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UPON THE PARETO OPTIMALITY OF GROUP BEHAVIOR[1]

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ABSTRACT

In this paper some of the conditions for and obstacles to Pareto optimality of group behavior are considered. Special emphasis is put upon those obstacles which arise from the imperfection of man's knowledge yet these obstacles are not given inclusive attention. It is found to be convenient to discuss these questions for "societies" which involve limiting states of inter-dependence since they give insights into the non-limiting cases.

The term "society" is used to designate any group which is independent of another. It follows that nearly every collection of existing individuals cannot form a "society" and that a more general formulation is required. Formulations are given which enable us to conceptualise some conditions which render it impossible for a group to <u>ensure</u> Pareto optimality for itself. It is suggested that such conditions will commonly prevail.

UPON THE PARETO OPTIMALITY OF GROUP BEHAVIOR (1)

C. Tisdell

The Pareto optimality concept has been extensively used in economies and a number of related disciplines. The assumption of the Pareto optimality of group behavior has been widely employed in game theory constructs, some related theories of group behavior and at least, implicitly, in a number of economic models which are designed to explain the behavior of a collection of economic agents. It has also been suggested that the whole society should strive to attain Pareto optimality in order "to maximize its welfare". There is no doubt that the concept has and does exert considerable influence in the positive and normative areas of economic thought, despite the fact that many writers have noted its serious limitations. (2)

This essay involves another attempt to consider the limitations of the Pareto optimality hypothesis of group behavior. In it, I shall attempt to develop some arguments in general terms in the hope that these might help with the conceptualisation of some of the basic problems associated with the Pareto optimality hypothesis of group behavior. (3) To begin to clarify some of the factors which may decide whether group behavior is Pareto optimal or not, it is possibly of assistance to classify groups according to differences in the "extent" of the interdependence of their members.

Taking any group of individuals, the following possibilities exist:

(1) No (possible) action of any member can influence the satisfaction of any other member. (Complete independence)

(2) Every (possible) action of at least one member affects the satisfaction of another member or other members but no other member can affect his satisfaction. (One-way dependence). Obviously, this condition cannot apply to

every member of the group.

(3) Every (possible) action of any member affects the satisfaction of every other member. (Complete interdependence)

(4) This leaves cases which are a mixture. They may involve independence or one-way dependence for some acts and interdependence for others.

If the following assumptions are satisfied, a group's behavior is certain to be Pareto optimal:

(a) Group possibility (1) is the only one for the society.

(b) Each member of the group has perfect knowledge of all the alternative courses of action which he can take.

(c) Each member has perfect foresight of the different outcomes (payoffs) which would stem from his alternative courses of action and which would affect his satisfactions.

(d) No member changes his preference ordering of the courses of action as time elapses.

(e) Each member has a complete preference ordering over his possible courses of action which meets the usual consistency requirements and is also consistent with his ordering of the outcome which attach to the acts.

(f) Each member chooses his most preferred course of action.

If the preceding assumptions are satisfied, then no member of the group will ever feel disappointed with his course of action nor will he ever have occasion to diverge from his planned course. To us, there hardly seems to be the remotest probability that a group has or will ever fulfill these conditions. We note that if condition (f) is violated and all the others hold, that Pareto non-optimality follows by logical necessity. However, if condition (a) continues to hold, violations of any other combination of conditions do not necessarily result in Pareto non-optimality. Pareto non-optimality may arise from inadequate knowledge of possible courses of action, from imperfect knowledge of outcomes, from a change in preference orderings with time, from inconsistencies or gaps in preference orderings but none of these factors are by necessity inconsistent with the occurence of Pareto optimality. Also, it can be perceived that members of a group which satisfies the above assumptions would have no motive for communicating with one another.

Implicit in the above set of assumptions are some very peculiar assumptions about the decision process itself. It is treated as though it is an effortless one involving no alternative cost. There are no costs involved in deciding upon ones preferences and there is no need to search for knowledge; it is freely and fully available without cost.

Let us now suppose that group possibility (2) is the only possible one for society. In this case, every (possible) action of at least one member affects the satisfaction of another member or other members but no other member can affect his satisfaction. We also suppose that conditions (b) to (f) above will hold. Under this set of conditions, the group's joint action need not be Pareto optimal. If each of the independent members has a uniquely preferred course of action, then Pareto optimality is certain to occur given the immediately preceding set of conditions. We are, of course, assuming that the dependent members of the group can predict the actions of the independent members. However, if at least one member, who is himself independent but upon whom some members of the group depend, has a number of most preferred courses of action (among which he is indifferent), Pareto optimality of group behavior is not assured. In this case, the independent member will be indifferent about his "best" courses of action and need not select the one which is preferred by other members. Our comments upon the violation of conditions (b) to (f) in the group independence case apply also to this one. If condition (f) is the only one violated then Pareto non-optimality is certain to occur but any other combination of can result in Pareto non-optimality but need not.

The complete interdependence case (3) is of more interest to us than the preceding ones since it more closely approximates the conditions of our own society. In this case, we assume that a different choice by any member will affect the satisfaction of every other member. In this circumstance there are new obstacles to the attainment of Pareto optimality and in some cases these seem to be intractable. Nevertheless, complete knowledge need not entail indeterminancy problems in this case.

To illustrate this, let us make the following assumptions:

(a) Group possibility (3) is the only possible one for the society.

(b) Each member of the group has perfect knowledge of all the possible courses of action which can be taken by the group.

(c) Each member has perfect foresight of the different outcomes (payoffs) which would stem from all the alternative courses of action of the group and which would affect his satisfaction.

(d) No member changes his preference ordering of the group's courses of action as time elapses.

(e) Each member has a preference ordering over the group's alternative courses of action which meets the usually consistency requirements, is complete, and is consistent with his orderings of the possible outcomes which attach to these courses.

(f) Where it is within his power, each individual will select a (group) course of action which he prefers to one which he does not.

Let us suppose that our group consists of n members, that a_i represents the possible courses of member i and U_i represents the satisfaction of member i in an ordinal fashion. Further, let us suppose that

$$U_1(a_1^o, a_2, \ldots, a_n) > U_1(a_1, a_2, \ldots, a_n) \qquad a_1 \neq a_1^o$$

$$U_2(a_1, a_2^o, \ldots, a_n) > U_2(a_1, a_2, \ldots, a_n) \qquad a_2 \neq a_2^o$$

$$\vdots$$

$$U_n(a_1, a_2, \ldots, a_n^o) > U_n(a_1, a_2, \ldots, a_n) \qquad a_n \neq a_n^o$$

Since each member of the group has the power to select his own course of action, the i-th player will select course a_i^o if he chooses consistently with his preferences. The joint action of the group will be $[a_1^o, a_2^o, \ldots a_n^o]$, no individual will have any incentive to diverge from his chosen course of action and the joint action will be Pareto optimal. Any divergence of joint action from $[a_1^o, a_2^o, \ldots, a_n^o]$ will make some individual worse off. Of course, there is no implication in this situation that $U_i(a_1^o, a_2^o, \ldots, a_n^o) \geq U_i(a_1, a_2, \ldots, a_n)$ for any member.

No doubt the above set of conditions can be applied so that a stable and Pareto optimal solution can be obtained for groups having more complex preference patterns. By a process of systematically eliminating dominated courses of action, a group of super-rational beings can in some cases reach a stable and Pareto optimal group strategy by independently choosing their own courses of action. To illustrate this, we suppose a group of two members and assume that each member has two possible courses of action. Player one's alternative courses of action are a_1' and a_1^* and player two's alternative courses of action are a_2' and a_2^*. We also assume that

$$U_1(a_1', a_2) < U_1(a_1, a_2) \qquad a_1 \neq a_1'$$

$$U_2(a_1, a_2') < U_2(a_1, a_2) \qquad \begin{cases} a_1 \neq a_1' \\ a_2 \neq a_2' \end{cases}$$

and this implies that

$$U_1(a_1', a_2) < U_1(a_1^*, a_2) \quad \text{and}$$
$$U_2(a_1^*, a_2) < U_2(a_1^*, a_2^*)$$

and is <u>consistent</u> with

$U_2(a_1', a_2') > U_2(a_1, a_2^*)$. Acting independently the players should choose the joint strategy (a_1^*, a_2^*). Player two will reason that player one will never adopt a_1' and will act accordingly. The position (a_1^*, a_2^*) is obviously stable since no player can have any incentive to diverge from it <u>knowing</u> the other player has selected either the strategy a_1^* or a_2^*, as the case may be. Stability follows since

$$U_1(a_1', a_2^*) < U_1(a_1^*, a_2^*) \quad \text{and} \quad U_2(a_1^*, a_2') < U_2(a_1^*, a_2^*).$$

The position is also Pareto optimal. Any joint action involving a_1' would make member one worse off and member two will be made worse off by a joint action which involves a_1^* and a_2' in place of a_2^*. Hence, each alternative joint course of action to (a_1^*, a_2^*) would make at least one member worse off.

It is evident that in some cases involving a larger group and a larger number of strategies that stable Pareto optimal solutions can be obtained by the systematic elimination of dominated strategies. Sometimes an iterative procedure can be involved in deciding upon the dominated strategies. Member one may be expected to eliminate a number of strategies and this may lead member two to eliminate some strategies which in turn would cause member one to eliminate further strategies.

In the last set of situations, it is assumed that each individual will know the other members' preference well enough to be able to eliminate their dominated strategies as possibilities. Thus we have implicitly slipped in an extra assumption to the ones contained in the last (a) - (f) group above. It is clear that there are some empirical cases in which this condition is not approximated, and that individuals can sometimes gain an advantage by giving others a false impression of their preferences.

If the last set of conditions (a) - (f) hold, if

$$U_i(a_1^o, a_2^o, \ldots, a_n^o) > U_i(a_1, a_2, \ldots, a_n)$$

where $i = 1, \ldots, n$ and $(a_1, a_2, \ldots, a_n) \neq (a_1^o, a_2^o, \ldots, a_n^o)$, if each member of the group is aware of this last fact and if communication is costless and sufficiently rapid, then one might expect all members of the group to agree to the joint strategy $(a_1^o, a_2^o, \ldots, a_n^o)$. This joint strategy will be stable in the sense that if every other individual of the group adopts his a^o-th course of action, then each individual will find it to his advantage to adopt his a^o-th course of action. The joint strategy $(a_1^o, a_2^o, \ldots, a_n^o)$ is Pareto optimal. One fears, however, that there would be cases in which

$$U_i(a_1^o, a_2^o, \ldots, a_n^o) > U_i(a_1, a_2, \ldots, a_n)$$

where i = 1, ..., n and $(a_1, a_2, \ldots, a_n) \neq (a_1^o, a_2^o, \ldots, a_n^o)$, and yet all members may not be aware of this fact and no member may be willing to reveal his most preferred joint strategy for fear that others will not reveal theirs, that their most preferred joint strategies are different and that they may be able to use his small piece of information to their advantage. In this case, ignorance of preferences may persist and members of a group may fail to adopt a joint action which they all consider to be the very best course.

In all the group situations considered so far, the Pareto optimal solutions for them are "self-enforcing". No member can obtain an advantage by diverging unilaterally from the group solution. But there are interdependence group situations in which this is not so for any possible Pareto optimal solution. In these cases, at least one individual has a unilateral interest to diverge from any Pareto optimal solution. This is so in the following example: The last set of (a) - (f) conditions are assumed to hold and the group is supposed to consist of two members and each is supposed to have two alternative courses of action. The preference orderings which the two individuals have over the possible joint courses of action are indicated by the numbers in the following matrix:

	a_2^1	a_2^2
a_1^1	7,4	3,6
a_1^2	2,5	8,2

All possible joint courses of action except (a_1^2, a_2^1) are Pareto optimal. But individual one has an incentive to diverge unilaterally from the Pareto optimal solution (a_1^1, a_2^2) and individual two has an incentive to diverge unilaterally from the Pareto optimal solutions (a_1^1, a_2^1) and (a_1^2, a_2^2). Hence, it is implied in this case that every agreement which is designed to ensure a particular Pareto optimal solution must rely upon the good faith of the parties for its fulfillment since there is always an "incentive" for at least one party to dishonor it by his unilateral action. If there is any doubt about the good faith of the parties, then uncertainty is inescapable in this type of situation.

In the class of group situations now under discussion, alternative Pareto optimal solutions involve a conflict of interest and this conflict may result in group action which is less than Pareto optimal even if conditions are comparatively ideal. If the following conditions are satisfied Pareto optimality of group behavior is not assured:

(i) The last set of conditions (a) - (f) hold.

(ii) Members of the group can bargain about the joint course of action which they will pursue as from time, T_o.

(iii) There are a number of Pareto optimal courses of joint action and these involve a conflict of interest among members of the group.

(iv) In the absence of any agreement regulating their joint action, the group may adopt a joint action which is less than Pareto optimal.

(v) There are "no" technical limitations upon the ability of group members to contact one another. There is no cost of communicating.

(vi) Yet there is a time limit upon the bargaining process. If no agreement upon a joint course of action is reached by T_o, then agreement is a fait accompli and a less than Pareto optimal set of joint actions can occur in the interval which commences at T_o.

(vii) All members of the group are aware of the group's alternative Pareto optimal courses of action.

Members of the group may fail to reach agreement for the following reasons:

(i) In bargaining, individuals make assessments of the willingness of other individuals to make concessions. Sometimes they misjudge this. Bargainers sometimes believe that by holding out that they will gain a particular concession but they are sometimes mistaken and as the result of their failure to reach agreement, Pareto non-optimality remains a possibility. A great deal of uncertainty surrounds the outcome of a bargaining process and the process sometimes leads to results which are less than Pareto optimal.(4)

(ii) Again, we have not considered the possibility that some members of a group may be able to predict the behavior of other members of the group with varying degrees of accuracy. No doubt, the predictive ability which each member <u>feels</u> that he has will have an impact upon his willingness to make concessions and so upon the possibility of a Pareto optimal agreement. If in the last matrix example which we discussed above, each member believes that he can perfectly predict the other's course of action, then at least one member must be wrong and members will be unable to agree upon a Pareto optimal course of action because no joint action can ensure <u>each</u> member as "much" as he believes that he can obtain alone. Although this is an extreme example, it illustrates the point that the prevailing opinion of group members about their ability to predict the behavior of others may be such as to rule out the possibility of a Pareto optimal agreement. Indeed, the prevailing beliefs may sometimes be such that even the minimum demands of all members cannot be satisfied simultaneously. This is so in the extreme example just considered.

(iii) Again, it is not always justifiable to assume that a bargaining process is an emotionally neutral one for members of a group. If individuals actually bargain this may bring into play emotions which are absent otherwise and these new factors may render a Pareto optimal agreement impossible. We might crudely make allowance for this new element by varying our utility figures for joint strategies so that they depend upon whether these are a bargained outcome or not. To illustrate this, let us suppose that the circumstances pertaining to the last matrix above are relevant. We assume that the utility figures in that matrix refer either to joint actions which are not the result of agreement or to an emotionally neutral bargaining case. To save the reader the effort of referring back, that matrix is

	a_2^1	a_2^2
a_1^1	7,4	3,6
a_1^2	2,5	8,2

But if bargaining is not a neutral process, a different utility matrix may be relevant to bargained courses of action. For instance, individual one may have the following ordering of "bargained" joint actions:

	a_2^1	a_2^2
a_1^1	7	2
a_1^2	1	9

This matrix indicates that individual one would most of all like to obtain an agreement to joint strategy (a_1^2, a_2^2), and that he would prefer the bargained outcome (a_1^2, a_2^2) to the unbargained outcome (a_1^2, a_2^2). The bargained outcome (a_1^1, a_2^2) is less preferred than the unbargained one. The same is true of the bargained outcome (a_1^2, a_2^1). We expect that individual one will never agree to joint actions (a_1^2, a_2^1) or (a_1^1, a_2^2).

Individual two might have the following ordering of bargained joint strategies:

	a_2^1	a_2^2
a_1^1	2	7
a_1^2	5	1

If this is so, individual two will be unprepared to agree to joint actions (a_1^1, a_2^1) or (a_1^2, a_2^2). Hence, there is no joint strategy to which both members are prepared to agree. Our allowance for bargaining emotions renders agreement impossible in the above case if each member insists upon being a little better off in a bargained situation than in the least nasty situation which he can ensure himself without any agreement.

(iv) Furthermore, if we wish to consider all possibilities, we should not assume that members of a group proceed upon the principle that all men [or even some men] are absolutely trustworthy. In most circumstances, one might expect members of a group to be attentive to the probability of being double crossed and to consider this probability along with its consequences before entering into an agreement. Indeed, members of a group may sometimes believe that in reaching an agreement that they are substituting one uncertain situation for another. In the limiting case where nobody is prepared to fulfill the agreement at all, the position is not materially different to that in the absence of the agreement. Otherwise, the agreement may alter the probability of different joint actions without necessarily creating certainty.

As a result of distrust members may reach no agreement at all or curiously enough they may agree to a Pareto sub-optimal joint course of action. To illustrate the last possibility, we suppose that the group conditions which were relevant for the first matrix above hold. However, we now suppose that each member of the group of two has three alternative courses of action and we use a different set of numbers to indicate preferences. Preferences are indicated in the following matrix:

	a_2^1	a_2^2	a_2^3
a_1^1	5,3	6,5	0,8
a_1^2	3,5	8,0	1,0
a_1^3	4,4	5,3	3,5

The Pareto optimal courses of action in this case are (a_1^1, a_2^2), (a_1^1, a_2^3) and (a_1^2, a_2^2). We can safely exclude the possibility that parties will agree to either of the last two joint actions. However, we are not in a position to conclude that they will agree to (a_1^1, a_2^2). If member one breaks the agreement, member two may find that he is faced by the joint course of action (a_1^2, a_2^2) rather than (a_2^1, a_2^2). If member two feels that there is a sufficiently high probability of one's breaking the agreement, he will not agree to a_2^2. Similarly, individual one may not be prepared to agree to a_1^1. On the other hand, it is conceivable that members of the group may agree to (a_1^3, a_2^1). Consequently, respective parties by maintaining their part of the agreement avoid the possibility of (a_1^1, a_2^3), (a_1^2, a_2^2) and (a_1^2, a_2^3). Of course, the parties could do this without agreement. Individual one can do this by adopting a_1^3 and individual two can do it by adopting course a_2^1. How then can they benefit by an agreement? One possibility is that the agreement changes the subjective probabilities of the joint acts favorably for the members. The agreement may change individual two's subjective probability distribution over (a_1^i, a_2^1), $i = 1, 2$ and 3 and individual one's subjective probability distribution over (a_1^3, a_2^j) where $j = 1, 2, 3$.

If we were to move outside the world which we have just constructed and were to allow the possibility that members of a group may not be cogniscant of all their possible courses of action or of all the outcomes stemming from them, we should raise a whole new set of problems. In this new world the flexibility of agreements will be important. One possibility is that agreements will be for much shorter durations so as to permit new agreements to be drawn up as time goes on and possible courses of action and outcomes become clearer. In this world which closely approximates our own, no one can be sure of the Pareto optimal courses of action and everyone stumbles forward in a myopic condition. This is the type of world which our theories must ultimately come to grips with. Indeed, it would be no surprise if Pareto optimality was the exception rather than the rule in such a world for its fulfillment would have to be accomplished by partially blind actors.

Instead of now delving into the world which has just been mentioned, let us proceed to a discussion of the "mixed" group cases (4) in order not to lose symmetry. Under this heading have been included all of those cases which do not fall under the other three but which contain a mixture of their elements. There are, of course, a large number of sub-cases included under heading (iv) but since they raise few new difficulties which stand in the way of Pareto optimality, a systematic consideration of them all will not be attempted. One possibility is that each member of the society can choose either to be independent of or to be dependent upon the actions of all other members. There are some courses of action open to each member of the society which will make him independent and some which will make him dependent. However, it could well be that all the Pareto optimal strategies for the society require the members to engage in strategies which make them interdependent. Yet, members may not agree to adopt these strategies since they may feel that the risk of their being double crossed is such that they would prefer independent strategies. The situation here is similar to one which was discussed under heading (iii). Indeed, all of the problems of attaining Pareto optimality which were discussed under the first three headings arise for some type (iv) cases.

The term society has been used to designate any group whose satisfaction is independent of the actions of any set of individuals outside of it. Given this usage, then an existing human population does <u>not</u> form a society if at a future date members of it will be affected by the actions of individuals born in the interim. Indeed, there may be human groups such that the term society can only be strictly applied to an existing collection of individuals <u>plus</u> a set which will be born over an infinite or at least very large span of time. This situation can arise if one generation of individuals tends to live for some time along with the next and even later generations so that a chain of interdependence arises. Where a society is like this, explicit allowance ought to be made for the fact that all "players" are not alive at the "beginning" of the game. Presumably, since unborn players cannot take part in the earliest agreements and cannot express their preferences for some time, these factors might be expected to impose additional restrictions upon the applicability of the Pareto optimal theory of group behavior. We must take account of the fact that all groups do not exist in "splendid isolation" and this requires a more general formulation.

Using the formulation which will soon be presented, we shall first consider for any group which exists at the beginning of a period t whether or not this group can ensure Pareto optimality for itself after allowing for all behavioral possibilities, including the behavioral possibilities of individuals _not_ in the group or not yet born. Let I represent any non-empty subset of the set of all individuals $\{1, 2, \ldots, n\}$ which exist at the beginning of period t. The set I may consist of just _one_ individual, any combination of individuals or all individuals who exist at the beginning of t. We suppose that there are $r = 1, \ldots, k$ individuals in group I and _do not_ exclude the possibility that $k = 1$. To be general, we assume that group I need not be faced by a single set of alternative couses of action which will be open to it to choose from but may be faced by a number of possible sets. The group I may not have just one set of alternative courses of action open to it but may have a number of possible (probable) sets open to it. Its actual course must always be confined to the set of alternative courses which do arise for it. Formalizing, let B represent the set of alternative courses of action which may be open to group I.

$B = \{B_1, B_2, \ldots, B_h\}$ where any element such as B_1, represents one set of alternative courses of joint action (for the entire future) which may be open to group I. Each element of the sets $B_1, B_2, \ldots, B_h$ represent a course of action which may arise for group I's choice. Where each b value represents a course of action (for the entire future) which may arise for group I's choice,

$$B_1 = \{b_{11}, b_{12}, \ldots, b_{1v(1)}\}$$

$$B_2 = \{b_{21}, b_{22}, \ldots, b_{2v(2)}\}$$

$$\ldots \quad \ldots\ldots\ldots\ldots$$

$$B_h = \{b_{h1}, b_{h2}, \ldots, b_{hv(h)}\}$$

Each element of the sets $B_1, B_2, \ldots, B_h$ can be expressed as a vector which indicates one combination of actions which the members of group I may be able to undertake over their entire future. An element such as b_{11} may be expressend as $[b_{11,t}, b_{11,t+1}, \ldots, b_{11,t+\pi}]$ where $t + \pi$ extends up to the time when the last member of group I is unable to act. The element b_{11} indicates a combination of acts each of which is performed in consecutive periods.

Let $H = B_1 \cap B_2 \cap \ldots \cap B_{h-1} \cap B_h$. Then, set H represents the set of courses of action which are certain to be available for group I's choice. Only for elements contained in the set H is group I certain to have a choice. If the set H is empty, then there is no course of action which group I is certain to have the opportunity of choosing.

Let $K = B_1 \cup B_2 \cup \ldots \cup B_{h-1} \cup B_h$. Then K represents all of the b values which group I <u>may</u> have an opportunity of choosing. We suppose that there is associated with each b value which may arise for group I's choice, a set of possible courses of action which may be pursued by all individuals not included in I. For any given value of b, we represent this set by $\xi(b)$ and recognize that the elements of the set may vary with b. $\xi(b) = \{\xi_1(b), \xi_2(2), \ldots \xi_{m(b)}(b)\}$ where $b \in K$ and where any element such as $\xi_1(b)$ represents one possible course of action which may be pursued by group $\bar{I}$ given that group I adopts course b.

Although this raises a number of metaphysic problems, it will be assumed that there is for each individual in group I an ordering which ranks ordinally the actual "satisfaction" which he would derive from each of the possible $[b, \xi_z(b)]$ combinations. ξ_z represents any element of $\xi(b)$ and b can assume any value in the set K. This "true satisfaction" ordering for the r-th individual is represented ordinally by $U^r(b, \xi_z(b))$ where $b \in K$ and $\xi_z(b) \in \xi(b)$ and this ordering is assumed to be transitive and complete.

Given these conditions, then it is impossible for group I to select a course of action which will <u>ensure</u> Pareto optimality for it, unless there is at least one value of b, say b*, such that

$$[U^1(b^*, \xi_z(b^*)), U^2(b^*, \xi_z(b^*)), \ldots, U^k(b^*, \xi_z(b^*))]$$

$$\nless [U^1(b, \xi_z(b)), U^2(b, \xi_z(b)), \ldots, U^k(b, \xi_z(b))]$$

or more compactly,

$$[U^r(b^*, \xi_z(b^*))] \nless [U^r(b, \xi_z(b))] \tag{1}$$

where r = 1, ..., k and where $\xi_z(b^*)$ can assume the value of every element in the set $\xi(b^*)$ and $(b, \xi_z(b))$ can assume every value for every possible b value except b*; <u>and</u>

$$b^* \in H. \tag{2}$$

Group I cannot ensure Pareto optimality for itself if there is no $b \in K$ which satisfies condition (1) and (2).

Let B^* represent the set of all b values which satisfy expression (1). Then, group I cannot select a course of action, i.e., a b value which will ensure Pareto optimality for it (after allowing for its behavioral environment) unless the set $B^* \cap H$ is non-empty. The non-emptiness of both B^* and H is necessary for the non-emptiness of $B^* \cap H$. Of course, both B^* and H may be non-empty and yet $B^* \cap H$ may be empty. Given our assumptions about U^r, it can be shown that B^* is certain to be non-empty if each individual in group I is indifferent about the $\xi_z(b)$ elements in each set $\xi(b)$ where $b \in K$. Of course, this last condition is automatically satisfied if there is only one element in each of the $\xi(b)$ sets. If any of the individuals in group I are not indifferent about the elements in each of the $\xi(b)$ sets then B^* can be empty. But even if B^* is not empty, e.g. because there is just one element in each of the $\xi(b)$ sets, H may be empty. However, it will not be empty if there is just one set of possible alternatives. Indeed, if this is so and B^* is not empty, then $B^* \cap H$ is non-empty.

Let P represent the set of b values which can never be Pareto optimal for group I. Then P will be empty or non-empty depending upon the particular circumstances of the case. The group I is faced by the following position: it will be certain <u>not</u> to achieve Pareto optimality if it selects a b value in the set P; it will be certain to obtain Pareto optimality if it selects a b value in the set $B^* \cap H$; and if it selects any other value in the set K, i.e., any element in the set $K - (B^* \cap H \cup P)$, it will only have a chance <u>between</u> zero and unity of realizing Pareto optimality. While either the set $B^* \cap H$ or the set $D = K - (B^* \cap H \cup P)$ may be empty both cannot be simultaneously empty. If both these sets are empty, it is implied that all elements of K belong to set P since $K = P \cup D \cup (B \cap H)$. This would imply that every possible course of action, i.e., every $(b, \xi_z(b))$ combination, is less preferred than another possible one in the set K and so would involve a logical contradiction.

Now, there are a number of possible ways in which the above construction can be interpreted. What can the above sets represent? One interpretation is that they refer to some theoretical objective estimate which could supposedly be derived given what is known at the beginning of period t. By assumption, whatever actually occurs is always consistent with the objective estimate. A grave difficulty for this interpretation is that there may be unique events in very general social systems and these may render it impossible to discover some sets of theoretical possibilities. Unfortunately, the theoretical objective set cannot necessarily be established by a consensus of opinion. But even if we could approximate the objective set in particular instances, I would not be surprised to find that K is frequently comprised of more than one set of alternatives and that there is often more than one $\xi_z(b)$ value associated with each b value. Also, given our assumptions about each of the U^r orderings, I should not be surprised to find that $B^* \cap H$ is often objectively empty. If this does happen to the case, it clearly limits the applicability of all economic theories (in particular some welfare theories) which are based upon the implicit assumption that $B^* \cap H$ is never empty. Again, we can re-interpret our construction so as to relate it to a subjective domain. Even if individuals in group I were to reach a consensus as to what they think is possible, then they may perceive a number of possible sets in K and $B^* \cap H$ may be empty within this subjective domain. Our conceptions may be of interest in both the subjective and objective spheres.

Now, group I is most unlikely to decide upon an inflexible course of action for its whole future for to do so would be to ignore information which may come to hand in the future. It may only decide upon an inflexible joint action for period t and may leave its later courses of action (more or less) open. It is then interesting to pose the question of what are the conditions under which group I can choose (an inflexible or single path) course of action for t and be certain that this chosen course of action is necessary for its Pareto optimality.

Let $B_t = \{B_{t1}, B_{t2}, \ldots, B_{tq}\}$ represent the set of the sets of alternative courses of action which may be open to group I during period t. We represent each of these sets $B_{t1}, B_{t2}, \ldots, B_{tq}$, as

$$B_{ts} = \{\beta_{s1}, \beta_{s2}, \ldots, \beta_{s,p(q)}\}$$

where $s = 1, \ldots, q$ and each element, such as β_{11}, represents one course of action which may be available to group I for period t. Let

$$S = \sum_{s=1}^{q} B_{ts}$$

Then, S represents the set of all courses of action which may be open to group I for period t. β_t represents any element of S and is one course of action which may be available to group I for period t. Let

$$Z = \prod_{s=1}^{q} B_{ts}$$

Z represents the set of all courses of action for period t which are certain to be available for group I's choice. There is associated with each element, β_t, which is contained in set S, a set

$$\theta(\beta_t) = \{\theta_1(\beta_t), \theta_2(\beta_t), \ldots, \theta_{m(\beta_t)}(\beta_t)\}$$

where any element, such as $\theta_1(\beta_t)$, represents a course of action other than β_t which may possibly be pursued by group I and $\bar{I}$ given β_t. The set $\theta(\beta_t)$ represents all courses of action which may possibly be pursued by group I and $\bar{I}$ given β_t. Let $\theta_g(\beta_t)$ represent any element of $\theta(\beta_t)$. Then only if there is at least one value of β_t, say β^*_t, such that

$$[U^r(\beta^*_t, \theta_g(\beta^*_t))] \nless [U^r(\beta_t, \theta(\beta_t))] \tag{3}$$

where $r = 1, \ldots, k$ and where $\theta_g(\beta^*_t)$ can assume the value of every element in $\theta(\beta^*_t)$ and $(\beta_t, \theta_g(\beta_t))$ can assume every value for every possible β_t value except β^*_t <u>and</u> such that

$$\beta^*_t \in Z , \tag{4}$$

does there exist at least one β_t which must be consistent with Pareto optimality for group I.

Let β^* represent the set of all β_t values which satisfy condition (3). If $\beta^* \cap Z$ is empty, then there is no β_t which is certain to be consistent with Pareto optimality for group. If it is non-empty then there is at least one β_t value which is certainly consistent with Pareto optimality for group I. It is clear that the conclusions which we reached for the previous construction apply to this one, mutatis mutandis. For example, $\beta^* \cap Z$ is empty if either β^* or Z is empty, or if each individual in group I is indifferent about the elements in each of the sets $\theta(\beta_t)$ and if there is only one set in the set S, then $\beta^* \cap Z$ is non-empty. I shall not bother the reader with further details upon this point but it should be noted that this construction permits of both objective and subjective interpretations in a similar way to the last one and for both these interpretations, I speculate that $\beta^* \cap Z$ is often empty.

In the model just considered, period t is assumed to be of some fixed but undefined length. For the period, group I is assumed to pursue an inflexible course of action. Its inflexibility may be the result of its own decision or it may arise because its knowledge does not change during period t, or if it does change, group I may be technically "unable" to vary its course of action of that period so as to make allowance for it. We are challenged by the fact that in some circumstances the degree of flexibility is a consciously imposed restriction. It is itself often the result of a decision and that decision will in part depend upon expectations about the availability of knowledge in the future. Until we obtain a clearer understanding of these flexibility decisions and relationships, many of our constructions of group behavior and group behavioral possibilities will be insubstantial. Once we recognize that knowledge is not a fixed thing and realize that man is not completely fixed in a single path which he must follow, we can hardly fail to see that the flexibility of (group) decisions is of great significance. Although I am not going to deal with them in this essay, flexibility questions deserve more than a perfunctory treatment.

Even if the members of group I form a closed group, i.e., a group in which the members may reach joint agreements among themselves but will not attempt to reach agreements with individuals outside the group, and if $\beta^* \cap Z$ is objectively non-empty, members of the group may not act to attain Pareto optimality for themselves because the factors mentioned for the society case at the beginning of this essay may operate to rule out a Pareto optimal "agreement". Of course, even if $\beta^* \cap Z$ is objectively non-empty, members may not perceive this or their perceptions may diverge to such an extent that they are unable to agree upon its non-emptiness and upon other facts. They may be unable to reach a consensus or to do so would be costly. This raises the question of what effect communication problems have upon the likelihood of Pareto optimality.

While I do not intend at this stage to get involved in a detailed discussion of communication questions, it might be noted that when an individual desires a change in social structure (i.e., in the probable actions of a group of individuals), and this could be a change in which all relevant individuals are willing to agree, he may fail to communicate about it, (i) because the cost to him of the communications may offset his probable gains to such an extent as to deter him or (ii) because, even if costs do not offset to this extent, the individual may feel that it is very probable that some other individual or individuals will initiate communication and bear the major burden or major part of the costs involved in communicating about the change. In the latter case, the individual fails to communicate because he is prepared to take a risk upon the likelihood that some other individual or individuals will initiate and foster communication about the desired changes. If every individual in the group is either in circumstance (i) or (ii), a desired change may fail to materialize. In case (i), a straightforward externality barrier arises, in the second case externality plus a chance element is involved. We can obviously extend these thoughts so that they apply to sub-groups rather than to individuals.

There are, as we have noted, other risks tied up with the communication of preferences. Professors Arrow[5] and Morgenstern[6] have pointed out that it is sometimes to an individual's advantage to give a distorted impression of his preferences rather than to reveal his "actual" ones. If the individual reveals his actual preferences either his gains may be less as a result of an imposed system of allocation which varies action with expressed preferences or he may give his "opponents" strategic information. It should also be observed that an individual will not always be aware of his "best" or "approximately best" distortion. Since the individual must often distort or reveal his preferences with some uncertainty, he may fail to reveal his actual preferences when that revelation would be to his advantage. As noted, a whole group may fail to improve its situation because of this problem.

Professor Morgenstern[7] has indicated that this distortion factor raises some serious problems for the testability of theories based upon the Pareto optimum hypothesis since individuals may not consider an interviewer or observer neutrally. The reaction of the individual will be conditioned by his perception of what the interviewer or observer is likely to do with the material. Is the observer going to try and induce individuals in the economy to act differently on the basis of the material or is it likely to come into the hands of someone who may act differently given the preference material? (One sees an analogy between some problems here and some quantum physics problems.) Also, if information upon the preferences of an individual is sought, one must be aware of the fact that time and effort is involved in expressing them. If the individual, who is the subject of the research, has preferences about how he wishes to spend his time, then if he does co-operate in expressing them, to say a "neutral" observer, his expression of them may only be perfunctory or may be designed to satisfy the observer in as short a time as possible. No doubt, there are limits to the time which any of us will spend on this matter. Indeed, to express our complete ordering over all of society's possibilities would involve a great deal of time and I, for one, would not care to co-operate. On the other hand, the individual may be prepared to express his preferences upon a few possibilities and theories need to be designed so as to take some account of this. In line with this, I would also suggest that an individual often has a limited vision of his own preferences. To visualize them completely and consistently involves effort (e.g. a search for possibilities and the effort of consistently ordering them) and the individual is obliged to reach some "decision" upon this matter. Since we know so little about this phenomenon, it is hazardous to venture an opinion as to whether it places practical limitations upon the Pareto optimality approach. This matter cannot be decided either way by abstract thought alone.

Again, an individual's expressed preference ordering may be incomplete since one may be expecting him to order preferentially some alternatives which he is not genuinely in a position to compare. Our knowledge of our own preferences depends upon our previous experience. As between two possibilities, one of which he has never experienced, an individual may not know which one he perfers. I believe that this situation is more common than is realized and becomes very important when we are considering preferences for large systems, e.g. those for a society, country or large collection of individuals. Of course, even within our framework of limited knowledge, there may be some consistency; one may not know whether one prefers A to B but yet may be sure that A or B is preferred to C. Hence, upon the rationalist approach, one should never choice C in preference to A or B. Yet on the basis of his limited experience, the individual may venture to give a complete preference ordering - he may, for instance, suggest "his most probable" ordering.

This brings us face to face with the metaphysical problem of whether an individual always perceives his true preferences or the actual satisfaction which he would derive from his alternatives. I believe that it can be safely conjectured that an individual because of his limited experience will not always know his "true" preferences and in some circumstances neither will nor can discover them. In his life-time, an individual and his society only has one path which he and it _will_ follow and he cannot experience all the paths which might have been open to him and his society. It seems impossible to escape doubt about the closeness which an individual's perceived preference ordering bears to his "true" one. Consequently, if an individual has or may have a distorted view of his "true" preferences, this raises an obstacle to the testing of an objective Pareto optimality theory. Hence, it seems, that if we press the Pareto optimality theory towards its logical limit, we reach the disconcenting point of view that there are a number of circumstances in which it is not testable, not realistic and does not yield substantial economies in thought or in its application.

To conclude: Although problems of co-operation can create barriers to Pareto optimality for a group, the elimination of such barriers does not ensure Pareto optimality. No matter how avidly a group may seek to co-operate so as to _ensure_ Pareto optimality for itself, the realization of this goal may be impossible for no one may know which course of action will be Pareto optimal. In circumstances where this is so, any one who advises a group to act Pareto optimally is requiring them to perform miraculous deeds. There are some welfare and normative economic, game and social theories which do require some miraculous transformations of the social universe before they can be applied and which ought to be treated with great skepticism. But besides being skeptical, we need to, and can, develop theories which make allowance for the limitations of man's knowledge. Although one may employ a limited Pareto optimum concept in developing such theories, and in this case one should specifically clarify its limited nature, I cannot, in view of the problems which have been raised, escape the conclusion that we ought to examine and search for other bases of social behavior.

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FOOTNOTES

1. I wish to thank Mr. D. Butt and Professor T. W. Swan of the Australian National University and Professors W. J. Baumol and O. Morgenstern of Princeton University for commenting upon this and related topics.

2. Professor Morgenstern has recently published a thought provoking article upon this subject. See O. Morgenstern, "Pareto Optimum and Economic Organization", pp. 573-586 in Systeme und Methoden in den Wirtschafts- und Sozialwissenschaften, N. Kloten, W. Krelle, H. Müller and F. Neumark (eds.), J.C.B. Mohr, Tübingen, 1964. Other contributions are, however numerous. Since I do not wish to compile a long bibliography on this subject, let me mention just two other interesting articles. They are: H. A. Simon, "A Behavioral Model of Rational Choice", Quarterly Journal of Economics, Vol. 79, 1955, pp. 99-118, and M. Shubik, "Information, Theories of Competition and the Theory of Games", Journal of Political Economy, Vol. 60, 1952, pp. 145-150.

3. By the Pareto optimality hypothesis of group behavior, I mean the postulate that a group will organize itself and act so that it will be impossible to make any member of the group better off without making another member worse off.

4. Sometimes, a bargainer is disappointed that he did not yield earlier. It is because he miscalculates "the extent to which he can press his advantage" that Pareto optimality sometimes fails to occur.

5. K. J. Arrow, *Social Choice and Individual Values*, 2nd Edn., John Wiley and Sons, New York, 1963.

6. O. Morgenstern, *ibid*.

7. *Op. cit*.