

ANOTHER VIEW OF
COMMENTS ON ADELMAN'S

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Research Memorandum
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ABSTRACT

Adelman's recent study, "Long Cycles - Fact or Artifact?", represents an attempt to apply spectral analysis to time series data in order to ascertain the relative contribution of a long-swing component to the historical time path of economic activity in the United States. While the spectral analytic technique may be appropriate for such a study, Adelman's use of this technique leaves much to be desired. This paper has been prepared in an attempt to point out some of the subtleties involved in the application of spectral analysis and to suggest that Adelman's rejection of the long-swing hypothesis is not well-founded. Although the paper is concerned primarily with the long-swing hypothesis, the comments on hypothesis testing and filtering techniques are applicable to a wider class of problems and illustrate the flexibility of the estimation technique.

With respect to Adelman's paper, the basic conclusion which emerges is that the estimates presented by Adelman contain virtually no information about the long swing. In an attempt to rectify this deficiency, two alternative estimation procedures are introduced. The alternative estimates which are obtained suggest that the case for the long-swing hypothesis is mixed; some of the evidence examined being favorable and some unfavorable. Without further analysis it is impossible to formulate a definite conclusion about the existence of long cycles in the historical pattern of development of the United States economy.

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ANOTHER VIEW OF THE LONG SWING:

COMMENTS ON ADELMAN'S STUDY OF LONG CYCLES*

I. INTRODUCTION

Professor Irma Adelman's recent investigation [2] of the existence of long swings is one of an increasing number of economic studies in which spectral analysis has been used. We agree with her that the statistical methods used in earlier studies might produce fictitious long cycles and that spectral analysis may be appropriate to investigate the importance of long cycles. However, the application of spectral analysis involves a number of subtle points, and the technique can very easily be misused. It is the purpose of this paper to indicate several points at which it would appear that Adelman's analysis is questionable.

The basic questions which we would like to raise in connection with Adelman's study are concerned with the formulation of the long-swing hypothesis in terms of the spectrum and with the interpretation of the spectral estimates presented in her paper. It is argued in Section II that Adelman has not adequately specified a criterion according to which it is possible to determine whether long swings "exist". We offer several alternative criteria which are appropriate for a spectral analytic test for the existence of long swings. In Section III we argue that Adelman's interpretation of the spectral estimates is inappropriate and that her estimates, properly interpreted, provide virtually no information about the long-swing hypothesis. In Section IV we present our own estimates based on the data used in Adelman's study and our tentative conclusion as to the importance of the long swing.

*

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II. FORMULATION OF THE LONG-SWING HYPOTHESIS

The long-swing hypothesis has been formulated alternatively in terms of the levels, deviations from trend, and rates of growth of an economic variable. In each of these forms the hypothesis is concerned with the existence of a class of fluctuations which are longer in duration than the business cycle but of a shorter-term nature than the secular trend [10, Chapter 2]. Following Adelman, we shall confine our discussion to the deviation-from-trend variant of the hypothesis. In formulating the long-swing hypothesis in such a way that it is amenable to spectral analysis, there are at least two important specifications to be made: (1) the frequency band which contains the long swing, and (2) the criteria which may be used to judge the relative importance of the long-swing frequency band.

A tentative specification of the long-swing frequency band may be made by referring to the lower and upper limits of the duration of the long swing which have been suggested by previous investigators. Following Abramovitz [1, p. 412], Adelman apparently assumed 10 years and 20 years as the lower and upper limits. It might be argued that the duration of the long swing should not be as short as 10 years on the ground that 10 years is nearly the average duration of the so-called major cycle. This argument will not be emphasized in this paper. Instead, we would like to call attention to the literature in which movements of duration longer than 20 years are considered to be long swings. For example, Kuznets [10, chapter 7 and p. 423] seems to include movements of longer duration in the long swing, taking 20 years as the average duration rather than the maximum duration.

Taking 10 years as the lower limit to the long-swing duration, we shall adopt 1/10 cycle per year (abbreviated as c/y) as the upper limit of the long-swing frequency band. A determination of the lower limit of the frequency

band involves a number of subtle points, and any choice would be somewhat arbitrary. The basic motivation underlying an attempt to divide long-term movements into trend and long swings suggests that the upper limit of the frequency band which contains the trend should be taken as the lower limit of the long-swing frequency band. Although the spectral density is strictly defined only for a stationary stochastic process, the frequency representation of a trend can be analyzed by using the pseudo-spectrum [7, especially section 6]. The pseudo-spectrum of a linear trend is concentrated in the very low frequencies [5, Figure 8.1, p. 131], but a precise specification of the frequency band which contains the trend is to a certain extent arbitrary. One might take $1/n$ c/y as the upper limit of the frequency band for trend, where n is the number of years covered by the data, since the band of frequencies $[0, 1/n]$ c/y contains most of the power contributed by the trend.

It might be argued, however, that $1/n$ c/y is too low for the lower limit of the long-swing frequency band. When time series data run over 60-80 years, as most of the data available for a test of the long-swing hypothesis do, $1/60$ or $1/80$ c/y as the lower limit of the long-swing frequency band means 60 or 80 years as the upper limit of the duration of the long-swing, and this upper limit might be too long.¹ Yet a crucially important point of disagreement between Adelman and the authors is that we feel very strongly that the lower limit cannot be as high as $1/20$ c/y because it excludes Kuznets' long cycles from the study. These considerations suggest a tentative specification of the long-swing band as the interval between $1/10$ c/y on the one hand and $1/30$ to $1/40$ c/y on the other.

It is now necessary to consider in terms of spectral analysis what is meant by the "existence" of a cycle in a given frequency band. Adelman

¹ However, Kuznets [11, p. 25] does not think so.

nowhere explicitly stated the criterion of "existence" which she used.² In one sentence which could be interpreted as a presentation of an existence criterion [2, p.453], Adelman appears to suggest what might be called a one-sided test which involves a comparison of the power contributed by the long-swing band with the contribution of higher frequencies. It does not appear to us, however, that she used this criterion to derive her conclusions, because her conclusions, if based on this existence criterion, are not entirely consistent with the estimates. In another sentence Adelman [2, p.450] appears to suggest a wholly inadequate criterion, namely, that a cycle of a given frequency will be said to exist if the amplitude of a sinusoidal component of this frequency is different from zero and hence the component contributes something to the overall variance of the series. That this criterion is inadequate may be seen by considering the spectrum of a sequence of independent random variables (white noise) which is a positive constant over the entire frequency axis. Most economists would agree that no cycles exist in an independent random series, but a strict adherence to this existence criterion leads to the statement that cycles of all durations exist.

Several criteria, which may be used to determine whether a band of frequencies $[f_1, f_2]$ is important, immediately suggest themselves.³ First, cycles of frequencies $[f_1, f_2]$ can be said to exist if the spectrum exhibits a relative peak in this frequency band. If no relative peak exists in the band, then we reject the hypothesis that the series contains an important cyclical component in this frequency band. A test based on this existence criterion might be called a two-sided test since it involves a comparison of the average

² In private correspondence with one of the authors Adelman indicated that she used the criterion which we shall refer to below as a two-sided test. Our repeated readings of her paper failed to uncover any statements expressing this criterion.

³ The notation $[f_1, f_2]$ is used to denote the band of frequencies the lower and upper limits of which are f_1 c/y and f_2 c/y, respectively.

power contributed by the band $[f_1, f_2]$ with the contribution of contiguous bands on either side of the given band. Second, in the particular case of long swings, Kuznets seems to place great importance on the condition that the long-term movements, if meaningful, should not be swamped by the short-term movements [10, chapter 2]. This condition for the "existence" of long swings may be interpreted to mean either (1) that the variance contribution of frequencies centered on the long swing, $[f_l \pm \delta f]$, should exceed the contribution of any higher frequency band of equal length, or (2) that the contribution of $[f_l \pm \delta f]$ should exceed the contribution of $[f_l \pm \delta f, 1/2]$ to the variance of the series. Both of the variants of this criterion involve a comparison of the average contribution of the long-swing band with the contribution of higher frequency bands and will therefore be referred to as one-sided tests. With respect to the first variant of the one-sided test, it seems reasonable to emphasize a comparison of the contribution of the long-swing band with that band of frequencies which contains the three-to four-year business cycle. In the rest of the present paper the first version of the one-sided test will be referred to simply as the one-sided test.

Execution of tests based on these criteria with the use of statistical data requires a use of estimates of spectral densities. The fact that the spectral estimates are themselves random variables suggests that the existence criterion should be formulated in terms of sampling theory. The sampling theory of spectral estimates suggests that the distribution of an estimate can be approximated by a χ^2 distribution. Since the approximate distribution of the spectral estimates relies on near-normality of the underlying stochastic process, the significance tests should be taken as rough guides only. It should be emphasized that a test based on any of the above criteria involves a significance test of the difference between two estimates of the spectral density.

In connection with spectral estimation and hypothesis testing, the "outlier problem" might be mentioned. It has been suggested that methods used to eliminate short-run fluctuations (business cycles) from time series may well be responsible for the observed regularities in the adjusted series. The reason for this, it has been conjectured, is that while the smoothing technique reduces the amplitude of short-run fluctuations, the procedure smooths the extreme values, or outliers, which are contained in the original series.⁴ The question thus arises as to what adjustment, if any, should be made for outliers which are associated with severe depressions or vigorous booms. Although smoothing devices intended to reduce short-run fluctuations are not used in the estimation of the spectrum, the averaging of outliers does occur in the estimation of the autocorrelation function which is then Fourier transformed to obtain the spectral density function. For this reason the outlier problem is not eliminated by the use of the spectral analytic technique but rather emerges in a somewhat different form.

The question of whether or not outliers should be adjusted can only be made after having specified the way in which the extreme values have been generated and the way in which they influence the future course of events. If the extreme values are assumed to have been generated endogenously, then no special adjustment is required. Indeed, the use of the word outlier is a misnomer in this instance. On the other hand, if the outliers are exogenously generated, it is necessary to specify the way in which the outlier influences the future course of events. For example, if the future time path of the system is independent of the outlier, then the problem of adjustment is considerably different from the case in which the system responds to the outlier in a causal way. These comments

⁴ Kuznets [10, pp. 361-65] introduces this argument and then proceeds to defend the long-swing hypothesis against this criticism. Fels [4] applies a somewhat similar argument to the postwar period.

should be sufficient to indicate that only through model building will it be possible to deal effectively with the outlier problem. Since this is beyond the scope of the present paper, it is merely mentioned at this point.

III. THE IMPLICATIONS OF ADELMAN'S ESTIMATES

In an attempt to test the validity of the long-swing hypothesis, Adelman analyzed a number of series covering the period 1889-1957. She concludes that "the filtered spectra ... offer no evidence for the existence of a long cycle component in the business fluctuations of the U.S. economy since 1890" [2, p.459]. That this conclusion inevitably follows from the estimates of the power spectra is far from obvious. In this section the logic underlying Adelman's rejection of the long-swing hypothesis is critically examined. In order to focus attention on the estimation procedure employed by Adelman, only her rather narrow specification of the long-swing frequency band, namely $[1/18, 1/10]$ c/y, is considered. The discussion in the present section provides the motivation for the alternative estimates presented in the following section.

Adelman's rejection of the long-swing hypothesis is based on the estimated spectra of series which have been filtered in such a way as to eliminate or greatly attenuate the power below $1/18$ c/y. Before considering the implications of the filtered estimates, it is necessary to consider in some detail the rationale which underlies the filtering process. This can best be accomplished by referring to Figure 3.1 in which an estimate of the spectrum of the

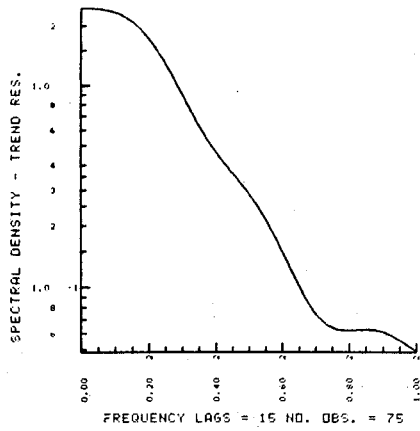


FIGURE 3.1
Spectral Density of GNP,
Truncation Point = 15.

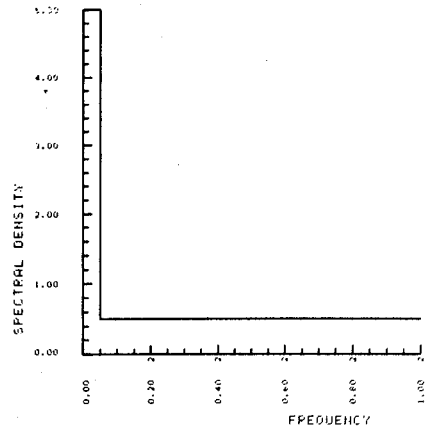


FIGURE 3.2
Spike at the Origin.

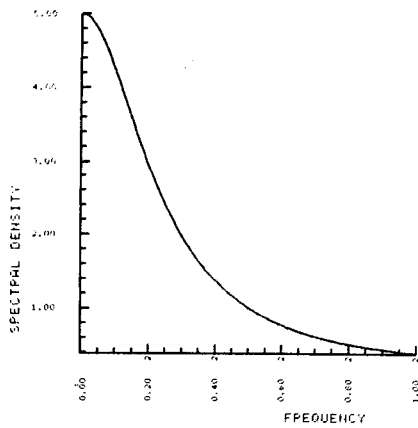


FIGURE 3.3
Typical Spectral Shape.

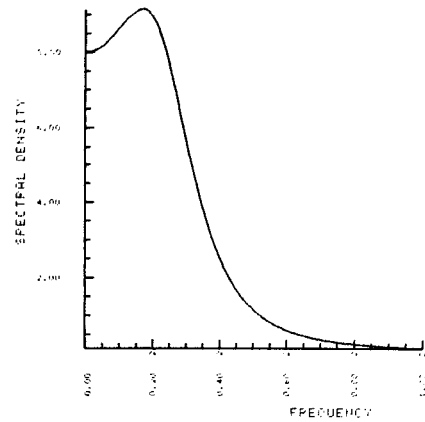


FIGURE 3.4
Long-Swing Spectrum.

residuals from a log-linear trend of constant dollar Gross National Product is shown.^{5,6} From this estimate alone it is impossible to identify unambiguously the "true" spectrum of the underlying generating process. Any one of at least three distinct generating processes could have given rise to the estimate shown in Figure 3.1. The spectra of these alternative processes which are consistent with the estimated spectrum include⁷ (1) the spectrum which is constant except for a sharp spike in the interval $[0, 1/n]$ c/y (Figure 3.2), in which case the long-swing hypothesis should be rejected; (2) the typical spectral shape⁸ (Figure 3.3), in which case the long-swing hypothesis should be rejected on the basis of a two-sided test but not rejected on the basis of a one-sided test; and (3) the long-swing spectrum (Figure 3.4) in which a relative peak emerges in the interval $[1/18, 1/10]$ c/y,

⁵ This estimate differs from the unfiltered spectral estimate shown in Adelman's Figure 2 only in that it was normalized by dividing by the estimated variance of the series and was estimated at the points $j/300$ c/y ($j = 0, 1, \dots, 100$) whereas Adelman's spectral estimate was obtained at the points $j/2m$ c/y ($j = 0, 1, \dots, m$) where m denotes the truncation point employed in the calculation of the autocovariance function. These differences are not particularly important at this point.

⁶ In all of the following figures the horizontal (frequency) axis is plotted on a linear scale with $1/300$ c/y as the unit of measurement. The vertical axis is plotted on a log scale when estimates of the spectral density are shown and on a linear scale otherwise. The numbers on the axes are of the following form: one digit before and two digits after the decimal point followed by another digit which may be preceded by a minus sign. This last digit is the exponent of ten which determines the actual value of the number. For example the number $0.60\ 2$ should be read as 0.60×10^2 or 60.

⁷ These alternative specifications of the true spectrum are meant to apply only to the interval $[0, 1/10]$ c/y and not to the entire frequency axis. Specifically, the business-cycle frequency band, $[1/10, 1/2]$ c/y, is not being considered here.

⁸ A number of investigators using the technique of spectral analysis have suggested that the general contour of the estimates shown in Figure 3.3 is typical of a large number of economic variables, and Granger [6] has called this the "typical spectral shape". However, the small peak centered on the business cycle frequency is not considered to be a part of the typical spectral shape.

in which case the long-swing hypothesis should not be rejected on the basis of either a one-sided or two-sided test.⁹

The inability of the spectral estimates shown in Figure 3.1 to discriminate among the alternative spectra shown in Figures 3.2 - 3.4 arises as a result of the procedure by which the estimate is obtained. With a finite number of observations it is possible to estimate only the average power contained in a set of frequency bands of finite width [3].¹⁰ The estimates obtained from a finite realization of a process can be thought of as estimates of a weighted average of the true spectrum. The weighting function which is used in averaging the true spectrum is called the spectral window, the term having been chosen to suggest that the estimate is obtained from the true spectrum by "looking" at it through this window. The spectral window, centered on 0, 1/30, 2/30, and 3/30 c/y, which was used to estimate the spectrum of the GNP series is shown in Figure 3.5. It will be noted that the window emphasizes the power contributed by the frequency on which the window is centered and suppresses the contribution of frequencies distant from the center frequency.

⁹ A fourth possibility might be mentioned at this point, namely, the spectrum which exhibits a relative peak in the interval $[1/30, 1/18]$ c/y. With Adelman's definition of the long-swing frequency band this spectrum is not consistent with the long-swing hypothesis (at least on the basis of the two-sided test), but is consistent with the hypothesis when a less restrictive specification of the long-swing band such as that advanced in the previous section is employed. The estimates presented in the following section are relevant for this wider definition of the long-swing frequency band.

¹⁰ An analogy to this averaging is provided by the problem of estimating a continuous probability density function from a finite sample, in which a histogram is often used.

It is now apparent why the spectral estimate shown in Figure 3.1 is incapable of discriminating among the three alternative spectra enumerated above. By looking at any one of the hypothetical spectra of Figures 3.2 - 3.4 through the spectral window of Figure 3.5, the spectrum of Figure 3.1 could be obtained. The resolution of the spectral window is simply not fine enough to identify unambiguously the true spectrum of the generating process. A natural procedure to discriminate among the different possibilities is to use a more sharply focussed spectral window. This is explored in the following section.

It is this problem of identifying the true spectrum which Adelman hoped to surmount by introducing a filter which rejects, at least partially, the power below $1/18$ c/y which is contained in the series. What is never made entirely clear is the way in which this particular filter is supposed to resolve the problem.¹¹ In view of the fact that after filtering the series it is possible to recover at most only that portion of the spectrum above $1/15$ c/y,¹² the information about the long-swing hypothesis contained in the filtered spectrum is certainly limited. Adelman suggests that the filter is appropriate for testing the hypothesis that the power contained in the frequency band centered on $1/15$ c/y is due primarily to a concentration of power in the interval $[0, 1/18]$ c/y. Perhaps this hypothesis is meant to identify the true spectrum as that shown in Figure 3.2 which is, of course,

¹¹ Adelman frequently refers to the "leakage" problem, but it appears to us that what concerns her is what might be called "smudging" which results from the smoothing within the main lobe of the spectral window rather than leakage through the side lobes of the window.

¹² The reason for this is that at frequencies lower than $1/15$ c/y the frequency response (or gain) of the filter is changing more rapidly than the spectral window so that the estimates are not well-behaved in the low-frequency end of the spectrum. It is shown below that even the estimate centered on $1/15$ c/y is misleading.

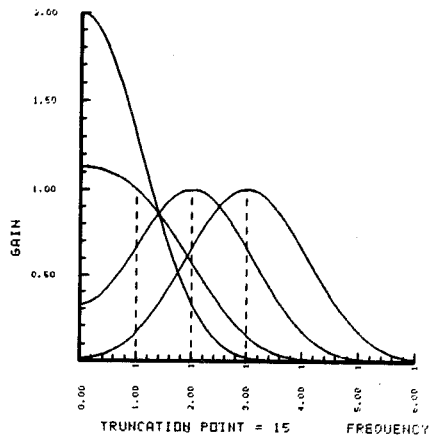


FIGURE 3.5

Normalized Windows Centered on 0,
1/30, 2/30, and 3/30 c/y.

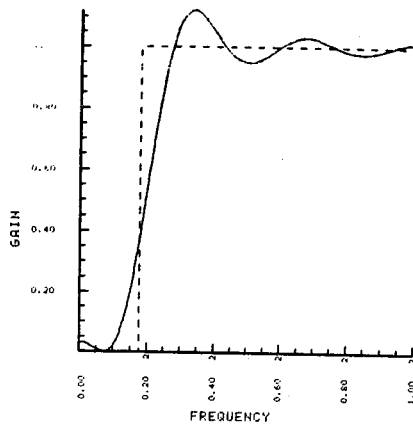


FIGURE 3.6

Gain of Low-Reject Filter.

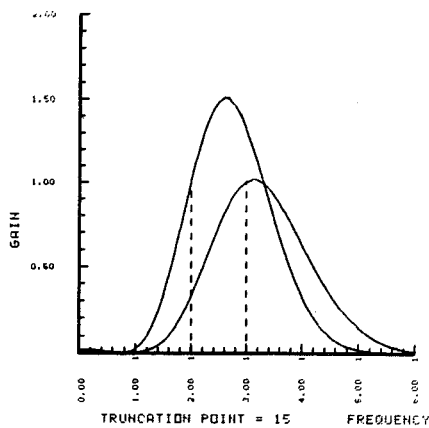


FIGURE 3.7

Recolored Windows Centered on 2/30
and 3/30 c/y.

inconsistent with the long-swing hypothesis. But the recolored estimates presented by Adelman are not appropriate for a test of this hypothesis for the simple reason that they, too, are unable to discriminate among the alternative hypothesis of Figures 3.2 - 3.4.

If the spike-at-the origin hypothesis were true, one would expect the recolored estimate centered on 1/15 c/y to be less than the unfiltered estimate. The unfiltered and recolored estimates for Gross National Product centered on 1/15 c/y and 1/10 c/y set out in Table 3.1 indicate that this is precisely the case. The unfiltered estimates, denoted by $\hat{S}(f)$ in the table, exceed the

TABLE 3.1

Unfiltered and Recolored Estimates of the Spectral
Density of Gross National Product

Number of Lags	15		20		25		30		
	f	$\hat{S}(f)$	$\tilde{S}(f)$	f	$\hat{S}(f)$	$\tilde{S}(f)$	f	$\hat{S}(f)$	$\tilde{S}(f)$
f	1/15	1/10	1/15	1/10	1/15	1/10	1/15	1/10	
$\hat{S}(f)$	1.74	0.92	1.82	0.73	1.85	0.59	1.86	0.49	
$\tilde{S}(f)$	1.00	0.48	1.16	0.47	1.29	0.43	1.41	0.40	

recolored estimates, $\tilde{S}(f)$, in all cases. However, it is not valid to conclude from this, as does Adelman, that the observed concentration of power around 1/15 c/y is the result of smoothing a "true" spectrum which is constant except for a sharp spike near the origin. The reason for this is that the reduction in power which results from the filtering-recoloring process is exactly what one would expect if the "true" spectrum were of any one of three alternative shapes shown in Figures 3.2 - 3.4. The recolored estimates shown in Table 3.1 are subject to

another difficulty which derives from the particular low-reject filter that Adelman used in her analysis. Specifically, the recolored estimates centered on 1/15 c/y are misleading in the sense that they do not adequately reflect the power contributed by this frequency band.

The inadequacy of Adelman's filter, the gain of which is shown in Figure 3.6, can best be illustrated by considering what might be called the recolored spectral window. The recolored window is obtained by combining the spectral window (Figure 3.5) and the gain of the low-reject filter (Figure 3.6). Since the spectrum of the filtered series is the product of the original spectrum and the gain of the filter, the filtering process can be thought of as changing the shape of the spectral window used in the estimation of the spectrum. After filtering the series it is as if the true spectrum is visible through a double window, one window corresponding to the gain of the filter and the other to the spectral window.¹³ The recolored windows centered on 1/15 and 1/10 c/y are

¹³ A more rigorous development of the relationship between the recolored spectral window and the regular window might be helpful to the mathematically inclined reader. Let $\{x(t)\}$ denote the original series and $\{y(t)\}$ denote the filtered series. Then

$$(1) \quad S_y(f) = G(f) S_x(f)$$

where $S_x(f)$, $S_y(f)$ are the spectra of the original and filtered series, respectively, and $G(f)$ denotes the gain of the filter. The estimates of the spectra of $\{x(t)\}$, $\hat{S}_x(f)$, and $\{y(t)\}$, $\hat{S}_y(f)$, are given (approximately) by

$$(2) \quad \hat{S}_x(f_0) \approx \int_0^{1/2} S_x(f) K(f_0, f) df \quad 0 \leq f_0 \leq 1/2$$

and

$$(3) \quad \hat{S}_y(f_0) \approx \int_0^{1/2} S_y(f) K(f_0, f) df \quad 0 \leq f_0 \leq 1/2$$

where $K(f_0, f)$ denotes the spectral window centered on f_0 . These expressions indicate that the estimate is obtained by smoothing the true spectrum as

shown in Figure 3.7. Although the recolored window centered on $1/15$ c/y more sharply suppresses the low-frequency contribution to the spectrum than does the regular spectral window, it is not well-behaved in the sense that the contribution of the band $[1/15, 1/10]$ c/y is accentuated. As a result of the distortion introduced by the filtering process and reflected in the recolored window, the estimate centered on $1/15$ c/y is virtually meaningless. Since only the estimates centered on frequencies higher than $1/15$ c/y are reliable after the filtering, the information about the long swing contained in the recolored estimates is negligible.

It is reasonable to conclude from this discussion that neither the unfiltered nor filtered estimates presented in Adelman's study of long cycles are useful for a test of the long-swing hypothesis when it is formulated in terms of residuals from a log-linear trend. More specifically, the estimates themselves provide no basis for a rejection of the long-swing hypothesis. It is true, as Adelman intimates by alluding to the possibility of inadequate trend removal, that the true spectrum might consist of a spike at the origin,¹⁴ in which case the long-swing hypothesis should be

discussed in the text. Substitution of (1) into (3) yields

$$(4) \quad \hat{S}_y(f_0) \approx \int_0^{1/2} G(f) S_x(f) K(f_0, f) df \quad 0 \leq f_0 \leq 1/2$$

which Adelman calls the filtered spectrum. The recolored estimates of the spectrum of $\{x(t)\}$, $\tilde{S}_x(f_0)$, is obtained by dividing $\hat{S}_y(f_0)$ by $G(f_0)$, i.e.,

$$(5) \quad \tilde{S}_x(f_0) \approx \int_0^{1/2} S_x(f) \tilde{K}(f_0, f) df \quad 0 \leq f_0 \leq 1/2$$

where the recolored spectral window $\tilde{K}(f_0, f) = G(f) K(f_0, f)/G(f_0)$. This is the sense in which the recolored estimate is obtained by looking through a double window, one given by $G(f)/G(f_0)$ and the other by $K(f_0, f)$. The normalized Parzen window, $K(f_0, f)$, is shown in Figure 3.5, the gain of the low-reject filter, $G(f)$, is shown in Figure 3.6, and the recolored window, $\tilde{K}(f_0, f)$, is shown in Figure 3.7. On all this see Hext [8].

¹⁴ This follows from the discussion in the preceding section of the pseudo-spectrum of a linear trend. It might be pointed out that the concentration of power near the origin is not necessarily the result of inadequate trend removal. It is possible to construct a stationary autoregressive process with a constant mean, the spectrum of which has a peak at the origin. For example, the spectrum of the generating process

rejected. But it can be argued with equal plausibility that the true spectrum exhibits a relative peak in the long-swing frequency band. Both the typical spectral shape (at least on the basis of a one-sided test of the hypothesis) and the long-swing spectrum are consistent with the long-swing hypothesis. Without a more detailed exploration of the power at frequencies lower than $1/10$ c/y, it is therefore impossible to reject the long-swing hypothesis.

IV. ALTERNATIVE TESTS OF THE LONG-SWING HYPOTHESIS

In an attempt to resolve the ambiguity of the estimates presented in Adelman's study, the results derived from two alternative estimation techniques are described in this section. First the implications of estimates obtained by increasing the truncation point used in the estimation formula are considered. Then a filtering experiment is described in which a pair of filters designed specifically for a study of the long-swing hypothesis is employed. The estimates presented in this section are appropriate for the specification of the long-swing frequency band described in Section II, namely, $[1/40, 1/10]$ c/y. Only the results obtained from the series marked with an asterisk in Adelman's paper are presented here.¹⁵

The truncation point. As suggested in the previous section, a natural procedure for discriminating among the alternative hypotheses embodied in Figures 3.2 - 3.4 is to increase the number of lags used in the estimation of the spectrum.

$x_t - a x_{t-1} = \epsilon_t$ ($0 < a < 1$), where ϵ_t is a sequence of independent random variables, is characterized by a peak at the origin [6].

¹⁵ These series include Gross National Product, Number of Manhours Worked for the Production of Real Gross Product, Capital Stock, Output per Unit of Labor Input, Output per Unit of Capital Input, Gross Private Domestic Investment, Total Consumption Expenditure, Wholesale Price Index of all Commodities, and Total Population Residing in the United States. For source references, see Adelman [2, pp. 25-26].

The results obtained by using 20, 24, 28 lags in the estimation of the spectrum of residuals from a log-linear trend fitted by least squares¹⁶ are shown in Figures 4.1 - 4.3 for the Gross National Product series. It is clear from these figures that a relative peak emerges in the interval $[1/30, 1/25]$ c/y as the number of lags used in the estimation of the spectrum of the GNP series is increased. However, this result is not common to all the series under consideration as is evidenced by Table 4.1. In this table the location of the relative peak in the low-frequency end of the estimated spectrum is shown. It will be noted that the relative peaks in the estimated spectrum fall in the long-swing frequency band in the cases of GNP, Gross Private Domestic Investment and the Wholesale Price Index. However, for all the other series the peaks lie below the long-swing frequency band. For these latter series one is led to conclude either (1) that the evidence is unfavorable to the long-swing hypothesis on the basis of a two-sided test; or (2) that the trend elimination is incomplete and therefore no conclusions can be derived on the basis of a two-sided test at this time.¹⁷

Thus far the variance of the spectral estimates has been ignored in connection with the one- and two-sided tests of the long-swing hypothesis. With respect to the one-sided test, the estimates are unequivocally favorable to the long-swing hypothesis even after the variance of the estimates is taken into consideration. The ratio of the spectral estimates centered on the long-swing frequency band to those centered on the business-cycle frequency band range from 10 to 1 to 50 to 1.

¹⁶ The residual series which are used in the estimation of the spectrum are the logarithms of the trend relatives.

¹⁷ In an attempt to explore this latter possibility, the spectra of the residuals from a log-cubic trend were estimated. It was found that a relative peak emerges in the interval $[1/33, 1/15]$ c/y in all cases except Population. Applying the significance tests described below, it was found that the estimate centered on the long-swing frequency is greater than the estimate centered on 0 c/y at the 5 per cent level, again except for the case of Population.

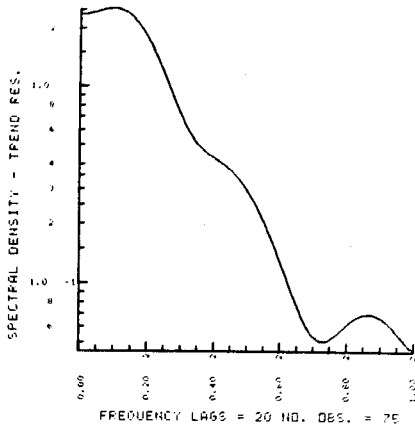


FIGURE 4.1
Spectral Density of GNP,
Truncation Point = 20.

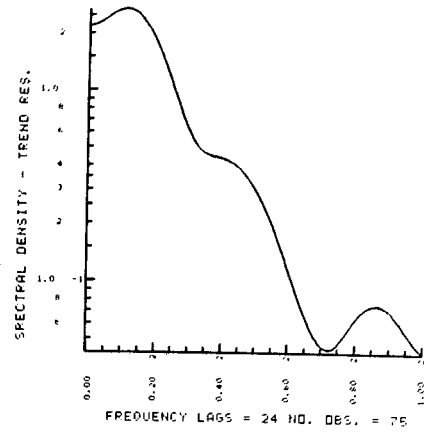


FIGURE 4.2
Spectral Density of GNP,
Truncation Point = 24.

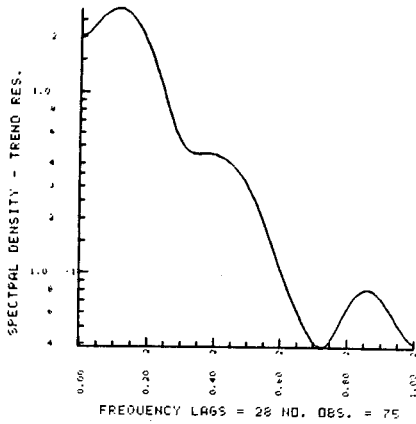


FIGURE 4.3
Spectral Density of GNP,
Truncation Point = 28.

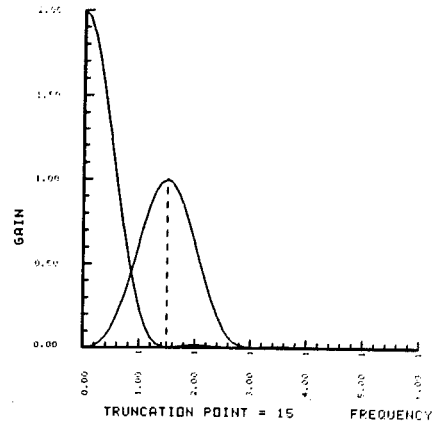


FIGURE 4.4
Recolored Windows Centered
on 0 and 1/20 c/y.

TABLE 4.1

Location of Low-Frequency Peaks in the Estimated
Spectrum

Series	Number of Lags	Peak Frequency (c/y)
1. GNP	20	1/33.3
	24	1/28.5
	28	1/28.5
2. Consumption	20	0
	24	1/66.6
	28	1/40
3. Investment	20	1/33.3
	24	1/33.3
	28	1/33.3
4. Manhours Worked	20	0
	24	0
	28	1/200 - 1/100
5. Capital Stock	20	0
	24	0
	28	0
6. Output per Unit of Labor Input	20	0
	24	0
	28	0 - 1/200
7. Output per Unit of Capital Stock	20	0
	24	0 - 1/66.6
	28	1/50
8. Wholesale Price Index	20	1/50
	24	1/33.3
	28	1/33.3
9. Population	20	0
	24	0
	28	0

What is less clear is the effect of sampling variability upon the verdict in terms of the two-sided test. This test has been conducted simply by locating the relative peaks in the estimated spectrum. Although this method is a direct consequence of our earlier discussion of the two-sided test in Section II, an attempt to locate precisely a peak in the spectrum is not necessarily so meaningful from the statistical point of view. An observed peak in the spectrum may very well be the result of sampling variability of the spectral estimates. Moreover, the spectral estimates centered on neighboring frequencies are correlated, and at the present stage of statistical theory there is no simple way to perform a significance test in such cases. It in fact turns out to be the case that all the estimates in the interval $[0, 1/14]$ c/y are correlated, so that a significance test conducted in the usual way is virtually meaningless.¹⁸ In an attempt to perform an adequate two-sided test, a special pair of filters were considered.

A filtering experiment. In view of the tentative results on the location of peaks in the spectrum it was decided that a comparison of the estimates centered on 0 and $1/(20 + 4\alpha)$ ($\alpha = 0, 1, 2$) c/y would be particularly desirable. Two filters were used to make the spectral estimates centered on these two frequencies as uncorrelated as possible. From the residual series $\{x_t\}$, two filtered

¹⁸ A substantial increase of the truncation point is necessary to achieve the independence between the spectral estimates at zero frequency and the long-swing frequency, f_ℓ . This can be seen by considering the equi-variable bandwidth of the Parzen window (as defined by Jenkins [9]) which is $1/m$ c/y where m denotes the truncation point. Only estimates which are separated by $2/m$ c/y are (almost) independent. A minimum value for m corresponding to f_ℓ may be obtained, namely, $2/m \leq f_\ell$ or $m \geq 2/f_\ell$. For $f_\ell = 1/20$ c/y, m must be greater than or equal to 40. It is felt that the variance of the spectral estimates would be unbearably large with the use of such a large value of m .

Since the two estimates under consideration are more completely separated as a result of the filtering, a significance test has been applied to the ratio of the spectral estimates centered on zero and f_{α} . Although the assumptions underlying the mathematical derivation of the significance test procedure are not exactly satisfied, the significance test still provides some indication of the extent to which the results of the estimation can be trusted. The null hypothesis is that the ratio of the two estimates is equal to one. Two-tailed tests at the 20 per cent and 10 per cent significance levels have been conducted using the F-distribution.¹⁹ The results of the significance test are (1) the deviations of observed ratios from one are insignificant in all the series when $f_{\alpha} = 1/20$ and $1/24$ c/y at the 10 per cent significance level;²⁰ (2) the estimates centered on $1/28$ c/y are significantly greater than those centered on zero c/y at the 10 per cent level in the case of the Wholesale Price Index, Gross National Product, and Gross Private Domestic Investment; and (3) no other time series reveals $\hat{S}(f_{\alpha})$ significantly greater than $\hat{S}(0)$ even when $1/28$ c/y is chosen as the center of the long-swing frequency band.

V. CONCLUSION

This paper has been confined intentionally to an evaluation of the results presented by Adelman. As a result the scope of the study is extremely limited even when viewed as a purely statistical study of the importance of long cycles. First, the time series which were analyzed are limited with respect to both historical time periods and the level of aggregation involved. Second, only deviations from a logarithmic trend were investigated. The growth-rate variant of the long-swing hypothesis,

¹⁹ See Jenkins [9] for a discussion of the distribution of the spectral estimates obtained with the Parzen window.

²⁰ In the case of the Wholesale Price Index we obtain significance at the 20 percent level when $f_{\alpha} = 1/24$ c/y .

which has been studied in various forms by Kuznets and Abramovitz, has not been considered. Third, an important aspect of the long swing, the alledged coincidence (except for a reasonable lead or lag) among various economic variables, has not been explored. For these reasons the empirical results presented in this paper are only tentative although hopefully suggestive.

Despite these severe limitations it is apparent that Adelman's outright rejection of the long-swing hypothesis is unwarranted. It was shown that the estimates presented in her paper are simply inadequate for a rigorous statistical test of the hypothesis. In an attempt to rectify this difficulty, alternative estimates were obtained. Although the analysis was confined to a subset of the time series analyzed by Adelman, it was found that the case for the long-swing hypothesis is mixed. On the basis of a one-sided test, there can be no doubt about the validity of the hypothesis. The long-term components are considerably more important than the short-term movements in explaining the variance of the time series which were considered.

The results of the two-sided test of the long-swing hypothesis were considerably less favorable. Only three of the nine series which were considered provide evidence which tends to confirm the existence of long swings. Even in these cases the statistical significance of the peak in the long-swing interval is not impressive. The fact that the spectrum of the Population series (even after log-cubic detrending) possesses the "typical spectral shape" is particularly discouraging. These findings seem to indicate that a definite conclusion about the validity of the long-swing hypothesis cannot be made on the basis of existing evidence.

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