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PLAYING AN n-PERSON GAME, AN EXPERIMENT

by

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Appendix of players' explanations
translated from Hebrew by I.M. Lask

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1. Laboratory experiments for game theory.*)

Let me start by confessing that I am not a social scientist, that I am not an expert in laboratory experiments and, moreover, that the experiment described in this paper was neither intended originally to be a scientifically well planned experiment, nor, in fact, was it executed in accordance with the high rigor now achievable by the best available procedures. Yet, in spite of its many faults, the experiment does exhibit interesting results which are directly related to game theory and are highly important as such.

Many social scientists are under the impression that laboratory experiments can contribute very little to game theory.

Aside from pointing out technical difficulties in replicating a game situation in the laboratory (see, e.g., R.D. Luce and H. Raiffa[1] [10]) - difficulties that no one can deny - it is claimed that such experiments contribute nothing to game theory because they exhibit descriptive phenomena, whereas game theory is normative[2].

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(1) Numbers in square brackets refer to the references at the end of the paper.

(2) "Normative" is used in this paper in the conditional sense of "telling people what to do if they want to achieve certain given goals".

*) An abstract of this paper appeared in [11].

To be precise, an abstract theory is neither normative nor descriptive as such. It is the way one interprets it and/or applies it to human beings that may have normative or descriptive aspects. The present author holds the view that it is wrong to say that game theory can be interpreted and applied only in the normative sense. The author also thinks that even the normative aspects of game theory can benefit from and be tested in laboratory experiments.

Apart from differences of the level of abstraction and the degree of generality - the borderlines of which are difficult to determine - game theory differs from other theories in social sciences in the specific clientele to which it applies. The"people" treated by social scientists are, in general, average subjects belonging to a specific group characterized by profession, trade, sex, ethnic relations, etc., whereas at least some of the "players" in game theory are expected to be highly intelligent(1). Game theory is interested in the behavior of wise people and does not set a priory limits to the thinking abilities of its subjects. This does not mean to say that game theorists have a monopoly on wisdom, nor that they have succeeded in characterizing the wisest behavior. Even the present experiment, I believe, will show that the opposite is true.

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(1) Thus, a theory which intends to analyze how best to exploit mistakes of the opponent is still within the realms of game theory. We also include the study of cases in which communication is not perfect, and cases in which the game situation is not fully known to the players; though the study of the latter situation has hardly begun as yet.

Thus, an army or a firm (regarded as players) that has a staff of experts who use computers to evaluate mathematically various alternatives and who determine the operations of the army or of the firm, can very well be analyzed descriptively by the methods of game theory. Even the behavior of "average" people can be explained by game theoretical considerations if one takes into account what possible deviations from the theoretical predictions may result from what conceivable "mistakes"; this has been observed by I.C. Harsanyi in [7].

In principle, it is very easy to test the normative applications of a theory. (See R.J. Aumann [1]). One chooses a group of reasonably intelligent subjects, explains the issues, suggests the predictions of the theory as recommendations, states exactly what goals are achieved by these recommendations and checks whether the subjects indeed follow one's recommendations. If they do - what else can be demanded?(1) This test certainly supports the assumption that the subjects consciously are willing to accept the said goals if they are well informed about them. Such procedures are, of course, different from procedures for checking a descriptive application of a theory, where informing the subject of the

(1) Care should be taken to assure that the subjects follow the advice because they are convinced by the goals and not because of the good looks of the experimenter. This can be checked by the subjects' accounts during the experiment and in a final, detailed interview . If several conflicting goals exist, one may present several advices based on such goals and then check which of them is followed.

theoretical predictions is inadmissible.(1)

It has been said above that game theory is interested in the behavior of wise people. This implies that special precautions should be taken during the experimental sessions , designed for normative as well as for descriptive purposes. The subjects should be intelligent enough and the problems should be simple enough to assure that the necessary calculations and reasonings can be satisfactorily carried out. For somewhat complicated problems, experts equipped with computers should be available to help the subjects if they wish so. The rewards should be important enough to the subjects to encourage them to exert their best judgment. There should be no time limit imposed.(2) The subject should feel free, and even be

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(1) For descriptive purposes one might be interested in finding out whether the subjects possess such goals without being specifically informed of them. In such cases, of course, the usual experimental procedures should be used.

(2) Even an abundant time allowance, if specified, may bias the considerations either by creating the feeling that the problem is extremely difficult, so that the subject might give up a serious analysis of the alternatives, or by making the subject believe that it is so boring to play the game that the reward does not deserve his efforts.

encouraged to consult his friends and, sometimes, even experts . A detailed record of the negotiations and an exhaustive interview should be taken. This recommendation can hardly be overemphasized. One should know as much as possible what went on in the mind of the subject; why did he prefer one alternative to another; what were his rational and irrational motives, was he subjected to extraneous influences, etc. (See R.J. Aumann [1], where many of the above qualifications are treated).

An experiment may prove very interesting if the subjects consciously or subconsciously do not conform to the predictions of the theory. If their motives can be detected, it may be found that either the theory overlooked some very relevant aspects of the real life situation - this has happened in our experiment - or that the subjects are governed by completely different motives and, therefore, deserve a new chapter in game theory. To give a simple example: Consider the case of three players who are to share $100 among themselves. The mode of division is decided by majority vote, but one player has four votes, another has three votes and the remaining player has only two votes. Now, in spite of the window dressing, the three players are equally "strong", because precisely every pair of them forms a minimal winning coalition. Suppose that in an experimental replication of this game (such experiments were actually performed by W. E. Vinacke and A. Arkoff [21]) the subjects consistently split, say, in the ratio of 4:3:2 . Then, either the subjects did not realize that the division of power was as presented above - in which case they were simply stupid and the experiment

contributes nothing whatsoever to game theory - or they realized the above aspect of the game (and this is quite easy to determine in an interview) but, nevertheless, acted differently. In the latter case, surely the subjects were guided by other motives - call them "ethical motives" - which were predominating. For such people a new branch in game theory should be invented, in which, perhaps, preferences will depend on the way of presenting the circumstances.

2. Normative and descriptive aspects of the bargaining set. The kernel.

An n-person cooperative game $(N ; v)$ is defined by a set $N = \{1,2,\ldots,n\}$, the elements of which are called players, and a function $v = v(S)$ - the characteristic function - which attributes a real number $v(S)$ to each non-empty subset S of N. Such an S is called a coalition and the number $v(S)$ is known as the value of the coalition. For simplicity we assume[1]

(2.1) $v(i) = 0$, $i = 1,2,\ldots,n$, $v(S) \geqslant 0$ for each coalition S.

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[1] We shorten the notation and write $v(i)$ instead of $v(\{i\})$, $v(ij)$ instead of $v(\{i,j\})$, etc. Similarly, we shorten the notation for sets of coalitions and write $\{123\}$ instead of $\{\{1,2,3\}\}$, $\{12, 34\}$ instead of $\{\{1,2\},\{3,4\}\}$, etc. The first condition in (2.1) is merely a normalization of the characteristic function. The second comes to avoid some trivial modifications of the theory.

When such a game is being played, presumably the players partition themselves into coalitions, and each coalition formed distributes its value among its members. We also assume that each player gets at least the amount he can obtain by playing alone, i.e., as a 1-person coalition (individual rationality). Thus, an outcome can be represented by an <u>individually rational payoff configuration</u> (i.r.p.c.)

$$(2.2) \qquad (x;\mathcal{B}) = (x_1,x_2,\ldots,x_n;\ B_1,B_2,\ldots,B_m)\ .$$

Here, $\mathcal{B} = \{B_1,B_2,\ldots,B_m\}$ is a partition of N into coalitions and it is called the <u>coalition structure</u> (c.s.). The <u>payoff vector</u> (p.v.) $x = (x_1,x_2,\ldots,x_n)$ is an n-tuple which represents the payments to the players. It satisfies :

$$(2.3) \qquad \sum_{i\in B_j} x_i = v(B_j), \qquad j = 1,2,\ldots,m\ ; \qquad x_i \geqslant 0, \quad i=1,2,\ldots,n.$$

In R.J. Aumann and M. Maschler [2], several sets of i.r.p.c's, called the <u>bargaining sets</u>, were introduced. Their purpose is to suggest an answer to the following question: Suppose a coalition structure $\mathcal{B}$ is formed in a game $(N;v)$. How would or should the players share the proceeds? Among the various bargaining sets suggested, the bargaining set $\mathcal{M}_1^{(i)}$ attracted the main attention, because it possesses the property that for every c.s. $\mathcal{B}$ there exists a p.v. x, such that $(x;\mathcal{B})$ belongs to $\mathcal{M}_1^{(i)}$ (see M. Davis and M. Maschler [4] and B. Peleg [17]). (See also [12], where the various definitions of the bargaining sets are summarized and computed for many of the games treated in this paper).

We refer the reader to the above mentioned papers for reviewing the exact definitions and the mathematical properties of the bargaining sets. Here we shall stay content by analyzing some numerical examples in order to throw light on the relation of the theory to actual human behavior. Consider the game $A \equiv (\{1,2,3\}; v)$, where $v(i)=0$, $i=1,2,3$, $v(12)=60$, $v(13)=70$, $v(23)=90$ and $v(123)=0$. There is no question of the outcome if the c.s. $\{123\}$ or $\{1,2,3\}$ form, but if, say, $\{1, 23\}$ forms, one may ask for the "proper" share of 90 among the players 2 and 3. Suppose, e.g., that the i.r.p.c. (0, 30, 60 ; 1, 23) is being considered, then player 3 has an <u>objection</u> against player 2 : he can point out that he can offer player 1 5 units (which surely are preferred to the zero payment) and he will benefit by receiving 65 units. He does not need the consent of player 2 for such an objection (- call it "objection (a)"). This serves, perhaps, to show that 30:60 is not the "right" split, and player 2 should grant him more. Similarly, player 2 has an objection against player 3. He can claim that he can split, say (35:25) with player 1 ("objection (b)"). Unfortunately, for each i.r.p.c. with this c.s. , at least one player has an objection. A closer inspection, however, shows that the above objections are of different kinds. Objection (a) is <u>not justified</u> in the sense that it can be countered by player 2 , who can offer player 1 15 units, which is more than the 5 units offered by player 3 , and still protect his 30 original units (in fact, he receives 45 units). He does not need the consent of player 3 to achieve this. On the other hand,

player 2's objection (b) is justified because player 3 cannot offer player 1 at least 25 units and still keep his 60, nor can he obtain 60 by playing alone. The only i.r.p.c. , with the same c.s., for which no player has a justified objection against any other is (0,40,50; 1,23) .

Now, is this all relevant to the question of how would or should the players share the proceeds? This depends on the purpose of the players in playing the game. If, say, they simply want to get maximal proceeds, then the above discussion is not relevant but the answer is then very simple: The coalition structure 1,23 cannot be formed because player 2 wants 90 units and so does player 3 , and obviously they both cannot achieve their goals. It seems that there is no escape from concluding that the purpose of the players is to reach some kind of a "stable" compromise. Thus, normatively, we can say that if the players want to reach a stability characterized by the non-existence of justified objections with respect to the outcomes, and if the coalition structure 1,23 is formed, then the participants should split in accordance with (0,40,50; 1, 23) . It is easy to see that the only i.r.p.c.'s which possess the above kind of stability are :

$$
(2.4) \qquad \begin{cases} (\ 0,\ 0,\ 0\ ;\ 1,\ 2,\ 3\ ) \\ (20,40,\ 0\ ;\ 12,\ 3\ \ ) \\ (20,\ 0,50\ ;\ 13,\ 2\ \ ) \\ (\ 0,40,50\ ;\ 1,\ 23\ \ ) \\ (\ 0,\ 0,\ 0\ ;\ 123\ \ \ ) \end{cases}
$$

I.r.p.c.'s which possess this kind of stability constitute the bargaining set $\mathcal{M}_1^{(i)}$.

The same model can be interpreted descriptively; namely, that the players will negotiate various outcomes, that justified objections will be considered good reasons to change such outcomes, whereas unjustified objections will be discredited. Such negotiations will continue until the players hit some i.r.p.c.'s in $\mathcal{M}_1^{(i)}$, which will end the arguments(1). Such an interpretation can be tested experimentally. Similarly, the goals stated in the normative interpretation can be tested by examining whether people obey the above recommendations after it is explained to them what goals are thus achieved(2).

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(1) It is an open problem to find an intuitively convincing general method (or methods) of bargaining which may lead the players to i.r.p.c.'s in the bargaining set.

(2) The author recalls a very hot argument , which lasted about 45 minutes, with a prominent businessman in Minnesota, who refused to accept a (75,25,0;12,3) outcome in a game $(\{1\ 2\ 3\};v)$, where $v(i)=0$, $i=1,2,3$, $v(12)=v(13)=100$, $v(23)=50$, $v(123)=0$ in dollars. "I am in business all my life", he said, "and I am telling you that you cannot do business this way. You should give a guy a break". What apparently he meant was that, since player 2 was a partner of player 1 , he should have been allowed more. It seemed to him that player 2 was exploited to the maximum by the above i.r.p.c. - a situation

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The bargaining set suggests several possible outcomes, but it does not specify which one is preferable. Obviously only three outcomes in (2.4) are "reasonable", unless extreme cases are also considered. One can conjecture that all the outcomes in (2.4) will appear with given fixed probabilities. A more intuitive conjecture is that "secondary effects" will take place; namely, that the players will be willing to sacrifice a few units in order to "assure" their participation in a 2-person coalition. If this is true, then perhaps the outcomes will oscillate at random around the three main outcomes in the bargaining set. Still another conjecture may result from the fact that the "strong" player 3 has a distinct advantage: He, and only he, can not only assure his participation in a 2-person coalitiom, but he also can exclude each player from the 2-person coalition if he decides to offer the other player, say, 61 units[1]. From such point of view, player 2 "knows" that he will also be in the coalition since, obviously, it pays player 3 to pay him rather than to pay player 1. His

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(Continued from page 10)

"unheard of" in business circles - and he suggested that player 1 be given 50, player 2 - 25, and that the remaining 25 be split somewhere between 1:1 and 2:1 ratio, adding sarcastically that I would soon lose my neck doing business my way.

[1] Of course, he will resort to such an extreme offer only in a hypothetical (and improbable) situation, if, say, player 1 offers player 2 58 units and refuses to accept any amount from player 3.

only object, then, is to milk player 3 as much as possible. The conjecture which follows is that in real situations the coalition $\{2\ 3\}$ will form and player 3 will be the one who gets less than his share in the bargaining set. We do not know how to make a general theory out of any of these heuristic arguments, but, at least, such conjectures seem legitimate for the 3-person games.

The three i.r.p.c.'s in $\mathcal{M}_1^{(i)}$ in which a 2-person coalition is formed are characterized by the property that each player receives his quota[1] provided he manages to participate in a 2-person coalition. (The quotas are real numbers $\omega_1, \omega_2, \omega_3$, satisfying $\omega_i + \omega_j = v(ij)$, $i,j = 1,2,3$, $i \neq j$.

A change in the value $v(123)$ does not affect the first four i.r.p.c.'s in (2.4). If[2] $v(123) \geqslant v(12), v(13), v(23)$, two possibilities arise: If $v(123) \leqslant [v(12)+v(13)+v(23)]/2$, say, $v(123) = 92$ in our case, then (14, 34, 44; 123) is the unique i.r.p.c. in $\mathcal{M}_1^{(i)}$ with $\{123\}$ as the c.s. It is obtained by the players yielding equal amounts from their quotas. If $v(123) \geqslant [v(12)+v(13)+v(23)]/2$, the game has a non-empty core[3],

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(1) The term is due to L.S. Shapley [19], although he uses it with a slightly different meaning.

(2) We shall only be interested in such cases.

(3) The term is due to D.B. Gillies [6]. See also L.S. Shapley and M. Shubik [20] for information on recent studies of the core.

the core being the set of payoff vectors $x = (x_1, x_2, x_3)$ satisfying $x_1+x_2+x_3 = v(123)$, $x_i + x_j \geqslant v(ij)$, $i,j = 1,2,3$, $i \neq j$, and $x_i \geqslant 0$, $i = 1,2,3$. In this case, $(x; 123)$ belongs to $\mathcal{M}_1^{(i)}$ if, and only if, x belongs to the core of the game.

The bargaining set takes a completely different form if one player has a negative quota. Consider, e.g., the game $B \equiv (\{1\ 2\ 3\}; v)$, where $v(i) = 0$, $i = 1,2,3$, $v(12) = 20$, $v(13) = 30$, $v(23) = 100$, $v(123) = 0$. In this case, the bargaining set consists of

$$(2.5) \qquad \begin{cases} (0,\ 0,\ 0 \ ;\ 1,\ 2,\ 3) \\ (0,\ 20,\ 0 \ ;\ 12,\ 3\ ) \\ (0,\ 0,\ 30 \ ;\ 13,\ 2\ ) \\ (0,\ \alpha,\ 100-\alpha\ ;\ 1,23) \quad \text{for } 20 \leqslant \alpha \leqslant 70 \\ (0,\ 0,\ 0 \ ;\ 123\ ) \end{cases}$$

Of these i.r.p.c.'s , only the outcomes in the continuum $(0,\ \alpha,\ 100-\alpha\ ;\ 1,\ 23)$, $20 \leqslant \alpha \leqslant 70$, are really interesting. They are characterized by the property that no objection to them is possible. I.r.p.c.'s which have this property will be said to belong to the <u>quasi core</u> of the game[1].

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[1] "Quasi" , because we do not require that $x_1 + x_2 + x_3 = v(123)$. Also, we specifically assign the c.s. for elements of the quasi core. The same p.v. may belong to the quasi core with respect to one c.s. but not with respect to another.

If $v(N) \geq v(12), v(13), v(23)$; i.e., $v(N) \geq 100$ in our case, the game has a non-empty core and $(x;N)$ belongs to $\mathcal{M}_1^{(i)}$ if and only if x is in the core of the game.

We see that even for a fixed coalition structure the bargaining set $\mathcal{M}_1^{(i)}$ may contain an uncountable infinity of i.r.p.c.'s. No intuitive criterion is known for selecting one of them as distinguished from the others, but some attempt has been made to narrow down the bargaining set $\mathcal{M}_1^{(i)}$. Thus, in M. Davis and M. Maschler[5], the <u>kernel</u> of a game has been defined. It is a subset of $\mathcal{M}_1^{(i)}$ which contains elements for each c.s. In the case of a 3-person game it consists of a unique point in each c.s. The kernel $\mathcal{K}$ possesses many mathematically interesting properties (see M.Maschler and B.Peleg[15]). In the case of 3-person games its predictions sound intuitively reasonable, but in the general case nobody, including the inventors, believes that its predictions are preferable to other i.r.p.c.'s in the bargaining set. At any rate, we list here the predictions of the kernel for those c.s.'s in 3-person games which occur in our experiment, when $\mathcal{K}$ is a proper subset of $\mathcal{M}_1^{(i)}$.

We assume that the players are named in such a way that $v(12) \leq v(13) \leq v(23)$. Let $\omega_1, \omega_2, \omega_3$ be their quotas. Clearly, $\omega_1 \leq \omega_2 \leq \omega_3$.

For the c.s. $\{1, 23\}$, $(0, \omega_2, \omega_3 ; 1, 23)$ is in the kernel[1]. The situation is more complicated if the c.s. $\{123\}$ is considered.

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(1) It is a proper subset of $\mathcal{M}_1^{(i)}$ only if $\omega_1 < 0$.

Case A . If $\omega_1 \geq 0$, three procedures may occur.

Procedure 1. If $v(123) \geq 3\, v(23)$, the players ignore the 2-person coalitions and $\left(v(123)/3 , v(123)/3 , v(123)/3 ; 123\right)$ is in the kernel.

Procedure 2. If $2\, v(12) + 2\, v(13) - v(23) \leq v(123) \leq 3\, v(23)$, then players 2 and 3 decide to act as one player. They play against player 1 , leaving him the amount $\alpha = \left[v(123) - v(23)\right]/2$. Knowing this amount, they now share the remaining $\beta = \left[v(123) + v(23)\right]/2$ in such a way that player 2 receives $(\beta + w_2 - w_3)/2$ and player 3 receives $\left[\beta + w_3 - w_2\right]/2$. Here, $w_2 = \text{Max}\ (0 ; v(12) - \alpha)$ and $w_3 = \text{Max}\ (0 ; v(13) - \alpha)$.

Procedure 3. If[1] $2\, v(23) - v(12) - v(13) \leq v(123) \leq 2\, v(12) + 2\, v(13) - v(23)$, the kernel prediction is $(\omega_1 + d/3, \omega_2 + d/3, \omega_3 + d/3 ; 123)$, where $d = v(123) - \omega_1 - \omega_2 - \omega_3$.

Case B . If $\omega_1 < 0$, only the first two procedures occur. The first occurs if $v(123) \geq 3\, v(23)$, and the second occurs if $v(23) \leq v(123) \leq 3\, v(23)$.

We refer the reader to [5] , where an intuitive justification for these procedures and their common features are discussed.

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(1) The possibility that $v(123) < v(23)$ will not interest us, because when this occurred in the experiment, the c.s. $\{123\}$ never formed.

3. The procedures of the experiment.

Subjects. In all, 38 pupils (29 boys and 9 girls) participated in a contest consisting of 123 plays. These were students of an 11-th grade class of the Hebrew University Secondary School, Jerusalem, Israel. The class can be classified, intellectually, as highly above average, because this school admits students on a selective basis, and because the class consisted of students who had chosen the scientific curriculum with concentration on mathematics and physics.

The author was the mathematics teacher of the class and in this capacity he had taught about half of the students from grade 7, and most of the remaining students from grade 9. During these years, the author experimented in teaching some extracurricular chapters of mathematics and in penetrating more deeply into the standard subject matter. Although Game Theory was never mentioned, it is quite possible that their having this particular teacher was a contributory factor to their conforming to the predictions of the bargaining set theory; their having been taught for many years by the author could have conditioned the class to think (subconsciously) the way their teacher did. The author never interfered with the actual plays, nor did he express in the presence of the students any prediction or judgment of the outcomes. The teacher enjoyed popularity among the students, which enabled him to encourage enthusiasm amd a faithful participation during the contest which lasted for about six months.

Motivating the experiment. No significant budget was available to be distributed as rewards. Also, it is doubtful whether monetary rewards

could have been of much importance to these highschool kids[1]. At any rate, other incentives had to be provided in order to assure that the students exert their best judgment when playing the games. This was attempted on three levels :

(1). Idealistic grounds. The subjects were told repeatedly that this experiment was extremely important for scientific discovery, that we trusted their faithful efforts and that we expected major discoveries from this experiment. These aspects were also emphasized by a speech made by the head master of the school prior to the experiment. Such (exaggerated) arguments make an impression on Israeli children, because ideologies of all kinds play an extremely important role in almost every phase of life in Israel (including the phase of participation in youth movements).

(2). Spirit of sportmanship. If you are participating in a contest, where every participant is your friend with whom you share other activities such as studies, premilitary training and youth movement activities, you do not want to appear stupid, or to be laughed at. You try to outwit others and - at the same time - not to lose your neck.

(3). Trying to be the champion. Three prizes were offered to the first winners of the contest. These proved to be extremely important towards the end, when many belated accounts were filed in.

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(1) This does not mean that children's attitude to points must in any way be similar to the attitude of adults towards monetary payoffs. Only an experiment can verify such a claim.

Procedures. After the introductory speech by the head master, the author explained the rules of the games and insisted that every student copy them in his notebook [1]. An example of an intentionally trivial play was demonstrated on the blackboard and questions were answered. All the questions aimed at a better understanding of the rules of the games. The instructions were :

"You are requested to take part in an experiment, designed to study how coalitions are formed in human relations. In this experiment you will have to play many games, and your sincere efforts to do your best are very important to the success of this research.

We shall distribute cards among you, which will describe the various games and will list the players in each game. To each game there will correspond one card, and the cards will resemble the one drawn on the blackboard. (See figure)[2].

a. David Shoshani	
b. Ruth Avni	
c. Jacob Carmel	
a + b	60
a + c	80
b + c	40
a + b + c	150
Decision.	
Time of negotiation.	

[1] One pupil was absent. He received the instructions some other time.
[2] The names are fictitious

Here are the explanations and rules for playing in the contest. Please copy them in your notebooks.

(1) Several students are listed on each card. They will be called "players". A letter is assigned to each player: Student a , Student b , etc.

(2) Those coalitions which are possible in each game are written on each card. For example, a + b, a + c , a + b + c .

(3) A number is written next to each coalition, which tells the total number of points which the participants can get if they decide to form the coalition. This number is the <u>value</u> of the coalition.

(4) In order that a coalition will form, its members have to agree in advance on the number of points that each of them will receive. The sum of all the points which the members of a coalition get is, of course, the value of this coalition.

(5) A player can participate in at most one coalition in a game. If he does not succeed in joining a coalition, he will get no points in that game[1].

(6) Each player should try to obtain a <u>maximum number of points</u> in each game.

(7) The negotiations have to be carried on in the presence of all the participants of the game, but it is permissible and advisable that each player plan his own strategies beforehand, and also consult friends or acquaintances, before negotiations start. There is no requirement to finish the negotiations in one session.

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(1) Talking about a 1-person coalition was avoided.

(8) At the end of the negotiations, write on the card those coalitions which were formed and the number of points received by each player. For example: A coalition a + b was formed; a received 20 points, b received 40 points. Player c did not receive any point.

(9) Also, list the amount of time that the negotiations took. Indicate the number of meetings and the length of each one of them.

(10) On the back of the card, each player should write those reasons that brought him to agree or disagree to the various offers. Also, the players should describe the course of the negotiations, such as offers and counter-offers. Each player should also indicate if he planned his strategies alone, or consulted friends and acquaintances, how much time he dedicated to this (do not include the time of actual negotiations here) and to what extent he listened to the suggestions given to him[1].

(11) Each player will participate in many games and the total number of points that he will get, will determine his rank in the contest.

(12) At the end of the contest, the following prizes are offered to the players who get the highest scores:

A coupon for buying books worth I.P.[2]	10	(at that time about $6) for first place .
A coupon for buying books worth I.P.	5	(at that time about $3) for second place .
A coupon for buying books worth I.P.	3	(at that time about $2) for third place .

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(1) None of the students offered information on the items of this sentence. It has been decided not to insist on this information in order to make the description less awkward.

(2) Israeli Pounds.

(13) Remember: Each game has an object of its own: In each game you will have different partners from those in previous games. Therefore, there is no place for agreements such as: "I will help you now and you, in return, will help me later".

(14) In order to avoid the possibility that some players will try to harm a player who already has received many points, all the players must keep in strict secret the number of points that they or their partners have received. These should not be revealed until the end of the contest.

(15) Remember: There is no time limit to the negotiations.

(16) Remember: You are going to play many games. Even if you did not do well in some games, you will be able to collect points later on.

In each game try to get the maximum. Two points are better than one, but even if you get nothing, you have no reason not to try again in the next game.

Good luck.

Before answering questions which you may have, let us turn to the blackboard and see how a game may be played.[1] Player b in this game may start by offering c a split 20:20 . Player a is in danger of obtaining 0 . He therefore may propose a 30:30 split with b . If player c wants to avoid being left out, he may, for example, offer player b a 5:35 split. At this point, player a may suggest, say, a 70:40:40 division, and if all agree, then the coalition a + b + c is formed with this split.

Are there any questions? "

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[1] The game on the blackboard was the game mentioned earlier.

Comments. Clearly, not all the recommendations of Section 1 were fulfilled. Some items in the instructions are specific to this experiment. As im the pioneering experiment by G. Kalisch, J.W. Milnor, J. Nash and E.D. Nering [9], the games were presented directly in the characteristic function form, all of them normalized by (2.1). Since no facilities were available to record the negotiations and to review the players after each play, we asked the players to write by themselves their own personal accounts. Naturally, there were many omissions and all we could get was often only a general feeling about the negotiations.(1) We did insist, however, on the students playing each game at least a day after the cards were distributed.

We waited until most of the cards in one set were returned before we distributed another set, so that, in general, each player was occupied with one game. Due to technical difficulties (e.g, long plays or illness of a student), it happened occasionally that a student was occupied with as many as three plays.

Attempts were made to make the games independent of each other. This is why we rotated the players in the teams and asked that the score be kept secret. From occasionally questioning the students we learned that not only was the score concealed, but most students did not even bother to list it.(2) Even the first winners did not know that they were ranking high prior to being told so.

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(1) Yet, because of this self-writing, we could observe a few unexpected reasonings which we could not have revealed by a questionnaire - since we were unaware of them.

(2) Only one student came to me at the end of the contest, protesting that I had made a mistake in computing the score.

The subjects' conduct. Israel is a country with many parties based on clashing ideologies. Coalitions are a must and bargaining is a hard fight. No wonder that the students were enthusiastic about the contest. Some were skeptical and thought that the outcomes were arbitrary and that you could not learn from them about real coalition formation. Others repeatedly asked whether the outcomes showed consistency and turned out to be as predicted.(1) One student (student no.12) was apathetic and almost never tried to win points(2). Occasionally, a student tried to behave erratically in order to "prove to the teacher that the theory is wrong". In general, however, the contest was conducted in good spirit except for occasional complaints of "unfairness", "dishonesty", "double crossing", etc. The general opinion in the class was that in these plays all tricks were legitimate.

It was insisted that negotiations be carried on in the presence of all the players(3). Most negotiations took place during recesses or after school time, but quite a few students were reprimanded because they continued some heated debates during the lessons.

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(1) The author promised that the experiment would be evaluated at the end of the contest.

(2) This student had difficulties at home, in the class studies, and among his friends. He scored much worse than the rest of the students.

(3) Even if a 2-person coalition formed in a 4-person game, the two members were required to stay until the other coalition settled.

Negotiation time ran from 3 seconds to 40 minutes[1] with an average of 10 minutes per play. Most plays ended in one session, but several took as many as three meetings. ("Recess periods are quite short - you know.")

There was a feeling in the class that the games did not reflect real life, because - so they thought - trying to be fair and not to deprive a friend biased the outcomes. The author believes, quite contrarily, that in most cases the students acted as competitively as they were capable of. At least, this is the impression one gets from reading the accounts. Perhaps one reason for this discrepancy is that the plays were indeed frustrating. One never knows, descriptively speaking, whether one succeeds in achieving "the maximum" and one tends to justify any necessary compromise by claiming that one is trying to be fair.

In order to introduce a random selection of the students for the games, the names of the students were written on cards which were then shuffled. After the shuffle, we assigned numbers to the cards in a consecutive order. Henceforth, care was taken to group the students in such a way that at first no two of them met twice in the same play and later - no three of them.[2] For the sake of the psychologist who may be interested in this experiment let me put on record that players 1, 8, 9, 11, 20, 27, 28, 29, 32 were girls. The rest were boys.

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(1) This was the students' rough estimate. No one measured this time exactly.

(2) The whole procedure should certainly be planned better in other experiments. For example, a random bias could well be propagated by such a rotation.

Game No.	Players			Values of the coalitions				Outcome of the play	Time of negotiation in minutes	Deviation from $\mathcal{M}_1^{(i)}$
	1	2	3	12	13	23	123			
1	1	2	3	50	100	100	0	(0, 26, 74; 1,23)	6	[1, -1]
2	4	5	6	50	100	100	100	(0, 40, 60; 1,23)	40	[15, -15]
3	7	8	9	50	100	100	110	(30, 30, 50; 123)	15 = 5 + 10	**[10, 10, -20]**
4	10	11	12	50	100	100	125	(35, 35, 55; 123)	30 = 10 + 10 + 10	[10, 10, -20]
5	13	14	15	50	100	100	150	(50, 50, 50; 123)	7 = 5 + 2	[0, 0, 0]
6	16	17	18	40	80	80	0	(20, 20, 0; 12,3)	14 = 7 + 7	[0, 0]
7	19	20	21	40	80	80	80	(30, 0, 50; 13,2)	10	[10, -10]
8	22	23	24	40	80	80	90	(0, 20, 60; 1,23)	25	[0, 0]
9	25	26	27	40	80	80	100	(32, 32, 36; 123)	30	[12, 12, -24]
10	28	29	30	40	80	80	110	(30, 30, 50; 123)	35	[0, 0, 0]
11	31	32	33	60	70	90	0	(0, 35, 55; 1,23)	30 = 5 + 25	[-5, 5]
12	34	35	36	60	70	90	90	(0, 45, 45; 1,23)	15 = 10 + 5	[5, -5]
13	2	5	8	60	70	90	110	(30, 40, 40; 123)	3	[10, 0, -10]
14	3	6	9	60	70	90	150	(50, 50, 50; 123)	30	[0, 0, 0]
15	4	7	10	60	80	100	0	(25, 0, 55; 13,2)	5	[5, -5]
16	11	14	17	60	80	100	100	(0, 43, 57; 1,23)	?	[3, -3]
17	12	15	18	60	80	100	110	(10, 50, 50; 123)	10	**[-20/3, 40/3, -20/3]**
18	13	16	19	60	80	100	120	(35, 35, 50; 123)	7	[15, -5, -10]
19	23	20	26	60	80	100	150	(46, 50, 54; 123)	10	[0, 0, 0]
20	24	21	27	20	30	60	0	(0, 28, 32; 1,23)	2	[0, 0]
21	25	22	28	20	30	60	40	(0, 25, 35; 1,23)	5	[0, 0]
22	29	32	35	20	30	60	55	(0, 23, 37; 1,23)	4	[0, 0]
23	30	33	36	20	30	60	60	(5, 15, 0; 12,3)	20	[5, -5]
24	31	34	37	20	30	60	100	(20, 30, 50;123)	10	[0, 0, 0]
25	38	2	4	30	30	80	0	(0, 40, 40; 1,23)	1	[0, 0]
26	1	4	8	30	30	80	20	(0, 40, 40; 1,23)	2	[0, 0]
27	2	6	10	30	30	80	70	(0, 40, 40; 1,23)	5	[0, 0]
28	3	7	11	30	30	80	80	(0, 40, 40; 1,23)	5	[0, 0]
29	5	9	12	30	30	80	100	(10, 45, 45; 123)	15 = 5 + 10	[0, 0, 0]
30	32	37	38	70	90	120	0	(20, 0, 70; 13,2)	5	[0, 0]
31	35	36	28	70	90	120	0	(35, 35, 0; 12,3)	20 = 15 + 5	[15, -15]
32	30	33	36	20	30	60	60	(10, 0, 20; 13,2)	15	[10, -10]
33	14	18	22	0	50	50	50	(20, 0, 30; 13,2)	15	[20, -20]
34	15	19	23	0	50	50	60	(15, 15, 30; 123)	10	[5, 5, -10]
35	16	20	24	0	50	50	100	(25, 25, 50; 123)	5	[0, 0, 0]
36	25	29	33	0	50	50	120	(36, 36, 48; 123)	20	[0, 0, 0]
37	26	30	34	0	50	70	0	(0, 12, 58; 1,23)	3	[0, 0]
38	27	31	35	0	50	70	60	(0, 15, 55; 1,23)	30	[0, 0]
39	28	32	36	0	50	70	70	(2, 18, 50; 123)	10	[2, -2, 0]
40	37	2	7	0	50	70	80	(5, 25, 50; 123)	10 = 5 + 5	[0, 0, 0]
41	38	3	8	0	50	70	100	(20, 39, 41; 123)	16	[0, 0, 0]

Table 1. 3-person games of various kinds, general information.

There is a remarkable degree of consistency with the predictions of $\mathcal{M}_1^{(i)}$ (column d). Indeed, even if we reject games 25 - 28 , where symmetry considerations alone are sufficient to determine the outcome, only 5 out of the remaining outcomes deviate from $\mathcal{M}_1^{(i)}$ by more than 5 points[1] per player. The accounts in four out of the five exceptional plays show that extraneous factors (fights[2], prestige considerations, etc.) could cause the deviations[3] . Such factors were not present so pronouncedly in other plays.

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(1) It appears from the accounts that usually 5 points were regarded as the first noticeable amount worth fighting for.

(2) The reason for a fight would be an obnoxious conduct of one of the players, such as unwillingness to compromise, or not keeping promises. In game no.32, a fight over 3 points caused the strong player to switch to another coalition and thus lose at least 12 points. After completing game 7 , player 2 complained to me that she had discovered that a secret meeting had taken place prior to the playing of the game, in which players 1 and 3 had decided on the outcome. This was against the rules which required the presence of all the players during the negotiations. (The accounts, of course, do not reveal the conspiracy.)

(3) Note that four out of the five exceptional games were super-additive. We shall later see that in such games other complications arose. These will be discussed in Section 5 .

One cannot learn from the sketchy accounts what really went on in the minds of the players while playing, but several accounts seem to show that a pattern of justified objections or of objections which were countered in the spirit of the bargaining set theory was followed (+ in column f). In other plays it is clear that other procedures were adopted[1] (- in column f) .

It is interesting to note that the coalition of the two strongest players was predominating (column k). This is not too surprising for the games with a non-empty quasi core, but even out of the remaining 12 games, only 2 ended up with the strongest player being excluded from the 2-person coalition[2] (column i) . In this connection, it should be noted that the strong player received more than his quota only in 2 plays (one of which had a non-empty quasi core). A possible explanation of these phenomena is provided in Section 2 .

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(1) 0 in column f indicates that no justified objections could be raised with respect to the listed offers. A blank space means that owing to an incomplete account no conclusions could be deduced.

(2) In several games the two weakest players were in a symmetric role. When this was not the case, and the game had an empty quasi core, 3 games ended up with the two strongest players forming a coalition and 2 ended up with the weakest and the strongest players forming a coalition.

Finally, it should be noted that whenever the kernel narrowed a continuum of elements of $\mathcal{M}_1^{(i)}$ down to a single element, the deviation from the kernel was very small (column e) . We have no descriptive explanation for this observation, except noting that a quota of a player (predicted by the kernel) has, perhaps, a general appeal.

Second class. In 9 out of the 27 super-additive games (see Table 3) v(123) was equal to v(23) . When playing these games, a 3-person coalition formed only once (with the weakest player receiving merely a token of two points) . This indicates that the students were apparently far from being generous in seeking the welfare of their partners.

When playing games in which v(123) was greater than v(23) , a 3-person coalition was formed except in one game.

Pareto optimum outcomes occurred in all but three of the plays (column ℓ), but 8 outcomes were not coalitionally rational[1] (n.c.r.); i.e., a proper subcoalition of the coalition which was formed could have made more.

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(1) This concept appeared first in I. W. Milnor [16] . (The term was introduced in [2] .) It is also a special case of ψ- stability (see [10]).

a. Game No.	b. Values of the coalitions 12	13	23	123	c. Outcome of the play	d. Deviation from $\mathcal{M}_1^{(i)}$	e. Deviation from $\mathcal{K}$	f. Obj. and c. obj.	g. Order of quota	h. W u W	i. S u S	j. Core	k. Dev. of S from $\mathcal{K}$	l. P.O.	m. N. c. c.
2	50	100	100	100	(0, 40, 60; 1,23)	[15 , -15]	same	-	+			0		+	
7	40	80	80	80	(30, 0, 50; 13,2)	[10 , -10]	same	-	+			0		+	
12	60	70	90	90	(0, 45, 45; 1,23)	[5 , -5]	same	+	(+)			0		+	
16	60	80	100	100	(0, 43, 57; 1,23)	[3 , -3]	same	+	+			0		+	
23	20	30	60	60	(5, 15, 0; 12,3)	[5 , -5]	same	-?	+			-		-	
28	30	30	80	80	(0, 40, 40; 1,23)	[0 , 0]	[0 , 0]	0	+			+		+	
32	20	30	60	60	(10, 0, 20; 13,2)	[10 , -10]	same	+	+			-		-	
33	0	50	50	50	(20, 0, 30; 13,2)	[20 , -20]	same	-	+			-		+	
39	0	50	70	70	(2, 18, 50; 123)	[2 , -2 , 0]	[2 , 8 , -10]	0	+		+	(+)	-10	+	+
3	50	100	100	110	(30, 30, 50; 123)	[10 , 10 , -20]	same	+	+	+		0	-20	+	+
8	40	80	80	90	(0, 20, 60; 1,23)	[0 , 0]	same		+			0		-	
17	60	80	100	110	(10, 50, 50; 123)	[-20/3,40/3,-20/3]	same	-	(+)		+	0	-20/3	+	+
4	50	100	100	125	(35, 35, 55; 123)	[10 , 10 , -20]	same		+	+?		-	-20	+	+
9	40	80	80	100	(32, 32, 36; 123)	[12 , 12 , -24]	same	+	+	+		-	-24	+	+
13	60	70	90	110	(30, 40, 40; 123)	[10 , 0 , -10]	same	+	(+)	+		-	-10	+	+
18	60	80	100	120	(35, 35, 50; 123)	[15 , -5 , -10]	same		(+)			-	-10	+	+
5	50	100	100	150	(50, 50, 50; 123)	[0 , 0 , 0]	[50/3,50/3,-100/3]	+	(+)	+		+	-100/3	+	
10	40	80	80	110	(30, 30, 50; 123)	[0 , 0 , 0]	[20/3,20/3,-40/3]	-?	-	+		+	-40/3	+	
14	60	70	90	150	(50, 50, 50; 123)	[0 , 0 , 0]	[50/3,-10/3,-40/3]	-	(+)	+		+	-40/3	+	
19	60	80	100	150	(46, 50, 54; 123)	[0 , 0 , 0]	[16 , 0 , -16]	0	+	+		+	-16	+	
24	20	30	60	100	(20, 30, 50; 123)	[0 , 0 , 0]	[0 , -5 , 5]		+	+?		+	5	+	
29	30	30	80	100	(10, 45, 45; 123)	[0 , 0 , 0]	[0 , 0 , 0]	0	+		+	+	0	+	
34	0	50	50	60	(15, 15, 30; 123)	[5 , 5 , -10]	[10 , 10 , -20]	-	+	+		-	-20	+	+
35	0	50	50	100	(25, 25, 50; 123)	[0 , 0 , 0]	[0 , 0 , 0]	0 !	+	+		+	0	+	
36	0	50	50	120	(36, 36, 48; 123)	[0 , 0 , 0]	[1 , 1 , -2]	0	+	+		+	-2	+	
40	0	50	70	80	(5, 25, 50; 123)	[0 0 , 0]	[0 , 10 , -10]	+	+		+	+	-10	+	
41	0	50	70	100	(20, 39, 41; 123)	[0 , 0 , 0]	[5 , 14 , -19]		+	+		+	-19	+	

Key: Obj. and c. obj. = Objections and counter objections observed. S = the strongest player.
W u W = The two weakest players united. S u S = The two strongest players united.
P.O. = Pareto optimum. N.c.c. = Not coalitionally rational. same = same as in column d.

Table 3. Super - additive 3-person games of various kinds.

Three games, with $v(123) > v(23)$, had an empty core, and the core of four more games consisted of a unique point. We note that none of the outcomes of playing the games of the latter kind fell in or near the core (column j) .

Whenever the core occupied a significant portion of the Pareto optimum possible outcomes, the players chose an outcome in the core and therefore also in $\mathcal{M}_1^{(i)}$. This, of course, is not too much of a support to either of the core or of the bargaining set theories. It is much more interesting to explain , if possible, why particular points in the core were chosen in each case. The kernel theory is obviously insufficient for this purpose, because of the many large deviations from the kernel (column e) .

On the whole, the 27 outcomes in Table 3 exhibit an embarrassing situation: Either they are quite near the kernel or they strongly deviate from it. They also seem not to conform to almost any available solution concept. In 6 cases, even the order of the quotas is preserved only in the weak sense (column g) . We shall explain these strange results in Section 5 . Meanwhile, let us point out that one phenomenon appeared consistently in all the plays in which a 3-person coalition was formed. The accounts show that in 11 - 13 out of 18 plays, the two weakest players united in their fight against the strongest player, whereas only in 4 plays did the two strongest players unite in their fight against the weakest player (columns h and i) . Consequently, perhaps,the strongest player managed only once to obtain more than hisshare in (column k).

5. Three-person games (part II).

Consider the game $\mathbf{A} \equiv (\{1\ 2\ 3\}; v)$, where $v(12) = 50$, $v(13) = 50$, $v(S) = 0$ otherwise. The bargaining set $\mathcal{M}_1^{(i)}$ consists of (0, 0, 0 ; 1,2,3) , (50, 0, 0 ; 12, 3) , (50, 0, 0 ; 13, 2), (0, 0, 0 ; 1, 23) and (0, 0, 0 ; 123) . If we modify this game by requiring that $v(123) = 50$, then the last i.r.p.c. in the previous bargaining set is to be replaced by (50, 0, 0 ; 123) . Thus, the only profitable payoff vector is (50, 0, 0) . This is also the only p.v. in the core of the game and it is also the "central point" in each solution of J. von Neumann and O. Morgenstern (see [22]).

Yet, the students paid scarcely any attention to (50, 0, 0) . 13 plays were conducted for game **A** and 24 plays for game $\mathbf{B} = (\{1\ 2\ 3\}; v)$, where $v(12)=v(13)=60$, $v(23)=10$, $v(S)=0$ otherwise, which is similar to **A** but its core is empty[1]. Almost every player had the opportunity to play the role of the strong player (see Table 4) .

Chronologically, the following pattern roughly emerged during the plays (see Table 5): In the first round, many plays of game A ended up in a wildly competitive manner; i.e., by the strong player obtaining 49 points and one weak player receiving a single point (see check-marks in column c). Occasionally, the two weak players tried to exert pressure on the strong one, threatening that they would rather take no points than be satisfied with one point or so.

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(1) Its bargaining set consists of (0,0,0 ; 1,2,3), (55,5,0 ; 12, 3), (55,0,5 ; 13, 2), (0,5,5 ; 1, 23), and (0,0,0 ; 123) . If we put $v(123)=60$, the last i.r.p.c. is to be replaced by (53 1/3, 3 1/3, 3 1/3 ; 123) .

Game No.	Players			Values of the coalitions				Outcome of the play	Time of negotiation in minutes	Deviation from $\mathcal{M}_1^{(i)}$
	1	2	3	12	13	23	123			
42	13	17	21	0	50	50	0	(0, 1, 49; 1,23)	15	[1, -1]
43	22	30	3	50	50	0	0	(0, 0, 0; 1,2,3)	7	[0]
44	11	36	25	50	50	0	0	(30, 0,20; 13,2)	2	[-20, 20]
45	9	4	16	50	50	0	0	(40, 0,10; 13,2)	30 = 15 + 15	[-10,10]
46	14	34	20	50	50	0	0	(49, 0, 1;13,2)	5	[-1, 1]
47	32	19	27	50	50	0	0	(40, 0, 10;13,2)	10	[-10, 10]
48	26	13	37	50	50	0	0	(35, 0, 15;13,2)	7	[-15, 15]
49	29	21	1	50	50	0	0	(49, 1, 0;12,3)	0.05	[-1, 1]
50	31	5	15	50	50	0	0	(25, 0, 25;13,2)	3	[-25, 25]
51	35	2	12	50	50	0	0	(0, 0, 0,1,2,3)	9 = 1 + 5 + 3	[0]
52	38	17	23	50	50	0	0	(49, 1, 0;12,3)	1	[-1, 1]
53	18	33	7	50	50	0	0	(25, 25, 0;12,3)	3	[-25,25]
54	10	28	24	50	50	0	0	(25, 0, 25;13,2)	10	[-25,25]
55	6	22	11	60	60	10	0	(40, 0, 20;13,2)	5	[-15,15]
56	8	36	14	60	60	10	0	(40, 20, 0;12,3)	3	[-15,15]
57	23	10	38	60	60	10	0	(45, 15, 0;12,3)	7	[-10,10]
58	3	25	16	60	60	10	0	(48, 12, 0;12,3)	3	[-7, 7]
59	30	4	32	60	60	10	0	(45, 15, 0;12,3)	5	[-10,10]
60	20	27	13	60	60	10	0	(40, 20, 0;12,3)	2	[-15,15]
61	37	29	5	60	60	10	0	(35, 25, 0;12,3)	10	[-20,20]
62	1	31	26	60	60	10	0	(0, 5, 5;1,23)	3	[0, 0]
63	15	21	12	60	60	10	0	(40, 20, 0;12,3)	12 = 10 + 2	[-15, 15]
64	2	17	33	60	60	10	0	(20, 0, 40;13,2)	5	[-35, 35]
65	7	28	18	60	60	10	0	(30, 0, 30; 13,2)	5	[-25, 25]
66	24	23	35	60	60	10	0	(45, 15, 0;12,3)	1	[-10, 10]
67	17	8	15	60	60	10	0	(38, 0, 22;13,2)	10	[-17, 17]
68	5	10	3	60	60	10	0	(0, 5, 5;1,23)	5	[0, 0]
69	19	2	20	60	60	10	0	(0, 5, 5;1,23)	10	(0, 0]
70	4	11	18	60	60	10	0	(40, 20, 0;12,3)	5	[-15, 15]
71	16	1	14	60	60	10	0	(45, 15, 0;12,3)	8	[-10, 10]
72	33	24	37	60	60	10	0	(35, 25, 0;12,3)	3	[-20, 20]
73	25	13	23	60	60	10	0	(29, 0, 31;13,2)	5	[-26, 26]
74	28	26	35	60	60	10	0	(31, 29, 0;12,3)	5	[-24, 24]
75	36	38	31	60	60	10	0	(55, 5, 0;12,3)	10	[0, 0]
76	27	29	34	60	60	10	0	(35, 25, 0;12,3)	5	[-20, 20]
77	21	9	32	60	60	10	0	(45, 15, 0;12,3)	5	[-10, 10]
78	12	7	6	60	60	10	0	(0, 5, 5;1,23)	5	[0, 0]

Table 4. Extreme 3-person games.

Table 5a. A descriptive analysis of the outcomes of the extreme 3-person games (part a).

Key: Dev. = Deviation. mod. = modified. c. = coalition.
coop. s. f. = cooperative standard of fairness.

a. Game No.	b. Outcome of the play	c. Dev. from $\mathcal{M}_1^{(i)}$	d. Dev. from mod. game 2-person c.	e. Dev. from mod. game 3-person c.	f. Dev. from coop. s. f. 2-person c.	g. Dev. from coop. s. f. 3-person c.
42	(0, 1, 49; 1,23)	[1, -1] ✓	[-11½, 11½]	[-7⅔, -7⅔, 15⅓]	[-15⅔, 15⅔]	[-10 11/18, -10 11/18, 21 5/9]
43	(0, 0, 0; 1,2,3)	[0]				
44	(30, 0, 20; 13,2)	[-20, 20]	[- 7½, 7½]	[- 3½, 1⅔, 1⅔]	[- 3⅓, 3⅓] ✓	[2⅔, - 1⅓, - 1⅓]
45	(40, 0, 10; 13,2)	[-10, 10]	[2½, - 2½] ✓	[6⅔, - 3⅓, - 3⅓]	[6⅔, - 6⅔]	[12⅔, - 6⅓, - 6⅓]
46	(49, 0, 1; 13,2)	[-1, ½, ½] ✓	[11½, -11½]	[15⅔, - 7⅔, - 7⅔]	[15⅔, -15⅔]	[21⅔, -10 2/18, -10 2/18]
47	(40, 0, 10; 13,2)	[-10, 10]	[2½, - 2½] ✓	[6⅔, - 3⅓, - 3⅓]	[6⅔ - 6⅔]	[12⅔, - 6⅓, - 6⅓]
48	(35, 0, 15; 13,2)	[-15, 15]	[- 2½, 2½] ✓?	[1⅔, - ⅔, - ⅔]	[1⅔, - 1⅓] ✓?	[7⅔, - 3 2/18, - 3 2/18]
49	(49, 1, 0; 12,3)	[- 1, 1] ✓	[11½, -11½]	[15⅔, - 7⅔, - 7⅔]	[15⅔, -15⅔]	[21⅔, -10 2/18, -10 2/18]
50	(25, 0, 25; 13,2)	[-25, 25]	[-12½, 12½]	[- 8⅓, 4⅓, 4⅓]	[- 8⅓, 8⅓]	- 2⅔, 1 2/18, 1 2/18
51	(0, 0, 0; 1,2,3)	[0]				
52	(49, 1, 0; 12,3)	[- 1, 1] ✓	[11½, -11½]	[15⅔, - 7⅔, -7⅔]	[15⅔, -15⅔]	[21⅔, -10 2/18, -10 2/18]
53	(25, 25, 0; 12,3)	[-25, 25]	[-12½, 12½]	[- 8⅓, 4⅓ 4⅓]	[- 8⅓, 8⅓]	[- 2⅔, 1 2/18, 1 2/18]
54	(25, 0, 25; 13,2)	[-25, 25]	[-12½, 12½]	[- 8⅓, 4⅓, 4⅓]	[- 8⅓, 8⅓]	[- 2⅔, 1 2/18, 1 2/18]
55	(40, 0, 20; 13,2)	[-15, 15]	[- 2½, 2½] ✓	[3⅓, - 1⅔, - 1⅔]	[0 , 0]	[6⅔, - 3⅓, - 3⅓]
56	(40, 20, 0; 12,3)	[-15, 15]	[- 2½, 2½]	[3⅓, - 1⅔, - 1⅔]	[0 , 0] ✓	6⅔, - 3⅓, - 3⅓]
57	(45, 15, 0; 12,3)	[-10, 10]	[2½, - 2½]	[8⅓, - 4⅓, - 4⅓] ✓??	[5 , - 5]	[11⅔, - 5⅔, - 5⅔]
58	(48, 12, 0; 12,3)	[- 7, 7]	[5½, - 5½] ✓ !!	[11⅓, - 5⅔, - 5⅔]	[8 , - 8]	[14⅔, - 7⅓, - 7⅓]
59	(45, 15, 0; 12,3)	[-10, 10]	[2½, - 2½] ✓	[8⅓, - 4⅓, - 4⅓]	[5 , - 5]	[11⅔, - 5⅔, - 5⅔]
60	(40, 20, 0; 12,3)	[-15, 15]	[-2½, 2½] ✓?	[3⅓, - 1⅔, - 1⅔]	[0 , 0] ✓?	[6⅔, - 3⅓, - 3⅓]
61	(35, 25, 0; 12,3)	[-20, 20]	[-7½, 7½]	[- 1⅔, ⅔ , ⅔]	[- 5 , 5]	[1⅔, - ⅔ , - ⅔] ✓
62	(0, 5, 5; 1,23)	[0, 0]				
63	(40, 20, 0; 12,3)	[-15, 15]	[-2½, 2½]	[3⅓, - 1⅔, - 1⅔] ✓	[0 , 0]	[6⅔, - 3⅓, - 3⅓]
64	(20, 0, 40; 13,2)	[-35, 35]	[-22½, 22½]	[-16⅔, 8⅓, 8⅓]	[-20 ,20]	[-13⅓, 6⅔, 6⅔]
65	(30, 0, 30; 13,2)	[-25, 25]	[-12½, 12½]	[- 6⅔, 3⅓, 3⅓]	[-10 , 10]	[- 3⅓, 1⅔, 1⅔]
66	(45, 15, 0; 12,3)	[-10, 10]	[2½, - 2½] ✓	[8⅓, - 4⅓, - 4⅓]	[5 , - 5]	[11⅔, - 5⅔, - 5⅔]
67	(38, 0, 22; 13,2)	[-17, 17]	[- 4½, 4½] ✓?	[1⅓, - ⅔, - ⅔] ✓?	[- 2 , 2]	[4⅓, - 2⅓, - 2⅓]
68	(0, 5, 5; 1,23)	[0, 0]				
69	(0, 5, 5; 1,23)	[0, 0]				
70	(40, 20, 0; 12,3)	[-15, 15]	[- 2½, 2½]	[3⅓, - 1⅔, - 1⅔]	[0 , 0] ✓	[6⅔, - 3⅓, - 3⅓]
71	(45, 15, 0; 12,3)	[-10, 10]	[2½, - 2½] ✓	[8⅓, - 4⅓, - 4⅓]	[5 , - 5]	[11⅔, - 5⅔, - 5⅔]
72	(35, 25, 0; 12,3)	[-20, 20]	[- 7½, 7½]	[- 1⅔, ⅔, ⅔]	[- 5 , 5] ✓	[1⅔, - ⅔, - ⅔]
73	(29, 0, 31; 13,2)	[-26, 26]	[-13½, 13½]	[- 7⅔, 3⅔, 3⅔]	[-11 , 11]	[- 4⅓, 2⅓, 2⅓]
74	(31, 29, 0; 12,3)	[-24, 24]	[-11½, 11½]	[- 5⅔, 2⅔, 2⅔]	[- 9 , 9]	[- 2⅓, 1⅓, 1⅓] ✓
75	(55, 5, 0; 12,3)	[0, 0] ✓	[12½, -12½]	[18⅓, - 9⅓, - 9⅓]	[15 , -15]	[21⅔, -10⅔, -10⅔]
76	(35, 25, 0; 12,3)	[-20, 20]	[- 7½, 7½]	[- 1⅔, ⅔, ⅔]	[- 5 , 5] ✓	[1⅔, - ⅔, - ⅔]
77	(45, 15, 0; 12,3)	[-10, 10]	[2½, - 2½] ✓	[8⅓, - 4⅓, - 4⅓]	[5 , - 5]	[11⅔, - 5⅔, - 5⅔]
78	(0, 5, 5; 1,23)	[0, 0]				

Table 5b. A descriptive analysis of the outcomes of the extreme 3-person games (part b).

Key: W&W = The two weakest players. S = The strongest player. comp. = compete. suff. = sufficient. pt. = point. $v_1 = [v(123) + v(23)]/2$. $v_2 = 2\,v(123)/3$.

a. Game No.	h. W&W comp. on S's favors	i. W&W unite. No lottery	j. W&W threaten a lottery against S	k. W&W's joint demand	l. S's offer insuff. to break unity	m. S's offer suff. to break unity	n. W&W settle by lot	o. v_1 or v_2 asked and obtained	p. Remarks
42	+				20	20			side effects ✓
43		+							
44						20			
45	(+)		+			20			double cross
46	+						(+)		a lot on 1 pt.
47		+			5	10			
48		(+)				15			
49	+								
50			+	25			+	[0,0,0] ✓	
51			+?	25	15				disagreement ✓
52	+								
53			+	25	20		+	[0,0,0] ✓	
54			+	25			+	[0,0,0] ✓	
55			+	25		20			
56		+				20			
57			+				+		
58		+				12			double cross
59	(+)				15	20			
60		+		40	10	20			
61			+	35	15		+		
62			+?	20					disagreement ✓
63			+	40			+		
64			+	30	30		+	[0,0,0] ✓	
65			+	30	15	15	+	[5,-2½,-2½] ✓	
66									
67			+	25	20		+		
68		+?			20				disagreement ✓
69			+	35					disagreement ✓
70						20			
71			+			15			
72						25			
73									side effects ✓
74			+	40			+		
75	+	+		30		10			
76			+?	40	20	25			
77		+?				15			
78			+	40					disagreement ✓

It did not take long before this last threat was reenforced by a new argument: The weak players[1] 2 and 3 jointly demanded a certain sum, the receiver of which, they announced, would be chosen by lot[2]. This was an excellent procedure to by-pass the artificial prohibition of forming the 3-person coalition. Indeed, by an appropriate lottery, one can distribute $v(12)$ among the three players in any chosen way[3]. Such a procedure requires, of course, a "super-additivization " of the characteristic function; namely, replacing $v(123)=0$ by $v(123)=50$ in game A and by $v(123)=60$ in game B . Henceforth, this modification will be assumed.

In some cases, the weak players demanded $v(23) + [v(123)-v(23)]/2$, as if they acted as a single player. In other cases they demanded only $[v(123) - v(23)]/2$, as if one of them simply dropped out of the game[4]. Sometimes, the weak players even had"the nerve" to demand $2/3 \cdot v(123)$, as if they were demanding the rights of two out of three players.[5] (See column k.)

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(1) Note that, unlike the rest of the games, the strong player in game 42 was player 3 .

(2) They meant a lottery assigning equal probabilities.

(3) Assuming, of course, that the points represent transferable utility units.

(4) The two cases are indistinguishable in game A .

(5) Occasionally other demands were also heard (games 63, 65, 75).

For a while, the strong player remained helpless, feeling that he ought to accept the majority terms (column o) . Later, he realized that he could fight back. This he did by trying to convince a weak player to defect from the partnership with the other weak player. (Column ℓ shows the reported offers which were insufficient for causing the defections.) In general, player 1 managed to draw a weak player to his side. (This happened in about 12 plays[1].) Occasionally, a compromise was reached, whereby the negotiated offer was given to the winner of the lottery. (This happened in about 6 plays[2].)

Five plays ended up in disagreement, whereby the weak players had to divide the value $v(23)$ between themselves , and the strong player received no payment[3].

Having played and discussed so many games, most members of the class expressed (in various conversations) the opinion that the best thing to do for the weak players is to threaten jointly, and the best thing to do for the strong player is to offer a weak player a sufficient amount - roughly equal to the expectation of the weak player from the lottery - in order to cause him to defect. The class also expressed the belief that the strong player is capable of

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(1) These are check-marked in columns d and f , starting from game 55 .

(2) These are check-marked in columns e and g .

(3) See column p .

causing the defection, if a profitable outcome is to be reached. I must admit that I share this opinion with the class in spite of the bargaining set theory. I would behave in the same manner whether in the role of the strong or in that of the weak player.

If an experiment convincingly contradicts a theory , one either rejects the theory or modifies it, if possible. Since this experiment seems to be in conflict with several solution theories, it seems that the fault lies with the game representation and not with the theories. This idea was advanced in [13], where it was argued that even if a game is given in a characteristic function form, such characteristic function may not be the "right" function to describe the game. This is the case if a standard of fairness[1] is known to exist among the players.

In our case, a modification of the characteristic function can be determined if the following assumptions are expected to hold for the players:

(i) The outcome should be Pareto optimum and individually rational. (Group and individual rationality. J. von Neumann and O. Morgenstern [22]).

(ii) If two negotiation groups negotiate a share of α units, if the first group can control β units and the second can control γ units, $\alpha \geq \beta + \gamma$, then an equal split of the extra profits will result; namely, the first group will receive $\beta + (\alpha - \beta - \gamma)/2$ units and the second group will receive $\gamma + (\alpha - \beta - \gamma)/2$ units (assumption on the standard of fairness).

[1] Detailed discussions and exact definitions of the terms which we now introduce are provided in [13].

Thus, although $v(23)=0$ or 10, in our games, the coalition $\{2\ 3\}$ can act as a single negotiation group, bargaining with player 1 over the share of $v(123)$. By (ii), they will receive $u(23) \equiv v(23) + [v(123) - v(23) - 0]/2$. We argue that, realizing this, the coalition $\{2\ 3\}$ should regard its power as $u(23)$ instead of $v(23)$. By the same token, player 1, as a 1-person coalition should regard its power as $u(1) \equiv [v(123) - v(23)]/2$. By applying the above considerations to all the coalitions, one arrives at the constant sum representations of games A and B in terms of the power of the various coalitions: $u(1)=25$, $u(2)=0$, $u(3)=0$, $u(12)=50$, $u(13)=50$, $u(23)=25$, $u(123)=50$ for game A, and $u(1)=25$, $u(2)=0$, $u(3)=0$, $u(12)=60$, $u(13)=60$, $u(23)=35$, $u(123)=60$ for game B. The parts of the bargaining sets based on these new characteristic functions[1], in which the p.v. is Pareto optimum, are, respectively:

$$(5.1)\qquad \begin{cases} (37\ 1/2,\ 12\ 1/2,\ 0 \quad ;\ 12,\ 3) \\ (37\ 1/2,\ 0,\ 12\ 1/2 \quad ;\ 13,\ 2) \\ (33\ 1/3,\ 8\ 1/3,\ 8\ 1/3\ ;\ 123) \end{cases},$$

and

$$(5.2)\qquad \begin{cases} (42\ 1/2,\ 17\ 1/2,\ 0 \quad ;\ 12,\ 3) \\ (42\ 1/2,\ 0,\ 17\ 1/2 \quad ;\ 13,\ 2) \\ (36\ 2/3,\ 11\ 2/3,\ 11\ 2/3\ ;\ 123) \end{cases}.$$

These will be called the modified[2] $\mathcal{M}_1^{(im)}$'s, and $(\{1\ 2\ 3\}, u)$ will be called "the modified game".

(1) These characteristic functions do not possess the normalization $u(i)=0$, $i=1,2,3$.

(2) The upper index is motivated by the requirement that the outcome is an imputation in the sense of R.D. Luce and H. Raiffa [10].

Strictly speaking, assumption (ii) is not a standard of fairness in the sense of [13], because it lacks information on the share of an amount among three players. We shall regard it as an imperfect form of a standard of fairness. Actually, if the members of $\{2\ 3\}$ realize it, they can enter the game as <u>two</u> negotiation groups, bargaining with the third negotiation group (i.e., player 1) over the share of $v(123)$. Let us, therefore, add another assumption on the standard of fairness:

(iii) <u>If three negotiation groups negotiate a share of α units, if, separately, they can control β, γ and δ units, respectively, $\alpha \geq \beta + \gamma + \delta$, then an equal split of the extra profits will result; namely, they will receive $\beta + (\alpha - \beta - \gamma - \delta)/3$, $\gamma + (\alpha - \beta - \gamma - \delta)/3$ and $\delta + (\alpha - \beta - \gamma - \delta)/3$, respectively.</u>

Now, a 2-person coalition is allowed two options: it can declare itself either as a single negotiation group or as two negotiation groups. It will certainly take that alternative which renders it the largest power. In our games, the coalitions $\{1\ 2\}$ and $\{1\ 3\}$ will prefer to act as a single negotiation group, because $50 > 2/3 \cdot 50$ and $60 > 2/3 \cdot 60$, whereas the coalition $\{2\ 3\}$ will prefer to act as two negotiation groups, because $50/2 < 2 \cdot 50/3$ and $10 + (60-10)/2 < 2 \cdot 60/3$.

With this standard of fairness, the games are represented, in terms of the power of the coalitions, by :

$w(1) = 16\ 2/3$, $w(2) = 0$, $w(3) = 0$, $w(12) = 50$, $w(13) = 50$, $w(23) = 33\ 1/3$, $w(123) = 50$ and $w(1) = 20$, $w(2) = 0$, $w(3) = 0$, $w(12) = 60$, $w(13) = 60$, $w(23) = 40$, $w(123) = 60$. The corresponding bargaining sets are :

$$(5.3)\qquad \begin{cases} (33\ 1/3,\ 16\ 2/3,\ 0 \ ;\ 1,2\ 3) \\ (33\ 1/3,\ 0,\ 16\ 2/3 \ ;\ 13,\ 2) \\ (27\ 7/9,\ 11\ 1/9,\ 11\ 1/9 \ ;\ 123) \end{cases}$$

for game A, and

$$(5.4)\qquad \begin{cases} (40,\ 20,\ 0 \ ;\ 12,\ 3) \\ (40,\ 0,\ 20 \ ;\ 13,\ 2) \\ (33\ 1/3,\ 13\ 1/3,\ 13\ 1/3 \ ;\ 123) \end{cases}$$

for game B. We shall call these bargaining sets - the bargaining sets $\mathcal{M}_1^{(im)}$'s derived from the <u>cooperative standard of fairness</u> (see [13])[1].

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(1) Another possibility is that the players in a coalition, say, $\{2\ 3\}$, act as two players, but demand $v(23)+2[v(123)-v(23)]/3$; i.e., demand to split the extra profits in a 2:1 ratio (in addition to their own value). If every coalition acts this way one obtains Harsanyi's <u>upraised game</u> (see [7]). The upraised game coincides in game A with our representation based on the cooperative standard of fairness. For game B it yields $z(1) = 16\ 2/3$, $z(2) = 0$, $z(3) = 0$, $z(12) = 60$, $z(13) = 60$, $z(23) = 43\ 1/3$, $z(123) = 60$, and the resulting bargaining set is: $\{(38\ 1/3,\ 21\ 2/3,\ 0\ ;\ 12,\ 3),\ (38\ 1/3,\ 0,\ 21\ 2/3\ ;\ 13,\ 2),\ (31\ 1/9,\ 14\ 4/9,\ 14\ 4/9\ ;\ 123)\}$. It seems to us that

./.

Clearly, an attempt to explain the players' conduct by observing that each outcome falls within, say, 5 points from one of the various alternatives is not too convincing, because any quota-preserving outcome, chosen at random, has a good chance of being in the neighborhood of one of the seven alternatives. Instead, we shall try to read from the students' written accounts which of the alternatives the players had in mind, and then check if the numerical outcomes fell indeed within the neighborhood of the bargaining set. Accordingly, we make the following rules :

1. When it did not occur to the weak players that they could unite against the strong one, we check-marked [1] column c of Table 5, (see also column h). (The case of the naive weak players.)

(Continued from bottom of page 41.)

either the subjects did not possess this standard of fairness, or they were not sophisticated enough to realize this possibility. At any rate, the coalition $\{2\ 3\}$ never asked for a sum as high as 43 1/3. For this reason, we shall not explore this variant, although, a priori, it is as reasonable as the cooperative standard of fairness.

(1) We also check-marked game 75 in this column. In this game a very low offer caused a weak player to defect, whereupon the other weak player retaliated by competing wildly. (A similar situation arose in games 45 and 59, but the offer was high and the retaliator managed to save for himself a substantial amount.)

2. Any outcome which was not Pareto optimum was marked by "disagreement" in column p . These five plays will be excluded from the subsequent considerations.

3. If the weak players jointly demanded either $v(23)+\frac{1}{2}\left[v(123)-v(23)\right]$ or $2v(123)/3$ and the strong player accepted the term, and if a decision on the distribution was a lot, we check-marked[1] column o. (The case of the naive strong player.)

4. If a lottery was executed, then, obviously, a 3-person coalition was formed.

5. If no lottery took place, we considered the situation as if a 2-person coalition was formed. This is an ad-hoc rule, because each weak player could assign a certain subjective probability (say, equalling $\frac{1}{2}$) that the strong player would choose him as a partner and this would be a choice by lot without an actual lottery being performed.

6. If the weak players jointly demanded at first $2v(123)/3$, then they certainly saw the possibility of acting as two negotiation groups. We regarded the game as belonging to a cooperative standard of fairness and check-marked it in columns f or g , according to whether a 2-person coalition or a 3-person coalition was formed.

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[1] A joint demand of $2v(123)/3$ was accepted only in game 64. We also check-marked game 65, where the accepted joint element was $v(123)/2$.

7. If the weak players jointly demanded at first $v(23)+\frac{1}{2}[v(123)-v(23)]$ or less, then they were not as sophisticated, and the game was considered as governed by the modified characteristic function (columns d or e check-marked).

8. Whenever no joint demand was reported (or made), we had to base the decision on the accepted offer of the strong player. Only in this case we had to refer to the bargaining sets (5.1) -- (5.4). Since we had already decided on what coalition formed, this was a choice of one of two alternatives only. We check-marked the corresponding column.

9. Two games (nos. 43 and 73) remained unchecked. In game 43, the weak players - out of solidarity - simply refused any offer and did not ask for any, feeling that it was not right that one of them should be deprived. In game 73, a weak player wanted to prove to the teacher that "the accepted theory is wrong". He therefore insisted on receiving no points. This made the game a 2-person game, in which the players shared the sum $v(13)$ almost equally. (Column p check-marked).

Conclusions.

(1) The maximum deviation from $\mathcal{M}_1^{(i)}$ in the case of the naive weak players is 1 point per player.

(2) All the cases of disagreement fell in $\mathcal{M}_1^{(i)}$. This is not too surprising.

(3) The deviations from the modified $\mathcal{M}_1^{(im)}$ and from $\mathcal{M}_1^{(im)}$ based on the cooperative/standard of fairness/ in the checked places were extremely small. A deviation which is greater than 5 points per player occurred only in

games 57 and 58 [1] . In most cases the deviation is much less than 5 points. This correlation is quite surprising. If the explanation is true, then this means that the players modified the characteristic function in their minds (according to their abilities of sophistication) and then - subconsciously - tended to agree only to outcomes near the bargaining set of the altered games [2] .

(4) A learning process took place. This is indicated by the check-marks: At first, many naive players (weak as well as strong) existed. When the weak players jointly fought against the strong player, they started by asking $v(23) + \frac{1}{2}\left[v(123) - v(23)\right]$. It was only later that they realized the possibility of demanding more. However, disagreements increased as the joint demand grew (see column p) .

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(1) The deviation was $5\frac{1}{2}$ points in game 58 and $8\frac{1}{2}$ points in game 57.

(2) One should have checked by a detailed interview whether all the players had in mind the same modified characteristic function at the end of the play, or, perhaps various standards existed, whereupon a player accepted an outcome if it was higher than his "expectation" (based upon "his" characteristic function) . In the latter case, an acceptance of such payoff by one player may have caused the players to view the game differently. We shall later encounter such a phenomenon.

If such subconscious activities took place, they should have been traced also in the 3-person superadditive games discussed in the previous section.

These games suit our purpose even better than the ones just analyzed, because (1) we know exactly which coalition formed; (2) in 18 out of 27 games, /the/ two modifications of the bargaining set coincide [1]. This reduces the number of alternatives. On the other hand, most students' accounts could not disclose convincingly which standard of fairness the players had in mind. Table 6 lists the various alternatives and a + mark shows where good correlation is found. Out of the 27 outcomes, 19 fall within 5 points per player from one of the bargaining sets. Two plays deviate because of extraneous influences /(games 17 and 33)/ In four games the weak players demanded and received $2v(123)/3$ and, essentially, shared it equally [2]. Two other outcomes can be regarded as if actually a 2-person coalition was formed in the modified bargaining set and the weakest player received a token of 2 or 5 points.

Thus, the above considerations seem to be reasonable explanations of what actually happened during the contest. These should be regarded as conjectures which require to be tested in further experiments.

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(1) The modified game 36 (in the imperfect standard of fairness) is not super-additive.

(2) Despite the fact that in two of these games the weak players were not symmetric.

a. Game No.	b. Outcome of the play	c. Deviation from $\mathcal{K}$	d. Deviation from modified game	e. Deviation from coop. s. f.	f. Dev. from equal share of v(123)	g. A special deviation
2	(0, 40, 60; 1,23)	[15,-15]	[$2\frac{1}{2}$,-$2\frac{1}{2}$] +	[$2\frac{1}{2}$,-$2\frac{1}{2}$] +		
7	(30, 0, 50; 13,2)	[10,-10]	[0 , 0] +	[0 , 0] +		
8	(0, 20, 60; 1,23)	[0, 0] +	——	——		
12	(0, 45, 45; 1,23)	[5,- 5] +	[$2\frac{1}{2}$,-$2\frac{1}{2}$] +	[$2\frac{1}{2}$,-$2\frac{1}{2}$] +		
16	(0, 43, 57; 1,23)	[3,- 3] +	[-2 , 2] +	[-2 , 2] +		
23	(5, 15, 0; 12,3)	[5,- 5] +	——	——		
28	(0, 40, 40; 1,23)	[0, 0] +	[0 , 0] +	[0 , 0] +		
32	(10, 0, 20; 13,2)	[10,-10]	——	——		
33	(20, 0, 30; 13,2)	[20,-20]	[$7\frac{1}{2}$,-$7\frac{1}{2}$]	[$3\frac{1}{3}$,-$3\frac{1}{3}$] +		
3	(30, 30, 50; 123)	[10 ,10 ,-20]	[$1\frac{2}{3}$, $1\frac{2}{3}$,- $3\frac{1}{3}$] +	[$1\frac{2}{3}$, $1\frac{2}{3}$,- $3\frac{1}{3}$] +		
4	(35, 35, 55; 123)	[10 ,10 ,-20]	[$1\frac{2}{3}$, $1\frac{2}{3}$,- $3\frac{1}{3}$] +	[$1\frac{2}{3}$, $1\frac{2}{3}$,- $3\frac{1}{3}$] +		
5	(50, 50, 50; 123)	[$16\frac{2}{3}$,$16\frac{2}{3}$,-$33\frac{1}{3}$]	[$8\frac{1}{3}$, $8\frac{1}{3}$,-$16\frac{2}{3}$]	[$8\frac{1}{3}$, $8\frac{1}{3}$,-$16\frac{2}{3}$]	[0, 0, 0] +	
9	(32, 32, 36; 123)	[12 ,12 ,-24]	[$5\frac{1}{3}$, $5\frac{1}{3}$,-$10\frac{2}{3}$]	[$5\frac{1}{3}$, $5\frac{1}{3}$,-$10\frac{2}{3}$]	[$-1\frac{1}{3}$,$-1\frac{1}{3}$, $2\frac{2}{3}$] +	
10	(30, 30, 50; 123)	[$6\frac{2}{3}$, $6\frac{2}{3}$,-$13\frac{1}{3}$]	[0 , 0 , 0]+	[0 , 0 , 0] +		
13	(30, 40, 40; 123)	[10 , 0 ,-10]	[$1\frac{2}{3}$, $1\frac{2}{3}$,- $3\frac{1}{3}$] +	[$1\frac{2}{3}$, $1\frac{2}{3}$,- $3\frac{1}{3}$] +		
14	(50, 50, 50; 123)	[$16\frac{2}{3}$,$-3\frac{1}{3}$,-$13\frac{1}{3}$]	[$8\frac{1}{3}$,$-1\frac{2}{3}$,- $6\frac{2}{3}$]	[$8\frac{1}{3}$,$-1\frac{2}{3}$,- $6\frac{2}{3}$]	[0, 0, 0] +	
17	(10, 50, 50; 123)	[$-6\frac{2}{3}$,$13\frac{1}{3}$,- $6\frac{2}{3}$]	[$-16\frac{2}{3}$,$13\frac{1}{3}$, $3\frac{1}{3}$]	[$-16\frac{2}{3}$,$13\frac{1}{3}$, $3\frac{1}{3}$]		
18	(35, 35, 50; 123)	[15 ,-5 ,-10]	[5 ,-5 , 0]+	[5 ,-5 , 0]+		
19	(46, 50, 54; 123)	[16 , 0 ,-16]	[6 , 0 ,- 6]	[6 , 0 ,- 6]	[-4, 0, 4] +	
24	(20, 30, 50; 123)	[0 ,-5 , 5]+	[$-1\frac{2}{3}$,$-6\frac{2}{3}$, $8\frac{1}{3}$]	[$-4\frac{2}{9}$,$-7\frac{2}{9}$, $12\frac{4}{9}$]		
29	(10, 45, 45; 123)	[0 , 0 , 0]+	[$-6\frac{2}{3}$, $3\frac{1}{3}$, $3\frac{1}{3}$]	[$-7\frac{2}{9}$, $3\frac{2}{9}$, $3\frac{4}{9}$]		
34	(15, 15, 30; 123)	[10 ,10 ,-20]	[$3\frac{1}{3}$, $3\frac{1}{3}$,- $6\frac{2}{3}$]	[0 , 0 , 0]+		
35	(25, 25, 50; 123)	[0 , 0 , 0]+	[0 , 0 , 0]+	[$-5\frac{5}{9}$,$-5\frac{5}{9}$, $11\frac{1}{9}$]		
36	(36, 36, 48; 123)	[1 , 1 ,- 2]+	——	[$-2\frac{1}{3}$,$-2\frac{1}{3}$, $4\frac{2}{3}$] +		
39	(2, 18, 50; 123)	[2 , 8 ,-10]	[$-6\frac{1}{3}$,$-\frac{1}{3}$, $6\frac{2}{3}$]	[$-10\frac{2}{9}$,$-4\frac{2}{9}$, $14\frac{4}{9}$]		
40	(5, 25, 50; 123)	[0 ,10 ,-10]	[$-6\frac{2}{3}$, $3\frac{1}{3}$, $3\frac{1}{3}$]	[$-11\frac{1}{9}$,$-1\frac{1}{9}$, $12\frac{2}{9}$]		[2,$-4\frac{1}{2}$,$2\frac{1}{2}$] +
41	(20, 39, 41; 123)	[5 ,14 ,-19]	[$1\frac{2}{3}$,$10\frac{2}{3}$,-$12\frac{1}{3}$]	[$-3\frac{2}{9}$, $5\frac{1}{9}$,- $1\frac{8}{9}$] +		[0, 0 ,0] +

Key: coop. s. f. = cooperative standard of fairness. Dev. = Deviation.
A special deviation = Deviation from the modified game, considering it as if a 2-person coalition was formed.

Table 6. A descriptive analysis of the outcomes of the super-additive 3-person games.

6. 4-person games with flat[1] 2-person coalitions.

Games 79 - 95 are games of the type described by the heading of this section. Most of these games were played during a 3-month teachers' strike. Because of the strike the cards had to be mailed to the pupils' homes. Each student who received a card had to visit his partners, show them the card and organize a meeting for playing the game. Under such conditions it was almost impossible for the students to plan their negotiations in advance, nor did they have the opportunity of discussing their games with other friends. Consequently, it seems that the students did not exhibit a high degree of sophistication in these plays as compared with the foregoing ones. Nevertheless it appears that some subconscious factors existed and that the outcomes reveal interesting phenomena.

Four-person games with flat 2-person coalitions can be characterized, in the normalization (2.1), by $v(N)$, $N = \{1\ 2\ 3\ 4\}$, and by the 3-quota of the players. The 3-quota[2] is a vector $(\omega_1, \omega_2, \omega_3, \omega_4)$, whose components satisfy: $\omega_i + \omega_j + \omega_k = v(ijk)$, whenever i, j and k are disjoint.

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(1) A coalition is called "flat" if its value is equal to the sum of the values of its members, taken as 1-person coalitions. In our normalization - if its value is equal to zero.

(2) The term is used here in the sense of B. Peleg [18]. It is slightly different from G.K. Kalisch's 3-quota (see [8]).

The quotas in our games were always non-negative. For such games, the bargaining set prescribes a quota share for the members of a formed 3-person coalition. If the core is empty and if the game is super-additive, the bargaining set $\mathcal{M}_1^{(i)}$ prescribes for the grand coalition an equal deviation from the quota for all the players. If the core is not empty, the bargaining set simply prescribes the core for the grand coalition (see[12]). The kernel coincides with the bargaining set, except in the case of a formation of the grand coalition in a game having a non-empty core. In this case, the kernel prescribes a unique payoff vector which is computed in [5].

Table 7 lists the general information concerning the games, the players, the time of negotiation, the outcomes and the deviations from $\mathcal{M}_1^{(i)}$. The deviations from $\mathcal{K}$ differ from the deviations from $\mathcal{M}_1^{(i)}$ only in games 89 and 90 ; they are [-1, 18 1/3, -1 2/3, -15 2/3] and [1, 2, -7, 4] , respectively. One sees immediately that the deviations from the bargaining set are, in general, quite large for these games. This shows that a different explanation of the various outcomes should be sought.

Game No.	Players 1	2	3	4	Values of the coalitions 123	124	134	234	1234	Outcome of the play	Time of negotiation in minutes	Deviation from $\mathcal{M}_1^{(i)}$	Remarks
79	32	34	38	25	100	70	50	90	0	(15,45,40,0;123,4)	30	$[1\frac{2}{3}, -8\frac{1}{3}, 6\frac{2}{3}]$	
80	24	26	29	31	100	70	50	90	110	(33,33,34,0;123,4)	1	$[19\frac{2}{3}, -20\frac{1}{3}, \frac{2}{3}]$	
81	2	9	11	15	100	70	50	90	100	(15, 39, 36, 10;1234)	23=5+15+3	$[2\frac{1}{2}, -13\frac{1}{2}, 3\frac{1}{2}, 7\frac{1}{2}]$	n.c.r.
82	18	20	25	30	70	60	80	90	0	(0,30,30,30;1,234)	3	[10, -10, 0]	
83	6	8	16	21	70	60	80	90	100	(20,20,30,30;1234)	7	[10, 0, -10, 0]	n.c.r.
84	23	27	28	33	70	60	80	90	95	(10,30,30,0;123,4)	2	[0, 10, -10]	
85	3	4	12	14	100	110	90	120	0	(23,45,0,42;124,3)	10	[3, -5, 2]	
86	10	17	19	22	100	110	90	120	140	(22,29,42,47;1234)	35=20+10+5	[2, -21, 12, 7]	n.c.r.
87	1	5	7	13	100	110	90	120	130	(7,41,41,41;1234)	15	$[-10\frac{1}{2}, -6\frac{1}{2}, 13\frac{1}{2}, 3\frac{1}{2}]$	n.c.r.
88	21	30	37	35	60	80	100	120	0	(0,35,35,50;1,234)	10	[15, -5, -10]	
89	36	38	20	6	60	80	100	120	130	(4,40,40,46;1234)	10	$[-3\frac{1}{3}, 10, -3\frac{1}{3}, -3\frac{1}{3}]$	n.c.r
90	34	18	11	13	60	80	100	120	150	(16,27,38,69;1234)	3	[0, 0, 0, 0]	
91	14	13	12	9	100	120	60	80	0	(40,55,0,25;124,3)	5	[0, -5, 5]	
92	1	3	5	6	120	60	80	100	0	(40,40,40,0;123,4)	8	[20, 0, -20]	
93	10	15	16	17	80	100	120	60	0	(0, 0, 0, 0;1,2,3,4)	15= 5 + 10	[0]	
94	18	22	19	26	60	80	100	120	0	(10,0,45,45;134,2)	5	[10, 5, -15]	
95	24	27	29	31	60	120	80	100	0	(40,40,0,40;124,3)	2	20, 0, -20]	

Key: n. c. r. = not coalitionally rational.

Table 7. 4-person games with flat 2-person coalitions.

Table 8 distinguishes between games in which a 3-person coalition formed and games in which the grand coalition formed.(1)(2)

Only two out of 8 possible outcomes(3) were not Pareto optimum (column e); however, attending the core occurred only when the core was relatively large (1 out of 5 cases, see column f). In general, the order of the 3-quota was preserved by the outcomes, but usually only in the weak sense (column g). A demand for coalitional rationality did not impress the players (see "Remarks" in Table 7).

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(1) Note that games 87 and 89 were listed twice in Table 8. In one case (starred) they were listed as if a 3-person coalition had been formed. It seems that in these games, the coalition $\{2\ 3\ 4\}$ formed first, and later, the difference $v(1234) - v(234)$ was distributed among the four players. In game 89, a 3-person share of (38, 38, 44) was listed by the players, and then it was crossed out in favor of the (4, 40, 40, 46) payoff.

(2) Game 93 ended up in total disagreement. The accounts show that two players erroneously believed that if they stuck together and demanded a high payoff, they would eventually get their terms.

(3) The remaining games are not super-additive and, hence, are excluded from the count.

a. Game No.	b. 3-quota				c. v(N)	d. Outcome of the play	e. P.O.	f. Core	g. Order of quota	h. Dev. of key player	i. Split for others	j. Remarks
79	13 1/3	53 1/3	33 1/3	3 1/3	0	(15, 45, 40, 0; 123,4)						
80	13 1/3	53 1/3	33 1/3	3 1/3	110	(33, 33, 34, 0; 123,4)	-	-	+	1 1/3	45,40	
82	10	20	40	30	0	(0, 30, 30, 30; 234,1)			(-)	2/3	33,33	
84	10	20	40	30	95	(10, 30, 30, 0; 123,4)	-		(+)	0	30,30	
85	20	50	30	40	0	(23, 45, 0, 42; 124,3)			(+)	0	30,30	
88	0	20	40	60	0	(0, 35, 35, 50; 234,1)			+	3	45,42	quota share
91	40	60	0	20	0	(40, 55, 0, 25; 124,3)			(+)	-10	35,35	
92	20	40	60	0	0	(40, 40, 40, 0; 123,4)			+	5	40,55	quota share
94	0	20	40	60	0	(10, 0, 45, 45; 134,2			(+)	0	40,40	
95	20	40	0	60	0	(40, 40, 0, 40; 124,3)			(+)	10	45,45	
									(+)	0	40,40	
87*	20	50	30	40	130	(0, 40, 40, 40; 234,1)			(+)	0	40,40	interm. outcome
89*	0	20	40	60	130	(0, 38, 38, 44; 234,1)			(+)	-2	38,44	interm. outcome
93	60	0	20	40	0	(0, 0, 0, 0;1,2,3,4)						disagreement
81	13 1/3	53 1/3	33 1/3	3 1/3	100	(15, 39, 36, 10; 1234)	+		+	1 1/3	39,36,10	
83	10	20	40	30	100	(20, 20, 30, 30; 1234)	+	-	(+)	0	20,30,30	
86	20	50	30	40	140	(22, 29, 42, 47; 1234)	+	-	-	2	29,42,44	
90	0	20	40	60	150	(16, 27, 38, 69; 1234)	+	+	+	0	16,27,69	
87	20	50	30	40	130	(7, 41, 41, 41; 1234)	+		(+)	-10 2/3	41,41,41	
89	0	20	40	60	130	(4, 40, 40, 46; 1234)	+	-	(+)	- 3 1/3	40,40,46	

Key: P.O. = Pareto optimum. Dev. = Deviation. interm. = intermediate outcome. key = key player.

Table 8. 4-person games with flat 2-person coalitions. Conditional consent of key player.

4. Three-person games (part I).

The first 41 games consisted of 3-person games of various types. These are summarized in Table 1, where the reader can identify the players, the characteristic function, the outcome and the time of negotiation in each play[1]. Deviation from the nearest element in $\mathcal{M}_1^{(i)}$ is also indicated.

For our analysis, it is convenient to divide the games into two (overlapping) classes. One class consists of the games in which a 2-person coalition formed and the other consists of all the super-additive games (i.e., the games which have a super-additive characteristic function). Each game appears at least in one class, because no 3-person coalition formed in a non-super-additive game.

First class. In all the 23 plays (see Table 2), the outcomes preserved the order determined by the quota of the players[2] (column g). The evidence, however, is not conclusive to conclude that the players preferred an outcome in the quasi-core (see Section 2) whenever this was not empty (column h).

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(1) The table indicates situations where several meetings took place, and marks the length of such meetings. First term indicates first meeting, etc.

(2) In two plays the order was preserved only in the weak sense .

a. Game No.	b. Values of the coalitions 12	13	23	123	c. Outcome of the play	d. Dev. from $\mathcal{M}^{(i)}_1$	e. Dev. from K	f. Obj. and counter obj.	g. Order of quota	h. Quasi core	i. W&W	j. S&W	k. S&S	l. Dev. of S from quota	m. Remarks
1	50	100	100	0	(0, 26, 74; 1,23)	[1,-1]	same	+	+			(+)	(+)	-1	
2	50	100	100	100	(0, 40, 60; 1,23)	[15,-15]	same	-	+			(+)	(+)	-15	fight
6	40	80	80	0	(20, 20, 0; 12,3)	[0, 0]	same	+	+		+			0	
7	40	80	80	80	(30, 0, 50; 13,2)	[10,-10]	same	-	+			(+)	(+)	-10	partial comm.
8	40	80	80	90	(0, 20, 60; 1,23)	[0, 0]	same		+			(+)	(+)	0	
11	60	70	90	0	(0, 35, 55; 1,23)	[-5, 5]	same	-?	+				+	5	
12	60	70	90	90	(0, 45, 45; 1,23)	[5, -5]	same	+	(+)				+	-5	
15	60	80	100	0	(25, 0, 55; 13,2)	[5, -5]	same		+			+		-5	
16	60	80	100	100	(0, 43, 57; 1,23)	[3, -3]	same	+	+				+	-3	
20	20	30	60	0	(0, 28, 32; 1,23)	[0, 0]	[3,-3]	0	+	+			+	-3	
21	20	30	60	40	(0, 25, 35; 1,23)	[0, 0]	[0, 0]	0	+	+			+	0	
22	20	30	60	55	(0, 23, 37; 1,23)	[0, 0]	[-2, 2]	0	+	+			+	2	
23	20	30	60	60	(5, 15, 0; 12,3)	[5, -5]	same	-?	+	-	+			-5	
25	30	30	80	0	(0, 40, 40; 1,23)	[0, 0]	same	0	+	+			+	0	obvious
26	30	30	80	20	(0, 40, 40; 1,23)	[0, 0]	same	0	+	+			+	0	obvious
27	30	30	80	70	(0, 40, 40; 1,23)	[0, 0]	same	0	+	+			+	0	obvious
28	30	30	80	80	(0, 40, 40; 1,23)	[0, 0]	same	0	+	+			+	0	obvious
30	70	90	120	0	(20, 0, 70; 13,2)	[0, 0]	same	+	+			+		0	
31	70	90	120	0	(35, 35, 0; 12,3)	[15,-15]	same	-	(+)		+			-15	fight
32	20	30	60	60	(10, 0, 20; 13,2)	[10,-10]	same	+	+	-		+		-10	prestige
33	0	50	50	50	(20, 0, 30; 13,2)	[20,-20]	same	-	+	-		(+)	(+)	-20	
37	0	50	70	0	(0, 12, 58; 1,23)	[0, 0]	[2,-2]	0	+	+			+	-2	
38	0	50	70	60	(0, 15, 55; 1,23)	[0, 0]	[5,-5]	+	+	+			+	-5	

Key: Dev. = Deviation. Obj. and counter obj. = Objections and counter objections observed. W&W = The two weakest players formed a coalition. S&W = The weakest player and the strongest player formed a coalition. S&S = The two strongest players formed a coalition. partial comm. = partial communication existed during the play. S = the strongest player. same = same as in column d.

Table 2. Three person games of various kinds in which a 2-person coalition formed.

It seems that the players "refute" many solution concepts and criteria of present-day game theory, but before "concluding" that they acted stupidly, let us examine the outcomes more closely. One phenomenon is quite interesting: In all the cases where a 3 or 4-person coalition formed, at least one player deviated only very slightly from the bargaining set. In 2/3 of the cases where a 4-person coalition formed, at least two players deviated very slightly from the bargaining set. This suggests a more dynamic explanation for the outcomes. Indeed, an agreement on the payments is never reached simultaneously. After some negotiations a player, whom we shall call "the key player", may declare that he will be satisfied with a certain amount α. If this amount seems reasonable to the other players, and if they are willing to allow him this amount, then they face the problem of sharing the rest. This is a new 3-person game. Taking this standpoint and using our knowledge gained in the previous games, we can distinguish between several possibilities.

A. The key player conditionally accepts the amount α.

"Conditionally" means here that it is understood that he will accept α, provided that a certain coalition S (of which he is a member) is formed. In other words, the players first decide to form the coalition S, and only after such a decision has been reached do they bargain for their shares. An acceptance of α in this case means that the key player reserves himself the right to change his mind if the negotiations break down and another coalition is proposed.

If S is a 3-person coalition, and if the remaining two players want this coalition and agree to give the key player the

amount α, then these two players and the excluded player in $N - S$ face a 3-person game in which the two players have decided to form a coalition. Since none of these two players can make more than 0, either by himself, or, together with the excluded player[1], both players are of <u>equal bargaining strength</u> and, by the bargaining set theory, they may share $v(S) - \alpha$ any way they wish[2]. According to the kernel - they should share $v(S) - \alpha$ equally.

If S is the grand coalition, then the remaining three players face a 3-person game in which they all share $v(S) - \alpha$ but, again, any two of them (and, of course, each single player) cannot guarantee themselves more than a zero payment. Thus again, they will share $v(S) - \alpha$ either any way they wish, as predicted by $\mathcal{M}_1^{(i)}$, or equally, as predicted by $\mathcal{K}$. Since $\mathcal{K}$ is a more sensitive test in this case, we shall compare the outcomes with the kernel.[3] There remains the problem of deciding who is the key player, and how much he can demand.

If the game is not super-additive, or if the outcome is not Pareto optimum[4], then the key player may be satisfied with his share in $\mathcal{M}_1^{(i)}$. Another possibility is that the key player sacrifices

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(1) The key player cannot take part in these arguments, because his amount α is valid only if S is formed; i.e., if the two remaining players participate.

(2) Provided that the payoff is individually rational.

(3) One does not need the kernel theory in order to decide that if "all things are equal" the players will share the proceeds equally.

(4) This happened in our games whenever a 3-person coalition was formed.

a certain amount of his "proper" share in order to participate in a 3-person coalition. If the game is super-additive and the outcome is Pareto optimum[1], then other possibilities may arise; namely, the characteristic function may undergo some modifications due to the presence of a standard of fairness.

By definition, the key player is the player, in the coalition that formed, whose payment was the first to be determined /(explicitly or implicitly)/ Unfortunately, it is impossible to identify /with certainty/ this player from the short written accounts of the students. Let us, therefore, conjecture that the key player is a player whose deviation from $\mathcal{M}_1^{(i)}$ is relatively small and see if such a player can be chosen in such a way that the remaining players share their payoffs almost equally. This is attempted in columns h and i of Table 8 . (The hypothetical kay player is underlined in column d) .

Whenever a 3-person coalition is formed (including games 87* and 89*), the outcomes are remarkably consistent. In 9 out of 12 games a player deviates from $\mathcal{M}_1^{(i)}$ by not more than[2] 5 points, and the remaining partners share the rest almost equally.[3]

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(1) This happened in our games whenever a 4-person coalition was formed.

(2) In most games the deviation is considerably smaller.

(3) The only exceptions are game 91 in which the payoff is nearly a quota-share; i.e., nearly in $\mathcal{M}_1^{(i)}$, and games 88 and 94 , in which the key player deviates by 10 points from $\mathcal{M}_1^{(i)}$, but his partners still share the rest equally.

This explanation is not supported by the experiment in the cases where a 4-person coalition formed. Except for games 87 and 89 , no equal share occurred among three players. And even in games 87 and 89 , it seems that the distinguished player (player 1) was the last to join the coalition and, hence, he could not have been the key player. The reason for this discrepancy is that perhaps, whenever a 4-person coalition formed, the key player did not condition the acceptance of his own payment by the formation of this 4-person coalition.

B. The key player unconditionally accepts the amount α.

It is conceivable that a player k declares that he will accept an amount α and join any coalition the other members of which will be willing to offer him this amount. If this amount appears reasonable to the remaining players and they agree to offer him this amount, or if the remaining players believe that this key player will not lower his demand, then they face a 3-person reduced game, $(N - \{k\} ; v^*)$, where $v^*(S) = \text{Max}\left(v(S), v(S \cup \{k\}) - \alpha\right)$ for $S \subset N - \{k\}$, except that $v^*(S) = v(S \cup \{k\}) - \alpha$ if $S \cup \{k\}$ is under negotiation and $v^*(S) = v(S)$ if S is under negotiation. In other words, if a coalition S, $S \subset N - \{k\}$, is used for threatening purposes, its members can count both on the original value of their coalition and on the original value of the coalition which contains them and the key player.(1)

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(1) A similar phenomenon occurs in the kernel theory, and it is used to compute the kernel for various coalition structures. (See the sections dealing with the method of deleting a player in [5], [3] and [15]).

Instead of guessing who is the key player, let us test each player in a formed many-person coalition as if he was the key player, assuming that his α is the amount he actually received in the outcome. (Of course, the key player could have been the player who was excluded when a 3-person coalition was formed, in which case we cannot guess the amount α he could persistently have demanded, unless it is described in the written accounts(1). Let us, therefore, hope that such player was not the key player).

For each candidate for a key player, let us compute the reduced game for the other players and check if the payments of these players considerably deviate from the bargaining set.(2) This is done in Table 9 , column c . Neglecting, as usual, the deviations up to 5 points per player, we see in 16 out of 18 games (including games 87* and 89*) that there exist "key players" who could support our present explanation(3)(4) (these are marked in column c).

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(1) This never happened, except for game 93 , in which two such players existed and caused a breakdown of the negotiations.

(2) A deviation from the kernel of the reduced game would yield similar conclusions.

(3) Game 87 could be replaced by game 87* ; hence the only exception is game 89* .

(4) Column e lists those players for whom there is some evidence in the accounts to the effect that they could be the key players. This column supports the check-marks in column c (one exception only) and does not support column d in Table 8 (5 exceptions out of 8 cases). However, since these findings are based on my subjective interpretation of the students' accounts, they should be considered with caution.

a. Game No.	b. Dev. from $\mathcal{M}_1^{(i)}$. Players ordered by their quotas	c. Deviation of reduced game from $\mathcal{M}_1^{(i)}$ Player 1 as key player	Player 2 as key player	Player 3 as key player	Player 4 as key player	d. Strong player	e. Possible key player
79	$-8\frac{1}{3}$, $6\frac{2}{3}$, $1\frac{2}{3}$	[$-7\frac{1}{2}$, $7\frac{1}{2}$]	[$-2\frac{1}{2}$, $2\frac{1}{2}$] ✓	[5, -5] ✓		2 ✓	
80	$-20\frac{1}{3}$, $\frac{2}{3}$, $19\frac{2}{3}$	[-4, 4] ✓	[$9\frac{1}{2}$, $-9\frac{1}{2}$]	[20, -20]		2	
82	-10, 0, 10		[-5, 5] ✓	[5, -5] ✓	[10, -10]	3 ✓	<u>2</u> or 4
84	-10, 10, 0	[10, -10]	[5, -5] ✓	[-5, 5] ✓		3 ✓	
85	-5, 2, 3	[$-3\frac{1}{2}$, $3\frac{1}{2}$] ✓	[$\frac{1}{2}$, $-\frac{1}{2}$] ✓		[4, -4] ✓	2 ✓	<u>2</u>
88	-10, -5, 15		[0, 0] ✓	[$12\frac{1}{2}$, $-12\frac{1}{2}$]	[10, -10]	4	3 or 4
91	-5, 0, 5	[-5, 5] ✓	[$-2\frac{1}{2}$, $2\frac{1}{2}$] ✓		[0, 0] ✓	2 ✓	<u>2</u>
92	-20, 0, 20	[0, 0] ✓	[20, -20]	[10, -10]		3	
94	-15, 5, 10	[10, -10]		[10, -10]	[$2\frac{1}{2}$, $-2\frac{1}{2}$] ✓	4 ✓	3 or <u>4</u>
95	-20, 0, 20	[0, 0] ✓	[20, -20]		[10, -10]	4	
87*	-10, 0, 10		[5, -5] ✓	[-5, 5] ✓	[-10, 10]	2 ✓	
89*	-16, -2, 18		[7, -7]	[17, -17]	[10, -10]	4	
93	- - - - -	- - - - -	- - - - -	- - - - -	- - - - -	1	2 and 3
81	$-13\frac{1}{2}$, $3\frac{1}{2}$, $2\frac{1}{2}$, $7\frac{1}{2}$	[$-12\frac{2}{3}$, $4\frac{1}{3}$, $8\frac{1}{3}$]	[-2, -1, 3] ✓	[$3\frac{2}{3}$, $-12\frac{1}{3}$, $8\frac{2}{3}$]	[5, -11, 6]	2 ✓	
83	-10, 0, 0, 10	[0, 0, 0] ✓	[10, -10, 0]	[$6\frac{2}{3}$, $-3\frac{1}{3}$, $-3\frac{1}{3}$]	[10, 0, -10]	3 ?	<u>1</u> or 2
86	-21, 7, 12, 2	[-19, 12, 7]	[-5, 5, 0] ✓	[2, -9, 7]	[2, -14, 12]	2 ✓	<u>2</u>
90	0, 0, 0, 0	[0, 0, 0] ✓	[0, 0, 0] ✓	[0, 0, 0] ✓	[0, 0, 0] ✓	4 ✓	<u>4</u>
87	$-6\frac{1}{2}$, $3\frac{1}{2}$, $13\frac{1}{2}$, $-10\frac{1}{2}$	[-10, 10, 0]	[$-12\frac{2}{3}$, $11\frac{1}{3}$, $1\frac{1}{3}$]	[-6, -2, 8]	[$-9\frac{1}{3}$, $-5\frac{1}{3}$, $14\frac{2}{3}$]	2	
89	$-3\frac{1}{3}$, $-3\frac{1}{3}$, 10, $-3\frac{1}{3}$	[10, -5, -5]	[0, 0, 0] ✓	[-5, 10, -5]	[-5, 10, -5]	4	

<u>Table 9.</u> 4-person game with flat 2-person coalitions. An unconditional consent of the possible key players.

It is interesting to realize that the strongest player[1] never had any positive deviation from the bargaining set, and that the remaining players nearly always deviated positively (see column b); moreover, the strongest player could be considered the key player only in 10 or 11 games (marked cases in column d) .

We have offered two possible explanations for the outcomes of games 79 - 95 . The first seems quite convincing but it holds only for the cases in which a 3-person coalition formed.

Of the two different explanations, A and B , which are offered here, the latter seems to be supported by more outcomes; yet one should place a reserved confidence in it, because it involves a choice out of three or four possibilities. Moreover, we cannot help feeling that the equal share for two players in almost all the games in which a 3-person coalition formed is not coincidental. We would have felt more at ease if the written accounts had identified/with certainty/ the key player in each case, and determined whether he did or did not condition his consent. In future experiments of this type care should be taken to ascertain these points, and we conjecture that both explanations might then be supported.

One important aspect has certainly emerged in these experiments; namely, the possibility that <u>the mere determination of the payment to one player may change the original characteristic function.</u> This aspect becomes far more apparent in the experiments treated in the next section.

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(1) I.e., the player whose 3-quota is the highest.

7. 4-person quota games.

Let $(\{1\,2\,3\,4\}, v)$ be a 4-person game whose characteristic function satisfies: $v(ij) = \omega_i + \omega_j$ whenever $i \neq j$, and $v(S) = 0$ otherwise[1]. Here, $(\omega_1, \omega_2, \omega_3, \omega_4)$ is a fixed 4-tuple being the quota (or the 2-quota) of this game. It is reasonable to assume that if all $v(ij)$ are positive, then the coalition structure which forms consists of two 2-person coalitions. If the quota is non-negative, it is easy to verify that the only payoff vector for such coalition structure, which belongs to $\mathcal{M}_1^{(i)}$ is[2] $(\omega_1, \omega_2, \omega_3, \omega_4)$; i.e., it is independent of the particular coalition structure and every player receives his quota.

Games 96 - 113 were 4-person quota games (with the 3 and 4-person coalitions declared not permissible). Their quotas were all non-negative (see Table 10). Out of the 18 plays, only 4 plays ended up with the formation of a single 2-person coalition. In three of these games both players, who remained single-person coalitions, had a zero quota and, therefore, had no incentive to form together a 2-person coalition. The reason why a 2-person coalition failed to form in the 4th exceptional game (game 113) will be explained later.

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(1) Or one can regard every 3 or 4-person coalition as being not permissible. The resulting change in the bargaining set is obvious.

(2) This is no longer true if, say, $n = 6$ (see [14] Example 3.1).

Game No.	Players 1	2	3	4	Values of the coalitions 12	13	14	23	24	34	Outcome of the play	Time of negotiation in minutes	Deviation from $\mathcal{M}^{(i)}_1$	Quotas
96	1	9	18	27	100	120	80	80	40	60	(75,25,36,24;12,34)	10	[5,-5,-14,14]	70,30,50,10
97	2	10	19	28	80	120	80	80	40	80	(60,20,60,20;13,24)	3	[0,0,0,0]	60,20,60,20
98	3	11	20	29	20	20	40	20	40	40	(20,10,10,20;23,14)	5	[10,0,0,-10]	10,10,10,30
99	4	12	21	30	40	40	90	20	70	70	(35,10,10,55;23,14)	5	[5,0,0,-5]	30,10,10,60
100	5	13	22	31	80	30	90	90	150	100	(15,75,15,75;24,13)	15	[5,5,-5,-5]	10,70,20,80
101	6	14	23	32	70	80	90	90	100	110	(40,42,48,50;23,14)	30=10+20	[10,2,-2,-10]	30,40,50,60
102	7	15	24	33	30	100	90	90	80	150	(15,15,80,70;34,12)	15	[-5,5,0,0]	20,10,80,70
103	8	16	25	34	100	120	120	140	140	160	(40,60,80,80;12,34)	5	[0,0,0,0]	40,60,80,80
104	17	26	35	37	50	30	70	60	100	80	(15,35,20,60;34,12)	?	[5,-5,0,0]	10,40,20,60
105	1	20	10	37	50	80	70	130	120	150	(20,30,80,70;34,12)	10	[20,-20.0,0]	0,50,80,70
106	2	11	21	31	40	40	100	0	60	60	(40,0,0,60;14,2,3)	5	[0,0]	40,0,0,60
107	3	32	22	12	80	80	80	0	0	0	(75,0,5,0;13,2,4)	7	[-5,5]	80,0,0,0
108	4	13	23	33	90	90	140	20	70	70	(65,25,25,45;12,34)	?	[-15,15,15,-15]	80,10,10,60
109	5	14	24	34	90	90	90	20	20	20	(55,10,35,10;24,13)	?	[-25,0,25,0]	80,10,10,10
110	6	15	25	35	80	160	80	100	20	100	(40,40,89,11;34,12)	6=5+1	[-30,30,-1,1]	70,10,90,10
111	7	16	26	27	90	90	90	20	20	20	(70,20,10,10;34,12)	2	[-10,10,0,0]	80,10,10,10
112	8	17	18	28	160	70	70	90	90	0	(70,90,0,0;12,3,4)	5	[0,0]	70,90,0,0
113	9	36	29	38	100	70	140	30	100	70	(75,25,0,0;12,3,4)	5	[5,-5]	70,30,0,70

Table 10. 4-person quota games.

The accounts show that there were extraneous influences (fight) at work in game 108; hence this game will be excluded from our subsequent considerations.

Contrary to the predictions of the bargaining set, only 9 out of the 17 plays ended up within 5 points per player from the quota. The remaining 8 plays ended up in such a way that two players formed a coalition and obtained their quota (within five points per player), whereas the players in the remaining coalition received varying amounts.

The explanation of this seemingly strange phenomenon becomes clear at once, if one observes that in all the 17 plays, the first coalition which formed ended up in a nearly quota split. Moreover, whenever two 2-person coalitions formed, the deviations from the quota in the first coalition to form were not greater than the deviations in the second coalition.(1) Clearly, after a 2-person coalition is formed, the game reduces to a 2-person game for which every individually and group rational outcome is in the bargaining set.

These results exhibit in a strikingly convincing way the phenomenon observed in the previous section - namely, that partial agreements may change the nature of the game.

In [19], L.S. Shapley describes a von Neumann-Morgenstern solution to n-person quota games(2) ($n \geqslant 3$) . Our outcomes show

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(1) The coalition structures in the payoff configurations in Table 10 are listed in the order of their formation.

(2) Our games become Shapley's quota games by super-additivization.

exactly the same behavior as found in his solution which, if n is even and if the quotas are non-negative, consists of all the imputations in which (arbitrarily chosen) $n-2$ players receive their quotas and the remaining players divide their part in any arbitrary manner.

Our experiments suggest an intuitive interpretation for Shapley's solution: As long as at least four players remain, the game has a unique quota and there exists a pressure on each player to receive his quota. Sooner or later, more and more players receive their quotas, until two players remain and the resulting reduced 2-person game is such that any (non-negative) split is a quota split.(1)

Von Neumann-Morgenstern solutions usually exhibit very interesting standards of behavior. One can only admire that such standards also take into consideration the possibility that not all the payoffs are determined at once.

As a matter of fact - our outcomes exhibit another interesting phenomenon(2): <u>In all the reduced 2-person games, the player having the largest quota never received more than his quota and never received less than his partner.</u> Thus, although the reduced game is an ordinary 2-person non-constant-sum game, it has a history which tells that in

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(1) Similar interpretations can be given to Shapley's solutions for games with an odd number of players and/or quotas containing a negative component.

(2) For this observation I am indebted to a gentleman whose name, unfortunately, I do not remember.

the past one player was stronger. The history indicates a quota share. On the other hand, the game has also a present life which indicates that the players are equally strong and therefore they should share the proceeds equally. The resulting outcome therefore ought to be somewhere between these extreme distributions , in order to compensate for the regret of the strong player who, without any fault of his own, became weaker , and, perhaps, also to compensate for the guilty conscience of the weak player who is now exploiting his unfortunate partner. An agreement in the reduced game of play 113 came to naught on precisely these grounds: The quotas of the players were 0 and 70 . The player with the highest quota (a boy) suggested a 50:20 split; but the other player (a girl) did not see why 35:35 should not be the right split. Apparently the "regret function" was so effective that the boy "did not see any sense in receiving less than 50". The reduced game in play 109 almost broke down on similar grounds. The players had already written an account to this effect and then compromised and erased their account.

Making use of such "psychological" factors, the players out - smarted game theory in plays 100 and 101. The weakest players in these games were smart enough to realize that <u>if they waited until a 2-person coalition was formed, they would eventually receive more than their quotas</u>. The quotas in games 109 and 111 were (80,10,10,10) and the weak players decided to choose, by lot, a player who was to negotiate his share with player 1 without being sabotaged by the rest of the players.

Although the existence of a quota was not mentioned to the players, the class was intelligent enough to realize how the characteristic function was derived. (On many occasions the students told me that they knew how I had chosen the values of the coalitions.)

8. Different types of stability.

Consider the 4-person game whose characteristic function satisfies $v(ij) = 50$, for all i,j , $i \neq j$, $v(234) = 120$, $v(S) = 0$ otherwise[1]. It is reasonable to assume that either the coalition-structure $(234, 1)$ will form or the players will divide into two 2-person coalitions. The bargaining set $\mathcal{M}_1^{(i)}$ for these coalition structures is

$$(8.1) \qquad \begin{cases} (\ 0,\ 40,\ 40,\ 40 \quad ;\quad 234,\ 1\ ) \\ (\ \alpha,\ 50-\alpha,\ 25,\ 25 \quad ;\quad 12,\ 34\ ) \\ (\ \alpha,\ 25,\ 50-\alpha,\ 25 \quad ;\quad 13,\ 24\ ) \\ (\ \alpha,\ 25,\ 25,\ 50-\alpha \quad ;\quad 14,\ 23\ ) \end{cases},$$

where $2\frac{1}{2} \leq \alpha \leq 10$. ($\alpha = 2\frac{1}{2}$ for the outcomes in $\mathcal{K}$) .

It has been observed by H. Kesten[2] that the first i.r.p.c. in (8.1) would not be stable if we strengthen our stability requirements and demand that every objection by a non-empty <u>set of players</u> K against a non-empty set of players L has a counter objection, where $K \cap L = \phi$, $K, L \subset \{234\}$. (See [2] for details). Indeed, set $\{2\}$ can object against set $\{34\}$ by joining player 1 and offering him 5 units, and there is no way in which <u>both players</u> 3 and 4 can simultaneously protect their share of 80 units. As a matter of fact, no payoff configuration having (1, 234) as a coalition structure can be stable in this strong sense.

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(1) One may assume that, with the exception of $\{234\}$, the 3 and 4-person coalitions are not permissible. This causes only obvious modifications in the bargaining set.

(2) Oral communication.

Assuming that the players want at least the type of stability which is reflected by $\mathcal{M}_1^{(i)}$, one can present the following a priori arguments for and against (0, 40, 40, 40 ; 1, 234) . Player 2 may reason as follows: "If I am quick enough, I can approach player 1, offering to share our 50 points in such a way that I receive, say, 45 points. But if I reason this way, then, presumably, players 3 and 4 reason in a similar fashion. We therefore compete for player 1's favors and, assuming that we are of equal bargaining power (whatever that means), I shall win with a probability equalling 1/3 . If I lose, I obtain 25 . Thus my expectation from this adventure is $1/3 \cdot 45 + 2/3 \cdot 25 < 40$. Hence it seems better for me to stick to (0, 40, 40, 40 ; 1, 234) which, moreover, has the advantage of yielding a symmetric payoff to the members of the 3-person coalition ... But there is every reason to believe that player 1 will not stay put. He will probably approach, say, player 3 and offer him, say, 45 points, promising not to listen to other offers. Will not such a binding offer cause player 3 to defect ? Thus, I am in a dangerous situation and I must compete in order to gain player 1's favors ... and here we go again",

How will the players behave under such circumstances? Will they prefer the more stable and less symmetric outcome, or will they stick together against pressures exerted by player 1 ? One may even wonder whether the players will choose any outcome in the bargaining set in view of these conflicting arguments.

Games 114 - 123 are ten games of this type [1] (see Table 11).

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(1) Except that v(234) is equal to 111 in half of the games. The bargaining set in this case is similar to (8.1), except that $7 \leq \alpha \leq 13$. ($\alpha = 7$ for outcomes in $\mathcal{K}$.)

Game No.	Players 1	2	3	4	Values of the coalitions 12	13	14	23	24	34	234	Outcome of the play	Time of negotiation-minutes	Deviation from $\mathcal{M}_1^{(i)}$
114	1	11	22	33	50	50	50	50	50	50	120	(5,45,25,25;12,34)	2	[0,0,0,0]
115	16	12	23	34	50	50	50	50	50	50	111	(10,25,25,40;14,23)	15	[0,0,0,0]
116	3	13	24	35	50	50	50	50	50	50	111	(12,0,38,0;13,2,4)	3	[0,0]
117	4	14	25	36	50	50	50	50	50	50	120	(5,25,25,45;14,23)	15	[0,0,0,0]
118	5	37	15	26	50	50	50	50	50	50	111	(5,45,25,25;12,34)	15	[-2,2,0,0]
119	6	28	27	38	50	50	50	50	50	50	120	(9,25,25,41;14,23)	1	[0,0,0,0]
120	7	17	29	19	50	50	50	50	50	50	111	(10,25,25,40;14,23)	5	[0,0,0,0]
121	8	19	20	31	50	50	50	50	50	50	120	(0,40,40,40;234,1)	10	[0,0,0]
122	9	10	21	32	50	50	50	50	50	50	111	(10,25,40,25;13,24)	5	[0,0,0,0]
123	35	24	22	8	50	50	50	50	50	50	120	(9,41,25,25;12,34)	15=7+8	[0,0,0,0]

Table 11. 4-person games of a special kind.

The school year was nearly over when these games were played, and it was clear that they were among the last to be played. The students were impatient and kept asking when the outcomes of the contest would be announced and the prizes distributed. Efforts were made to return old and forgotten cards and everyone tried to increase his or her score in the last minute. Thus, during these games most students fought pretty hard to win points.

It is quite significant to observe that all the ten plays ended up in $\mathcal{M}_1^{(i)}$, except for play 118 where the deviation $[-2, 2, 0, 0]$ is negligible.

Only one play (no.116) ended up with an unreasonable coalition structure[1] $(13, 2, 4)$.

In eight plays two 2-person coalitions were formed and only in one play (no.121) the coalition $\{234\}$ emerged. This shows that although the coalition $\{234\}$ offers in $\mathcal{M}_1^{(i)}$ a symmetric share to its members and despite the fact that in other coalition structures two of its members suffer heavy losses compared to the small gain that one of them may obtain by joining player 1, yet this coalition hardly ever formed and the players preferred the more stable payoff

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(1) From the accounts we gather that player 4 in this game probably did not understand his position. At first he offered player 1 a generous amount which was rejected simply because player 1 could not take him seriously. After the coalition $\{13\}$ was formed, player 4 asked for 40 out of the 50 points that he could make with player 2 and was, of course, refused.

configuration in which two 2-person coalitions formed[1]. The accounts reveal that it was quite clear to the players that one of them would eventually join player 1, and the fear to obtain merely 25 points played an important role in the negotiations. The coalition involving player 1 was always the first to form. The initiative for such coalition came sometimes from player 1 and sometimes from his partner. On several occasions, the two players who were left alone tried to break the coalition involving player 1 by suggesting the coalition $\{234\}$ and offering player 1's partner an amount exceeding the amounts which they specified for themselves. (Their claim was that even then they would be better off because they would receive more than 25 points.) Such offers were always rejected on the ground that this asymmetric payoff was not stable and must break down during further negotiations.

These outcomes show that a desire for stability enters as an important factor in the process which leads to a final settlement and that justified objections of one player against two players are quite effective[2].

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(1) An interesting reason was advanced in one case (play 117) for rejecting $\{234\}$. The defecting player remarked that not only did he obtain more points by defecting, but he also caused two other players to gain fewer points. Under the conditions of the contest this is a legitimate consideration. However, it was mentioned in no other case.

(2) The coalition $\{234\}$ was negotiated in most plays and there were plenty of occasions for the players to realize that it would not lead to a very stable outcome.

9. Concluding remarks.

We have attempted in this paper to provide explanations for the various outcomes of the experiment. Although the bargaining set alone did not suffice to explain the outcomes for many types of games, we believe that by incorporating other ideas with the bargaining set theory, a more or less satisfactory solution could be reached. Thus, the present analysis contains several conjectures which can be tested by statistical methods.

In addition, the outcomes exhibit interesting features which deserve further theoretical and experimental study. It seems that standards of fairness which may exist among various groups must needs be taken into account when a game is to be described by a characteristic function. Also, it becomes fairly clear that a normative analysis of playing a game should take into consideration changes in the characteristic function (and in the set of active players) which result from intermediate agreements. Thus one can see, on the one hand, that game theoretical tools may be used to describe human behavior and, on the other, that more topics will have to be evolved in game theory before a satisfactory approximation to real situations can be achieved.

We have made no attempt to classify the players according to their methods of playing the various games. Neither have we tried to follow each player and check each player's "improvements" during the contest. Such tasks could perhaps be undertaken by psychologists. (1)

(1) The relevant details for such studies can be gathered from the tables and from the written accounts.

Still it would appear that various personality traits such as active versus passive participation in the play, readiness to compromise as against sticking to a single demand, various degrees of sophistication, etc., are vividly reflected in the negotiations. It seems not unreasonable to hope that such traits can be analyzed and that the interaction of people who possess those various traits can be studied by confronting the subjects with suitable cooperative games.

A P P E N D I X

This appendix contains the accounts written by the children at the close of each play.

In the translation we have done our best to preserve the unpolished and not always entirely consistent argumentation which is so typical of teenagers. We have been faithful to the original document with the exception of the following changes which were introduced:

1. Whereever names of players appeared in the text, they were replaced by the letters which represent those players.

2. A unified system of notations has been used in order to describe the coalition suggested and how its members were supposed to share the proceeds[1]. This was found necessary, in order to save the reader's time and prevent his frustration while deciding who received what during the many offers.

Additional notes and remarks have been added by the author, with the aim of clarifying what was actually happening during the negotiations. These notes are put in square brackets.

It need only be added that these accounts may be somewhat difficult to understand unless the reader compares them with the data on the corresponding games and their outcomes. The necessary information can be found in the various tables of the paper.

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(1) E.g. "a+b+c 20:30:40" means: a coalition $\{abc\}$, where player a receives 20, player b receives 30 and player c receives 40. " b→a+c 30:20:40 " means: Player b suggests that the coalition $\{abc\}$ forms, that he receives 30 and players a and c receive 20 and 40, respectively. "a/b+c" means: either coalition a+c or coalition b+c.

GAME 1. Course of play: a→c 40:60. b→c 38:62. a→c 36:64. And so on until b→c 26:74. a→c 25:75. b→a 25:25. c→a 60:40. b→c 38:62. a→c 35:65. b→c 26:74.
Reasons of player b: I agreed to receive 26 because that was the maximum I could get. Because if I had offered to a to receive less than 25 and I got more, then c would have offered [to a] more than me, but less than 25. So that in my next offer to c , or even to a when the time came, I would not have been able to demand more than 26 but I would receive less. Therefore I agreed to the above decision.
Reasons of player c: I knew in advance that the only weapons that a and b could use against me was a coalition between them for equal shares. I have to confess that my situation in this game was very easy because of the rejection from the beginning of the coalition of a+b+c . I tried to show a and b that the only possibility that would pay them is: [with c , when] c receives 74. With the first one that understood, with him I came to a coalition.
Reasons of player a: Since b did not agree to the 30 points I offered him [after the last offer listed above] and preferred to make a coalition with c , I was compelled to receive 0 points.

GAME 2. Course of play: b and c decided that the real solution is: a+b+c 30:30:40 , if there was no desire to deprive a or b through a coalition of the type a+c 40:60 for personal reasons. [This meaning: If there was no personal reason on account of which the players wish to discriminate against one of them.] But since a did not take into consideration that the game had to finish and refused to participate [in this coalition] , b and c made the above[1] coalition which paid them.
Reasons of player c: I was prepared to make the coalition a+b+c 30:30:40 because of unwillingness to deprive a/b for personal reasons. But my position which was stronger made it possible for me, after a refused, to form the coalition we formed, a coalition which paid better.
Reasons of player b: I came to the conclusion that it was necessary to compromise between the amount that could be obtained and the time being devoted. I also received the impression that c was in a better position if the game is seen as final, because of the actual conditions. [In other words: This player thought that theoretically speaking there was no reason on account of which this play ever had to finish, but since the participants were prepared to devote only a finite amount of time to negotiations it must finally come to an end.] So I aspired to a coalition a+b+c 30:30:40 . Because a did not see that the game must finish some time, he demanded that he should receive 40, and c was opposed to this, I could exploit the obstinacy of a to obtain a coalition b+c 40:60 .
[It seems that player a annoyed the others by his obstinate insistence on obtaining an exaggerated number of points. In spite of the requests of the author, this player refused to give a report on this game. It seems that he felt insulted on account of the quarrel that burst out and on account of the fact that his companions mocked him.]

(1) ["The above" here and elsewhere usually refers to the outcome of the play.]

GAME 3. Course of play: c→b 50:50. a→c 40:60. b→c 30:70. a→c 25:75. b→a 25:25. c→a+b 50:30:30. This process along general lines was repeated several times with slight changes. a and b wished for a joint coalition with c and offered c 40. c refused so they finally decided on a+b+c 30:30:50.
Reasons of player c: Since there was no possibility of a coalition with a or with b, I saw a coalition of the three of us as a prospect for a maximum of 50, since if I had tried for more it would have been more worthwhile for them to make a coalition between them.
Reasons of players a and b: Since a coalition between one of us and c did not become possible, and the maximum which we could obtain by setting up a coalition between us was 25, it paid us to set up a coalition in which we would obtain 30 points each.

GAME 4. Course of play: First offer a+b 25:25. Second offer a+b+c 35:35:55. This offer was accepted. [The report was written hastily and negligently. The game itself was conducted in three meetings, each of which took close on 10 minutes.]

GAME 5. Course of play: (1) [Several proposals for coalition were made:] a+c and b+c, with various point ratios. [Finally they considered proposals for dividing the points in the coalition a+b+c :] (2) a+b+c 26:26:98, (3) a+b+c 70:70:10 or a+b 25:25, (4) a+b+c 50:50:50 (the proposal that was accepted).
Reasons of player b: Since I saw that there was no possibility of any agreement at all with c, because a always would offer less until we reached a number of points smaller than 25 and then it would be possible to make an agreement a+b when each would receive 25 points. Therefore we agreed on a three-fold coalition with 50 for each. That is the maximum I can get.
Reasons of player c: The possibility which was before me of reaching a three-fold coalition, when there would be 98 points to my credit, came to nothing after a and b preferred to make a coalition between them. Not one of them agreed to enter into a coalition b+c or a+c, for the reason given by b (see above), so I entered into a coalition a+b+c, when each one had 50 points to his credit.
Reasons of player a: The goal from the beginning was to obtain 50, which to my mind was the maximum for a (and b). This was possible only by a bond with c who agreed (to my mind wrongly) to it. And the purpose was achieved.

GAME 6. Reasons of player c: I offered one of them 21 points, because then I receive 59 points. The second offered me 60 points with 20 points to him. My considerations at first were that I would give one of them more than he could receive with the second, but they did not agree and wanted to make a coalition both of them.
Reasons of player b: After all the attempts to obtain more with player c, I saw that he wanted too much and therefore any transaction with him was not certain. The best prospects were that a or b should divide with c a/b+c 21:59. I am not prepared to risk the 20 points for 1 point more.

Reasons of player a: From the beginning I understood that it was worthwhile for me to make a coalition with player c a+c 21:59 , but I did not take into account that someone else could agree on 20:60 . From this I drew the conclusion that instead of running a risk and not receiving anything we should set up a "unity": Player b and I.

GAME 7. Reasons of player a: I offered b to go into a joint coalition against c , but b did not stand against the pressure of receiving 41 from c . That was why we did not go together and I suggested a joint coalition to c with the division a+c 30:50. To this he agreed and b remained with nothing.
Reasons of player c: I offered b 41 on the assumption that a would offer me more. He offered and we decided to set up a coalition in which c would receive 50 and a would receive 30 .
Reasons of player b: I was in a coalition with a until c decided as a decision and last word to go into a coalition with me and give me 41 points. At the last moment a offered more points. Me they did not give a possibility for an additional offer, and as a result of the pressure of time (they had to leave) the results were noted down: coalition a+c .

GAME 8. Reasons of player c: After an attempt to reach a compromise which did not succeed, I saw this number as the maximum.
Reasons of player b: Since my proposal that we should set up a three-fold coalition and that each one should receive 30 points was rejected by c , and a showed signs of chopping and changing which endangered our position, I preferred this coalition.
[Player a did not give any report.]

GAME 9. Reasons of players a and b : Since a and b had the same situation, we also had the same reasons. We had three possibilities: coalition a+b or a/b+c or a+b+c . After negotiations we saw that a/b+c would never be possible because a coalition like that would be prevented by the third rival who would lower his number of points in the coalition until the number of points of a or of b would be less than 20 and then it would be more worthwhile for them to make a coalition between them. But since such a coalition is not worthwhile there remained a coalition of three. Since c obstinately wanted 40 points we made a compromise and decided on this division.
[Player c did not give reasons.]

GAME 10. Course of play: Player c was in the best situation, and therefore players a and b tried to put pressure on him and threatened that they would make a coalition of two when each would receive 20 . c then offered more to one and then the second increased her offer by 1 point, so that we saw that a coalition of two could not come about, and therefore we resolved on [a coalition of] three. At first a and b insisted on a+b+c 35:35:40 , but c did not agree, and we reached the acceptable compromise.

Reasons of player b: The game turned round the same axis all the time when a and I were each offering 1 point more, so that the game did not progress. If I had made a coalition with a , then I would not have obtained more than 20, and naturally if I were in coalition with c he would have put pressure on me, and at best I would have received 30.
[Players a and c did not give reasons.]

GAME 11. Course of play: Since the possibility of a+b+c did not come into account, therefore one of us had to give in, because as regards any situation at all the third party, who is not a party in the coalition, could offer a larger number of points, in such a way that another person would remain outside the coalition. Since there was no choice left, we had to unite in a coalition without hearing the offers of the third any more, although he was liable to offer a larger number of points. We have no special reason on account of which we chose expressly this coalition.
Reasons of player b: I agreed to give c 55 points and receive only 35 points because I had that possibility or else to leave the coalition altogether, since then a and c would have made a coalition. No choice remained except to satisfy c's demand for 55 points and remain satisfied with 35.
Reasons of player a: Since the decision about the coalition is arbitrary then even if I make a better offer it will not be accepted. Therefore we could not find a reason for me not joining the coalition.
Reasons of player c: By my reckoning I could have naturally gained 49 points. Since I had the best prospects, and also by a threat, I succeeded in obtaining more.

GAME 12. Course of play: a and b put pressure on c [with a threat of] an alliance of a+b 30:30 . c tried to strike up a coalition a+c , but was compelled to offer more than 30, something which did not pay him. Therefore he turned to an alliance b+c 45:45 , in order to prevent additional means of pressure from a and b .
Reasons of player b: c had many more prospects by extortion with a and therefore I had to extort as much as I could from him. I don't understand why he agreed to receive only 45.
Reasons of player c: The coalition a+b+c is not worthwhile since the alliance b+c has prospects of getting more. When a offers me more, for instance a+c 20:50 or a+c 10:60 , b could offer him more - which was actually done - and in that way press me to the alliance b+c . Therefore I agreed to this division.
Reasons of player a: There was an offer of a+b 30:30 . I agreed to this since I was in the hardest position. If I had made a coalition with c I would have received much less, so that if b offered me a+b 30:30 this was my maximum and I would agree.

GAME 13. Course of play: b+a+c 55:55:0. b+c 45:45. a+c 20:50. b+a 30:30 c+b 50:40 a+b 15:45. c+a 50:20 b+a+c 40:30:40.
Reasons of player b: In my earlier play I reached the conclusion that there was no sense in much bargaining, because in any case the final result would be illogical. Therefore I did my best to reach a compromise quickly that would be acceptable for all of them.
Reasons of player a: Since by my prospects I saw that the maximum I could get was 20, I agreed to the result.
Reasons of player c: Since I reached the conclusion that the maximum points I could obtain would be between 40 and 50, and in the course of the game I realized that in a coalition with b I would not get more than 45 points, and b and a insisted on a three-fold coalition in the form a+b+c 30:40:40, I agreed to a compromise.

GAME 14. Course of play: b+c 45:45. a→b+c 58:46:46. c→a+b 60:30:60. a→b+c 50:50:50. b agrees, c opposes and offers b to proceed to a restricted coalition on the conditions b+c 70:20. b agrees, a agrees [i.e., does not make a counter suggestion], c changes his mind and agrees to a+b+c 50:50:50.
Reasons of player a: The maximum I aspired to was 50. I saw the only choice to get this was by way of risk. I left the opening to b and c in order that I could offer them more. As I foresaw, my first offer (a→b+c 58:46:46) was not accepted and I offered it only in order to show them what was the most logical way at least for two among the participants (a and b). The continuation was clear according to the end of the play.
Reasons of player b: c appeared to be in the best position. Therefore in a coalition with her I could receive 45. Therefore I was happy to obtain 50 as the second possibility.
Reasons of player c: Since a did not agree under any condition to a joint coalition, when I had an advantage in it, I was compelled to agree to 50, although I saw that this was the minimum of points that I could obtain. The coalition with b, when I would have 20 and he would have 70 seemed to me less good.

GAME 15. Reasons of player a: Since b refused to make a coalition with me on a 30:30 division, therefore I went with c and received 25. The reason was that I and c did not have the patience or time to continue the argument with b.
[The other players did not give reasons.]

GAME 16. Course of play: Two meetings were held with a long time for thought between them. After the first meeting the final offer was a+b 30:30.
Reasons of player c: In no coalition could I obtain 60 points, for if I had done this with a he would receive 20 points and together with b he would receive 40 and it would be more worth their while to share between them b+a 40:20 in order to press me. I was compelled to give way, but on the least points, and I tried a+c 21:59 with a, but this was not stable either, for a coalition a+b 30:30 came about. I explained to b that he could receive more than 40 in a coalition of a [whose value is] 100 points because I did not

wish, since I was not able, to receive 60 points. This was worth his while, because he knew that I would not make a similar coalition with a , since I knew that a could always obtain more with b , and from that he knew that I would not break the coalition. I knew that he would not break it because he knew that if he tried to divide, say, a+b 15:45 with a , I could offer a more and he would jump at the bargain. The discussion was about how much I had to give in . I wanted to give him 41 and he wanted 45 and we compromised on 43 points.
Reasons of player b: See reasons of c .
Reasons of player a: I wanted to make a coalition with b or with b+c, so that c should receive little. But b agreed to a coalition with c.

GAME 17. Course of play: a+b 30:30. c+b 30:70 . a gives in. A sum of 100 is obtained. a joins with 10 points in a three-fold coalition so that the sum of b+c remains 100 and the division is a+b+c 10:50:50 .
Reasons of player b: Since the maximum that I can obtain is 50 points I could enter into a joint coalition with c on b+c 50:50 . But then a offers c a larger number of points and then I lose. Therefore, I agreed to enter a three-fold coalition in order to recompense a with 10 points (which is the maximum that he could obtain) and divide with c 50:50 (and 10 for a).
Reasons of player c: Since I can enter into a coalition with b only when each one receives 50 points, then a enters with 10 points.
[Player a did not state his reasons.]
GAME 18. [No reports were given.]

GAME 19. Course of play: We reckoned the prospects by mathematical ratio . [?]
Reasons of player b: I wanted to choose two possibilities: either a coalition with a of 30:30 or else a coalition with a+c . Since, if I had tried to make a coalition with c , a would have tried with all his forces to make a coalition which should be worthwhile for c and I would come out with just 0 . Seeing that the prospects for the largest number of points is in a coalition of three where c would be prepared to give up many points, because there exists a threat of a coalition b+a , therefore we arranged ourselves in a coalition a+b+c , where I have a maximum of possible points.
Reasons of player a: In this coalition I have the maximum of points out of all coalitions and, in addition, that this coalition was proposed by me.
[Player c did not state his reasons.]

GAME 20. Reasons of player b: I proposed to c to make a coalition where c would receive 31 and b 29. c argued that he would prefer to make a coalition with a in which he would receive 2 points or 1 point less, and therefore I suggested to him to receive 1 point more in a coalition, that is 32 points.

Reasons of player a: I gave in out of despair. I almost decided to commit suicide.
Reasons of player c: b offered me to make a coalition with him in a fashion that I would receive 4 points more than him. To be sure, I had perhaps prospects of obtaining more points, but I was already sick of keeping on bargaining.

GAME 21. Reasons of player b: b [would be able to obtain] a maximum of 20 [by making a coalition] with a . After pressure on c to give 25, since otherwise I would rest satisfied with 20 from a , it was agreed that I should receive 25. I could not raise it higher because at the beginning she agreed to give me only 21.
Reasons of player a: The game was clear. There were no prospects because a coalition of three was not worthwhile for b and c and I could not offer b or c any offers which would be more worthwhile to them than a coalition between the two.
[Player c gave no reasons.]

GAME 22. Reasons of player b: I agreed to the offer because this was the maximum that I could receive in the light of the obstinacy of c to receive at least 37 points. I had no more prospects to receive points if I would have been in coalition with a or with a and c together.
Reasons of player c: I ought to have given b only 20.5 . But since b and a could press on me together I decided to be satisfied with 37 points.
[Player a gave no reasons.]

GAME 23. Reasons of player c: I tried to partner them all in a coalition a+b+c , offering a+b [a total sum of] 25, a sum which was greater than the sum they could obtain by a coalition of their own. I even tried to make a coalition of b+c where b would receive 25 and c 35 , but a and b were obstinate in spite of the explanations and proofs that my offers would pay better. So they established the alliance between the two of them. I do not understand their reasons.
[Players a and b did not give their reasons.]

GAME 24. Reasons of player a: It was clear that from the first coalition I would not have reached 20 points. If I had demanded 20 points in the second, b would have offered c more and they would have prepared the third coalition (b+c). Therefore it was more worth my while to participate in the three-fold coalition and receive 20 points and not demand more, for otherwise b and c would agree to set up a coalition between them.(1)
Reasons of player c: The high value of the coalition a+b+c in relation to the other coalitions fixed it almost from the first that this coalition would be established. In order to prevent provocative actions against me, like a+b agreeing demonstratively [to set up a coalition], or else would try to influence me to give them more in their coalition with me, I decided on this situation which in any case [gives me a value] higher than the maximum value

(1) ["first coalition" and "second coalition" means here a+b and a+c, respectively.]

that I could obtain in a coalition with any other one. I did not demand more although there was a basis for this.
Reasons of player b: If I had gone with a I would have received a maximum of 15 , but in truth only 10. If I had gone with c I would have received a maximum of 30. But because we made a coalition of three I received the maximum that I could receive, and we also made a partner of a who could have lost points.

GAME 25. Course of play: b→c 40:40 , [the offer was adopted].
Reasons of player c: Since I saw that it was not worth making a coalition with a since the maximum that I could receive in this coalition was 30, I made a coalition with b and we divided half and half, because we had the same prospects with a .
Reasons of player b: Since I saw that a coalition with a would give me a maximum of 30 points and my prospects were equal to those of c , therefore I agreed to divide with c according to the decision.
[Player a gave no reasons.]

GAME 26. Course of play: There was no point in organizing any other coalition; since the prospects of b and c were equal and the maximum they could both get was 40, we decided what we decided.
[The players gave no separate reasons.]

GAME 27. Course of play: a→b 15:15. c offered [b] more, the more a offered b . a began to offer to c but b offered more in a coalition to c . The situation of b and c appeared to be equal and finally it was decided b+c 40:40 .
[The players gave no separate reasons.]

GAME 28. Course of play: a renounced his participation on account of his poor prospects. In view of the equal prospects of b and c , the coalition was divided in the ratio 1:1 .
[The players gave no separate reasons.]

GAME 29. Reasons of player a: From the beginning I had no prospects, for if the other two had decided to divide between them I could not have offered them any solution except that they should go into a three-fold coalition in which they would receive more than in a double coalition and I would also receive something.
Reasons of player c: b and c had a larger sum, but it would have been possible to increase it above 40 for each one. Both of them had an equal amount of points with a , and therefore the division was made between b and c in equal shares.
Reasons of player b: My considerations were like those of c . Since we had better prospects we decided to exploit a coalition of three in order to receive 5 points more. (In a coalition of two we would have received only 40 points each.)

GAME 30. Reasons of player a: Since b wanted to go into a coalition with c b+c 60:60 , I offered c 70 points, [that is] more than the previous offer, in order that he should get into a

coalition with me and so I would obtain the minimum of points that I could obtain - 20 points.
Reasons of player c: I had no possibility of finishing the game and receiving more than 70 points, for a and b could have put pressure on me. Therefore we decided not to change the proposal we had received.
Reasons of player b: I did not agree to the situation and made counter-offers, but they both decided that "a word is a word" and I had no choice.

GAME 31. Reasons of player a: It was the most I could get.
Reasons of player c: They tricked me. After we wrote a decision they changed it.
Reasons of player b: I wanted to set up a coalition with c , b+c 60:60 , but in the light of her treachery and breaking promises it was decided to establish a coalition a+b 35:35 with a, on condition that we did not withdraw from the decision.

GAME 32. Course of play. At first we decided that a coalition would be set up a+b 5:15 , but when the game went on c offered a 10 points in a coalition a+c . b did not agree to receive 25, so he was not taken in.
Reasons of player c: I offered b 25 - an amount greater than what he could have received in coalition a+b . He demanded more. I did not agree and formed a coalition a+c , since b did not agree even to a coalition of three because he aspired to receive more in the coalition b+c .
Reasons of player a: In the conditions produced in the game I had no possibility of gaining even one point, so I agreed to every offer that was made to me.
Reasons of player b: Since c did not agree to give me 28 and threatened a coalition of a+b , I agreed not to receive anything.

GAME 33. Reasons of player c: I offered a three-fold coalition a+b+c 10:10:30 . The offer was not accepted. a and b made a proposal to carry out a three-fold coalition a+b+c 15:15:20 . c did not agree and persuaded b to carry out a coalition of two with him, c+b 30:20 , without listening to the requests of a , in order not to reach the situation that a and b suggested. b was firm and wanted to keep the original proposal, therefore I persuaded a that it was worth his while to receive 20, or else that nobody receive any point. The proposal was accepted. b saw that he was not getting any points and agreed to give c 49 points. This offer was not accepted because then it would break the coalition[1] a+c and renew the proposal a+b+c 15:15:20, something which was not desirable for either a or c .
Reasons of player a: After c did not agree to coalition a+b+c 15:15:20, and I saw that c did not aspire to more than 30, therefore I agreed to a coalition a+c 20:30 on the assumption that c would not shift from his decision.
Reasons of player b: I offered c+b 49:1. c did not agree and entered into a coalition with a .

[1] [Namely; accepting the offer will cause other offers which would break the coalition b+c.]

GAME 34. Course of play: a+b+c 1:1:58. a+b+c 20:20:20. a+b 0:0. a+b+c 10:10:40. b+c 16:44. a+b+c 15:15:30.
Reasons of players a and b: We saw that c could choose one of us and that way the second would receive naught. But we could not know in advance which he would choose. Therefore we decided to threaten c with forming a coalition of a+b and so they would all receive naught. After a fall and rise of "the price" we agreed on a coalition a+b+c. At the beginning we wanted each of us to receive 20, but c did not agree and finally we agreed on what was decided.
Reasons of player c: Because of the threat of a+b I had to agree to this coalition.

GAME 35. Reasons of player c: In a coalition with one participant I could receive a maximum of 49. Therefore 50 in a coalition of three seemed preferable.
Reasons of player b: I offered a to enter a coalition of a+b [whose value is] = 0, in order that c should agree to a coalition of a+b+c [whose value is] = 100, in the distribution a+b+c 25:25:50. [25 is] the maximum of points that I could obtain. a agreed.
Reasons of player a: I agreed to the offer of b , because otherwise c demanded more and I did not agree to receive less than what we offered [namely] 25 points.

GAME 36. Reasons of players a and b: We offered c a distribution of 40 points for each, but since c aspired to obtain 60 points or else that all should receive 0 points, in this way we reached the present compromise at last.
Reasons of player c: Because in the coalition with a and with b I could not receive theoretically more than 50, I had to agree at last to this compromise.

GAME 37. Reasons of player a: I had no choice.
Reasons of player b: In order to attract c to come into a coalition with me I had to offer him an offer that was higher than what a could offer him.
Reasons of player c: I received 58 and agreed to this because if I had to go with a then I would receive less than 50. So that the points I gave to b did not at all change the general situation.

GAME 38. Course of play: c+a 49:1, a agrees. b+c 20:50, c does not agree. c+b 69:1, b does not agree. c+b 60:10, b does not agree. c+b 55:15, b agrees. At first sight it was clear that the coalition b+c would be the most worthwhile in this case for c who is the decisive one in this game. Since a agreed to 1 in coalition a+c a situation was created in which every offer made by b would be more worthwhile for c . But he was interested in a maximum of points and he offered b only 1 point. The latter did not agree and demanded 19 points, in such a way c offered b 15 or 0, that is he was prepared to rest satisfied with 49. In the light of this demand b was prepared to agree and they reached the

above decision.
Reasons of player c: In effect I am the one who makes the decision because I participate in all the coalitions [with a positive value]. For this reason I demand the maximum points in every case. a agrees, but I am interested in a coalition b+c; but b does not agree to receive 1. So we reach a resolution that, as far as I am concerned, is a mistake on his [player b's] part, because in any other case I would not have obtained such a number of points.
Reasons of player b: From the first it was clear to me that although c determines the coalition, still the coalition in which I appear is the worthwhile one for him, and therefore he is largely dependent on me. But a spoils it for me in a certain degree when he is prepared to give c 49 points. For this reason I can do nothing but reach a compromise in the form of the abovesaid decision.
Reasons of player a: My position in the game is not good. Since c put me in a state of 1 or nothing at all I agreed. At this stage I cannot offer more, so the initiative passes from me to b and c who decide between them.

GAME 39. Reasons of player a: It is better to receive 2 than 0.
Reasons of player b: c demanded at least 50. I agreed to 18 points at least. I could obtain this amount, so I agreed to a coalition a+c+b.
Reasons of player c: I was interested in 50 at least (coalition a+c). Coalition b+c and also coalition a+b+c made this possible. Coalition a+b+c was preferable since it reduced the number of points that b received while the number of points that I received did not change.

GAME 40. Course of play: a→c 1:49. b→c 20:50.
a→b+c 6:22:52. b→c+a 25:50:5.
Reasons of player a: This coalition is the only one that is reasonable for me, because a coalition between me and c would be broken by an opposing proposal of b at once, since b and c can come to a compromise between them on a coalition b+c 20:50. Since b insisted [on getting 25 points] I agreed to the above proposal which was accepted.
Reasons of player c: In this game I succeeded in receiving a large number of points because of my obvious advantage. I tried to get more than 50 points and suggested a coalition with b , but he opposed this and agreed only that he should receive 25 and in that way I should receive 45 points. Therefore we were compelled to set up a coalition of three [a 3-person coalition] and I received 50 points.
Reasons of player b: Since a offered c to receive 49 I could not offer c less than 50, and then I would receive 20. Therefore, in order that c should receive the 50 and I should profit, I suggested the coalition that was accepted.

GAME 41. Reasons of player a: I could not receive more points because of the obstinacy of b .
Reasons of player b: I gave up a point so that a should not muck it up.
Reasons of player c: I could actually have received more points, only a and b found this arrangement as the best and I decided to give in.

GAME 42. Reasons of player b: It was impossible to get more than a point because the other could always offer less.
Reasons of player c: The only possibility for establishing a coalition is that b or a should receive 1 point because any other offer of b or a would lead to a better offer on the part of a or b to c.
Reasons of player a: It is not correct that it was impossible to receive more. I said to b that he should want 10 or more and I would not go down, and in that way c would have to agree. That way I [we] could have fixed who loses more. Finally to be sure c was prepared to give me a number of points so that I should agree and he would not continue with b . But I decided to give the point to b, because it does not change much to receive a point or two.

GAME 43. Reasons of player a: I came with an offer $\overset{b+a}{\wedge}$ 1:49 to b. He refused. I made a similar offer to c - he refused. Between them they made an agreement b+c when each of them receive 0. Through this obstinate decision not one of them agreed to come to the following agreement a+b or a+c 30:20.
Reasons of players b and c: From the consideration that b or c would be deprived because the offer of a could come to b or c equally, we, b and c, decided to set up a coalition.

GAME 44. Reasons of player a: Since b and c wanted to make a coalition between them so that I should not profit, I made the suggestion to c with a promise not to accept a better offer from b.
Reasons of player c: Because the offer of a was an opportunity that was against the odds I agreed to accept it at once.
Reasons of player b: I preferred the coalition b+c in order to prevent a from receiving a high amount, but because of the stand of c I was unable to do anything.

GAME 45. Reasons of player c: Since me and b were in the same situation, where if we had made a toss-up between us only one could win, and since each of us had 50% loss, therefore we tried to "squeeze" as much as was possible if we would have divided. But since she [player a] stood on 25:25, we decided that we would threaten and if she would also not agree to change, we would decide on a coalition c+b. That was the consideration at first, but that changed when b did not agree and wanted a+b 30:20 with a. But she did not agree to this and went with me a+c 40:10 and that was how we finished.
Reasons of player a: Since at first the move was so that each of b and c offered me more until they reached a decision to "threaten" me so that I should lower the number of points I took to myself, and after I did not agree to lower to more than 25 points to one of them and after one of them (b) changed his mind and agreed to receive 20 from me the second (c) offered me 40 points and he would take 10, and then b offered me 49 points and 1 to him, but I did not agree to receive his proposal because I saw that he was prepared to change it if c would try to influence him. Therefore I agreed

to a+c 40:10 together with c.
Reasons of player b: I received nil because of unfairness of one of the others. At first I offered a to take himself 30 and I 20 provided that he did not receive a better offer. He agreed to the offer but when c came and offered a a better offer (a+c 40:10), a accepted the offer without keeping the promise.
Player a adds: Note: Even before I managed to receive his offer of 30:20 , I received an offer of 40:10 and therefore I promised him nothing and there is no basis for b's complaint.

GAME 46. Reasons of player c: Both b and c offered a 49 points. Since a did not know whom to choose he tossed up a coin and c received 1 point.
Reasons of player b: There was a possibility that b or c would receive 1 point or more [or so] . We tossed a coin and the luck was with c. If b had gone with c [and threaten] 0:0 , then a would have offered one of the two more. [Not too clear. The idea is that a can always break such a threat.] But because of the lack of time we did not do this.
Reasons of player a : I was afraid that b and c would make a coalition of 0:0 . They did not do this. If they had done this I would have offered one of them 5. I am sure he would have accepted.

GAME 47. Reasons of player a: I demanded 45 in a coalition with b or c , but b and c decided among themselves that it was not worthwhile for them to receive 5 and I should receive 45. I was compelled to lower the number of my points to 40, and with a promise from c not to listen any longer to offers I finished with this coalition. If I had demanded more, the members of the coalition[group] would not have made a coalition with me.
Reasons of player c: I offered with b to a that we should all go for 0 , for it was not worthwhile for me to accept a's offer to receive 1. a was prepared to offer me 5, but I did not find that enough either. Player b agreed to receive [the 5 points] but I persuaded him that it was not worthwhile for him for the sake of 5 points. Again we reckoned to decide on 0 for all, but a despaired and finally decided to give me 10.
Reasons of player b: Since I was unable to be sure whom a would choose for the coalition, I suggested that we should go together in a coalition of 0. But when a finally proposed the coalition a+c [with a distribution of 40:10] , I could not succeed to have any more influence in any form.

GAME 48. Reasons of player a: It was impossible to receive more in spite of a good situation, because b decided for personal reasons not to receive more than 0.
Reasons of player c: It is more worthwhile to receive 15 than 0. If I should demand more then a coalition a+b was liable to be set up. [Player b gave no reasons.] [Player b's passive attitude is demonstrated also in plays 42, 60, and 73, (where he acts as player a, c and b, respectively). He used to tell his friends that he acted this way in order to prove that the theory cannot predict the outcomes. After playing game 73, I approached him and explained that his conduct proves nothing, because the theory is concerned only with participents who wish to maximize their profits, and therefore his conduct will only force me to ignore his plays. Upon my request he promised to play better in the rest of the games.]

GAME 49. Reasons of player b: I offered 1 to [receive from] a on the assumption that this was the only arrangement that could be to his advantage. For if I had offered a less, c would have offered her more, and things would have gone on like that until we would reach the point where b or c would receive 1.
Reasons of player a: This was the maximum of points I could receive. Therefore I agreed to the first proposal of b.
Reasons of player c: I had no choice because the first proposal of b was to give a 49 points and I could not offer less.

GAME 50. Course of play: The proposal was: a+b 25:25 or a+c 25:25 or b+c 0:0 and obviously a would receive 0.
Reasons of player b: I was out for a solution of decision by luck (tossing a coin) between me and c, [to make sure] which should go into coalition with a, for it was clear that at the moment we would start a debate and a would offer one a certain amount we would reach the point that a would receive 50 and b and c 0. There was another choice of 0:0:0, but this did not seem worthwhile against the prospect of 50% gaining 25 points.
Reasons of player c: As above.
Reasons of player a: Since b and c decided either that there should be a coalition of [whose value is] 50 and one of them would get 25 or that they should make a coalition between them and then all of them would receive 0. Since I was interested in a maximum number of points, therefore I offered another [offer], but it was not acceptable. Therefore I agreed to the decision and received from it the maximum of points, in this case 25.

GAME 51. Course of play: a+b 49:1. c+a 0:50. c+b 0:0. a+c 10:40. a+c 12:38. a+c 20:30. b and c suggested to a that he should give one of them 25 and they would decide among themselves which should receive this. He did not agree and therefore it was decided b+c 0:0.
Reasons of player c: A threat together with b against a, that he would lose more by his refusal or should agree to 25:25 with b or c, with an agreement between us that we shall not accept other offers, otherwise we would arrive sooner or later at b+a 1:49, and by that we would reach an equal loss. a did not agree and therefore the coalition is b+c 0:0.
Reasons of player a: I suggested to c instead of 0 to give him 15, and he refused in the hope that b would gain ?! [Player a does not understand why player c refused his offer. It looks to him as if player c fights in order that player b will win.]
Reasons of player b: Since my prospects were equal to those of c, I decided to make the agreement mentioned in the description of the game. We resolved to do this in order to put pressure on a so that he should agree to give us 50% of the points. We knew that if he did not agree he would lose more by his refusal than we would from our refusal; and therefore it was decided to have a coalition b+c 0:0.

GAME 52. Reasons of player a: I got the maximum I could get.
Reasons of player b: I couldn't get more because c could always have gone lower by a point.
[Player c gave no reasons.]

GAME 53. Reasons of players b and c: Since one of us two would have to get zero we agreed that the chosen one of fate (between us) would receive a maximum number of points by threatening a. And that way we reached the present result.
Reasons of player a: The considerations of b and c were: It was better to receive 0 than to let me get a lot. So I had no choice except to enter into a coalition with one of them 25:25, as otherwise they agreed each one of them to receive 0 and not to give me 30 for example, and one of them would take 20.

GAME 54. Course of play: b and c offered a that one of them should make a coalition with him of 25:25. b and c decided which should take part in the coalition by lottery in which c won.
Reasons of player c: The decision about the lottery was useful. Since a was under strong pressure (b+c 0:0) and had to agree, because he was afraid he would lose this game whose conditions seemed to his benefit.
Reasons of player a: b and c put strong pressure on me till I had to agree to the lottery or lose the possible 25 points.
[Player b gave no reasons.]

GAME 55. Reasons of player a: I could have got out 45 but I was satisfied with 40. Otherwise they could have united and put me out.
Reasons of player c: Since the prospects with b were weak I agreed to the offer in order to avoid bargaining between a - b and a - c.
Reasons of player b: I made c an offer of a coalition with me in order to put pressure on a to give one of us 25 by lottery. a from the beginning and in principle did not want to go with me because I stood firm by my agreement with c and raised his offer to c so as to go with him up to 25, My reckoning was to continue and press a and afterwards to make a coalition with a if a gave in. But c gave in because a offered him 25 without listening to the better offer that I, b, would give him. [Apparently this player noted 25 by mistake instead of 20. The outcome was a+c 40:20.]

GAME 56. Reasons of player b: There was a suggestion by a to liquidate the game by the offer that was accepted. In any case, in an alliance with c I would not receive more than 10. So I agreed at once on condition that I would not change my mind. [In other words: a offered a+b 40:20 but made his offer conditional on b agreeing immediately. If b were to make some alternative offer, a reserved the right to make less satisfactory offers to b.]
Reasons of players c: They never offered me anything. So I was left outside.

Reasons of player a: The maximum I could receive was 40. Since b offered c a joint coalition, I agreed to a+b 40:20.

Game 57. Reasons of player a: I offered them 15 points and they made a lottery between them. The winner got it.
Reasons of player b: We, b+c, tried to threaten a, but it was useless. a agreed to allocate at most 15 points to a coalition with b or c and would not shift from his opinion. The decision fell in the form of a lottery.
[Player c gave no reasons.]

GAME 58. Reasons of player a: Being apprehensive about the decision b+c 5:5, I lowered the amount to 48 (in spite of the fact that as far as prospects were concerned I could have obtained more).
Reasons of player b: I offered c a coalition of b+c 5:5 and convinced him that if he came into a coalition with a, then he would not receive more than in a coalition with me (b+c 0:10). He gave up the partnership in the coalition and then I myself accepted the offer of a.
Reasons of player c: Player b at first said to me that it would be worth it for me to go along with him, and I thought that he was thinking this in an honest way, so that we would be able to receive more from a. But suddenly a offered b 12 points and both of them agreed, and I was sure that they would have done this also if I had given a more than 48 points. And after that player b himself said to me that the purpose was to trick me and agree with a.

GAME 59. Reasons of player c: I was prepared to receive any number of points as long as I would not get zero. Only my two companions in the coalition, the others, wouldn't listen at all to my proposals. They decided to make a coalition between them without listening at all to my proposals. In my opinion the decision was not fair!!
Reasons of player a: I offered b 15 points. b did not agree. I made the same offer to c. She did not agree either. So I offered c 20. c agreed but b agreed to receive 15. I fixed with him that I would go with him a+b 45:15 without reckoning what c would offer.
Reasons of player b: To make a coalition with c was not worthwhile for me to do, because the maximum of points that I would receive was 10 and so because of impatience to receive more from a I agreed to go and receive 15 points.

GAME 60. Reasons of player a: I offered b 10 points because that was the maximum she could receive in a coalition b+c. But her pressure was to go in a coalition b+c 5:5. I gave in to her offer and gave her 20 points in a coalition a+b 40:20.
Reasons of player b: a offered me 10 but I thought that by pressure and threatening with a coalition b+c I could get more, so I demanded 20. More than 20 I didn't dare because that was already a bigger risk, for a would find it hard to give in. She agreed.

Reasons of player c: Since somebody would receive 0 and I didn't have the energy to prevent that that somebody should be me and since b was a partner in that situation I wanted b to receive a maximum.

GAME 61. Course of play: There was an ultimatory offer of b and c to a, which was to give one of us by lottery 40 or else we would go b+c. a offered b 15 and having a long-range consideration was prepared not to give less. b asked for 30. a offered c 25 and not to go lower. Therefore we agreed by lottery to receive 25 points from a.
Reasons of player a: The ultimatory offer of b+c had no influence and I decided to be obstinate. But I didn't want to go too far and I agreed to the situation.
Reasons of player b: b+c tried to put pressure on a but did not succeed and so when a offered me 25 I agreed because otherwise he would have been in coalition with c.
[Player c gave no reasons.]

GAME 62. Reasons of players b and c: b and c had equal prospects for getting points. Therefore they decided that one of them would not lose through compulsion. Therefore we offered a 25 and one of us would get 35. Since a did not agree we decided not to give in and we decided on a coalition of b+c 5:5.
Reasons of player a: I did not agree to give b or c 35 as they demanded and therefore I agreed to receive 0.

GAME 63. Reasons of player b: I agreed with c that one of us would demand from a at least 20 points and if he did not agree, to make a coalition between us. The carrying out - who would make the coalition with a - was done by a lottery which was done with a consideration that it was better that each of us should have prospects to get 20 points than one should get 0 or 10.
Reasons of player a: I had to agree to the proposal a+b 40:20 because there was a threat of a+b [receiving] 10.
[Player c gave no reasons.]

GAME 64. Course of play: a+b 40:20. b+c 5:5, unless a agrees to a+b/c 20:40; i.e., putting pressure on a and a lottery. After the lottery [a proposed] a+b 30:30, and [the proposal] was rejected. Therefore a+c 20:40.
Reasons of player c: I made a lottery with b, and since I won we reached the present situation by threats on a.
Reasons of player a: I agreed to the offer because I didn't have patience to go on playing and arguing.
Reasons of player b: In an argument on each point I could receive at most 6 points. Therefore I decided that it would be worth my while to risk them in a lottery between me and c as against the 40 points which I could win.

GAME 65. Reasons of player a: I offered 15 to c, but b and c decided to offer 30 points at the most to me. For lack of choice I agreed to the offer and received 30 points.

Reasons of player c: I decided with b that whoever came out in the lottery would go into a coalition with a.
Reasons of player b: I lost in the lottery.

GAME 66. Course of play: a offered b a+b 45:15 and promised him not to shift from this offer, even if c should make a a more convenient offer.
Reasons of player a: I offered the offer so as to prevent joint pressure on the part of b and c.
Reasons of player b: I accepted the offer.
Reasons of player c: I offered a 55:5 but he did not agree.

GAME 67. Reasons of player c: Since my prospects and the prospects of b were equal, it was decided between us to go by a lottery with a for the maximum number of points a would be prepared to go with us for. Our offer [demand] was 25 to c or b, and we compromised at last on the amount given in the decision.
Reasons of player a: b and c wanted to extort 25 points from me by a lottery between them. I decided to get 40 points out and I didn't want to give in, but at last I compromised with c to share the difference between 20 and 25 between us. I put pressure on him and gave him an opportunity to sign a coalition b+c 5:5 , and he didn't want to do this. [Although a refers to an argument with c, it seems that a lottery between b and c was performed.]
[Player b gave no reasons.]

GAME 68. Reasons of players b and c: As a result of the failure of the means of pressure we applied on a the coalition finished in this way.
Reasons of player a: I decided not to offer them more than 20.

GAME 69. Reasons of player c: I offered a c+a 35:25 and similarly also b offered, but a did not agree. The pressure of b+c 5:5 did not influence him. The attempt to offer a [that I should receive] a small amount like 11 or 7 was not worthwhile, therefore I went into a coalition with b.
Reasons of player a: b and c wanted to influence me in a way that I should agree to a division of a+b/c 25:35 with one of them. But I didn't respond to this offer because I thought that they were only trying to put pressure on me to lower the number of points. In the end they went together.
Reasons of player b: We b and c offered a that he should receive 25, and one of us two by lottery should receive the remaining 35 points, and we put the pressure on him that if he would not agree we would make a coalition between us. With that calculation we did not succeed because he didn't agree to it. Therefore this coalition was set up.

GAME 70. Reasons of player b: Since a did not agree to give me more and promised not to accept a better offer I accepted his offer.
Reasons of player a: Because of lack of patience to conduct negotiations in which I would receive a larger number of points, I decided to promise b to receive 40 points from him, And so we did.

Reasons of player c: a and b did not want to hear my offer at all and decided to make a coalition alone.

GAME 71. Reasons of player a: At first I offered c 15, and afterwards b offered me more and so we went on until we came down from 10. Suddenly they offered that we should make a lottery, and in that way they would squeeze from me as much as possible. Seeing that, I saw the impending danger, I at once offered and just to b, who is more sensitive, 15. He agreed at once and I didn't intend to go down again and run a risk. Therefore we finished a+b 45:15.
Reasons of player c: They did not agree to accept me after a offered b 15 and did not go down. So I remained outside.
Reasons of player b: I got the maximum I could, after a offered me 15 points, while c could not offer more than 9 points.

GAME 72. Course of play: a offered b 35:25 and promised b not to change this offer even if c made him a more convenient one.
Reasons of player b: In the light of my situation in the game a's offer appeared quite convenient to me.
Reasons of player a: Since I was afraid in case they might gang up against me, I offered b 25 - an offer which was accepted at last.
Reasons of player c: I did not agree to the situation since both of them became obstinate. a did not want to go down to a more convenient offer and b did not want to go up to any other offer. Still, I did not have a choice.

GAME 73. Reasons of player b: In order that it should not be boring, changes have to be brought in from time to time. And since b or c received 0, why shouldn't it be b and only in order that a should receive less than half.
Reasons of players a and c: b was obstinate about getting 0 points, in order that he should be able to prove that the accepted theory [note the terminology!] is not correct. For this purpose he renounced his rights and allowed c to choose an amount which he would receive in coalition. [Meaning: He gave c an opportunity of fighting a with equal forces.]

GAME 74. Course of play: b and c offered a that he should give one of them 40 or that they should make it b+c 5;5. Finally they agreed to a+b/c 31:29, and the lottery was arranged between b and c. b won it.
Reasons of players b and c: Since our position was equal we decided to make a lottery which b won. The maximum which each of us could obtain was 29.
Reasons of player a: I was faced with a fact. I had no choice, I chose the lesser evil.

GAME 75. Reasons of player a: I was prepared to offer 10 to whichever would want to join me. It was decided to make a coalition like that with c, but b hurried up and offered me 55. I agreed on condition that the decision would not change [i.e., on condition that b would promise that his offer was irrevocable].

Reasons of player b: There was an agreement between me and c to impose pressure on a to divide the points to half, but since he agreed to take 10 from a, I had no choice except to offer a to go with him a+b 55:5.
Reasons of player c: I offered a 45 and I would get 15. b made a counter-offer: he 5 and a 55. For me it was not worthwhile to get less, therefore I stopped the discussion with the said results.

GAME 76. Reasons of player a: b and c tried to put pressure on me and said they would make a coalition between them if I did not give one of them 40. I didn't agree and offered one 20 on condition that if he accepts I would not listen to any more offers. At first he would not agree but when I offered 25 he agreed.
Reasons of player b: There was a prospect of receiving 35 (with big pressure), but then I would have a prospect of 50%, while here I had a possibility of 100% [to receive] 25, and therefore I agreed.
Reasons of player c: I wanted together with b to press on a, but b was broken and was seduced to a's proposal. I did not have any other choice, that means I had no possibility of appealing against a decision. The reason of b is most silly.

GAME 77. Reasons of player a: I offered b 15 points and did not agree to accept a better offer from c, on the assumption that if I accepted such an offer, b and c would force me by an agreement between them to give one of them more points.
Reasons of player b: Since I saw that a would not give one of us more than 15 points, not even if we threatened him that we would go b+c 5:5, I agreed to the offer. For if not, c would have accepted the same offer.
Reasons of player c: I have no reasons. Since the course of play was like this: a came to b with an offer to receive 15. b agreed, and for me there was no say or argument in the game - simply I didn't take part in it. If I had agreed to a coalition or not, in any case that thing did not change my situation, for a and b did not want arguments and in one minute they decided to make a coalition between them.

GAME 78. Reasons of player b: I reached an agreement with c that we should demand from a at least 40 points for one of us. a refused and we refused to go down in the number of points offered. Therefore I decided with c half and half.
Reasons of player c: We pressed on a to give 40 to one of us by lottery. Maybe this was a mistake to remain without compromise, but then as well nothing would have been assured for me. In any case this was a mistake if I wanted to win.
Reasons of player a: I just simply decided not to share in any coalition that would be offered me, since they tried to give me only 20 and to take themselves 40 points. I did not accept this. It is not correct that I did not agree only because of the number of points that were offered me, but because of a previous decision. [He wishes to say that it was not correct that he had a previous decision not to participate in any coalition. On the contrary. His refusal came on account of this "degrading" offer. This player was accused by

his friends in several plays of sabotaging the contest by not trying at all to win points. He now denies this.]

GAME 79. Reasons of player b: 45 was the maximum I could get. If they [one of them] had offered me more, then the second would offer me even more and after a little while the wheel would have turned. So I decided to agree, and to put an end to the hot negotiations.
Reasons of player a: I agreed to receive 15 only because b, c and d wanted to make a coalition without me. I preferred to receive 15 points instead of getting nothing.
Reasons of player d: a, b, c agreed among themselves to make the first coalition without any acceptable reason and did not listen to my offers.
Reasons of player c: I would not have been able to receive more than 40 points for fear of a coalition a+b+d.

GAME 80. Reasons of player c: This was the most convenient possibility in order to fix up - because all the members of the pair agreed at once and I did not see many prospects of getting more.
Reasons of player a: Like c.
Reasons of player b: Like c.
Reasons of player d: a+b+c agreed on 33 each and 100 together. In the coalitions in which I participate I can't offer more. And therefore I remained without points.

GAME 81. Reasons of player b: The coalition of a+b+d was not worthwhile because the number of its points was small. Since there was no way out of the arguments about the coalitions a+b+c or b+c+d, it was decided to have a coalition a+b+c+d with the points divided in proportion to the prospects of each of the participants. [I could not succeed in understanding what the calculation was that they made, or if they did actually make any.]
Reasons of player c: The coalition without b was not worthwhile. The other considerations like b.
Reasons of player a: There was no patience left [after three meetings had taken place], so I accepted the suggestion.
Reasons of player d: In order to prevent a coalition a+b+c I had to agree to a compromise like this.

GAME 82. Course of play: c→b+d 36:27:27, but a agreed to receive 2 points and offered a→b+d 2:29:29. Then c offered c→b+d 30:30:30. Since a could not offer such an amount, it followed that c+b+d was the most worthwhile coalition for all of them, each one receiving 30.
Reasons of player a: Since my position was not good I could not offer any one of the participants a better possibility than c offered to b+d and therefore the three of them decided to make a coalition. Any offer I would have offered, c could have offered a better coalition. I could not offer c to get as much as he wanted.
[Players b, c and d gave no reasons.]

GAME 83. Reasons of player a: I had something to do in the interval more interesting, and apart from that 20 points were enough for me considering my bad position and the tiny difference for the rest. Reasons of player c: I agreed to this because even with another coalition I would not have received more if we had divided equally. If there had been more time I would have gone on demanding. Reasons of player b: Since my position is not brilliant, and even in another coalition I would not have obtained more, I agreed. Reasons of player d: I offered b and c to make a coalition b+c+d 30:30:30. a offered to make a coalition d+c+a. Finally b and a offered to make a coalition of four [players] a+b+c+d 20:20:30:30.

GAME 84. Reasons of player b: I had quite a weak position. I wanted to finish quickly and c agreed to receive 30 points, and a agreed to 10. Otherwise the game would not have finished. Reasons of player c: My position was good. I wanted to get at least 30 and also to finish quickly. Reasons of player a: Since my position is the worst of all I accepted this proposal, for in any other coalition I would have lost. Reasons of player d: Since I offered more and they did not accept I had to agree to the present situation.

GAME 85. Reasons of player b: From the data I understood that my prospects in the game are the best and so I did not agree to get less than 45. I joined a coalition with a and d because they were the only ones who agreed to give me that number. Reasons of player a: In order to break the decision of b+c+d 45:35:40, I suggested 42 to d and was compelled to receive the rest. Reasons of player d: From examination of the values of the coalitions I decided that my prospects were to receive 40 points, so that after negotiations I suggested the coalition b+c+d 45:35:40. a offered me 42 points in a coalition a+b+d but I was apprehensive about agreeing because I was afraid of dangerous developments for me. After a promised not to accept any other proposal I agreed. Reasons of player c: I offered b+d+c 45:40:35 but they (a, b and d) decided to create a coalition without my participation, without hearing my proposals any more, so that a should not lose the 23 points by lowering. [That is, in the course of additional bargaining.]

GAME 86. Reasons of player a: There was a danger that b, c and d would make a coalition, because they had the highest sum. They began to offer all the sides to b until at last they reached a state where it was better for them to influence him by a coalition of a+c+d. That influenced the situation in the square coalition. Reasons of player c: I threatened b that we would enter into a coalition a+c+d, and by that he agreed to the square coalition. Reasons of player b: They mixed me up. Reasons of player d: My considerations were like those of a, because we went side by side [hand in hand] in our pressure on b.

Player b adds: At first we offered b+c+d 35:40:45, but they succeeded to raise the value of me up to 57 and then they made a coalition between them, and I joined as a fourth.

GAME 87. Reasons of player a: I achieved the maximum of points that I could. I could not offer more than c in case we were to make a coalition of three a+b+d, since c would have offered more than me and then there would have been the coalition b+c+d.
Reasons of player b: In my opinion this is the only situation which has any prospects of reaching a decision which would not be influenced by factors independent of the distribution of the forces, like time, readiness [to play], etc.
Reasons of player c: In my opinion I obtained a number of points that was relatively large for my situation which is not particularly good. I had a danger from a partnership of a in a coalition with b and d. Therefore I was happy to join in a coalition with c and d.
Reasons of player d: My opinion is the same as the opinion of b.

GAME 88. Course of play: b+c+d 35:40:45. a+b+c 2:29:29. a+c+d 4:43:53. b+c+d 35:35:50.
Reasons of player c: It was a choice between this coalition and another in which I received 0. Therefore I agreed.
Reasons of player b: In order to participate in a coalition in which I am assured the largest number of points I was compelled to renounce 5 points and give them to d so that he should agree to participate in a coalition b+c+d. [From this remark it can be seen that the offers of a did not make any particular impression on b.]
Reasons of player d: b+c can threaten b+c+a, and in that way to squeeze out more points. Any link with a would bring about a blackmail by the rest.
Reasons of player a: I offered d 53 points but c 43. They did not agree to the offer and preferred to make a coalition with b.

GAME 89. Reasons of player b: I received the largest number of points that I could get.
Reasons of player d: I received the largest number of points that I could get, even though I could have received another 1 or 2 [points].
Reasons of player c: I received the maximum points that I could receive.
Reasons of player a: I could not receive more.

GAME 90. Reasons of player a: I had no choice.
Reasons of player c: There were no good prospects for a coalition without d.
Reasons of player d: I agreed to give up 1 [point] in order to finish quickly.
Reasons of player b: I suggested we should make a coalition a+b+c+d 30:40:40:40. d did not agree and wanted to obtain 70 points. I had no choice except to receive 27 points.

GAME 91. Reasons of player d: I could not get more than 25 for if I had not agreed there would have been a coalition a+b+c 40:55:5. So I had to agree to 25.
Reasons of player c: There were no considerations. I suggested a coalition of c+b+a 5:55:40, because at first they didn't want to give b more than 50. But they agreed otherwise.
Reasons of player b: I would maybe have been able to get more because it was almost impossible to have a coalition without me. But in order to finish I agreed to this situation.
Reasons of player a: I could have pressed on d and received more than 40. But the unwillingness of c to play prevented me from that and I received only 40.

GAME 92. Reasons of player a: I couldn't receive more points because the other possibility we reached was 20 [points for me].
Reasons of player b: I agreed to this all because of the provocations of d.
[The other players did not give any reasons.]

GAME 93. Reasons of player a: I offered c to enter a coalition together with d but he fixed together with b not to enter into a coalition except on condition that b should take part in it. Since c continued in his (silly) obstinacy no coalition at all came about.
Reasons of player d: Like the reasons of player a.
Reasons of player b: Since my position was not good, there was a competition between me and c and at last only one of us would have received a small number of points. So therefore I agreed with c that both of us go together with a reasonable number for each one of us (30 [points]).
Reasons of player c: At first we tried to go with a and d. We agreed [on] a+c+d 40:40:40, but b offered d+a+b 45:45:10. a and d went with him. Then I offered c+a+d 30:50:40. a did not agree for some reason that d should get 40 and he [should get only] 50 and therefore we did not finish. We thought during the lesson, b and I, and found that if we were obstinate we could both receive at least 30 points [each one]. And why should they receive so much? And since a did not agree to get 20 we finished with 0:0:0:0.

GAME 94. Reasons of player a: I agreed to this coalition because otherwise b, c and d would have gone together b+c+d 40:40:40.
Reasons of player b: a, c, d didn't want to listen about my suggestions.
Reasons of player c: If I had gone with b and d I would have received a maximum 40. This way I received 45.
Reasons of player d: In the maximal coalition b+c+d I could have received 40. with the coalition a+c+d I received 45, and therefore I agreed.

GAME 95. Reasons of players d, b, a: This number of points was the maximal I could obtain from all the combinations - for every other combination would have led to a smaller sharing of points, only unless

somebody would receive a very small number of points, and he wouldn't agree to that.
Reasons of player c: I had the worst prospects of all and I didn't succeed to get into a coalition of [whose value is] 100.

GAME 96. The coalition a+b was established first.
Reasons of player a: We decided to split into pairs and each pair decided for itself about the number of points that each one in that pair would get. I got the maximum number of points I asked for and more than that nobody at all could offer me, for then he could receive more from another player.
Reasons of player d: It was clear that we needed to divide into pairs. b argued that it was not worth the while to make a coalition with me. So she reached an agreement with a, to his harm I think. For c no choice was left except to make a coalition with me. Here a long debate began because I knew that c was compelled to make the coalition with me so I fought patiently to obtain a maximum. I convinced c that he had no more prospects than b, therefore he ought not to receive 40 as he demanded. I offered 25 to me and 35 to him. After a debate we arrived at a compromise of c+d 36:24.
Reasons of player c: I proposed to a that we should enter into a coalition. b offered a more and so all the time we reached a situation where each offered the second more. a and b decided to enter into a coalition without listening to better offers and therefore no choice was left me except to enter into a coalition with d. Since I had more prospects than d I demanded that I should receive more than d - about in the proportion of the prospects I had - and therefore we entered into this coalition of c+d 36:24.
Reasons of player b: The negotiations were conducted as c wrote. I agreed to receive 25 points, for if I had not agreed there would have been a coalition a+c, and I would have been compelled to enter a coalition with d the sum of whose points [whose value] was 40 only. Therefore I came to a coalition with a.

GAME 97. The coalition a+c was established first. Course of play: d+c 40:40. d+a 40:40. a+c 60:60. d+b 20:20.
Reasons of player b: a+c decided to erect a coalition. If I had offered one of them even 1 point more, I would have reached a position in which it would have been better to go with d, [and accordingly] this was the decision.
Reasons of player c: This was the only possibility by which I could receive the largest number of points. Therefore I agreed to this.
[The other players did not give any reasons.]

GAME 98. The coalition b+c was established first.
Reasons of player a: After a coalition of b+c there was left the solitary possibility of arranging a coalition between me and d. I demanded half the amount and received it.
Reasons of player d: I agreed to compromise on half the amount since otherwise we could not have reached a compromise.

Reasons of player b: As I did not succeed in influencing d to make a coalition with me I was not able to obtain more than 10.
Reasons of player c: The maximum of points I could obtain was 20.[?]

GAME 99. The coalition b+c was established first. The second coalition was established shortly after.
Reasons of player d: (I) Under these conditions I could receive a maximum of points. (II) It was hard to finish the game, therefore we finished it this way.
Reasons of player a: I made a coalition with d because in the coalition with b I did not receive the larger number of points. (No better proposal was put to me.)
Reasons of player c: a and d did not agree to any proposal on my part, therefore I was compelled to create a coalition with b.
Reasons of player b: I and c had the same prospects. We reached the conclusion that by bargaining to shift out d+b [or d+c] and letting him [player d] obtain a coalition against one of us, we would lose up to 1:69. [In other words, we shall lose by competing for player d's favors.] Therefore we decided to share equally the 20 roughly in the same way that a+d divided between themselves the 90. [Namely, in a quota split. This I gather from the fact that a+d did not share the 90 equally and I see no other interpretation for the words "in the same way".]

GAME 100. The coalition b+d was established first.
Reasons of player a: Because I recognized that I was in the worst position I decided to leave the others to argue, and to enter into coalition with whoever would remain outside and get half the points.
Reasons of player d: Since b offered c 25 and would receive 65 I offered a counter-offer (in the coalition b+d) in such a way that he [player c] should receive 70. Since he did not agree and in another coalition I would have a small number of points, I made him a final offer [to receive] 75. He accepted and so we reached the above results.
Reasons of player b: I was astonished that this was so easy, because after all d could have offered c more and so at last I would have been compelled to go with d. Well then - did d think that I would go with c and receive less that way?
Reasons of player c: a's idea of leaving the rest to work and entering with the one left out in a coalition and receive half of the points, negated the possibility of preventing the coalition b+d, and compelled me, after attempts to obtain points [in a coalition] with d, to enter the coalition with a.

GAME 101. The coalition b+c was established first.
Reasons of player c: At the start it came out for me to be with a [in negotiations to be] in a coalition. Because of his vigorous stand [i.e., on account of his exaggerated demands] I gave up 2 points from my theoretical prospects, so as to ensure my participation in the coalition [with b].
Reasons of player b: At first I proposed a coalition b+d 40:60. Since c offered me 42 points I went with him, b+c.

Reasons of player a: I understood that after two of the others execute a coalition between them, they would in practice lose their advantage for me, since the one left over would not have any possibility for a coalition [otherwise, except to unite with me] . Therefore [when I remained alone with d] I demanded 40; not more - because she would not have agreed [to give me more] .
Reasons of player d: I didn't have a choice because I had already made a coalition with b 60:40, and he regretted it and made a coalition with c. I was compelled to make a coalition with a, and a demanded 40. I was compelled to be satisfied with 50 points.

GAME 102. The coalition c+d was established first. After less than half a minute the second coalition was set up.
Reasons of player c: I decided to reach a situation of equality with d with a surplus of 10 to my advantage, since our coalitions with a and b were almost equal with a small advantage to c.
Reasons of player d: I wanted to make together with c a coalition d+c 75:75, but since c threatened with the coalition a+c, I agreed to the present situation.
Reasons of player a: I tried to reach a coalition with c and offered him 60. But c succeeded in gaining more when he made a coalition with d. I was compelled to make the coalition with b, and we shared the number of points equally.
Reasons of player b: Since my position in the game was fixed by the others, (because I cannot offer a larger sum than the other players even to one) I was compelled to agree for lack of choice to receive 15 from a.

GAME 103. The coalition a+b was established first.
Reasons of player d: I offered b to go into a coalition b+d [whose value was] 140, 55:85; but a offered b a+b 40:60. And since d and c had the same prospects, therefore I agreed. c also agreed for the same reasons.
[Player c agrees to the reasons given by player d, and signs his number at player d's text.]
Reasons of player a: Since my position in the coalition [in the game] is the worst, I could not present exaggerated demands. The first proposal that c offered me was a+c 35:85. Therefore I offered a+b 40:60.
Reasons of player b: At first I offered d to give me 60 and he would take 80. He did not agree and offered me d+b 85:55. I agreed with him, for that is better than a+b 50:50. But when a found that with c he would receive a+c 35:85 he agreed to go with me a+b 40:60. The rest finished 80:80. It seems that the tactics was successful.

GAME 104. The coalition c+d was established first. Course of play: c→d 20:60. b→d 30:70. c→b 20:40. d→b 58:42. Final decision: c+d 20:60; a+b 15:35.
Reasons of player c: I had to prevent the coalition of 100 [i.e., the coalition b+d whose value was 100] which would bring about that I would receive 15 and for that reason I received 20.

Reasons of player d: The coalition of 100 would have brought me to a result of less than 60 and therefore I agreed to the situation.
Reasons of player b: Since a could receive a minimum of 15 in the coalition a+c, then I could receive in a coalition with him only 35. In a coalition c+b I could receive 40, but d could offer c 25. To be sure, I could have exploited the coalitions b+d and b+c [in order to try] to obtain more, but there would be no end to the game.
Reasons of player a: If the pair c+d had finally decided on any coalition at all I could have got 25 out of b. But I could not lower more than 5 points in his value (40) for then someone else would have offered him 35 points. [Player a recognizes by this that the quota of player b is 40, and his own quota is 10. He also understands that once a coalition c+d is established the advantage of player b over him is lost. At the same time he does not reach the conclusion that it is worth waiting until the coalition c+d is established or, as a matter of fact, until any other coalition of which a is not a member is formed.]

GAME 105. The coalition c+d was established first.
Reasons of player c: I offered b a coalition c+b 80:50. d could have offered her - in order to obtain more than 70 - to create a coalition d+b 75:45, but I had already offered her more. For this reason d agreed to make with me the coalition c+d 80:70.
Reasons of player d: I agreed to the aforesaid situation because in this situation I received the largest number of points that would be possible and whereas in the light of the negotiations that were conducted I had almost lost these 70 as well. Therefore I agreed.
Reasons of players a+b: b and a came to an agreement between them after c+d had finished the coalition. Because a was apprehensive that b would offer c a large number of points he agreed to a coalition a+b as it was set up.

GAME 106. Only the coalition a+d came about.
Reasons of player d: I could have received at the most 60, but in actual fact 59, because the other players were not prepared to receive 0 and give points. Therefore I demanded 60 from a in a coalition a+d [whose value is] 100. This because if I were to demand more than 60 he was liable to agree to another coalition and I was liable to lose my advantage.
Reasons of player a: In this game I could from any other side in the game receive at most forty and he [my companion in the coalition] could receive naught. But this thing would not be worthwhile to him and therefore he would offer 39, therefore I accepted the proposal of d to make a coalition of 100 together.
Reasons of player c: I had no offer whatsoever that would satisfy a and d.
Reasons of player b: I could not offer a more than 40 and d more than 60, and therefore I could not participate [in any coalition at all].

GAME 107. Only the coalition a+c came about.
Reasons of player a: c was the only one who was prepared to receive 5 points. Therefore I made a coalition with him.
[The other players did not give any reasons.]

GAME 108. The coalition a+b was established first.
Reasons of player a: To begin with I decided to make a coalition with d and at the end d cancelled his offer and therefore I saw that d does not keep his word and I made a coalition with b in order to prevent friction.
Reasons of player d: I offered a 75 points, and since he did not agree at all to make a coalition with me I was compelled to make the present coalition.
Reasons of player c: We caused a dispute between a and d and in that way b and c succeeded to receive more than the theoretical prospect.
Reasons of player b: In fact I didn't want to fix d, but after they began quarrelling I [we], upon an agreement with c, could demand from them [players a and d] any amount we wished.

GAME 109. The coalition b+d was established first.
Course of play: b+c+d made a lottery. c won and by virtue of winning he was promised that he would not be disturbed (on the part of b and d) in making a coalition with a.
Reasons of player d: I and b were in the same situation, therefore I could not demand from him more than 10.
Reasons of player b: My reasons are included in the course of play and in the reasons of d.
[Players a and c gave no reasons.]

[The following report was noted by the players and was afterwards struck out:] Outcome: b+d 10:10, a - 0, c - 0.
"Course of play: a offered c a+c 70:20. c did not agree and they did not reach any kind of compromise.
Reasons of player a: I was prepared to give 20 and receive 70. At first it seemed that I would receive this amount, but afterwards I could not give up any more."
[In spite of the dispute they compromised at last on the division b+d 10:10; a+c 55:35.]

GAME 110. The coalition c+d was established first.
Reasons of player d: Every offer I made to obtain a greater profit [by linking] with a would have been broken by the said condition [he wishes to say: would have been destroyed by the opposing proposals] and cause me to receive [in the final result only] 10 points from b.
Reasons of player c: I offered a the coalition c+a 90:70. He refused and offered a+c 75:85. Since d's offer was better I accepted it.
Reasons of player a: I obtained as much as I could obtain because b had the strength to push me aside, since c+d went together, and I did not wish to be obstinate. I could have received 60 but there was no time, or a mistake on my part also. [He thus recognizes that

his considerations were wrong, but tries to avoid confessing this. Hence the numerous excuses. The truth is that 70 points were coming to him and he failed and eventually received only 40 because he demanded at first 75 points.]
Reasons of player b: Since c and d made a coalition together, my prospects became equal to the prospects of a and therefore I demanded 40 points and was obstinate about it.

GAME 111. The coalition c+d was established first.
Reasons of player a: I had a dominant position and therefore I agreed to receive 70 points. At first I offered [demanded] 80 points, but they threatened me that I would receive 0, and therefore I agreed to receive 70 points.
Reasons of player b: I did not think about it [i.e., I did not plan this in advance] but we pressed a to give 20 to whoever won. Since it was like that, we decided to make a lottery and the lot came out to me to receive the 20. The others received 10:10. Of course a did a silly thing, for otherwise we would not have had any possibility of receiving 0. [He wants to say that player a should not have taken their threat too seriously, because this threat means that one of them must come out with no points, if they stick to the threat.]
Reasons of player c: The three of us, b c and d, had the same chances; therefore, for lack of choice, we decided on a lottery [which would fix] who should make a coalition with a . I did not have any luck therefore it came out for me to make the coalition with d.
Reasons of player d: The reasons are identical with the reasons of c.

GAME 112. Only the coalition a+b came about.
Reasons of player d: They fixed me. I could not offer more, and therefore I did not receive anything.
Reasons of players a+b: The maximum we could obtain was a → 70 and b - 90, and since it was like that, we made a coalition between us with the division a+b 70:90.
[Player c gave no reason.]

GAME 113. Course of play: The coalition a+b was formed. c+d did not reach an agreement and preferred to receive 0.
Reasons of player b: I had the possibility of going either with a or with d into a coalition of [whose value is] 100. I offered to whoever was prepared to receive b+a/d 25:75 [i.e., he stated that he would link himself with the first of them who would agree to receive 75] . a agreed and we undertook not to change the offer [i.e., not to accept other offers] .
Reasons of player a: At first d offered a+d 70:70, and then b offered that one of us should receive 75. I agreed on condition [that he would undertake] that he would not accept any other offer, since d tried to make b a better offer (d+b 50:50) and to me as well (a+d 80:60), and both of us did not agree to go on and receive another offer, so there was left the coalition a+b 75:25.

Reasons of player d: I offered c a coalition d+c 50:20. Since she did not agree I did not want to go on with the negotiations for I saw no reason to receive less than 50.
Reasons of player c: d+c remained in a coalition of 70 [with a value of 70]. I offered d that we should divide 35:35. d was obstinate and under no circumstances would agree to compromise on another division, so I was compelled to receive 0.

GAME 114. The coalition a+b was established first.
Reasons of player a: I could not receive more points because then there would have come about a coalition b+c+d 40:40:40, and I would receive 0.
Reasons of player b: I first offered a the coalition with a mutual promise not to accept better offers and so I gained another 5 points.
Reasons of player c: The proposal was b+c+d 40:40:40. The offer was almost signed and then b offered a a+b 5:45 after we between us had tried to understand that it was not worth while going with a, so I had to go with b 25:25.
Reasons of player d: My reasons are identical with the reasons of c.

GAME 115. The coaltion a+d was established first.
Reasons of player a: At first I offered d [that I should receive] 10, because he would have agreed. Indeed he did agree, but after a discussion he offered me 12. I went with him. But at once they agreed to give him more therefore I offered him 40 and we finished the affair. I was sure that if they had gone on I would have taken much more.
Reasons of player b: There was no special reason . We decided not to give a more than 10 and between us [the remaining pair of players] we divided 50 in the proportion 25:25.
Reasons of player d: I was offered 40 by b+c in the coalition b+c+d, but they would have gone back on themselves, therefore I agreed to go with a. I offered b+c that they should give me 50 but they did not agree and that also induced me to go with a. If I had not gone with him some other one would have gone with him. [He tries to cover up his guilty conscience by offering many excuses.]
Reasons of player c. Because of my inability to achieve a coalition with a I was compelled to accept this coalition.

GAME 116. Only the coalition a+c came about.
Reasons of player c: By this coalition I receive 1 point more than would have been possible in a coalition of three [in a 3-person coalition] .
Reasons of player a: I accepted the offer of c to receive 12 points, seeing that amount as the most reasonable.
Reasons of player d: I offered a 20 and a promise not to move. For unknown reasons he did not agree. Then I demanded 40 from b - he did not agree.
[Player b gave no reasons.]

GAME 117. The coalition a+d was established first.
Reasons of player d: In a coalition of three a 3-person coalition I could have obtained about 40. I agreed to obtain 45 in a coalition of two with a, who in any case had no prospects, with an undertaking that he would not accept any other offer. By this I also prevented the receipt of more points from b and c.
Reasons of player a: I made a coalition with d from the following reason that b+c+d wanted to make a coalition 40:40:40 and leave me out. I offered d 45 and by doing so I received more than 0.
Reasons of player c: We had already agreed on a coalition b+c+d 40:40:40 but all of a sudden d regretted it and agreed to a's offer and was not prepared to change his opinion. Nothing was left for us except to form the coalition b+c 25:25.
Reasons of player b: See the reasons of c.

GAME 118. The coalition a+b was established first.
Reasons of player a: I was in a bad position from the beginning and almost obtained nothing. I could not receive more than 5.
Reasons of player b: Among the three participants in the coalition a+b+c one would have profited 50 and more and the rest would have received less than 45 points. My prospects were one-third to receive more than 45, two-thirds to receive less. Naturally the logical way [is] to receive 45.
Reasons of player c: Since a+b decided on a coalition between them, no choice was left me except to form a coalition c+d with each one of us having 25 points.
Reasons of player d: The reasons are like those of c.

GAME 119. The coalition a+d was established first.
Reasons of player a: I got up the maximum I could.
Reasons of player d: I wasn't able to get more, because a could have made a coalition with b or c.
Reasons of player c: The first pair decided earlier, therefore I acted this way for lack of choice.
Reasons of player b: The first pair fixed a coalition therefore I fixed like this.

GAME 120. The coalition a+d was established first.
Reasons of player d: I offered a to go a+d 10:40 and explained to him that otherwise there would be a coalition b+c+d and a would be left out. He agreed and so all the pressure of the others did not influence either him or me too.
Reasons of player a: d offered me 10 and I didn't agree at first and wanted to make a coalition with b so that I should obtain 12. Then the three others threatened that they would make a coalition together and therefore I returned to the offer of d, for fear I would not receive anything.
Reasons of player b: The first offer was a+d 10:40. a accepted this but afterwards offered me a+b 12:38. Then d decided to cancel the coalition and make a three-fold coalition and after [d] had once again agreed to the first coalition, a did not agree to break it again even for three assured points (a+b 13:37). [These were sure points] because I wouldn't have broken this, for they could

not offer me anything more certain. To d we could not offer more than 40 for this was not a stable coalition, and so we had to divide between us (b+c) 50 points.
[Player c gave no reasons.]

Game 121. Reasons of player c: b+c+d decided to make a coalition in which each one receives 40 points. a offered me 45 points, but I did not agree because in the same degree the others could offer a a higher number of points and I would be out of the game, therefore I agreed to a coalition of b+c+d.
Reasons of player a: Since they did not accept my offer 5:45 and decided on a coalition b+c+d, for lack of choice I came out with 0 points.
Reasons of player b: If I had tried to go into a coalition with a there was the possibility that a would not choose me specifically but somebody who would offer him more and so I would have received 25 at the most. Therefore I agreed to the coalition b+c+d.
[Player d gave no reasons.]

GAME 122. The coalition a+c was established first.
Reasons of player a: If I had not offered one of the three of them 40 points, they would have made a coalition between them and divided equally with the points they had, therefore I had to offer one of them a number of points that was better than what he would have received in a coalition of the three of them.
Reasons of player c: I agreed to the offer of 10 because I did not see any possibility of getting more.
Reasons of player b: a offered c to make a coalition a+c 10:40. Naturally c accepted this offer and then no choice was left me and d except to divide the amount [the value] of the coalition equally. c did not even wish to receive an offer of 45 in the coalition of three [in the 3-person coalition], for he felt afraid that b or d would receive a similar offer from a.
Reasons of player d: a and c made a coalition between them and reached an agreement without agreeing to other offers. I was compelled to make a coalition with b and to divide equally with him.

GAME 123. The coalition a+b was established first.
Reasons of player b: I obtained 41 - more than I could have done in a coalition of three [in a 3-person coalition].
Reasons of player c: I offered a a coalition a+c 12:38. Since a remained uncertain we decided on b+c+d 40:40:40. But b offered a a+b 9:41 which was received and therefore came the coalition c+d 25:25.
Reasons of player a: It was forbidden to risk too much and go up above 9 even though it was possible (12 was offered [to me]).
[The excuses that players make in order not to admit mistakes in rejecting a first offer are sometimes impressive.]
Reasons of player d: Since the decision was scarcely influenced by my opinion at all, I had no choice except to receive 25 in a coalition with c.

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