observing that  $s'$  is a function of  $S$  by (5), we obtain

$$V'(S) = U'(S - s' + P) + 1 - U'(S - s' + P) = 1$$

Hence we have

$$(6) V(S) = S + C = S + V(0)$$

where  $C = V(0)$  is a constant.

The constant is determined by

$$V(0) = \lim_{S \rightarrow 0} V(S) = v \int_0^{\infty} V(P - x) f(x) dx$$

or by substituting (6):

$$\begin{aligned}
 V(0) &= v \int_0^P (P + V(0) - x) f(x) dx \\
 &= v \left[ (P + V(0))F(P) - \int_0^P xf(x) dx \right]
 \end{aligned}$$

From this we obtain

$$V(0) = v \frac{PF(P) - \int_0^P xf(x) dx}{1 - vF(P)} = v \frac{\int_0^P F(x) dx}{1 - vF(P)}$$

2.7 Inserting (6) in (3) we obtain

$$\begin{aligned}
 w(s) &= s + v \int_0^{S-s+P} (S - s + P - x + V(0)) f(x) dx \\
 &= s + v(S - s + P + V(0)) F(S - s + P) - v \int_0^{S-s+P} xf(x) dx
 \end{aligned}$$

Hence equation (4) takes the form

$$vF(S - s + P) + vV(0) f(S - s + P) = 1$$

or by substituting the expression found for  $V(0)$

$$(1 - vF(P)) (1 - vF(S - s + P)) = v^2 f(S - s + P) \int_0^P F(x) dx$$

For some purposes it is convenient to substitute  $v = (1 + i)^{-1}$  and  $Z = S - s$ , and write the equation as follows

$$(7) \quad \frac{f(Z + P)}{1 + i - F(Z + P)} = \frac{1 + i - F(P)}{P \int_0^P F(x) dx}$$

If this equation in  $Z$  has a unique root, it is easy to verify that (5) is the general unique solution to our problem.

2.8 We shall not undertake a general discussion of equation (7). This will be a tedious task, and hardly worth while in a short paper primarily designed to illustrate a new approach to an old problem. We shall, however, note the following:

- (i) The right-hand side of the equation will decrease with increasing  $P$ , from infinity to  $i$ .
- (ii) The left-hand term will be monotonic and decreasing in  $(Z+P)$  for several important classes of probability distributions. For instance, for  $F(x) = 1 - e^{-ax}$  the term will decrease from  $va$  to zero.

For a given value of  $P$  the following three cases are possible if  $F(x) = 1 - e^{-ax}$ .

- (i) The equation has a unique root  $Z \leq 0$ . In this case a dividend  $s = S - Z$  will be paid, if  $S \geq Z$ . This is, of course, in accordance with (5)
- (ii) There is a unique root  $Z \leq 0$ . In this case no dividend will be paid, which also is in accordance with (5)
- (iii) The equation has no real root. This will happen when  $v$  or  $P$  are small. The intuitive meaning of this case is that the discounted expected value of future dividend payments is so low that the company will not continue its underwriting.

### 3. Discussion of the Simple Solution

3.1 The problem which we have discussed has been studied in several different contexts during the last few years. This is really what we should expect, since the mathematical model we arrived at can be given a number of different interpretations. The model can for instance be applied to inventory situations in which the firm will have to go out of business if demand in any period should exceed stocks.

The model can be interpreted more generally as a "survival game" played against "Nature" or against a player using a random strategy.

The methods which were used in solving the problem are not entirely new. It is, however, only recently that these methods have been systematically studied, and brought together in a fairly self contained theory. The main contributions are due to Bellman, [1], who has introduced the term dynamic programming for this class of methods.



3.2 Shubik and Thompson [9] have studied a special case of our problem. In their model the random variable  $x$  can take only the values  $-1$  and  $+1$ , with probabilities  $p$  and  $1-p$  respectively. For this case they find a simple and elegant solution.

The results of Shubik and Thompson have been generalized by Miyasawa [6] who studied the case where  $F(x)$  is an arbitrary discrete probability distribution. Miyasawa's general solution is extremely complex. This seems to be unavoidable, since the simple solution (7) which we found for the continuous case is next to meaningless for discrete probability distributions. However, it may be premature to conclude that the continuous case is essentially simpler, since we have not carried out a full discussion of our solution. It is clear that (7) for some probability distributions may have multiple solutions, and that this will lead to a number of difficulties.

3.3. The discount factor  $v$  which was introduced in paragraph 2.1 has nothing to do with the interest rate which the company may earn on its funds. Interest is in fact completely irrelevant in our model. When an amount  $s$  has been declared as a dividend, it becomes the absolute property of the share holders (or policy holders), and can not be used to pay future claims against the company. It is this removal of the uncertainty which matters, not whether  $s$  is actually paid out or continues to be administered by the company.

$v$  represents a preference system for the timing of dividend declarations. The assumption that  $v < 1$  means that an early declaration is preferred to a later one, or more loosely that it is considered desirable to remove as much uncertainty as early as possible. The preference may of course be reversed if the later payments are considerably greater than the earlier ones.

The preference for earlier payments implies that an impatience element (Minderschätzung) must exist in the economy. This suggestion was first made by Böhm-Bawerk [2], and it seems to have been accepted by most writers on

economic theory. There has, however, been a considerable controversy over both the implications of the impatience assumption, and the mathematical formulation of this assumption. A penetrating, and essentially non-mathematical study of the problem made by Morgenstern [7] in 1934 sums up the classical position, and exposes it to a severe criticism.

3.4. The most systematic mathematical discussion of the problem appears to be a recent study by Koopmans [5].

Koopmans defines a utility function  $u(s_1, s_2, \dots, s_t, \dots)$  over an infinite sequence of payments. From some innocent looking assumptions about this function he proves that we must have

$$u(s_1, \dots, s_t + \sigma, \dots, s_{t+i}, \dots) > u(s_1, \dots, s_t, \dots, s_{t+i} + \sigma, \dots)$$

for any positive  $i$  and  $\sigma$ . This means that the existence of an impatience element can be derived as a mathematical consequence of some more basic assumptions about timing preferences.

3.5. The most critical of Koopmans' assumptions appears to be his Postulate 3, which in our terms can be stated as follows:

For any positive  $\sigma$  and for any payment sequence  $s_1, \dots, s_2, \dots, s_t, \dots$ , the following inequality must hold

$$u(s_1 + \sigma, s_2, \dots, s_t, \dots) > u(s_1, s_2, \dots, s_t, \dots)$$

It is reasonable to assume that an insurance company will prefer the dividend sequence  $(2, 2, 2, 2, \dots)$  to  $(1, 2, 2, 2, \dots)$ . If, however, the company considers a reduction in dividend as a minor catastrophe, the sequence  $(2, 1, 2, 2, \dots)$  may not be preferred to  $(1, 1, 2, 2, \dots)$ . It is, of course, possible to find reasons for dismissing such preferences as "irrational," but there will still be some doubt about the general validity of the postulate. If the postulate is strengthened, so that we require the inequality

$$u(s_1 + \sigma_1, s_2 + \sigma_2, s_3, \dots, s_t, \dots) > u(s_1, s_2, \dots, s_3, \dots, s_t, \dots)$$

to hold for any payment sequence, and for any positive  $\sigma_1$  and  $\sigma_2$ , it follows from a result by Debreu [4] that the utility function must be of the form

$$u(s_1, \dots, s_t, \dots) = \sum_{t=1}^{\infty} v^{t-1} u(s_t)$$

This is a very strong result, which we shall study in more detail in paragraph 4.2.

3.6. In most economic situations it is natural, or even necessary, to assume that an element of impatience exists. This does not seem to be the case in the situation which we have studied. We will not run into any difficulties by assuming that insurance companies are "patient", i.e., that  $v > 1$ . This will mean that immediate dividend payments are given little importance in relation to the probability of staying in business and being able to pay dividends in the future. For large  $v$  we would expect the company to approach the behaviour assumed in Lundberg's theory, i.e., to retain all underwriting profits in order to increase the probability of survival. This does not, however, appear directly as a limiting case in our model. Our results are valid only if the function  $w(s)$  introduced in paragraph 2.5 has a maximum in the interval  $0 < s < S$ . If the maximum occurs at one of the extremes, the optimal dividend policy may not be determined by (7).

3.7. The case  $v = 1$  has some special interest. In this case the company seeks the dividend policy which will maximize the expected value of the total amount of dividend paid over the time when the company stays in business. The optimal dividend policy is given by (7), and the expected value corresponding to this policy is:

$$(8) \quad V(S) = S + V(0) = S + \frac{\int_0^P F(x)dx}{1 - F(P)}$$

This appears to be related to some results found by Segerdahl [8] by an entirely different approach.

Segerdahl studies the probability distribution of the time when ruin occurs, and finds approximate expressions for the moments of this distribution. His expression for the first moment, i.e., the "mean expectation of life" of the company appears to be an approximation to the exact formula (8), apart from a proportionality factor. Segerdahl gives an exact result for the special case  $F(x) = 1 - e^{-x}$ , and this corresponds to our formula (8) above.

The assumption behind Segerdahl's model is that the sole objective of an insurance company is to stay in business as long as possible, although this necessarily means that no dividend can ever be paid. His purpose is apparently to study the possibilities of increasing the expected life of the company, i.e., increasing the last term of equation (8) by suitable reinsurance arrangements. It is interesting to note that a reinsurance arrangement which is optimal for Segerdahl's company also will be optimal for a company which seeks to maximize expected dividend payments over its life span.

#### 4. Generalization of the Model

4.1. Our simple solution implied that no dividend should be distributed if the surplus of the company was less than an amount  $Z$  determined by (7). If the surplus becomes greater than  $Z$ , the whole excess should immediately be paid out as dividend.

It is obvious that this dividend policy may lead to considerable fluctuations in payments from one period to another. Such fluctuations are considered as undesirable by most insurance companies. In practice, these fluctuations can be reduced by transferring the excess  $S - Z$  to a special

dividend reserve, and make payments from this reserve at the steady or steadily increasing rate which most companies seem to prefer.

However, a sophisticated company may consider fluctuations even in the transfers to the dividend reserve as undesirable. It is therefore worth while seeking to generalize the model so that it explicitly takes into account this preference for stable payments.

4.2. From the result of Debreu [4] referred to in paragraph 3.5 it follows that it will be difficult, if not impossible, to generalize the model by assuming that the discount rate  $v$  varies over time. It is, however, possible to introduce a utility function  $u(x)$ , and define  $V(S)$  by

$$(9) \quad V(S) = \max [u(s) + v \int_0^{\infty} V(S-s + p-x) f(x) dx]$$

which is a generalization of (2).

If  $u(s)$  increases more slowly than  $s$ , the company will attach a lower utility to an immediate dividend increase than to a maintained rate in the future. Generally speaking this means that the current dividend rate will not be increased before it is reasonably certain that the higher rate can be maintained in the future.

As in paragraph 2.5 we can write

$$w(s) = u(s) + U(S-s+P)$$

Differentiating, we obtain the condition

$$u'(s) - U'(S-s+P) = 0$$

which determines the optimal dividend  $s = s(S)$  when the special reserve amounts to  $S$ . Differentiating again, with respect to  $S$ , we obtain

$$u''(s) \frac{ds}{dS} - (1 - \frac{ds}{dS}) U''(S-s+P) = 0$$

or

$$\frac{ds}{dS} = 1 - \frac{u''}{u'' + U''}$$

$u''(s)$  will usually be negative (decreasing marginal utility of money), and the same will hold for  $U''$  provided that the probability distribution is of the kind one usually finds in insurance problems. Hence we will under very general conditions have

$$\frac{ds}{dS} < 1$$

This means that if  $S$  increases in a successful underwriting period, only a part of the increase will be paid out as dividend immediately, the remainder will be kept in the special reserve to safeguard future dividend payments. It therefore seems that the model (9) gives a more realistic representation of company objectives than the simple model studied in Section 2.

4.3. In our model we assumed that claims in any given period were independent of the amount of claims paid in preceding periods. This assumption has been made in practically all previous studies in the theory of risk.

In our model it is fairly easy to relax the independence assumption. If  $f(x_2|x_1)$  is the frequency function of claims in period 2, if claims in period 1 amounted to  $x_1$ , we can define

$$V(S) = \max_{s_1, s_2} [s_1 + v \int_0^{\infty} (s_2 + v \int_0^{\infty} V(S - s_1 - s_2 - x_1 - x_2) f(x_2|x_1) dx_2) f(x_1) dx_1]$$

and determine the dividend payments  $s_1$  and  $s_2$  which satisfy the equation.

We shall not at present discuss this general problem. It is not clear what kind of inter-period dependence one should look for in insurance, and it is doubtful if the problem can be discussed in a rational manner, unless we make specific assumptions about how the premium  $P$  is adjusted to

changes in the probability distribution.

4.4. In our model we have not considered reinsurance. It would obviously be desirable to take this point into consideration, and assume that at the end of each period the company has to make two decisions:

- (i) How much of the surplus from the last period should be paid out as dividend.
- (ii) How should the portfolio underwritten in the next period be reinsured.

If we assume that all kinds of reinsurance cover have their prices in the market, it is possible to give a formal solution to this problem. However, the market price must be determined by demand and supply of reinsurance cover, and it has been proved in a previous paper [3] that stable prices cannot exist in a reinsurance market. Hence the direct approach to the problem will not lead to a realistic solution.

The problem can be analyzed in terms of the general theory of n-person games, but we shall not discuss this possibility in the present paper.

### 5. A Numerical Example

5.1. In the following we shall study a simple numerical example in order to bring out more clearly the meaning of the results in the preceding sections.

We shall assume that

$$F(x) = 1 - e^{-x}$$

Formula (7) can then be written

$$\frac{e^{-(Z+P)}}{i + e^{-(Z+P)}} = \frac{i + e^{-P}}{P - 1 + e^{-P}}$$

In this case we have, since the net premium is equal to 1,  $P = 1 + \lambda$ , where  $\lambda$  is the safety loading. Hence the formula can be written

$$\frac{e^{-(Z + P)}}{i + e^{-(Z + P)}} = \frac{i + e^{-P}}{\lambda + e^{-P}}$$

or solved with respect to  $e^{-Z}$

$$(11) \quad e^{-Z} = i \frac{ie^1 + \lambda + 1}{\lambda - i}$$

5.2. From the formula (11) in the preceding paragraph we note the following:

- (i) If  $i = 0$ , the only solution is  $Z = \infty$ . As  $Z = S - s$  this means that the company will not pay any dividend as long as the surplus  $S$  remains finite.
- (ii) If  $\lambda < i$  there is no real solution. This means that the company will not stay in the insurance business, either because expected profits are too low, or because the company discounts future profits at a too high rate.
- (iii) If the equation has a finite positive solution  $Z$ , this can obviously be interpreted as the surplus which the company considers necessary before any dividends can be paid.

Table 1 gives the value of  $Z$  for some selected values of  $i$  and  $\lambda$ .

Table 1  
Surplus necessary before payment of dividends

$\lambda \backslash i$	0.1	0.15	0.20
0	$\infty$	$\infty$	$\infty$
0.01	2.17	2.61	2.91
0.02	1.34	1.81	2.13
0.03	0.76	1.29	1.64
0.04	0.29	0.89	1.26
0.05	-	0.55	0.94
0.06	-	0.23	0.66
0.07	-	-	0.41
0.08	-	-	0.17



## 6. Conclusions

6.1. It is generally recognized that subjective elements must play an important part in any theory of risk. There can be no universal or objectively "correct" answer to questions as to what reserve funds an insurance company should keep, or what reinsurance arrangements it should seek. It is, however, possible to find solutions which are optimal when the subjective elements are given, and spelt out in an unambiguous manner. This latter problem is not always easy. The subjective elements are usually referred to in a vague manner as "attitude to risk", "degree of prudence", etc., and cannot easily be expressed in an operational form.

6.2. In the preceding paragraphs we have shown that the attitude to risk can be completely determined by the two elements

(i) The discount factor  $v$  which is to be applied to future dividend payments.

(ii) The utility function  $u(s)$

It is, however, possible to use other elements, which may look different, but which must be mathematically equivalent to the two considered in this paper.

An insurance company can for instance apply the rule of thumb that it will keep a special reserve so that the probability of ruin in the next period is just equal to a certain number  $\pi$ . This company will have a well defined attitude to risk, or a "risk policy", defined by a single number  $\pi$ . However, it is clear that this risk policy can also be defined by a linear utility function  $u(s) = s$ , and a discount rate  $v$  determined by formula (7).

6.3. More generally we can assume that an insurance company divides profits earned between dividends and special reserves so that a generalized utility function  $U(s, \pi)$  is maximized. This function will define the company's risk policy, but this policy can also be determined by a pair  $(v, u(s))$ .

In general a consistent risk policy can be defined in many different ways, so that the policy-makers of an insurance company have a considerable choice of expressions when they want to spell out the objectives of the company. It appears, however, that definition by means of a discount factor  $v$  and a utility function  $u(s)$  is the most convenient form when it comes to determining the actions which are optimal under the given objectives.

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