A SPECTRAL ANALYSIS OF JAPANESE ECONOMIC TIME SERIES SINCE THE 1880's*

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ABSTRACT

Power spectra and cross-spectra of Japanese economic annual series since 1879 or 1887 are estimated. The power spectral estimates suggest that the long swings and business cycles are meaningful phenomena in many of the time series.

Reference cycles of long swings and business cycles are determined separately from the remodulated series of the rate of change of the individual annual series at the frequencies of 1/25 cycle per year and of 1/6.5 cycle per year. They provide valuable information on Japanese economic fluctuations since the 1880's. The Rostowian stages of growth are considered in terms of these reference cycles.

The cross-spectral estimates clarify the lead-lag relationships among the series. We find three key factors, loans and discounts of all banks, exports, and government consumption leading the other series and driving the rapid growth. Mechanisms of growth processes are explained by our lead-lag relationships. The transfer of labor forces among the industries and some characteristics of investment and private consumption are also considered by means of the cross-spectral estimates.

A SPECTRAL ANALYSIS OF JAPANESE ECONOMIC TIME SERIES SINCE THE 1880's

I. Introduction

The Japanese economy grew at a remarkably rapid rate during the long period after the Meiji restoration. This growth was not always smooth nor steady and it is with the fluctuating rhythm of this growth that we are mainly concerned in this paper. The period considered begins in the 1880's. By this time Japanese modern industrialization had begun. Most of the data are also available by this time.

As the period of the annual series used in this paper spans the years from 1878 to 1961, data cover only 84 years. A longer period would have been more satisfactory in making estimates, but experience has shown that this length of data can provide useful information. In general, spectral analysis of time series assumes that the basic series of which spectral estimates are being made is generated by a stationary stochastic process. However, the time series with which we are concerned in this study cannot be taken as realizations of a stationary process. They represent the historical facts of a given period, and are not samples taken from an unlimited time series without history. Consequently the spectrum should be considered as a pseudo spectrum*, although we shall use the term "spectrum" instead of "pseudo spectrum" for simplicity.

We estimate the power spectrum of the series of the rate of change of economic time series. The spectral estimates allow us to examine the existence of long swings, major cycles and minor cycles in

* M. Hatanaka and M. Suzuki [10].

The Basic Series Considered in This Paper (cont.)

13.	Import price index	IMP	1878-1961	[6] [2]
14.	Export price index	EXP	1878-1961	[6] [2]
15.	Terms of trade	TT	1878-1961	
16.	Money stock	MS	1878-1961	
17.	Cash currency	CC	1878-1961	[3] [1]
18.	Deposit currency	DC	1878-1961	[3] [1]
19.	Loans and discounts of the Bank of Japan	LDBJ	1884-1961	[1]
20.	Loans and discounts of all banks	LDAB	1884-1961	[3] [1]
21.	Deposits of all banks	DEPAB	1884-1961	[3] [1]
22.	Discount rate of the Bank of Japan	DRBJ	1884-1961	[1]
23.	Primary industry products	P1	1878-1961	[6] [2]
24.	Secondary industry products	P2	1878-1961	[6] [2]
25.	Tertiary industry products	P3	1878-1961	[6] [2]
26.	Secondary and tertiary industry products	P23	1878-1961	
27.	Labor force in primary industries	L1	1878-1961	[6] [7]
28.	Labor force in secondary industries	L2	1878-1961	[6] [7]
29.	Labor force in tertiary industries	L3	1878-1961	[6] [7]
30.	Labor force in secondary and tertiary industries	L23	1878-1961	
31.	Labor force, total	LT	1878-1961	[6] [7]
32.	Population	PO	1878-1961	[8]
33.	Production Index of Nagoya College of Commerce	PINCC	1868-1936	[4]

TABLE I - THE BANDS OF LOCAL PEAKS

[J] means that the J-th band has a local peak in the spectrum of the series of the rate of change

Set A (23 series)

Period	48	24	16	12	9.8	8	6.9	6	5.3	4.8	4.4	4	3.7	3.4	3.2	3	2.8	2.6	2.5	2.4	2.3
NNIc	[1]	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	[18]	19	20	21
PCc	1	[2]	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
DIc	[1]	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	[18]	19	20	21
GCc	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	[19]	20	21
IMc	[1]	2	3	4	5	6	7	8	9	10	11	[12]	13	14	15	16	17	[18]	19	20	21
EXc	1	[2]	3	4	5	6	7	8	9	10	11	12	[13]	14	15	16	17	18	19	20	21
BTc	1	[2]	3	4	5	6	7	8	9	[10]	11	12	13	14	[15]	16	17	18	19	20	21
GP	[1]	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	[18]	19	20	21
CP	[1]	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	[18]	19	20	21
PP	[1]	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	[18]	19	20	21
IMP	1	[2]	3	4	5	6	7	8	9	10	11	12	13	14	15	[16]	17	18	19	20	21
EXP	1	[2]	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	[19]	20	21
TT	1	[2]	3	4	5	6	7	[8]	9	10	11	12	13	[14]	15	16	17	18	[19]	20	21
MSc	1	2	3	4	5	6	7	8	9	10	11	12	13	14	[15]	16	17	18	19	20	21
CCc	1	2	3	4	5	6	7	8	9	10	11	12	13	14	[15]	16	17	[18]	19	20	21
Dcc	[1]	2	3	4	5	6	7	8	9	10	11	12	13	[14]	15	16	17	18	19	20	[21]
LDBJc	1	2	[3]	4	5	6	7	8	[9]	10	11	12	13	14	15	16	[17]	18	19	[20]	21
LDABc	[1]	2	3	4	5	6	7	8	9	10	11	12	[13]	14	15	16	17	18	19	20	21
DEPABc	[1]	2	3	4	5	6	7	8	9	10	11	12	[13]	14	15	16	17	18	19	20	21
Plc	1	[2]	3	4	5	6	7	8	9	10	11	12	13	14	15	[16]	17	18	19	20	21
P2c	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
P3c	[1]	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	[18]	19	20	21
P23c	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	[17]	18	19	20	21
No. of Series	10	7	1	1	0	0	0	1	2	1	0	1	3	3	4	2	2	8	3	1	1

year or, the 2m/j years per cycle. The period in the first row of Table I shows the length of years per cycle for $m = 24$.

The weighted first difference is used to pre-whiten the rate-of-change series in the estimation. The weights are chosen adequately for each series. The spectrum is calculated by recoloring the pre-whitened spectral estimate.

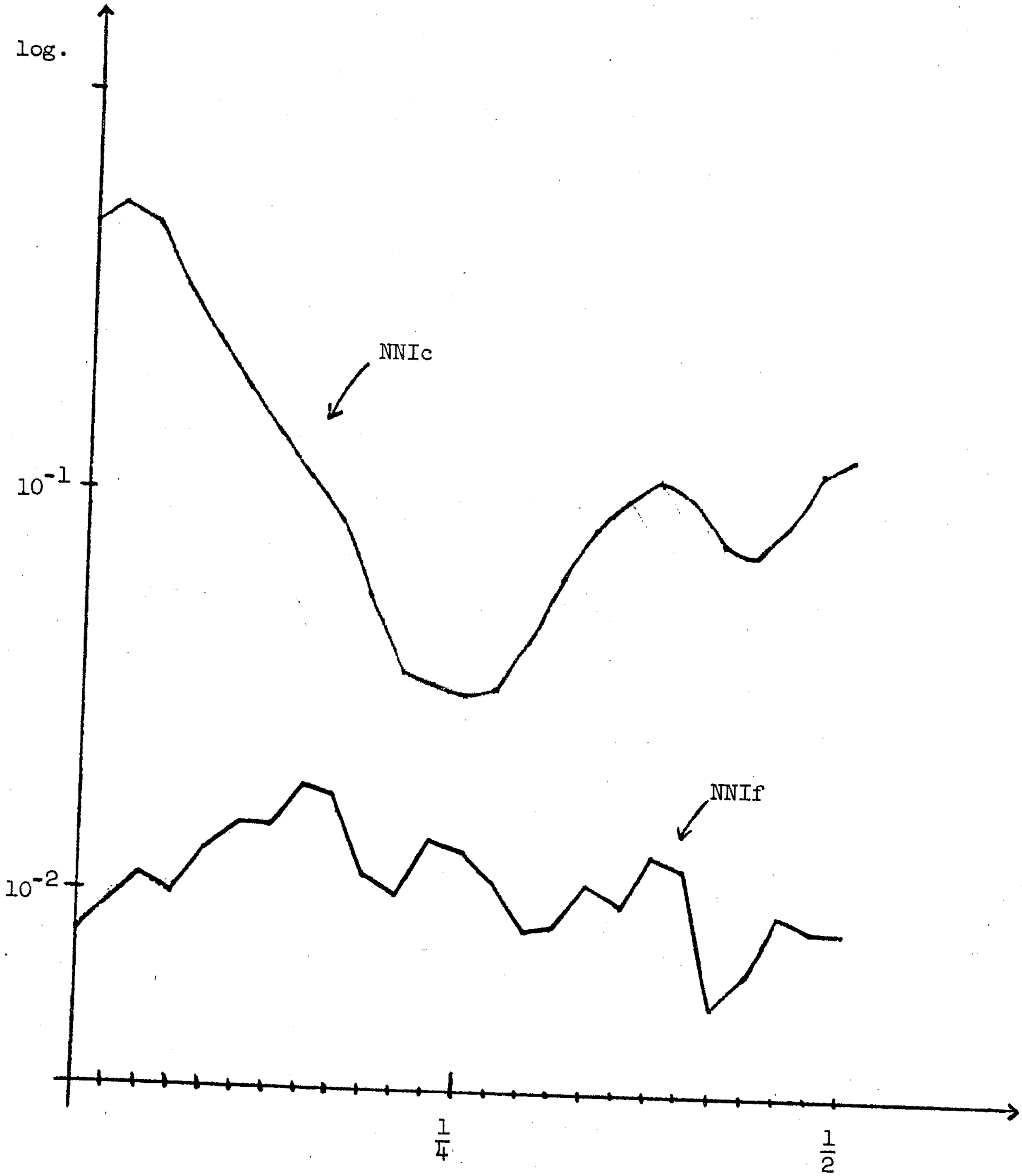
From the last row in Table I, we can determine how many series have their local peaks in each band. Table II shows the groups of the frequency bands in which the local peaks are concentrated.

TABLE II

THE GROUPS OF THE CONCENTRATED BANDS

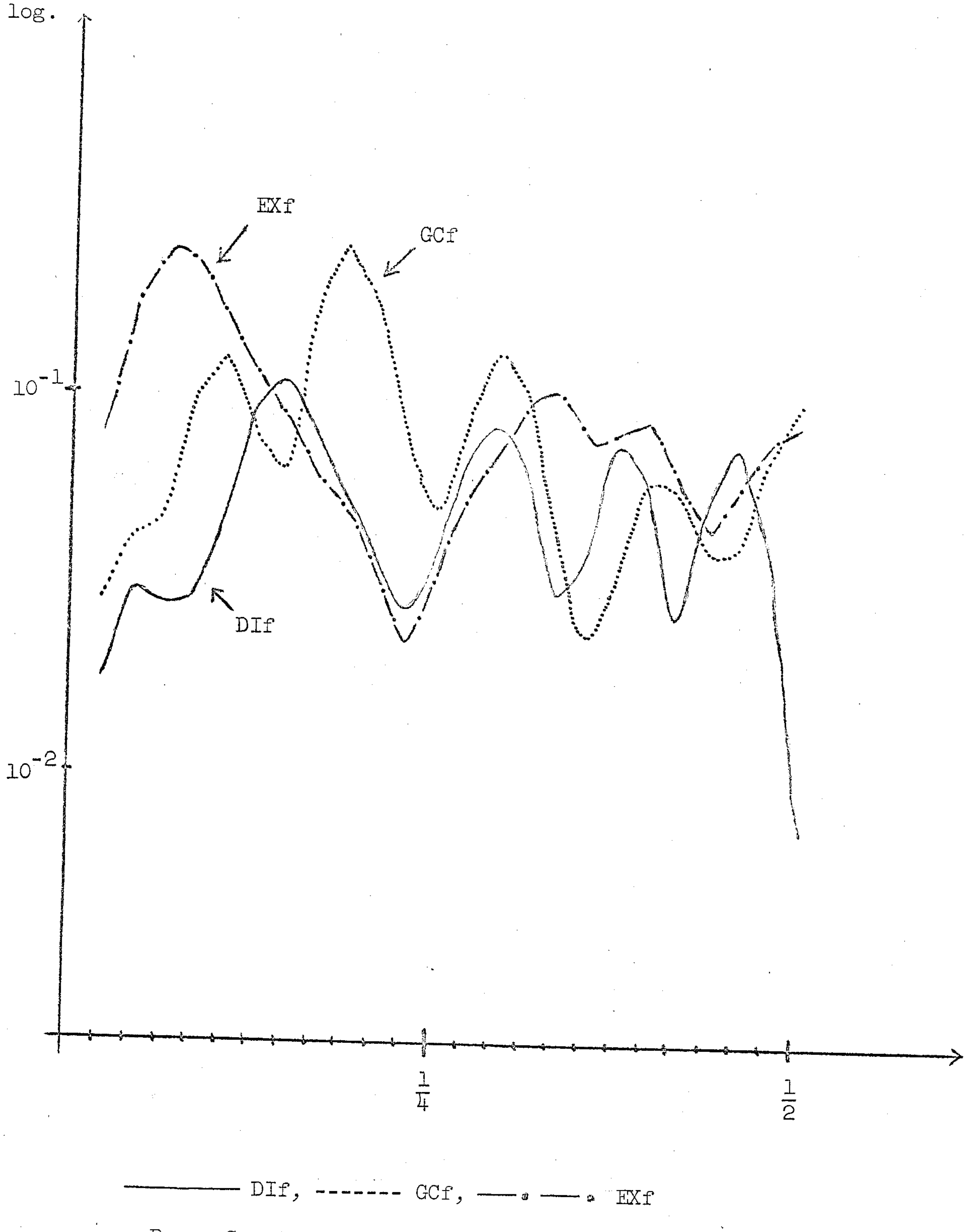
Set	Group	Band	Number	Period (y/c)
A	I	1,2	17	50-20
	V	17,18,19	13	2.9-2.5
B	I	2	14	32-19
	II	7,8,9	13	7.4-5.6
	III	11,12	12	4.5-4.0
	IV	14,15,16	10	3.4-3.0
	V	17,18,19	13	2.8-2.5

Groups I and II correspond to the long swings of more than 20 years and the business cycles of 8-6 years respectively. Groups III, IV and V as a whole correspond to the minor cycles.



Power Spectra of NNIc and NNIf, m = 24.

Figure 1



Power Spectra of DIf, GCf and EXf, $m = 24$

Figure 2

19th frequency bands. The series in fixed prices have, in general, smaller power spectra than the series in current prices and have some small peaks.

The spectra of domestic investment (DIf), government consumption (GCf) and exports (EXf) have very strong power and some sharp peaks as shown in Figure 2. These were the fundamental factors driving the Japanese economic growth.

Exports from Japan have been very unstable, because Japan has been in a position of a marginal supplier in world trade for a long time. Further, the strong fluctuations of exports and imports with different phases caused fluctuations in the balance of trade, the spectrum of which has strong power in the low frequency domain, and sharp peaks at 20 y/c, 3.7 y/c and 2.5 y/c . Private consumption (PCf) has a weaker spectrum than the others and only one sharp peak at 4.5 y/c. PCf was a factor of stabilizing NNIf or GDPf, both spectra of which are very weak and almost the same.

The spectra of indices of the general price (GP), the consumption price (CP), the product price (PP), the export price (EXP), and the import price (IMP) have similar shapes to that of NNIc but more strong powers than that. Also the spectra of money stock (MSc), cash currency (CCc), loans and discounts of all banks (LDABc) and deposits of all banks (DEPABc) have similar shapes to NNIc. But loans and discounts of the Bank of Japan (LDBJc) has a very different shape and is stronger than other monetary series. The spectrum of the discount rate of the Bank of Japan is almost white noise. The products of primary, secondary, and tertiary industries (P1f, P2f, P3f) and the products of

III. Reference Cycles of Long Swings and Business Cycles

i. The determination of reference cycles by digital filtering

Long swings, major cycles and minor cycles were found in the power spectra in Section II. To visualize these cycles, let us remodulate the series of the rates of change at the 25-year cycle and the 6.5-year cycle.

We filter x_t at the frequency ω_o by the following method. By multiplying x_t by $\cos \omega_o t$ and $\sin \omega_o t$ and applying a low pass filter, F, we have,

$$z_t' = F(x_t \cos \omega_o t)$$
$$z_t'' = F(x_t \sin \omega_o t) \quad .$$

Then the remodulated series at the frequency ω_o can be obtained by

$$y_t = z_t' \cos \omega_o t + z_t'' \sin \omega_o t \quad .$$

For the low pass filter, F, we use a simple moving average of 9 years applied twice in the remodulation at the frequency of 1/25 c/y, and of 7 years at the frequency 1/6.5 c/y. The transfer functions of these filters are:

$$L_1(\omega) = \frac{1}{81}(9 + 16 \cos \omega + 14 \cos 2\omega + 12 \cos 3\omega + 10 \cos 4\omega + 8 \cos 5\omega + 6 \cos 6\omega + 4 \cos 7\omega + 2 \cos 8\omega)$$

$$L_2(\omega) = \frac{1}{49}(7 + 12 \cos \omega + 10 \cos 2\omega + 8 \cos 3\omega + 6 \cos 4\omega + 4 \cos 5\omega + 2 \cos 6\omega) \quad .$$

Now let us find the reference cycle of the long swing by the NBER method from the series of the rate of change remodulated at the 25-year cycle.

First, we compute the percentage, C_t, of remodulated series undergoing cyclical expansion at time t over the total series for each group. As Japanese economic time series show a sharp upward trend in the whole period, the long swings of these series do not decline except for a few years. That is, the long swings of the rate-of-change series fluctuate in the domain of positive value except for a few years. It is natural under the strong upward trend to regard a year as recession if the rate of change is falling, even if it is positive. Needless to say, the phase of the rate-of-change series is different from the phase of the actual value series. The peak in the rate of change comes early in expansion and the trough early in recession while the actual-value series rises and falls. The turning year of the actual value series is the year in which the rate-of-change series cross the zeroline. Second, we get the reference cycle of long swing of the rate of change by accumulating $(C_t - 50)$, which is shown in Figure 3 for set A and set B. Third, we get the percentage of the series which have a negative rate of change at t and find the turning year at which the negative rate dominates the positive rate. The lines ab and a'b' show the hypothetical zero lines under which the reference cycles of rate of change take the negative value for set A and set B respectively.

The major business cycle component of the rate of change, which is obtained by the remodulation at 6.5 y/c fluctuates cyclically

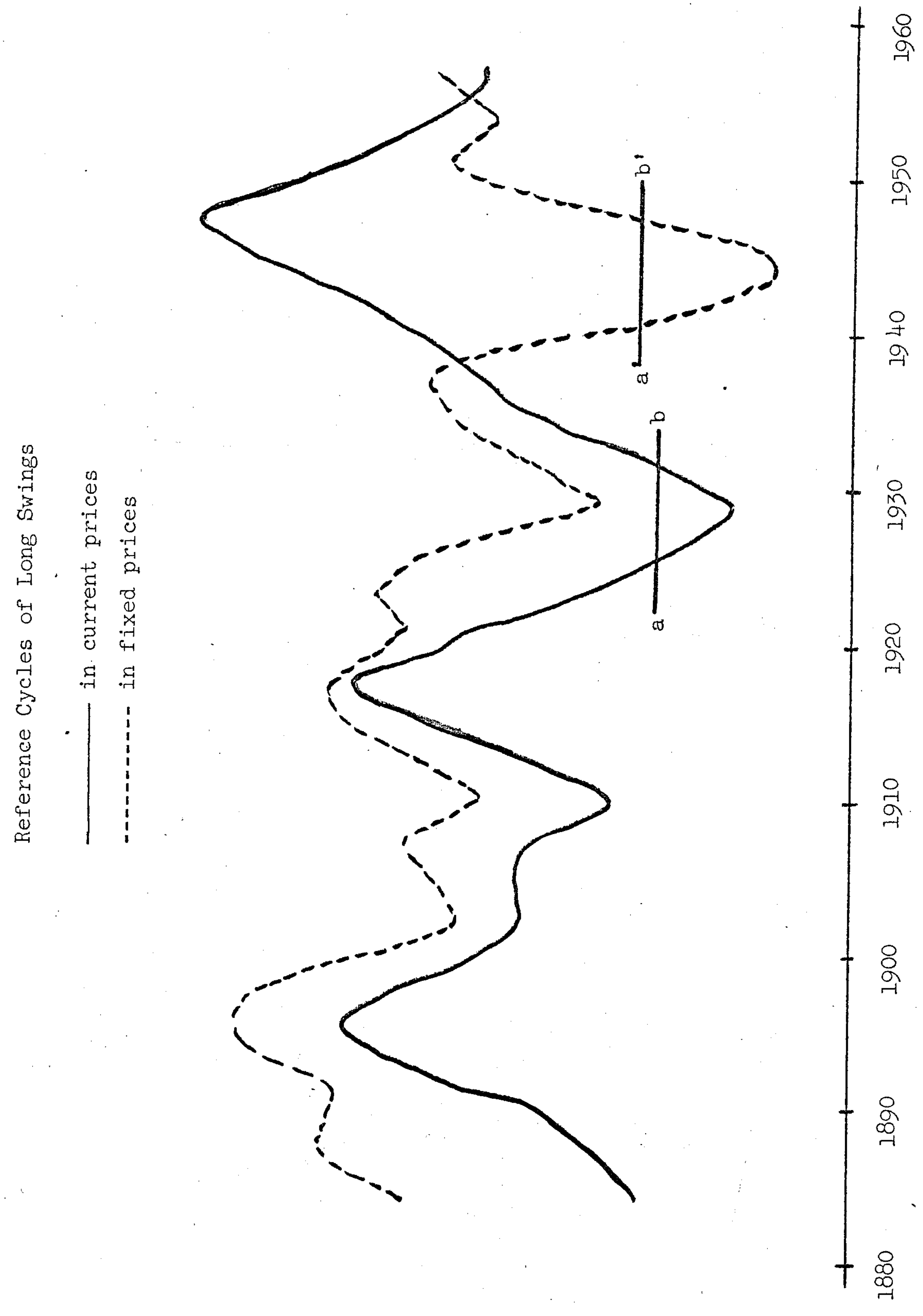

Reference Cycles of Long Swings
in current prices
in fixed prices
a
b
a
b'
1880
1890
1900
1910
1920
1930
1940
1950
1960

Figure 3

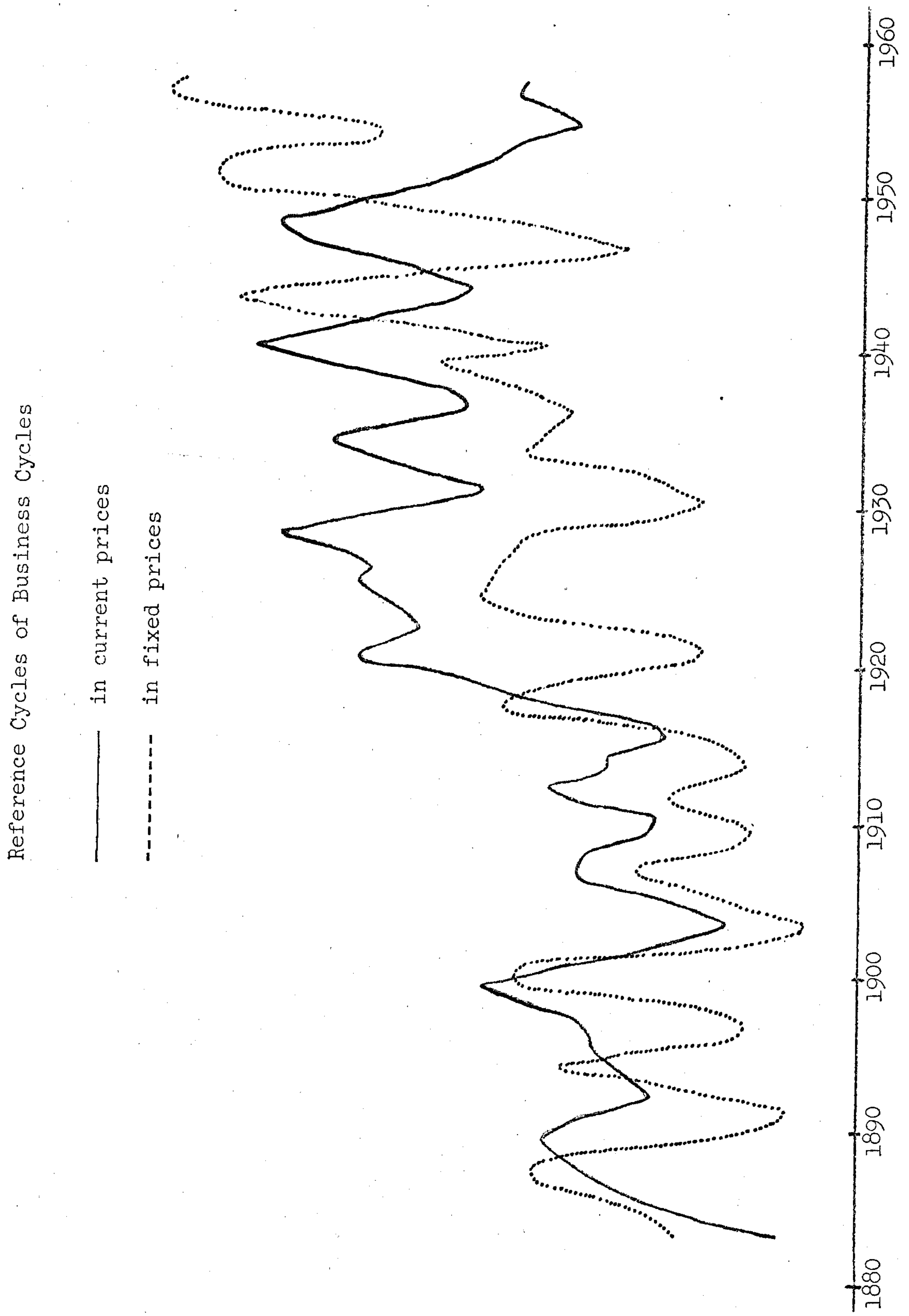

Reference Cycles of Business Cycles
in current prices
in fixed prices
1880
1890
1900
1910
1920
1930
1940
1950
1960

Figure 4

Long swings of the ratio of gross domestic investment to gross domestic product. (DIf/GDPf)

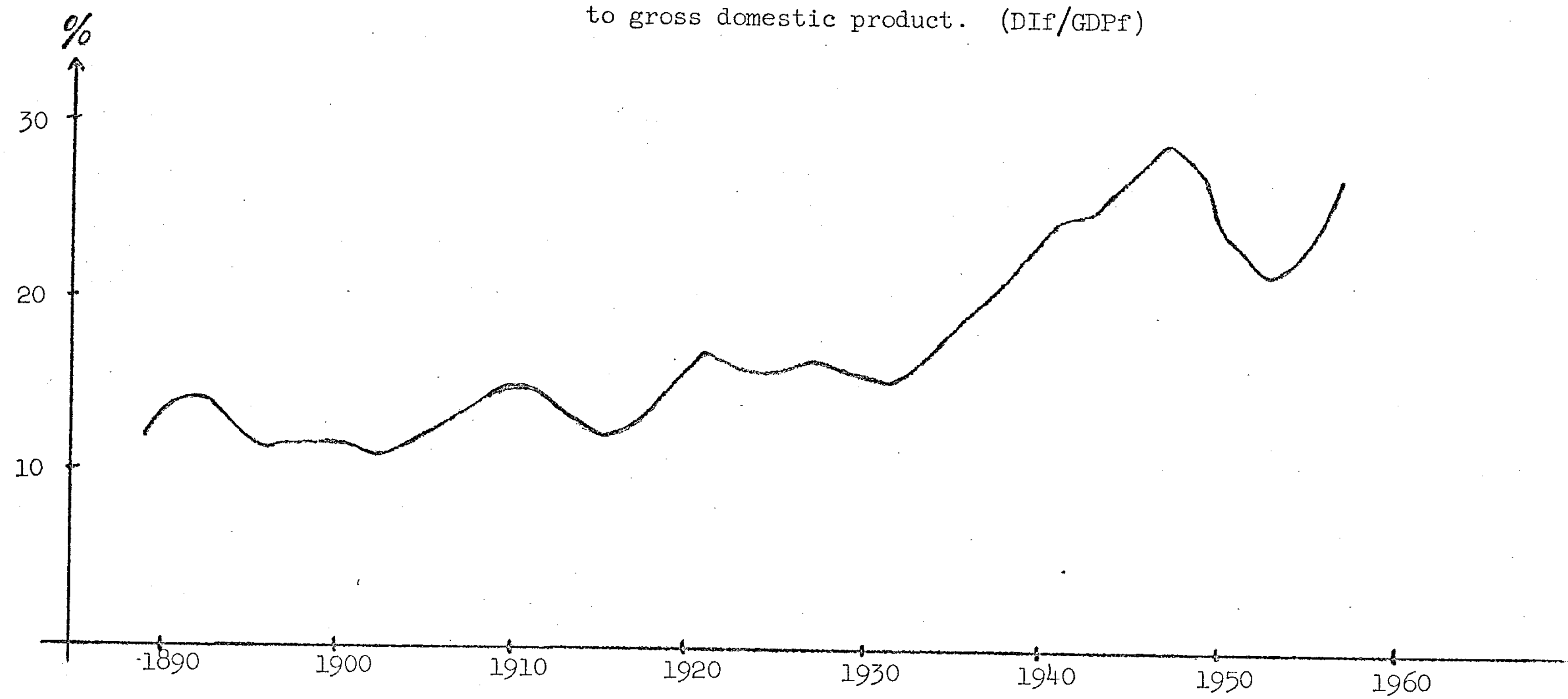


Figure 5

TABLE III

Period	Year	L.S.(c.p.)	L.S.(f.p.)	B.C.(c.p.)	B.C.(f.p.)
I	1884	trough	trough	trough	trough
	1886		peak		peak
	1889			peak	
	1891		trough		trough
	1892			trough	
	1894-95	peak	peak		peak
	1896				trough
	1899			peak	peak
	1902-03	plateau	trough	trough	trough
	1907		peak	peak	peak
	1909-10	trough	trough	trough	trough
II	1911			peak	peak
	1914				trough
	1916			trough	
	1917-18	peak	peak		peak
	1920-21		trough	peak	trough
	1922			trough	
	1923-24		peak		peak
	1926	cross zeroline			
	1928			peak	
	1929-30	trough	trough	trough	trough
	1932	cross zeroline			
	1933-35		peak	peak	peak
	1936			trough	trough
	1939				peak
	1940		cross zeroline	peak	trough
III	1943				peak
	1944		trough	trough	
	1946	peak			trough
	1947		cross zeroline		
	1948			peak	
	1951-52		peak		peak
IV	1954		trough	trough	trough

After the trough of 1910, the process trades were widely developed and Japan progressed with self-sustained growth. The stage of the take-off was complete in this first period of our long swings.

The stage of the take-off mentioned by Rostow is from 1879 to 1900.* We think the period of 1884-1910 is preferred as the stage of take-off from the point of view of our long swings. In the first period of long swings, there were 4 business cycles, the average period of which was 6.5 years.

The second period considered is 1910-1940. After the depression of 1910, or, after the take-off, Japan expanded her market in foreign countries and reached the peak of her expansion at the first World War, which brought an inflationary boom in the whole economy. After the first World War, the long swings slowed down towards the great depression of 1929. In this process, there was the disturbance of the Kanto Earthquake disaster and a depression of 1920-22 of the business cycle component which was accompanied by an inflationary boom caused by war. The long swings in current prices crossed the hypothetical zeroline in 1926 and 1932, but the long swings in fixed prices did not cross the zeroline. This means that the long swings of actual value in current prices declined in 1926 and reached a trough in 1932, while the actual series in fixed prices did not decline. After the great depression the drive to maturity progressed until 1940 with one and 3/4 business cycles, the average period of which was 6.3 years. The long swings in current prices grew from 1929 to 1947 with inflationary pressures on prices, while the long swings in fixed prices slowed down around the 1930's and

*Rostow [22], p. 38.

introduced to a great extent. Many durable consumer goods are being produced. The Japanese society now begins to grow towards the age of high mass-consumption.

Further details of business cycles will be discussed using monthly series in another paper.

The number of lags used in the estimation of the cross-spectrum is m = 15 in this section. We increased the degrees of freedom since the cross-spectral estimates appear, for purposes of interpretation, to have larger sample variances than the estimates of the power spectrum.

The middle point of the j-th band of frequency is given by 30/j years/cycle. We represent the j-th band by frequency j/30 for simplicity in the following section. The second band spans [20,12] y/c which induces the long swing component and the 5th band, [6.6,5.5] y/c which includes the business cycle component. As both the 2nd and 5th bands are correlated in the Parzen window, we cannot find the relationship among the series independently for both components.

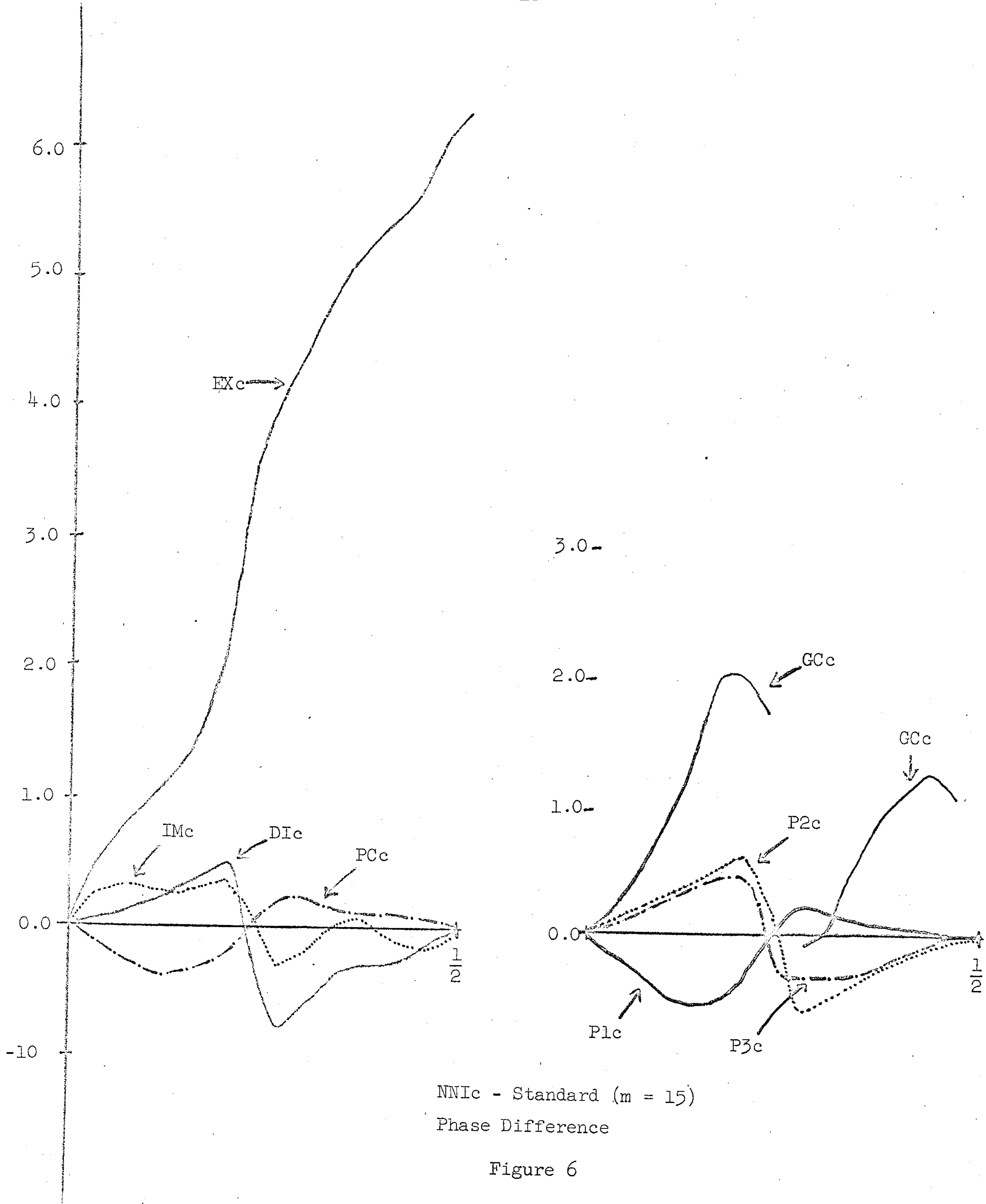
ii. The lead-lag relationships

Here we discuss the cross-spectrum of the rate-of-change series in current prices because of its high coherence and stable phase differences. We find this convenient and meaningful, although we cannot deduce the behavior of output in real terms from the series in current prices. In the following paragraph, the series are the rate-of-change series in current prices unless otherwise mentioned.

The phase difference shows the lead-lag relationship between two observed series which involves the causality lag and the feedback lag.*

If $Ph_{xy}(\omega) = a\omega$, where a is constant, x_t is said to have a fixed time lag and if $Ph_{xy}(\omega) = b$, x_t is said to have a fixed

* Granger, J., in association with Hatanaka, M., [9], Chapter 7.



NNIc - Standard (m = 15)
Phase Difference

Figure 6

Private consumption (PCc) lags behind all of the other series at low frequencies and leads IMc, DIc and NNIc at high frequencies. Products of the primary industry (P1c) have very small phase differences and high coherences with respect to PC. This results from the fact that a large portion of PC comes from primary industries.

Products of secondary and tertiary industries (P2c and P3c) have very small phase differences and high coherences with respect to DI. The larger part of secondary and tertiary industries were introduced in Japan after the Meiji Restoration and they could always develop with new investment.

It is very interesting to find the gaps of the phase diagram of NNI to DI, IM, GC, PC, P1, P2 and P3 at the frequency of 1/4. We should consider the lag relationship dividing the components at this frequency.

Figure 7, (1) shows the phase differences of money stocks (MS) to national income (NNI), cash currency (CC), deposit currency (DC), (where MS = CC + DC), deposits of all banks (DEPAB) and loans and discounts of the Bank of Japan (LDBJ). The phase diagram shows that MSc and CCc lag behind national income with the exception of the very high frequencies, while DCc leads NNIc with the exception of the middle domain around the 1/4 frequency.* DEPAB leads all of the other series at frequencies less than 1/4 and DCc leads all of the other series at frequencies higher than 1/3.3.

* This is the Japanese case of the problem considered in the economy of the United States by Friedman, M., and Schwartz, A. J., [5].

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The phase differences of deposits of all banks (DEPAB) to loans and discounts of all banks (LDAB) is shown in Figure 7 (2). LDAB leads DEPAB in the whole domain with a very high coherence. Consequently, LDAB also leads the other series. This means that LDAB would move independently with the behavior of DEPAB and that DEPAB would depend on LDAB. LDAB also leads EX and GC in almost the whole domain. Ohkawa and Rosovsky consider a credit expansion* to modern sectors as the leading factor for inducing economic growth. We may consider the loans and discounts of all banks (LDAB) as the representative of the whole credit expansion, although LDAB is not all of it. Such a credit expansion was one of the major factors causing the inflationary process in the whole period. Why was the credit created leading the other factors in face of danger of secular inflation? Ohkawa and Rosovsky state "credit creation cannot really be explained in endogenous terms within the cyclical mechanism. Rather it must be discussed in terms of the over-all expansionist atmosphere which prevailed during the entire course of Japanese industrialization."**

Loans and discounts of the Bank of Japan (LDBJ) come at the end of the whole series at low frequencies but the coherences to other series are small. It seems to behave as an independent factor aiming the adjustment of the whole economy in some period, or

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*It takes forms such as "a government subsidy designed to start or expand some particular branch of industry (shipbuilding would be a typical case), public, semi-public, or private bank loans, or simple government currency expansion for military or other purposes". Ohkawa and Rosovsky, [18], p. 17.

**Ibid, p. 19.

We chose the phase difference of two series which have a high coherence to calculate the length of the lag. The results are shown in Table IV, which can be read such that EX leads NNI with 1.43-year lag, or P1 lags behind NNI with 0.46-year lag, etc., in a six-year cycle component of the rate-of-change series in current prices. The small difference in the length of the lag is not significant, because of leakages.

Although the lengths of the lags are a little different, the long swings have almost the same relationship as the business cycles. This does not necessarily result from the Parzen window. The estimates by the Tukey-Hanning window with lag $m = 20$ also show almost the same relationships in both long swings and business cycles. We, therefore, discuss only the business cycle component. However, similar conclusions may be made about the long swings.

Let us try to explain the Japanese economy with the lead-lag relationship in Table IV. We take government consumption (GC), loans and discounts of all banks (LDAB) and exports (EX), as the key factors of the Japanese economic growth.

A part of the lead-lag relationship can be read as follows:

GC, LDAB, EX → DEPAB → P2, P3 → DI → IM → DC → NNI → MS → PC → CC → P1.

We start with our three key factors of GC, LDAB and EX. In the business cycle component, the three factors have the relationship GC → LDAB → EX, while in long swings they have the relationship LDAB → EX → GC. They drive growth and create a credit expansion. As a result of this expansion the products in secondary and tertiary industries (P2 and P3) increase as a whole. Now new equipment is which has already come when Japan begins to enjoy her improvement. Around the time private consumption begins to increase, the prices of production goods and consumption goods also begin to rise. Thus the rise of all prices weakens Japan's export position in world trade. The need for imports is now a clear ceiling to the expansion of output and leads to a balance-of-payments crisis.* Although the lack of data does not enable us to find precisely the lag-relation of the balance of payments, it might stand after the price indices. Now the Bank of Japan comes in to adjust the economy. Loans and discounts of the Bank of Japan increase with a lag of 1.54 years, a quarter of one business cycle, after the increase of deposits of all banks (DEPAB). This means that the actual value of LDBJ increases when the rate of change of DEPAB begins to decline. At this time, government consumption, loans and discounts of all banks, and exports are already declining.

iii. The transfer of labor forces

Now let us consider the relationship among the three industries. Here we use the rate-of-change series in fixed prices. (A-B) in Figure 9 means the phase difference of series A to series B.

(P3f - P2f) shows that secondary industry products (P2f) leads tertiary industry products (P3f) at low frequencies. (L2 - P2f) has a phase difference such as $a + b\omega$ at low frequencies and $a' - b'\omega$ at high frequencies. Namely, the labor force in secondary industries (L2) has both a fixed time lag and a fixed angle lag to P2f at low frequencies and both a fixed time lead and a fixed angle lag at high frequencies.

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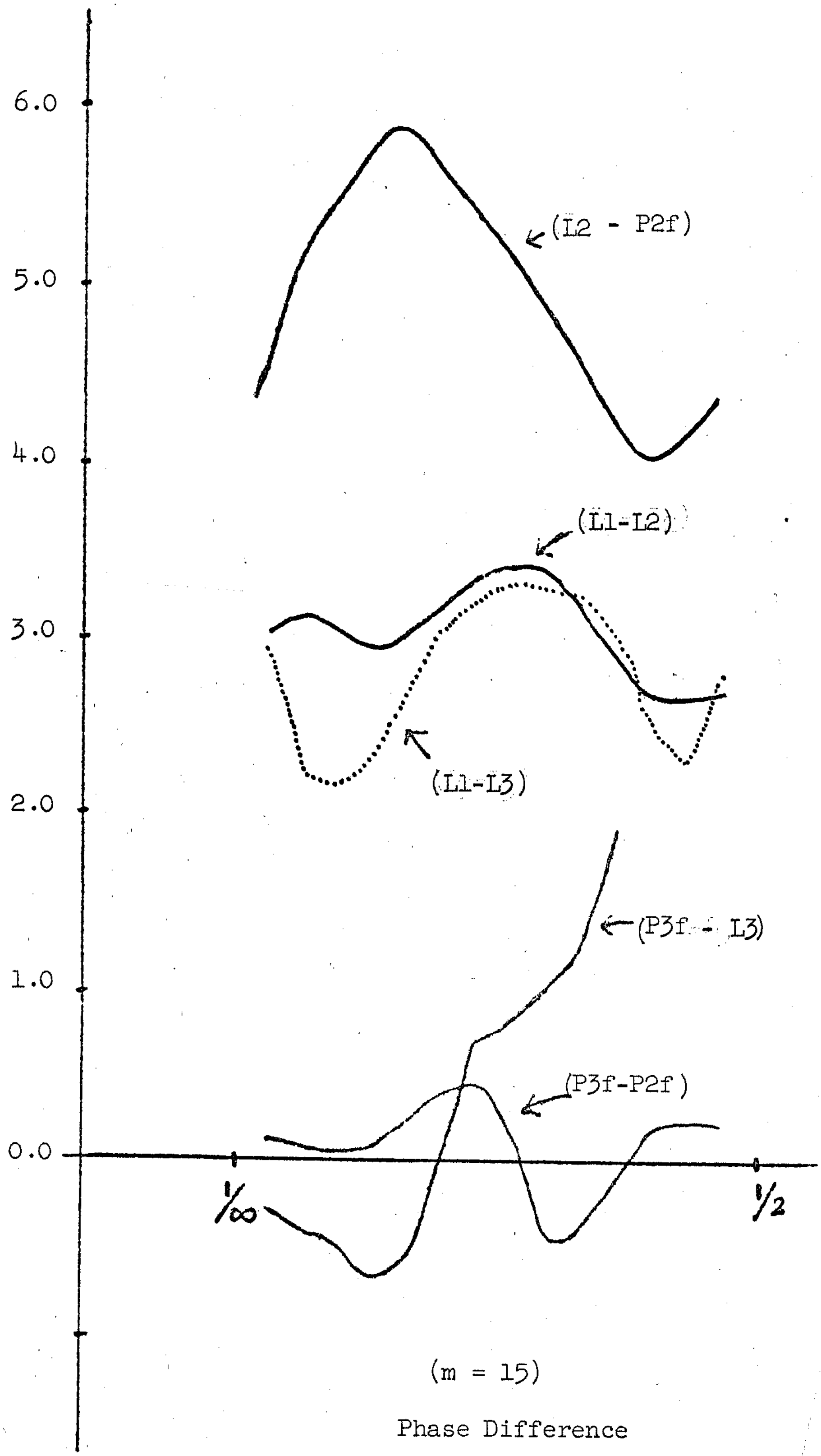
*Ohkawa and Rosovsky, [18], p. 19.



(m = 15)

Phase Difference

Figure 9

iv. Estimates of gain

Gain may be thought of as the absolute value of a complex-valued regression coefficient at each frequency. Now let us consider some characteristics of investment and private consumption by their gains on national income. Here we estimate the cross-spectrum of the actual-value series in fixed prices pre-whitened by a symmetric linear filter with weights of (0.25, - 0.45, 0.25).

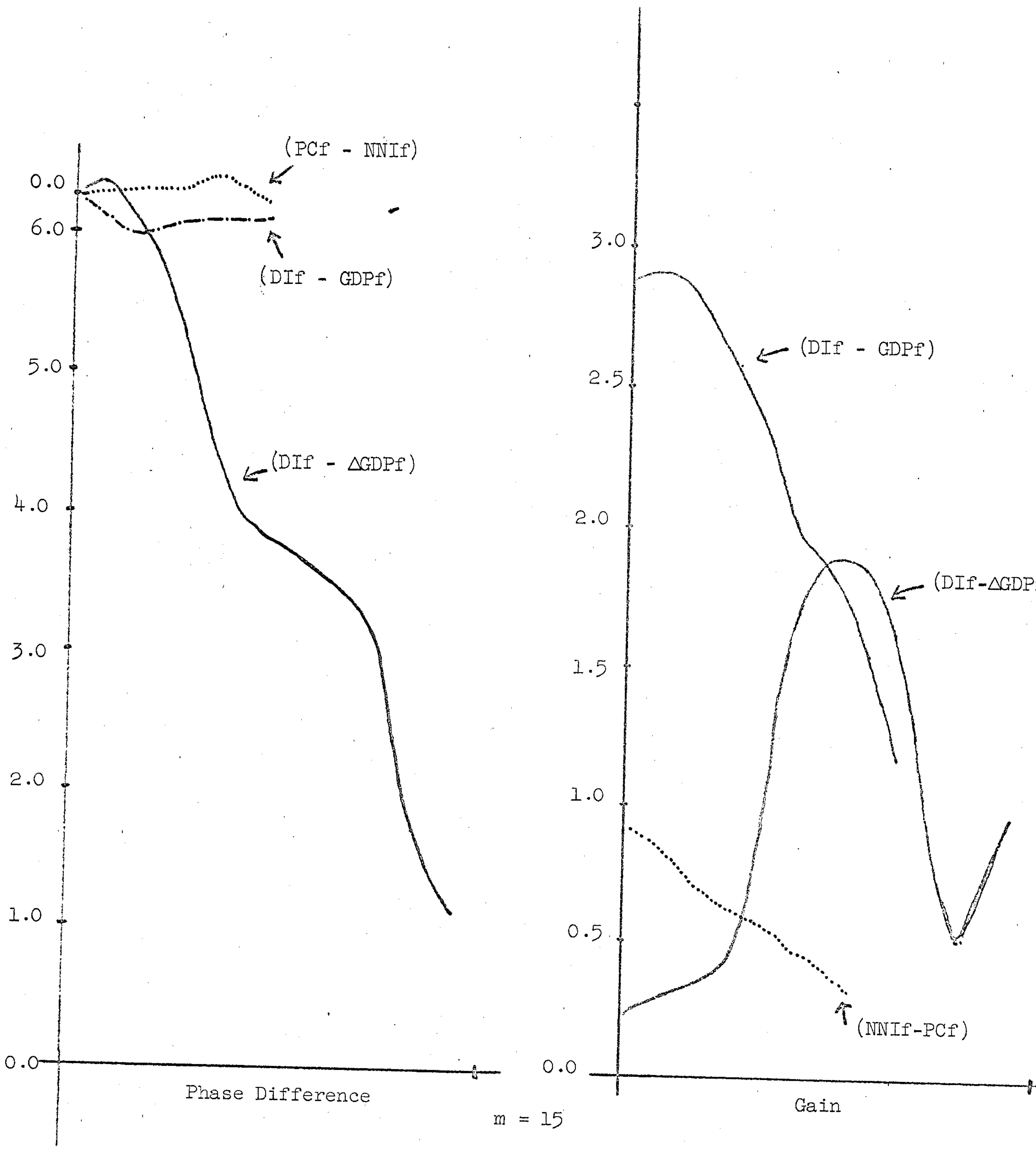
The phase differences of domestic investment (DIf) to gross domestic product (GDPf) and the first difference of GDPf ($\Delta GDPf = GDPf_{t+1} - GDPf_t$) in Figure 10 show that the investment, I_t, leads both Y_t and $\Delta Y_t = Y_{t+1} - Y_t$. The phase difference of DIf to GDPf is not shown at the high frequencies, because the coherence is very small. Private consumption (C_t = PCf) lags very slightly behind Y_t (net national income, NNIf) and the coherence is very small at high frequencies. We can write the above relationships as follows:

$$Y_t = aI_{t-k_1} , \quad \Delta Y_t = bI_{t-k_2} , \quad C_t = cY_{t-k_3} .$$

The lengths of lags at frequency 1/6 are

k_1 = 0.13 years, k_2 = 1.7 years, and k_3 = 0.14 years.

The characteristics of the coefficients a,b, and c are known from the gains. The gain of Y_t on I_t and the gain of C_t on Y_t have larger values at lower frequencies. The gain of ΔY_t on I_t has its largest value at frequency 1/4.3. The production coefficient of investment is largest in the 4.3-year cycle component and the lag from investment expenditure to the realization of product of investment is 1.6 years at that component.



m = 15

Figure 10

V. Concluding Remarks

Our research shows that power and cross-spectral estimates can provide quantitative and visual information for the analysis of the Japanese economic growth process.

However, the results described in this paper are not necessarily final conclusions of our research, since most of the data available at present are tentative and the method of spectral estimation is still to be completely developed.

where

$$\lambda(s) = 1 - 6(\frac{s}{m})^2 + 6(|\frac{s}{m}|)^3 \qquad s \leq \frac{m}{2}$$

$$= 2(1 - |\frac{s}{m}|)^3 \qquad s \geq \frac{m}{2} .$$

Due to $R_{xx}(s) = R_{xx}(-s)$, the power spectrum is given by:

$$F_{xx}(\omega) = R_{xx}(0) + 2 \sum_{s=1}^{m} R_{xx}(s)\lambda(s) \cos \omega s.$$

The cross-spectrum is in general complex-valued. Its real and imaginary parts are called respectively the co-spectrum and the quadrature spectrum. The co-spectrum is given by:

$$C_{xy}(\omega) = R_{xy}(0) + \sum_{s=1}^{m} [R_{xy}(s) + R_{yx}(s)] \lambda(s) \cos \omega s.$$

The quadrature spectrum is given by:

$$Q_{xy}(\omega) = \sum_{s=1}^{m} [R_{xy}(s) - R_{yx}(s)] \lambda(s) \sin \omega s.$$

Let us define from these a variety of spectral quantities as follows:

1. Coherence between x_t and y_t:

$$Coh_{xy}(\omega) = \frac{|F_{xy}(\omega)|^2}{F_{xx}(\omega) \cdot F_{yy}(\omega)}$$

APPENDIX B SOURCES OF DATA

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