ON THE FLEXIBILITY OF COST-PLUS PRICING *)

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INTRODUCTION

A significant number of firms express their product prices as percentages of a unit cost base. The accumulation of literature on this business practice indicates that cost-plus pricing is empirically interesting but analytically embarrassing. There are several characteristics of cost-plus pricing that have not been adequately discussed by price theorists. What determines both the magnitude of the cost markup and its variation over time? What motivation underlies the large variety of cost-plus pricing practices noted in case studies of business firms?

This paper interprets cost-plus pricing rules as rule of thumb reactions by firms to unfulfilled expectations of demand. The emphasis of this discussion of cost-plus pricing is not only the markup construction of the price but the conventional employment and modification of the markups by firms. In adapting to changing demand and cost conditions, the price decisions dictated by the cost-plus rules may be interpreted as reflecting specific assumptions about the demand-cost structures. The pricing rules are rules of thumb for situations of imperfect information. Further, it is demonstrated that the relative performance of various cost-plus rules depend upon not only the demand-cost environ of the firm but the accuracy of the firm's own characterization of the demand-cost structure.[1]

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*) The author is indebted for the comments of Professors W. Baumol, S. Goldfeld, and H. Shapiro and Mr. D. Needham on early drafts of this paper.

1 "Usually - and this is true of most of the literature - either subjective or objective demand-cost schedules are recognized but not both ... (Both) facts should somehow enter the scheme and the compatibility of the two ought to be demonstrated." (23, p. 479).

Section I of this paper discusses the interpretation of the flexibility of the cost-plus markup over time. Selected pricing rules are presented as prototypes for a number of empirical pricing practices discussed in pricing case studies. Section II describes a model of a hypothetical, single-product firm that is used in a digital sampling of the performance of the selected pricing rules. Section III is a brief summary of the performance of these rules in various demand-cost structures. Finally, more detailed properties of the pricing rules are presented in Section IV.

I. THE PRICING RULES

Rudimentarily defined, cost-plus pricing is the application of an absolute or percentage markup on the costs (variously defined) associated with a selected volume of product output that determines the unit price.

Given the number and complex interaction of factors influencing a price decision by a firm, it is remarkable that pricing case studies have indicated the use of this "simple" pricing process (see 9, 13, and 15). It is possible, however, to overstate the prevelance of this practice. Further the pricing process itself assumes subsidiary importance as a behavioral reaction in certain market structures. Nevertheless, a substantial literature has accumulated on the subject of cost-plus pricing.[2] Despite some ingenious interpretations (e.g. 2, chapters 6 and 7), the theoretical respectability of this practice is accepted with reluctance.

Unlike the reference just cited, there are other interpretations of cost-plus pricing that are largely irrelevant. The assertion that the intersection of the subjective marginal revenue and cost curves determines the cost-plus markup may impute "marginalism" to cost-plus pricing, but is of little value in understanding the unique characteristics of markup pricing. Another interpretative treatment of cost-plus pricing links a rigid markup to a firm's subjective specification of a kinky demand curve. But the condition that the price is expressed as a cost markup does not determine the position of the kink; again, the significance of the association is trivial. The assumption

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[2] For example, cost-plus pricing is a fundamental element in the inflation schema of (20).

of sluggish adjustments to the changing (subjective?) demand-cost environ of the firm has lead a recent study (14) to impute a "partial marginalism" to the variation of the pricing markups over time. Here also, there has been no demonstration of a unique relation between lagged or imperfect responses and the use of markup pricing.

Many discussions of cost-plus pricing at least implicitly recognize that these pricing policies are a class of rules of thumb used in a context of imperfect information. Nevertheless, if it is not possible to establish a unique response by the markup rules to the environmental stimuli of the firm, the linkage with imperfect knowledge is devoid of operational meaning. For example, if the price margin is determined by a Market Research Department's estimation of marginal revenue and cost, the essential "rule of thumb" is the particular statistical estimating technique used. But if a signal, such as a change in profits or volume sold, leads to "typical" modifications of either the markup or its use in the determination of the price (or both), then these markup pricing variates may offer a unique contribution to the interpretation of business behavior.

The following discussion will develop nine rules that seem to the author to represent many of the practices of firms using "flexible" markup policies. The classification is not exhaustive and is more pertinent for small business firms. Since the categories were derived from an interpretation of already existent studies,[3] no more than "casual empiricism" is claimed.

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[3] For example, see (13) and (15).

Rule 1: Rigid Full-Cost Markup

The price established by full-cost markup may be expressed as

$$P_t = ATC^{qt*}/(1 - M^f)$$

where ATC^{qt*} is the average total cost of the period t expected output; the asterisk denotes expected magnitudes. M^f is the full-cost markup applied to the cost base to determine the period price.[4]

It is interesting to note that the formula for the full cost markup is closely related to a variate of rate-of-return pricing: the ratio of expected (or actual) profits to sales.[5] Dividing both components of the profit/sales ratio by the expected (actual) volume, we obtain the full-cost markup format

$$M^f = (P - ATC^q)/P$$

The popularity of the profit/sales ratio is in part due to the ready availability of the data. The precise use of either ratio by firms is uncertain. It seems implausible that they are directly maximized since this may well lead to an "optimal" output rate less than the profit-maximizing volume. A more reasonable interpretation is one variate of the constrained revenue maximization hypothesis presented by W. Baumol (2, p. 69). In the present version of full-cost pricing, the sole reaction available to the firm (short of taking the product off the market) is the selection of each period's expected product volume.

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4 The determination of the expected output and the full-cost markup are explained shortly.

5 Chamberlain (7, p. 56) cites the case of a firm, which upon losing a large government contract, found that the profit/sales ratio had risen. This was due to the lower rate of return on the lost sales relative to the profitability of the civilian sales. Large manufacturing firms often employ a measure of return to financial capital which is a product of the profit/sales ratio and "turnover" (sales/investment).

In actual usage, the average total costs ordinarily provide a floor for prices in this rule. This characteristic has provoked a good deal of the criticism of the whole class of markup pricing. The allocation of a portion (or all) of the overhead costs appears to violate the marginalistic "Principle of Avoidable Costs." A markup high enough to include the overhead costs may reflect a favorable expectation of demand, but the case studies report a reluctance to put aside this floor even when such a demand expectation is clearly not warranted. The assertion that markup pricing "presupposes adequate demand" (8, p. 365) can rationalize nearly all that firms do. In fact, markups are sometimes used to examine the "adequacy" of demand. The decision to drop or substitute product lines often is based on a comparison of markup measures.[6]

Rule 2: adaptive full-cost markup

One definition of full-cost markup pricing implies a price that includes that "normal" profit sufficient to keep the firm in operation. Here, full-cost pricing appears to be a long-run phenomenon. Particularly in the case of small firms, however, there is evidence of a greater volatility in the short-run markups than is warranted by this interpretation. The markup revisions are normally in response to departures from the expected goals, for there is little evidence of autonomous changes in the markup in an aggressive design to test the market structure.

One interpretation of this short-run flexibility is an adjustment in the markup to bring it in line with the actual margin obtained. An adaptive expectation formulation is used for Rule 2. The expected markup is modified proportionally to the most recent discrepancy.[7]

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6 One of the critical deficiencies of this paper is the exclusion of the role of markups in multi-product operations.

7 "g" is one of two reaction parameters used in this paper. Both parameters (g and f) are drawn from the uniform distribution $U(\frac{1}{2}, 1/12)$. The values

$$M_t^{f*} = M_{t-1}^{f*} + g(M_{t-1}^{f} - M_{t-1}^{f*})$$

Rule 3: "safety" full-cost markup

With the restriction of non-negative markups, the average total cost function serves as a price floor in Rules 1 and 2. Rule 3 is the combination of a price ceiling with Rule 2. The construction of this price ceiling is described in Part a) of the Appendix. Anticipating the discussion briefly, the purpose of this additional constraint is to avoid the establishment of an "unreasonably high" price.

Assume the selected price ceiling is P_c and the unconstrained price for period t given by Rule 2 is P_t' :

$$P_t = P_t' \quad \text{if} \quad P_t' < P_c$$

$$P_t = P_c \quad \text{if} \quad P_t' > P_c$$

Rule 4: rigid variable-cost markup

Variable-cost markups refer to markups on estimated direct costs. It is a common practice for retail firms to apply markups to their wholesale costs. The distinction between costs that vary and those that are invariant to the production rate depends upon the relevant time horizon and the accounting practices in use. In some retail firms, little modification of the amount of labor applied is undertaken in the short run (16, pp. 36-38). Thus, markups on invoice cost (sometimes called "gross-margin" pricing) often may be considered roughly equivalent to markups on "variable" costs.

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7 (con't) of these parameters are randomly selected in the simulation and do not represent empirical evidence. They are used for a "sensitivity" analysis explained in Section IV.

The determination of the unit price by the variable-cost markup is

$$P_t = (1+M^v)AVC^{qt*}$$

where AVC^{qt*} equals the average variable costs at the expected output. As was the markup of Rule 1, the variable-cost markup (M^v) is fixed in the short run.

Rule 5: adaptive variable-cost markup

Similar to Rule 2, Rule 5 uses a proportional adjustment mechanism for the modification of the variable-cost markup

$$M_t^{v*} = M_{t-1}^{v*} + g(M_{t-1}^{v} - M_{t-1}^{v*})$$

Firms appear to be indifferent as to whether the markup is expressed as a function of the unit price

$$\text{(a)} \quad M = (P-C)/P$$

or the unit cost

$$\text{(b)} \quad M = (P-C)/C$$

Rules 4 and 5 use the (b) version. The consequence of the fractional adjustment method for the two base selections are not equivalent. For example, assume a stable demand line, but the cost base is shifted from C_1 to C_2.

Rule 5, using format (b), modifies the original price (P_1) for the cost shift to give

$$P_2^{(b)} = \left(1 + (1-g)\frac{(P_1 - C_1)}{C_1} + g\frac{(P_1 - C_2)}{C_2} \right) C_2$$

$$= P_1 \left(\frac{C_2 + g(C_1 - C_2)}{C_1} \right)$$

The use of format (a) would give the modified price

$$P_2^{(a)} = P_1 \frac{C_2}{C_1 - g\,(C_1 - C_2)}$$

Therefore, for a fractional reaction parameter ($0<g<1$) ,

$$P_2^{(b)} > P_2^{(a)}$$

Rule 6: "safety" variable-cost markup

The ceiling constraint, as used in Rule 3, is imposed on Rule 5 to define Rule 6.

Rule 7: volume adjusted markup

Often mentioned in conjunction with the short-run flexibility of markups is an adjustment similar in direction to the movement of the volume of the product sold during the recent past. Perhaps the simplest characterization of this process uses these assumptions:[8] 1) The firm operates with the expectation of constant marginal costs ($AVC^* = C$); 2) The departure from sales expectation is believed to be derived from a shift in the intercept of a linear demand relation.

Assuming the demand line is

$$P_t = a_t + bQ_t \ ,$$

the profit-maximizing variable-cost markup is

$$M_t^V = (a_t - C)/2C \quad .$$

At a given price, the change in the quantity demanded is

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[8] These assumptions seem plausible given the use of breakeven analysis as presented in 7 (pp. 242-244). The expectation of this practice appears to be that profit varies directly with volume, and the "contribution margin" (unit price minus unit variable costs) remains constant for the given price. (See also 18, p. 54)

$$Q_{t-1} - Q_t = (a_t - a_{t-1})/b$$

therefore, the adjustment of the profit-maximizing markup should be

$$M_t^v - M_{t-1}^v = b(Q_{t-1} - Q_t)/2C \ .$$

The modification of the variable-cost markup by Rule 7 is

$$M_t^{v*} = M_{t-1}^{v*} + h(Q^*_{t-1} - Q_{t-1}) \qquad h = b^*/2C$$

The selection of the expected demand slope b^* is explained in Section II. A more complex version of Rule 7 might assign a threshold to the impact of the Q discrepancy as a stimulus to the markup adjustment and asymmetrical reactions to the sign of the discrepancy.

Rule 8: inexact volume rule

This rule inserts the reaction parameter g into Rule 7, allowing for the possibility that the firm may over or under-react to the result dictated by the assumptions used in the construction of Rule 7.

$$M_t^{v*} = M^*_{t-1} + g'h(Q^*_{t-1} - Q_{t-1})$$

$$g' = g + 0.5 \ ; \ 0.5 < g' < 1.5$$

Rule 9: absolute target markup

A target goal of an absolute profit magnitude is used in Rule 9. According to W. Haynes (15, p. 55), among the small firms using target goals, the absolute measure was used more frequently than the profit/sales ratio. In Rule 9, the unit price selected is

$$P_t = ATC^{qt^*} + T/Q^*_t$$

where T is the absolute profit sought by the firm. The target profit goal (T) is cautiously somewhat less than the maximum profit (PR*) thought to be attainable by the firm.

$$T = g' PR^*$$

$$g' = (g/2) + .25 \quad ; \quad .25 < g' < .75$$

Quantity Rule

Obviously, a great deal of variation can enter from changes in the expected production rate (Q_t^*) that is used in the cost base of the markup product. The expected production rate is often expressed as a percentage of "capacity" with possible allowances for both secular and cyclical movements. The use of standard volume costing is more closely associated with full-cost markups.[9] If the actual past costs have been averaged (instead of past volumes) the presence of a fair amount of volume variability is a potential source of distortion. This refers not only to the absence of linearity in the cost structure, but the fact that an uneven distribution of past production rates can misstate the true cost at the "standard" volume. It is probable that some averaging of past volumes is undertaken by many firms. Indeed, many firms use absorption costing; an ex ante interpretation of this process demands that the projected costs be fully "absorbed" at the projected volume.

One of the simplest averaging forms is the distributed lag with geometrically decaying weights:[10]

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[9] This paper deals only with production to demand, rather than, for example, to inventories. Within this context, one could possibly explain the use of a fixed volume for a flexible markup application, if the variations in demand were believed to be characterized by a swiveling of a demand line through its intersection with a line of constant average variable cost.

[10] The use of this adaptive formulation to characterize the formation of expectations has been criticized by E. Mills (19, ch. 3). One objection is that the expectations cannot have the potentiality of being unbiassed unless the true variable (e.g. Q_t) is generated by an autoregressive scheme such as that cited above. To obtain the most efficient predictor, it is, of course, desirable

$$Q_t^* = (1-f) \sum_{i=1} f^{i-1} Q_{t-1}$$

which may be expressed in the format of the pricing rules

$$Q_t^* = (1-f)\, Q_{t-1} + f Q_{t-1}^*$$

The reaction parameter f is described in Footnote 7. Note that the expected value of the reaction parameter ($f = .5$) is consistent with the expectational assumptions used in the construction of Rule 7. That is, displacements from profit-maximizing price-quantity positions are characterized by shifts in the expected demand line intercept: The maximizing quantity for the model cited in Rule 7 is $(c-a)/2b^*$. For a shift in the demand line intercept (a), the optimal shift in the expected quantity is $-\Delta a/2b^*$. According to Rule 7,

$$-\Delta a = b^*(Q_{t-1} - Q_{t-1}^*)$$

therefore

$$Q_t^* = .5Q_{t-1}^* + .5Q_{t-1}$$

Rule Characteristics

This section will briefly remark on the dynamic properties of some of the basic rule forms. The discussion will use only the linear relations: $P = a+bQ$ (demand) and $TC = cQ+e$ (total cost).

With stable demand and cost relations, the conditions for the convergence of Rule 9 may be ascertained by inspection of Figure 1.

$$T' = g'PR^* + e$$

10 (cont)
to avoid a possible bias of specification. However, the question of interest here is whether or not the fractional adjustment mechanism is an adequate description of the real-world formation of expectations.

Figures: section I

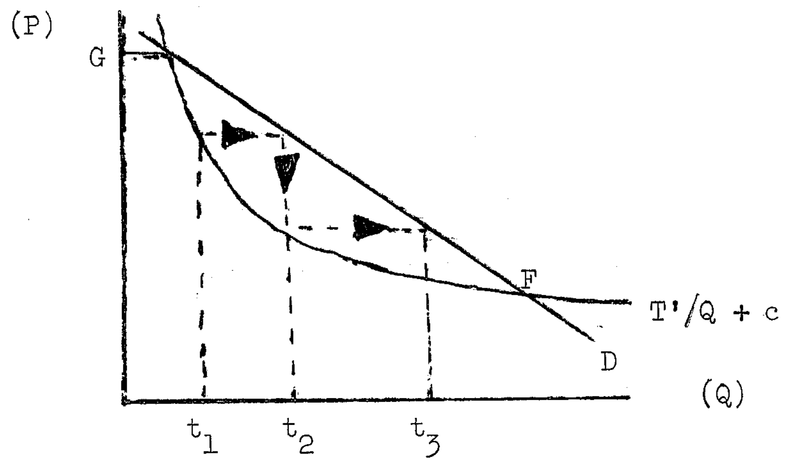

Figure 1

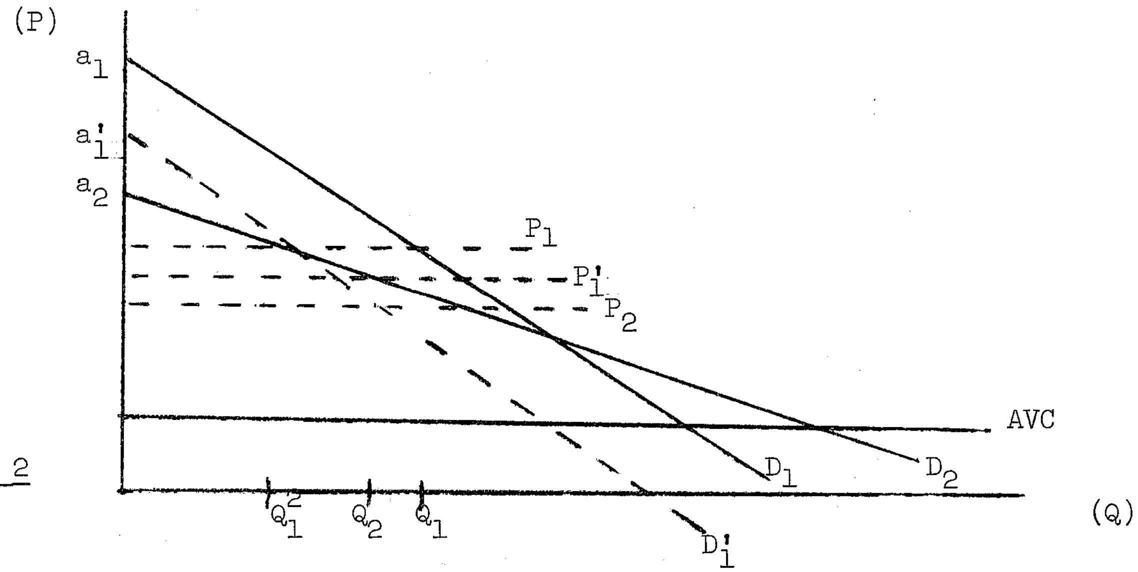

Figure 2

T' is the target magnitude desired in excess of the average variable cost c . In Figure 1, the firm moves toward the equilibrium intersection F as long as the price sequence initiates below P=G . For a firm deciding that it was confronted by a moderately steep, but stable, demand line, the selection of Rule 9 could be reasonable.[11] The textbook opinion that this rule may be unstable is largely determined by the use of the ubiquitous U-shaped average cost curve (e.g. 2, pp. 65-66).

The stability of Rule 1 depends a great deal upon the reaction parameter (f) of the quantity rule. If $Q^*_t = Q_{t-1}$, Rule 1 may be examined by the same procedure cited above for Rule 9. The (redefined) unit cost and target profit magnitudes are

$$C' = C/1-M^f \quad \text{and} \quad T'' = e/1-M^f$$

Given the possibility of erroneous specification of the expected profit attainable, a preference for stability would tend to favor a downward bias in the expected markup.

Where there exist rising or falling average variable costs, a cobweb model may be used to examine the stability of Rule 4, again taking the extreme case $Q^*_t = Q_{t-1}$. (The "supply" line is the markup multiple of the variable cost line.)

Rule 7 was specifically designed to react to such deviations from the expected demand line as an erroneous estimate of the intercept. However this rule may also partially correct unwarranted estimates of the expected demand line slope. For example, if $Q_t = Q_{t-1}$, Rule 7 will tend to correct in the right direction, as long as the ordinate of the intersection of the true demand line with the expected (but misspecified) demand line is below the initial expected price.

[11] The condition for a damped time path is $|T'/b| < Q^2_{t-1}$.

A simple example of Rule 7 that also exhibits the influence of the quantity reaction rule is shown in Figure 2. Demand has shifted from D_1 to D_2. P_1 and P_2 are the profit-maximizing prices for D_1 and D_2. The assumption that the shift involved only a change in the intercept is implicit in line D'_1. If an over-reaction (e.g. Rule 8) positioned the intercept of D'_1 at a_2, the correct price would be given for the illustrated case of the constant unit variable cost (AVC=C), for the maximizing price is the same for all demand lines through a given price axis intercept. Rule 7 assumes $a_2 - a_1 = b_1(Q_1 - Q_1^2)$; the error depends upon the magnitude of the neglected term $(b_1 - b_2)Q_1^2$. The adjusted price according to Rule 7 (P'_1) gives Q'_2 as the quantity actually demanded by D_2. If the demand line remains at D_2, the next adjustment will move in an incorrect direction. The results for $Q_t^* = Q_1$ and/or a demand return to D_1 are given by a similar inspection of the illustration.

The introduction of the fractional adjustment of the markup, in conjunction with the quantity reaction rule, leads to unwieldy analytical appraisals. The ratio form of the markup introduces nonlinearities in all but the most trivial of examples. The next section presents a model used for digital simulation of the rules.[12]

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[12] There are objections to the use of the term 'simulation' for this type of analysis. We use the term for the imitation by the computer of hypothetical demand-cost structures. Although certain restrictions on the demand-cost environ were based on empirical studies, the link to reality is indeed tenuous. Digital simulation is a more powerful technique than the diagrammatic analysis of section 1. Nevertheless, the limitations of this method should be noted. "...the underlying reality which (the simulated experiment) is designed to replicate may not really be composed of random phenomena or if random they may not have the same statistical characteristics as the models used in the experiment." - (4). For recent contribution to the simulation of rules of thumb, see (3).

II. A Hypothetical Firm

Discussions of markup pricing often indicate the emphasis placed on cost estimation accompanied by rather sketchy estimation of demand conditions. In part due to the use of increasingly sophisticated methods of cost accounting, the uncertainty concerning the former has tended to be underplayed.[13] Of course, there are firms who employ statistical methods of demand measurement. For other firms, an inability to support market testing or the lack of adequate data prohibits the use of these techniques. Yet some expectational apparatus should be imputed to these firms. Even the selection of the production process, whose cost is so assiduously analyzed, presupposes the estimation of demand conditions. (The classical illustration of this relationship is the automobile industry.)

For each firm there exists a variety of information pertinent to the demand environ that aids in congealing the expectations of the policy-maker. We assume that all the expectational data concerning demand may be expressed adequately in a single measure that is discussed shortly. The nature of the higher moments of the expectational distribution is neglected since the reactions to unrealized expectations are assumed to have been built into the pricing rules. The true cost structures are known by the firm.

Demand

The location of any demand line $P = a+bQ$ depends on the values assumed by the two parameters a and b. The slopes of the demand lines of the simulated firms are illustrated in Figure 3 ($b = -\tan\theta$; $0 < \theta < \pi/2$). The demand quadrant (D_1DD_2) is bisected into wedges A and B by the line CD. The simulated

[13] A distinction between the use of markup and imitative pricing has been predicated on the possibility of extracting reliable estimates of cost. However, in the case of industries with significant elements of oligopoly, it is not uncommon for firms to justify "traditional" markups as a device to secure similar patterns. "Margin leadership" could be a difficult matter to ascertain.

models present the firm with demand lines erected from the generating point D, which are swiveled (normal distribution) over either the A or B wedge.

The slope of the line E(A) (or E(B)) is the expected value of the lines generated in the A (or B) wedge. At the beginning of the simulation of a rule, the firm selects either E(A) or E(B) as the expected demand line. This choice is assumed to represent the expected value of the firm's distribution of demand expectations and is held throughout the simulated experiment. The demand wedge (A or B) from which the "actual" demand lines are generated may be unknown to the firm; therefore, the demand line selected by the firm may not coincide with the expected value of the population source of the "actual" demand lines. The selection of E(A) or E(B) will determine the initial cost-plus markup used by the firm. If the expected volume is not forthcoming in the initial period, the pricing rules will dictate the reaction by the firm to this "unrealized expectation".

Concurrent with the moving slope of the demand line, the price axis intercept (a) is varied by moving the abscissa of the generating point (D in Figure 3) along the line FF'. This line is devided into four equal segments as illustrated. The period transition matrix of the five generating positions along FF' is shown in Figure 4. The a_{ij} element of this matrix denotes the probability of moving the generating point of the demand line from the i^{th} position at time t to the j^{th} position at time $t+1$. To characterize the movement of the intercept (a) of a demand line in the form of a "business cycle" (for a given slope b), the average number of time periods from the "trough" to "peak" positions (1 to 5, or F to F') is thirty-three. One of the properties of this matrix is that all five positions of the generating point of the simulated demand lines are equally likely to occur in the "long run." (See 11, p. 358) However, the generating point of the "expected" demand line selected by the firm is fixed at D (position 3), as illustrated, for all simulations discussed in this paper.

Figures: section II

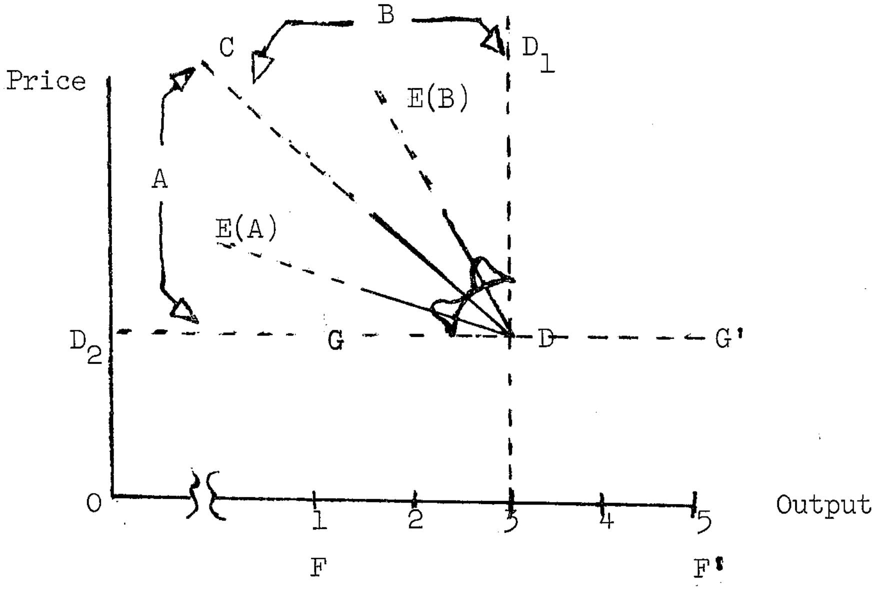

Figure 3

N.B.: OF=FF'

t \ t+1	1	2	3	4	5
1	2/3	1/3			
2	1/3	1/3	1/3		
3		1/3	1/3	1/3	
4			1/3	1/3	1/3
5				1/3	2/3

Figure 4

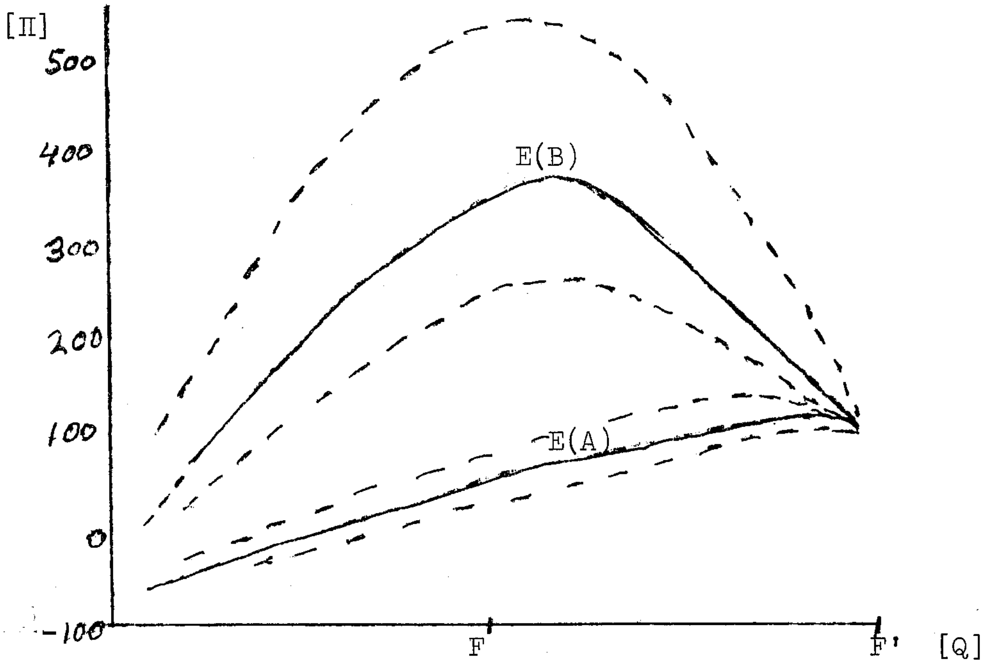

Figure 5

Illustrated in Figure 5 is the model of constant marginal cost when the generating point is located at G' (Figure 3). Profit is sketched against the quantity demanded by the demand lines whose slopes are either of the expected values E(A) or E(B). The standard deviations of slope variability at that generating position are indicated by the dashed lines. The effect of a positively sloped unit variable cost line is to make the profit plot more symmetrical. A negatively sloped cost line causes the profit function to fall more gently after its maximum value. For a generating point located at the G position (the "trough" position), the asymmetry of E(B) is reversed with the longest tail extending to the right. E(A) becomes more symmetrical. In this case, the standard deviations for both demand wedges cross over after G and begin to splay out from the expected-slope profit plot. Of course, at location G the profit curves are much lower with a more limited range of profitable output. Note that this is a highly variable demand environ.[14]

Cost

All cost structures used in the simulated experiments specify an upward kink in the average variable cost at output F' (Figure 3). We assume that the production facilities operate profitably only over the output range OF'. That is, the discontinuity in the marginal cost in conjunction with the models' demand renders unprofitable those outputs in excess of OF'.[15]

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[14] It will be possible for the demand lines to be generated so that less volume is demanded at a lower price and more at a higher price in succeeding periods.

[15] This kink in the cost curve determines the short-run 'capacity' of the firm. A similar concept of capacity is discussed in (6), p. 833ff.

In addition to the "capacity" constraint on Q_t and Q_t^* , the models used a minimum constraint on Q_t^* , as well as the lower bound of non-negativity on Q_t . The knowledge of the approximate position of the generating point of the demand lines presents an opportunity for the firm to narrow the range of output expected for positions of maximum return. In the case of linear demand, the marginal lines intersect at a point midway to the origin at the level of the generating point. The extension of the two marginal lines bounding the expectational range (of demand slope variability) border the range of maximum-profit outputs at that generating position.[16] A lower bound for Q_t^* was constructed from the assumption that the firm knew the minimum position of the generating point was located at G.

The relation of the estimated "fixed" to "variable" costs will affect the relative performance of certain rules. The results presented here used the same proportion of "fixed" to total costs (40% at G in Figure 3) for all cost structures. The model used both quadratic ($TC = cQ + dQ^2 + e$) and linear total cost functions. Non-zero values of d were obtained by swiveling the average variable cost line at output G . The end-points of the swiveled lines deviated from the G value by $\pm$ 10%.

Many empirical studies have indicated the presence of a linear total cost function for a wide range of output for a variety of firms. (See the exhaustive bibliography in 22; note also interpretative discussion in 21.) When introducing particular demand and cost assumptions, there is sometimes a tendency to play a shell game with "the firm" as the pea. For this paper, it is sufficient to note that the joint presence of a linear total cost relation and the use of markup pricing appears to be applicable for a number of firms.

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[16] For a firm with considerable penalty rates for deviation from planned production, an error in the determination of the optimum period output rate may be more expensive than an erroneous initial position of an adaptive pricing policy.

Certain model forms also "shocked" the variable cost function by a 20% step-up halfway through the simulated run for the remainder of the periods in that run. Discussion of the implication of cost variation for markup pricing is given in b) of the Appendix.

The Simulation Procedure

At the beginning of each simulated run for a particular rule, the cost structure (rising, falling, or horizontal average variable cost line), the "actual" demand sector (wedge A or B), and the expectation line (E(A) or E(B)) were selected. The initial markup chosen by the firm maximized the return believed available from the cost structure and the demand line expectation. Of course the "actual" demand sector was not known by the firm. The starting position of the generating point of the "actual" demand lines was randomly selected (with equal probability) from the five possible positions along the line GG'. The slope of the "actual" demand line for the first period was then drawn from the selected demand wedge (A or B). Generating a new demand line for each period, the behavior of the rule was then observed for twenty decision periods for each run. Each rule was sampled for forty runs under each model form.

We shall want to compare the performance of different rules in similar model forms. To facilitate the comparison, the following notation is used in the summary tables of Sections III and IV:

MODEL j(a,b,c,d) , j=1,...8

a=1; demand lines drawn from wedge A
 =2; demand lines drawn from wedge B
b=1; expected demand line = E(A)
 =2; expected demand line = E(B)
c=1; average variable cost line remained constant throughout the run
 =2; average variable cost line increased by 20% at the beginning of decision period 11
d=0;+, or -; the d coding refers to the direction of the slope of the average variable cost line

To avoid confusing sampling variability with the relative performances of the rules, all the stochastic variables were selected from the same sample.[17] A glance at the profit variability in Figure 5 confirms the necessity for this caution, particularly in the case of demand lines drawn from wedge B. Of course, the possibility exists that the sample selected uniquely favored particular rules.

III. The Results in Brief

In this section, as an overall index of performance, the cumulative profit position attained at the end of the twenty decision periods, averaged over the forty sample runs, was used to rank the nine selected pricing rules.

In Chart 1', the rules are ranked for the eight models of the case of constant average cost.[18] Models 1-4 are the experiments where the 'true' demand lines are generated from wedge A (see figure 3). In model 1, where the expected demand line selected by the firm is E(A), the constant price maintained by the rigid variable-cost markup (rule 4) performed best. The volume rules were the next most profitable. Among the full-cost variates (rules 1-3), the 'safety ceiling' rule 3 had a better end-of-run profit position than the unconstrained adaptive full-cost rule 2.

In model 2, the expected demand line E(A) again coincided with the generating wedge A, but the variable cost was increased by 20% halfway through the sample run. Here, the flexibility of the volume rules (7,8) proved superior to the rigidity of rule 4, which dropped into fifth place.

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[17] A more sophisticated experimental design for the minimization of confounding in a micro-model simulation is discussed in 5, ch. 7.

[18] Rules 5 and 6 are deleted from Table 1'. Given the constant unit variable cost, their performance was similar to that of rule 4. The ranking of the pricing rules for the case of increasing (Table 2') and decreasing (Table 3') average variable cost are tabulated for comparison with Table 1'.

In models 3 and 4, the expected demand slope selected by the firm was E(B). The only rules to obtain a profitable end-of-run position after this misspecification of the 'average' demand slope were the adaptive full-cost variates (2,3). Again, the 'safety ceiling' version (rule 3) was the more profitable.

In models 5-8, the 'true' demand lines were drawn from wedge B. The volume rules (7,8) performed exceptionally well in models 5 and 6, where an erroneous average demand slope (E(A)) was expected by the firm.

Table 1'

Rank	Model 1 (1,1,1,0)	2 (1,1,2,0)	3 (1,2,1,0)	4 (1,2,2,0)
1	rule 4	rule 7	rule 3	rule 3
2	7	8	2	2
3	8)	3	*8	*8
4	3	2	*7	*7
5	2	4	*4	*4
6	1	*9	*1	*9
7	9	*1	*9)	*1)

Rank	5 (2,1,1,0)	6 (2,1,2,0)	7 (2,2,1,0)	8 (2,2,2,0)
1	rule 8	rule 8	rule 4	rule 4
2	7	7	7	7
3	4	4	8	8
4	2	1	1	2
5	1	2	2	1
6	3	9	3	3
7	9	3	9	9

Table 2'

Rank	Model 1 (1,1,1,+)	2 (1,1,2,+)	3 (1,2,1,+)	4 (1,2,2,+)
1	rule 4	rule 4	rule* 3	rule* 3
2	2	2	*2	*2
3	8	7	*8	*8
4	7	8	*7	*7
5	3	3	*6	*6
6	1	*1	*4	*4
7	*5	*9	*5	*5
8	*9	*5	*9)	*9)
9	*6	*6	*1)	*1)

Table 3'

Rank	Model 1 (1,1,1,-)	2 (1,1,2,-)	3 (1,2,1,-)	4 (1,2,2,-)
1	rule 4	rule 6	rule 3	rule 3
2	9	3)	2	2
3	6	9)	*8	*8
4	3	2	*7	*7
5	2	5	*6	*6
6	8	4	*5	*5
7	7	7)	*4	*1)
8	5	8)	*9)	*4)
9	1	1	*1)	*9)

Tied ranks are indicated by the bracket) . Negative end-of-run profit values are noted by the asterisk * .

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When the demand lines were drawn from wedge B, the 'safety ceiling' rule 3 was less profitable than the remaining full-cost rules (1,2). The price ceiling prevented the firm from pursuing the more profitable opportunities available in the B demand wedge.

Several broad observations may be made concerning the relative performance of the pricing rules in the context of the constant marginal cost models. Perhaps the most resilient class of rules were the volume variates

7 and 8. It is interesting to observe that even the imperfect reactions of rule 8 performed as well as rule 7.[19]

At the other extreme, even in the absence of a quadratic cost function, the target rule (9) performed very poorly in the volatile environments of the simulated experiments.

On the average, the full-cost rules (1-3) were less profitable than the volume rules. The adaptive markups of the full-cost rules 2 and 3 proved to be more profitable than the fixed markup of rule 1. In fact, rules 2 and 3 were the only rules to maintain a positive end-of-run profit position for all of the models of Table 1'.

As apparent from Chart 1', the ranking of the cost-plus pricing rules depends upon both the actual and the expected demand conditions of the firm. The proliferation of cost-plus pricing variates are adaptations to specific demand-cost structures. Rules of thumb which are obviously inferior in static, perfect information models may be ranked more favorably in conditions of variability in the demand-cost environ and imperfect information.

IV. A Closer Look

The selection of the appropriate criteria of rules of thumb is a matter of some contention. It is uncertain whether some rule characteristics are desired as means to an "ultimate" end attributed to firms, or are desired as end-goals in themselves.

For Tables 1, 2, and 3, the following descriptive statistics are used: ARPROFIT is the end-of-run profit position (summed over the twenty periods) averaged over the forty sample runs.

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[19] In the next section, it is shown that the over-reactions of rule 8 ($g'>1.0$) tended to be less profitable than under-reactions ($g' < 1.0$) .

ARMISS is the square of the deviations of the actual profit obtained from the "true" maximum profit attainable, also summed over the twenty periods and averaged over the forty run sample. This measure is somewhat deceptive since the "true" maximum profit attainable is a function of the sampling variability. An optimal strategy may not necessarily prove to be the best performer in a particular sample. This measure is nevertheless included as some indication of a "fall from grace". The measure is comparable for different rules since the "true" maximum values were the same for all rules within a given model form. ARMISS/2 is the same measure as ARMISS but the period summation runs only over the last half of the periods of each run; the measure was doubled for comparability with ARMISS. In a limited way, this aids in distinguishing the various speeds of adjustment, for example, to misspecification of the expected demand wedges.

ARLOSS is the average run value of the sum of the negative profit values over the last half of each run. The run values are divided by the sum (again over the last half of each run) of the "fixed" production costs. A value less than one (1) indicates that the decision to produce was less costly than the cost of the zero production rate.

SDRLOSS is the standard deviation of the (40) run loss ratios.

ARDELTAP is the average change in profit (absolute value) for the last half of each run, also averaged over the forty runs. Other studies have used the least-squares regression of profit on time to measure the general movements of the profit behavior over time. An inspection of the simulated results found that the average difference gave as fair a picture as any other simple measure.

Table 4 presents the results of a regression analysis of the sensitivity of the rule performances to the reaction parameters g (price rules) and f (quantity rule). Undoubtedly, the linear form of the regression did violence to the results of the interaction of g and f. The dependent series was the run sums of the absolute value of the deviation of the actual profit obtained from the "true" maximum profit attainable. The length of data for each regression was therefore forty. The Q(F) and P(G) headers in Table 4 refer to the coefficients of the quantity and price reaction paramenters respectively (varied only inter-run) used as the explanatory variables of the regressions. The standard deviations are given directly below the regression coefficients. The values of the price parameter g were separated to determine if certain ranges of the pricing parameter g gave significantly different rule performances. For example, assume that we are interested in those values of g that are less than k. $(0 < k < 1)$

$$g_i^* = g_i < k$$

$$g_i = g_i > k$$

The series of the reaction parameters used in the simulation

$$R(G) = (g^*_1, g_2, g_3^*, g_4^*, \ldots, g_{40})$$

is decomposed into the two series

$$P(G) = (0, g_2, 0, 0, \ldots, g_{40}) \quad \text{and}$$

$$\text{P-SPLIT} = (g_1^*, 0, g_3^*, g_4^*, \ldots, 0)$$

which are used as separate explanatory series in the regression.

The particular split-outs of the g parameter are indicated below.[20]

REGRESSION	Q(F)	P(G)	P-SPLIT
1	F	$.25 < G < .75$	$.75 < G < .25$
2	F	$G < .5$	$.5 < G$

The TEST statistics in Table 4 indicate if the split-out series of the g parameter (P(G) and P-SPLIT) gave coefficient values that were statistically distinguishable (17, pp. 130-133). The null hypothesis is rejected for test values (approximately) exceeding 1.7 at the .10 level of significance.

As a whole, the charts indicate that while many of the markup rules are flexible with respect to the vagaries of particular models, again no one rule is adaptable to all of the situations presented here. Note especially the asymmetry of relative performance with the presence or absence of demand misspecification. The cross-classifications are most clearly obtained by familiarization with the tables. The text will focus on Tables 1 and 4 to point out some of the peculiarities of the simulated results.

With the exception of the cost step-up models (even-numbered), the average-cost markup rules are not particularly exciting in Table 1, as expected. Rule 4 is obviously sensitive to the expectation of demand. The volume rule (7) was fairly reliable; oddly enough, it performed better in the models with the incorrect specification of the demand slope, a situation for which it was not specifically designed. Section I has mentioned the reason for this partial success. More striking is the resilience of this rule indicated by the nearly similar performance turned in by the inexact volume rule (8). The average

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[20] Due to an oversight in the programming, the markup reaction parameter used in the simulation was $G = 1-g$. Therefore, instead of the speed of adjustment varying directly with the magnitude of the parameter, as in the text of Section I, the speed of adjustment varies <u>inversely</u> to the size of the parameter G of the regression analysis.

performance of Rule 8 should be similar to Rule 7; however, in most of the models, the discrepancy did not appear to be large in the SDRLOSS measure which is a limited measure of sampling variability.

Among the full-cost rules, it is somewhat difficult to take the rigid Rule 1 markup seriously. It performed well enough in models 5 and 6, for the simple reason that profits are sometimes available with higher prices at lower volumes. (The fact that this is sometimes overlooked is the evidence underlying the construction of the volume rule 7.) Like Rule 9, Rule 1 is rather unstable; when it moves in the right direction it is often for the wrong (expected) reason. A surprising performance in Table 1 was given by the adaptive full-cost rule (2), a formulation not explicitly designed for direct profit maximization. The resiliency of this rule (and the ceiling version, Rule 3) was continued in Tables 2 and 3. We have noted the good performances under the condition of demand misspecification (models 3 and 4). The effect of the price ceiling on Rule 3 was a two-edged sword, of course, both cutting losses and limiting profits. A peculiarity of Rule 3 is seen in models 5 and 6; the ARLOSS of Rule 3 is greater than the loss for Rule 2 contrary to the motivation of the addition of the price ceiling.

Looking at Table 4 (again, constant unit variable costs), the sign for the coefficient of F indicates that the discrepancy of the dependent variable tended to be minimized when the adjustment of the predicted volume was more sluggish. With regard to the margin reaction of Rule 2, the first regression form indicates that the greater discrepancy was associated with the more "extreme" reactions (P-SPLIT: $.75 < G < .25$). Only the split-out for 2 is presented for Rule 8; it appears that over-reactions of the volume rule were less profitable than under-reactions.

The results of models 5-8 are not shown in Table 4. When the demand lines were drawn from wedge B (Figure 3), the coefficient of multiple correlation was very low. A tentative explanation is that the sampling variability of the period maximum profit was of such a large magnitude (see Figure 5) that the explanatory power that could be attributed to the reaction parameters was swamped. It is also possible that the linear specification of the regressions was a poor approximation.

To minimize the tabular presentation, the results of the regression analyses of the quadratic cost environs are not shown. Briefly summarizing the excluded results: with the exception of Rule 5 in the case of increasing average variable cost and Rule 8 for models 3 and 4 of the decreasing cost experiment, the more sluggish modifications of the expected output rates were again the more profitable reactions. Where the initial position of the adaptive full-cost rule (2) misspecified the demand wedge, the more rapid mark-up adjustment ($G < .5$) was more profitable for the increasing cost environ, whereas the most extreme adjustment ($.75 < G < .25$) gave the poorest performances with a decreasing unit variable cost line. For models 3 and 4, when the expected demand line was steeper than warranted, the more rapid markup adjustments of the adaptive variable-cost rule (5) ($G < .5$) were most profitable. For the correct identification of the demand wedge, no significant range of the G values were found for Rule 5. Finally, as in Table 4, under-reactions were the best performers in the quadratic cost environs for the volume rule (8).

Obviously, pricing policies vary in their sensitivity to the particular formation of expectations and demand-cost relations assumed. The over-simplified representations of this paper are the crudest of benchmarks. While digital simulation may be easily adapted to more refined characterizations of the manner in which firms use markup policies, the selection of the environ remains a problem. There are no "typical" economic milieus, but, pragmatically, we must assume a manageable assortment.

TABLE 1

RULE	ARPROFIT	ARMISS	ARMISS/2	ARLOSS	SDRLOSS	ARDELTAP
			MODEL 1 (1,1,1,0)			
1	0.388E 03	0.112E 06	0.157E 06	0.352E-00	0.437E-00	0.159E 02
2	0.110E 04	0.628E 04	0.653E 04	0.131E-01	0.385E-01	0.157E 02
3	0.113E 04	0.477E 04	0.413E 04	0.491E-03	0.212E-02	0.139E 02
4	0.115E 04	0.236E 04	0.232E 04	0.725E-04	0.267E-03	0.118E 02
7	0.114E 04	0.317E 04	0.260E 04	0.154E-02	0.400E-02	0.166E 02
8	0.114E 04	0.290E 04	0.315E 04	0.292E-02	0.837E-02	0.163E 02
9	0.235E 03	0.104E 06	0.125E 06	0.212E-00	0.385E-00	0.251E 02
			MODEL 2 (1,1,2,0)			
1	-0.173E 03	0.109E 06	0.153E 06	0.612E 00	0.369E-00	0.192E 02
2	0.581E 03	0.126E 05	0.192E 05	0.127E-00	0.183E-00	0.191E 02
3	0.605E 03	0.832E 04	0.112E 05	0.730E-01	0.845E-01	0.131E 02
4	0.506E 03	0.162E 05	0.300E 05	0.224E-00	0.187E-00	0.422E 02
7	0.625E 03	0.836E 04	0.130E 05	0.110E-00	0.900E-01	0.320E 02
8	0.623E 03	0.810E 04	0.136E 05	0.114E-00	0.993E-01	0.325E 02
9	-0.286E 02	0.803E 05	0.778E 05	0.374E-00	0.406E-00	0.130E 02
			MODEL 3 (1,2,1,0)			
1	-0.192E 04	0.529E 06	0.544E 06	0.100E 01	0.	0.
2	0.626E 03	0.681E 05	0.236E 05	0.344E-01	0.156E-00	0.203E 02
3	0.686E 03	0.553E 05	0.138E 05	0.991E-02	0.195E-01	0.205E 02
4	-0.190E 04	0.522E 06	0.535E 06	0.990E 00	0.268E-01	0.215E 01
7	-0.609E 03	0.279E 06	0.274E 06	0.503E 00	0.992E-01	0.116E 03
8	-0.591E 03	0.272E 06	0.261E 06	0.483E-00	0.147E-00	0.104E 03
9	-0.192E 04	0.529E 06	0.544E 06	0.100E 01	0.	0.
			MODEL 4 (1,2,2,0)			
1	-0.192E 04	0.403E 06	0.293E 06	0.100E 01	0.	0.
2	0.163E 03	0.667E 05	0.208E 05	0.134E-00	0.204E-00	0.168E 02
3	0.232E 03	0.549E 05	0.130E 05	0.102E-00	0.128E-00	0.168E 02
4	-0.191E 04	0.400E 06	0.293E 06	0.100E 01	0.	0.267E-00
7	-0.832E 03	0.218E 06	0.154E 06	0.565E 00	0.128E-00	0.818E 02
8	-0.733E 03	0.210E 06	0.137E 06	0.547E 00	0.135E-00	0.793E 02
9	-0.192E 04	0.403E 06	0.293E 06	0.100E 01	0.	0.
			MODEL 5 (2,1,1,0)			
1	0.102E 04	0.802E 09	0.146E 10	0.	0.	0.137E 02
2	0.103E 04	0.802E 09	0.146E 10	0.179E-01	0.605E-01	0.118E 02
3	0.793E 03	0.802E 09	0.146E 10	0.228E-01	0.610E-01	0.103E 02
4	0.105E 04	0.802E 09	0.146E 10	0.319E-02	0.608E-02	0.106E 02
7	0.121E 04	0.802E 09	0.146E 10	0.405E-01	0.851E-01	0.135E 02
8	0.122E 04	0.801E 09	0.146E 10	0.379E-01	0.877E-01	0.133E 02
9	0.520E 03	0.801E 09	0.146E 10	0.149E-02	0.439E-02	0.143E 02
			MODEL 6 (2,1,2,0)			
1	0.104E 04	0.802E 09	0.146E 10	0.827E-03	0.316E-02	0.167E 02
2	0.767E 03	0.802E 09	0.146E 10	0.620E-01	0.148E-00	0.121E 02
3	0.374E 03	0.802E 09	0.146E 10	0.123E-00	0.155E-00	0.106E 02
4	0.120E 04	0.802E 09	0.146E 10	0.162E-02	0.367E-02	0.159E 02
7	0.128E 04	0.802E 09	0.146E 10	0.384E-01	0.858E-01	0.164E 02
8	0.130E 04	0.802E 09	0.146E 10	0.355E-01	0.861E-01	0.165E 02
9	0.494E 03	0.802E 09	0.146E 10	0.137E-01	0.703E-01	0.155E 02
			MODEL 7 (2,2,1,0)			
1	0.348E 04	0.803E 09	0.147E 10	0.981E-01	0.164E-00	0.855E 02
2	0.357E 04	0.803E 09	0.147E 10	0.150E-01	0.510E-01	0.569E 02
3	0.267E 04	0.803E 09	0.146E 10	0.240E-02	0.755E-02	0.303E 02
4	0.406E 04	0.803E 09	0.147E 10	0.219E-01	0.426E-01	0.883E 02
7	0.405E 04	0.804E 09	0.147E 10	1.000E-02	0.246E-01	0.110E 03
8	0.403E 04	0.804E 09	0.147E 10	0.106E-01	0.259E-01	0.110E 03
9	0.150E 04	0.790E 09	0.146E 10	0.171E-00	0.349E-00	0.238E 02

TABLE 1 CONT.

RULE	ARPROFIT	ARMISS	ARMISS/2	ARLOSS	SDRLOSS	ARDELTAP
			MODEL 8 (2,2,2,0)			
1	0.329E 04	0.805E 09	0.147E 10	0.158E-00	0.212E-00	0.107E 03
2	0.332E 04	0.803E 09	0.147E 10	0.387E-01	0.118E-00	0.596E 02
3	0.242E 04	0.803E 09	0.146E 10	0.720E-02	0.256E-01	0.289E 02
4	0.376E 04	0.804E 09	0.147E 10	0.888E-01	0.136E-00	0.134E 03
7	0.374E 04	0.804E 09	0.147E 10	0.385E-01	0.482E-01	0.136E 03
8	0.373E 04	0.804E 09	0.147E 10	0.411E-01	0.495E-01	0.138E 03
9	0.147E 04	0.791E 09	0.146E 10	0.187E-00	0.359E-00	0.268E 02

TABLE 2

			MODEL 1 (1,1,1,+)			
1	0.139E 02	0.903E 05	0.115E 06	0.345E-00	0.409E-00	0.153E 02
2	0.581E 03	0.160E 05	0.189E 05	0.239E-01	0.556E-01	0.945E 01
3	0.386E 03	0.257E 05	0.298E 05	0.525E-02	0.133E-01	0.433E 01
4	0.724E 03	0.858E 04	0.642E 04	0.126E-01	0.282E-01	0.167E 02
5	-0.174E 03	0.442E 06	0.776E 06	0.655E 00	0.160E 01	0.318E 02
6	-0.127E 04	0.126E 07	0.169E 07	0.129E 01	0.209E 01	0.531E 02
7	0.448E 03	0.404E 05	0.491E 05	0.124E-00	0.189E-00	0.291E 02
8	0.476E 03	0.402E 05	0.467E 05	0.103E-00	0.186E-00	0.295E 02
9	-0.232E 03	0.113E 06	0.139E 06	0.325E-00	0.450E-00	0.439E 01
			MODEL 2 (1,1,2,+)			
1	-0.447E 03	0.103E 06	0.141E 06	0.630E 00	0.353E-00	0.214E 02
2	0.226E 03	0.214E 05	0.297E 05	0.158E-00	0.191E-00	0.134E 02
3	0.484E 01	0.329E 05	0.442E 05	0.209E-00	0.110E-00	0.651E 01
4	0.313E 03	0.153E 05	0.199E 05	0.185E-00	0.113E-00	0.421E 02
5	-0.532E 03	0.513E 06	0.917E 06	0.840E 00	0.183E 01	0.442E 02
6	-0.164E 04	0.140E 07	0.196E 07	0.151E 01	0.243E 01	0.559E 02
7	0.201E 03	0.298E 05	0.280E 05	0.183E-00	0.134E-00	0.384E 02
8	0.189E 03	0.325E 05	0.313E 05	0.194E-00	0.158E-00	0.405E 02
9	-0.472E 03	0.103E 06	0.119E 06	0.526E 00	0.432E-00	0.789E 01
			MODEL 3 (1,2,1,+)			
1	-0.192E 04	0.414E 06	0.427E 06	0.100E 01	0.	0.
2	-0.101E 03	0.876E 05	0.622E 05	0.579E-01	0.154E-00	0.777E 01
3	-0.629E 02	0.791E 05	0.552E 05	0.336E-01	0.296E-01	0.740E 01
4	-0.183E 04	0.402E 06	0.415E 06	0.985E 00	0.340E-01	0.381E 01
5	-0.191E 04	0.412E 06	0.427E 06	0.100E 01	0.	0.331E-00
6	-0.173E 04	0.383E 06	0.367E 06	0.908E 00	0.108E-00	0.193E 02
7	-0.115E 04	0.399E 06	0.295E 06	0.666E 00	0.199E-00	0.787E 02
8	-0.112E 04	0.362E 06	0.311E 06	0.663E 00	0.252E-00	0.937E 02
9	-0.192E 04	0.414E 06	0.427E 06	0.100E 01	0.	0.
			MODEL 4 (1,2,2,+)			
1	-0.192E 04	0.337E 06	0.273E 06	0.100E 01	0.	0.
2	-0.401E 03	0.876E 05	0.622E 05	0.302E-00	0.214E-00	0.897E 01
3	-0.358E 03	0.800E 05	0.571E 05	0.280E-00	0.184E-00	0.857E 01
4	-0.190E 04	0.331E 06	0.272E 06	0.999E 00	0.789E-02	0.512E 00
5	-0.191E 04	0.335E 06	0.273E 06	0.100E 01	0.	0.331E-00
6	-0.174E 04	0.293E 06	0.227E 06	0.913E 00	0.104E-00	0.176E 02
7	-0.128E 04	0.423E 06	0.343E 06	0.771E 00	0.362E-00	0.815E 02
8	-0.124E 04	0.315E 06	0.217E 06	0.726E 00	0.242E-00	0.743E 02
9	-0.192E 04	0.337E 06	0.273E 06	0.100E 01	0.	0.

TABLE 3

RULE	ARPROFIT	ARMISS	ARMISS/2	ARLOSS	SDRLOSS	ARDELTAP
			MODEL 1 (1,1,1,-)			
1	0.159E 04	0.398E 06	0.245E 06	0.302E-00	0.439E-00	0.336E 02
2	0.194E 04	0.142E 06	0.463E 05	0.252E-01	0.656E-01	0.309E 02
3	0.205E 04	0.848E 05	0.602E 05	0.634E-03	0.293E-02	0.336E 02
4	0.234E 04	0.416E 06	0.134E 06	0.646E-02	0.241E-01	0.483E 02
5	0.185E 04	0.162E 06	0.630E 05	0.251E-01	0.815E-01	0.303E 02
6	0.206E 04	0.145E 06	0.410E 05	0.407E-03	0.183E-02	0.275E 02
7	0.189E 04	0.132E 06	0.340E 05	0.440E-02	0.123E-01	0.264E 02
8	0.191E 04	0.154E 06	0.339E 05	0.523E-02	0.166E-01	0.272E 02
9	0.211E 04	0.156E 08	0.236E 07	0.534E 00	0.406E-00	0.252E 03
			MODEL 2 (1,1,2,-)			
1	0.431E 03	0.390E 06	0.229E 06	0.668E 00	0.304E-00	0.213E 02
2	0.122E 04	0.136E 06	0.344E 05	0.136E-00	0.173E-00	0.347E 02
3	0.141E 04	0.834E 05	0.574E 05	0.539E-01	0.875E-01	0.319E 02
4	0.952E 03	0.408E 06	0.119E 06	0.423E-00	0.289E-00	0.469E 02
5	0.103E 04	0.160E 06	0.576E 05	0.213E-00	0.237E-00	0.359E 02
6	0.144E 04	0.137E 06	0.253E 05	0.499E-01	0.840E-01	0.256E 02
7	0.920E 03	0.152E 06	0.738E 05	0.258E-00	0.184E-00	0.507E 02
8	0.920E 03	0.176E 06	0.789E 05	0.270E-00	0.197E-00	0.508E 02
9	0.141E 04	0.146E 08	0.364E 06	0.494E-00	0.294E-00	0.101E 03
			MODEL 3 (1,2,1,-)			
1	-0.192E 04	0.755E 06	0.779E 06	0.100E 01	0.	0.
2	0.152E 04	0.344E 06	0.870E 05	0.432E-01	0.166E-00	0.408E 02
3	0.169E 04	0.342E 06	0.746E 05	0.863E-02	0.228E-01	0.423E 02
4	-0.191E 04	0.751E 06	0.774E 06	0.995E 00	0.200E-01	0.101E 01
5	-0.116E 04	0.543E 06	0.437E 06	0.522E 00	0.337E-00	0.475E 02
6	-0.771E 03	0.435E 06	0.333E 06	0.383E-00	0.272E-00	0.588E 02
7	-0.204E 03	0.383E 06	0.370E 06	0.439E-00	0.102E-00	0.157E 03
8	-0.167E 03	0.365E 06	0.312E 06	0.357E-00	0.152E-00	0.133E 03
9	-0.192E 04	0.755E 06	0.779E 06	0.100E 01	0.	0.
			MODEL 4 (1,2,2,-)			
1	-0.192E 04	0.554E 06	0.378E 06	0.100E 01	0.	0.
2	0.876E 03	0.313E 06	0.267E 05	0.117E-00	0.185E-00	0.301E 02
3	0.100E 04	0.315E 06	0.210E 05	0.860E-01	0.108E-00	0.307E 02
4	-0.192E 04	0.553E 06	0.378E 06	0.100E 01	0.	0.143E-00
5	-0.140E 04	0.442E 06	0.236E 06	0.671E 00	0.295E-00	0.317E 02
6	-0.110E 04	0.364E 06	0.191E 06	0.569E 00	0.254E-00	0.444E 02
7	-0.535E 03	0.287E 06	0.177E 06	0.496E-00	0.949E-01	0.974E 02
8	-0.533E 03	0.293E 06	0.168E 06	0.482E-00	0.106E-00	0.944E 02
9	-0.192E 04	0.554E 06	0.378E 06	0.100E 01	0.	0.

TABLE 4(0)

REGRES	R	CONSTANT	Q(F)	P(G)	(P-SPLIT)	TEST	MODEL
			RULE 2				
1	0.588E 00	0.149E 03	-0.118E 03	0.118E 03	0.203E 03		1
1		0.417E 02	0.628E 02	0.847E 02	0.552E 02	-0.118E 01	1
2	0.567E 00	0.156E 03	-0.145E 03	0.188E 03	0.153E 03		1
2		0.546E 02	0.620E 02	0.626E 02	0.171E 03	0.256E-00	1
1	0.757E 00	0.189E 03	-0.196E 03	0.199E 03	0.407E 03		2
1		0.517E 02	0.779E 02	0.105E 03	0.685E 02	-0.234E 01	2
2	0.719E 00	0.219E 03	-0.267E 03	0.359E 03	0.215E 03		2
2		0.707E 02	0.803E 02	0.810E 02	0.222E 03	0.816E 00	2

TABLE 4(0) CONT.

REGRES	R	CONSTANT	Q(F)	P(G)	(P-SPLIT)	TEST	MODEL
1	0.685E 00	0.291E 03	-0.662E 02	0.576E 03	0.109E 04		3
1		0.150E 03	0.226E 03	0.305E 03	0.199E 03	-0.198E 01	3
2	0.642E 00	0.289E 03	-0.213E 03	0.103E 04	0.984E 03		3
2		0.203E 03	0.231E 03	0.233E 03	0.638E 03	0.939E-01	3
1	0.676E 00	0.361E 03	-0.934E 02	0.485E 03	0.988E 03		4
1		0.142E 03	0.214E 03	0.288E 03	0.188E 03	-0.206E 01	4
2	0.627E 00	0.321E 03	-0.224E 03	0.964E 03	0.106E 04		4
2		0.193E 03	0.219E 03	0.221E 03	0.606E 03	-0.202E-00	4
			RULE 8				
2	0.231E-00	0.988E 02	-0.101E 02	0.151E 03	0.371E 02		1
2		0.358E 02	0.407E 02	0.112E 03	0.410E 02	-0.128E 01	1
2	0.461E-00	0.233E 03	-0.111E 03	0.193E 03	-0.948E 01		2
2		0.479E 02	0.545E 02	0.151E 03	0.549E 02	-0.170E 01	2
2	0.642E 00	0.139E 04	-0.750E 02	0.157E 04	0.893E 03		3
2		0.155E 03	0.177E 03	0.488E 03	0.178E 03	-0.175E 01	3
2	0.517E 00	0.135E 04	-0.169E 03	0.119E 04	0.564E 03		4
2		0.145E 03	0.165E 03	0.456E 03	0.166E 03	-0.172E 01	4

APPENDIX

a) Markups and "safety" margins

The expected profit is not always the only measure of interest to the firm. For example, there is the problem of sequential dispersion; a certain profitability must be maintained to enable the firm to remain eligible for "play". With the introduction of discounting, it is even more unlikely that future gains will offset the effect of present losses. The distinction that profit maximization may not be identical with the attempt to minimize losses has been brought out with clarity by W. Fellner (12). He cites the possible existence of a skewed profile of weights applicable to "favorable" as opposed to "unfavorable" surprises. The device of safety margins is used as a defense by which "the outcome of a venture may fall short of the 'best guess' without causing losses" (p. 152). He locates the safety margin at that rate of output for which the gap between the "best guess" average revenue and average variable cost is at a maximum, and so explains a possible preference for a variable-cost markup that would be higher than that which would maximize the "best guess" profits. (Unfavorable adjustments, in his example, are parallel downward shifts of the demand function or upward shifts of the cost function.)

Whether or not actual markups tend to be higher or lower than "best guess maximal" margins is an empirical question. The discussion of Section I would lead one to suspect such an inference since there is less of a tendency to explosive movements in the pricing rules that adopt lower initial markups. Fellner appears to be concerned with the case of a flexible price adjustment at the given safety-output rate, or the case of a fixed price and output selection with reference to upward shifts in the average cost. The nature of the "best" safety margins reflects whatever restrictions one assumes about the nature of the price and output flexibility.

Rules 3 and 6 use the construct of a safety range: In the case of the fixed-period price and flexible-period output environ of this paper, the range of prices that will avoid losses is determined by the "worst" (lowest) demand line expected (ARmin). The safety price range is bordered by that price level which maximizes the expected profits of ARmin and the price level at the intersection of the average total cost with ARmin. This range may or may not effectively constrain the selected "best guess" price. The generating point of the ARmin used in determining the price ceilings for Rules 3 and 6 was located at G (Figure 3); the slope of ARmin was one standard deviation below the slope value of the expected demand line (either E(A) or E(B)). The floor of the range, however, was left implicit in each rule, being ATC or AVC for Rules 3 and 6 respectively.

An interesting area of inquiry, which appears not to have been covered by existing case studies, is the examination of the reaction of firms to the potentiality of reductions in "best guess" profits as distinguished from possible "losses" (whether absolute or relative to the firms' minimum acceptable levels).

The consequence of unbounded prices is of particular importance for the class of rules known as "learning rules". A popular rule of this type used in simulation studies of the firm(s) is the following selection from (1, pp. 165-166).

$$P_{t+1} = P_t + g \mid \Delta P_t \mid \Delta\Pi / \Delta P_t$$

where g is a "scaling factor" to reduce violent fluctuations. They employed this more out of "directional" consideration than the result of any explicit relationship. (See also 3, pp. 41-43) A rule used in an earlier version of this paper on markups used the following assumptions: $Q = a+bP$ (demand) and $TC= cQ+e$ (total cost). The adjustment of the variable-cost markup was

$$M_t^{v*} = M_{t-1}^v + j \; \Delta\Pi_{t-1} / (\Delta M_{t-1}^v c^2)$$

The condition for convergence with a stable demand line is satisfied for $j < -1/b$. The selection of j was determined by the expected slope b^*. In a stable demand environ, even with allowance for misspecification of the expected slope, the rule did very well. The rule performed poorly in a moderately volatile demand environ because it ignores the lower bound on production.

b) <u>Markups and the cost base</u>

In reference to the "capacity kink" of the variable cost in Section II, if the marginal revenue curve is positioned in the range of the discontinuity of the marginal cost, analogous to the result of the kinky demand curve, minor fluctuations in the average cost intercept may not effect the profit maximizing price. However, the general efficiency of markups, in seeking the maximum short-run profit attainable, may be quite irregular in response to fluctuations in variable cost. If a firm is disturbed from a maximizing position by a rise in the unit cost base, the percentage nature of the markup will induce an excessive rise in the adjusted price. Of course, if the original markup was less than the maximizing margin, the firm may move closer to a position of maximum profits, a movement that may partially offset fluctuations in the absolute profit received. Statements such as "when price is set on an average cost basis, the firms are likely to raise price immediately by the full amount of the cost increase" (10, p. 278) are somewhat confusing. Without modification of the markup, the price increase would presumably exceed the amount of the cost increase. The actual speed of adjustment to a discrepancy between a desired and actual price due to a cost change is a matter of some contention. For a model which imputed delayed responses (desired price a linear function of direct cost) see (24).

Some studies of markup pricing have concentrated on industries characterized by the presence of significant oligopolistic elements. In these industries, a widespread acceptance of markup pricing is believed to enlarge the range of price variation that does not induce retaliatory actions. Excluding the possibility of direct collusion, the acceptance of price adjustments to variations in cost that are "believed" to be uniformly applicable depends upon the relationship selected to signal violation of the acceptance-indifference zone (i.e. effective "belief"). For a two-firm model, a signal such as the price ratio (or the absolute price differential) will assure passivity for an equal percentage (or absolute) cost change. However, for many plausible characterizations, such as a market-share signal, it is not possible to assert the absence of retaliation for similar cost changes without specification of the size of the indifference buffer-zone for variations of the signal.

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