# A SPECTRAL ANALYSIS OF THE LONG-SWING HYPOTHESIS



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## ABSTRACT

This paper is concerned with the existence of long swings in the rate of growth of output and related macro-economic variables. The usual method which is used to isolate long swings in economic time series is first to low-pass filter the series in order to attenuate the short-run fluctuations and then to analyze the filtered series. It is shown that inferences about the period of fluctuation of the original series based on estimates obtained from a filtered series may be misleading unless the effect of the filter is considered. Specifically, a major cycle with a periodicity of between eight and eleven years in the original series may appear as a long swing with a periodicity of between fifteen and twenty-five years in the filtered series. This suggests that the results of several earlier studies of the long-swing hypothesis must be interpreted with extreme caution.

The empirical results of this paper are presented in the form of estimates of the spectral density functions of the relative rate of growth of a number of macro-economic variables. These estimates indicate that the relative peaks which do emerge in the long-swing frequency band are in most cases extremely weak; in no case are they statistically significant. On the other hand, the major and minor business cycles stand out clearly in these estimates. These estimates, together with the observation that the usual filtering methods can "shift" a major-cycle peak in the spectrum into a long-swing peak, tend to cast considerable doubt on the existence of long swings in the rate of growth as fluctuations which are distinct from the major cycle.

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# A SPECTRAL ANALYSIS OF THE LONG-SWING HYPOTHESIS

E. Philip Howrey*

## I. INTRODUCTION

The long-term economic growth of the American economy over the past century has frequently been characterized as expansion at a relatively constant rate. It is widely accepted that for some purposes this is a useful abstraction from the rather wide fluctuations observed in the annual growth rate. In more detailed studies of the growth of the American economy, a certain amount of interest has centered on the problem of isolating regular fluctuations in the rate of expansion about the long-term average value. Although some of the variation in the rate of growth is thought to be accounted for by the ordinary business cycle, several studies have suggested that the rate of growth accelerates and decelerates in a fairly regular pattern of some twenty years duration. These long swings are thought to be distinct from and independent of the shorter business cycle.

This paper applies the technique of spectral analysis to the problem of determining the statistical significance of long swings in the rate of growth of output and other related macro-economic variables. Since the spectral analytic technique differs from some of the more traditional methods of characterizing a time series, a brief discussion of alternative methods of time-series

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analysis is presented in Section II. The possibility that filters designed to eliminate short-run fluctuations may impart a systematic fluctuation to the filtered series is also investigated in this section. The empirical results of the study are presented in Section III and the main conclusions are summarized in the final section.

## II. TIME-SERIES ANALYSIS AND FILTERING OPERATIONS

The long-swing hypothesis is concerned with the existence of fluctuations of duration ranging between fifteen and twenty-five years.[1] The hypothesis has been formulated alternatively in terms of the level, rate of growth, and deviation from trend of various economic variables. The usual method which is used to isolate long swings in a series is first to low-pass filter the series in order to attenuate the short-run fluctuations and then to mark off the peaks and troughs in the filtered series. This chronology of peaks and troughs, together with an estimate of the amplitude of the swings, is used to determine whether the original series contains a long-swing component. Apart from the subjectivity which is often involved in determining the peaks and troughs of the series, there are two points which should be considered in connection with this approach to the analysis of a time series. First, the determination of the "period" of a time series by counting peaks and troughs is but one of several alternative techniques which may be used for this purpose. Second, and more important, the inference of a period in the original series from estimates obtained from the filtered series may be misleading. These two points are developed in some detail in this section.

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[1] Although Kuznets [16, p. 423] suggests an average periodicity of twenty years for the long swing, there is less than universal agreement on the duration of these fluctuations. Abramovitz [3, p. 419], on the basis of his study of U.S. data, suggests an average duration of fourteen years for the long swing. For a discussion of some of the difficulties involved in an attempt to specify the duration of the long swing, see Hatanaka and Howrey [12].

The approach of this section is as follows. First, the assumptions and notation used throughout the paper are set out. Then four different measures of periodicity are introduced and illustrated with reference to a second-order autoregressive scheme. Finally, the effect of a frequently used low-pass filter on the periodicity of a series is considered. The major conclusion which emerges is that, in general, it is not valid to infer that the period of a fluctuation which is isolated in a filtered series is identical to the period of a fluctuation in the original (unfiltered) series.

## Assumptions and Notation

Throughout this section it is assumed that the series being analyzed is a realization $\{x_t;\ t = 1, 2, \ldots, n\}$ of a stationary stochastic process. The stationarity condition requires that the mean, variance, and covariance of the series be independent of time, i.e.,

$$(2.1)\qquad \begin{aligned} &E[x_t] = \mu && \text{(independent of } t\text{)} \\ &E[(x_t - \mu)(x_{t+s} - \mu)] = \gamma(s) && (s = 0, 1, \ldots; \text{ independent of } t). \end{aligned}$$

In addition, it is assumed that the realization of the process is normally distributed. This facilitates the computation of the expected value of the "period" of the series.

The empirical results of this paper are presented in the form of estimates of the power spectrum. The power spectrum of a real-valued stationary stochastic process $\{x_t\}$ is[2]

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[2] A good introduction to spectral analysis is given in Granger and Hatanaka [11] and Jenkins [13].

$$(2.2) \qquad f(\omega) = \frac{1}{2\pi} \sum_{s=-\infty}^{\infty} \gamma(s) \cos \omega s \qquad (-\pi \leq \omega \leq \pi)$$

where $\omega$ denotes angular frequency measured in radians per unit of time. The more familiar concept of periodicity (P) is easily translated into frequency by noting that $\omega = 2\pi/P$, so that low (high) frequencies correspond to long (short) cycles. The power spectrum provides a decomposition of the variance of the process since

$$(2.3) \qquad \gamma(0) = \text{var } x = \int_{-\pi}^{\pi} f(\omega)\, d\omega .$$

This follows because the power spectrum and the autocovariance function form a Fourier-transform pair. The definition of the power spectrum given in (2.2) is used to derive what is referred to below as the "spectrum period" of the process.

An example of a simple generating process which is used for illustrative purposes in this section is the second-order autoregressive process

$$(2.4) \qquad x_t + a\, x_{t-1} + b\, x_{t-2} = \epsilon_t \qquad (a^2 - 4b < 0, \quad b < 1)$$

where $\{\epsilon_t\}$ is a sequence of normal random variables with mean zero and variance $\sigma_\epsilon^2$. The inequalities guarantee that the characteristic roots of the difference equation are complex and less than unity in absolute value. The complete solution of (2.4) is given by[3]

$$(2.5) \qquad x_t = D^t(A \cos \theta t + B \sin \theta t) + \sum_{j=0}^{\infty} \xi_j\, \epsilon_{t-j+1}$$

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[3] This is discussed fully in Kendall [14].

where $D = \sqrt{b}$

$\theta = \cos^{-1}(-a/2D)$

$\xi_j = \frac{2}{\sqrt{4b - a^2}} D^j \sin j\theta$

and A and B are constants determined by the initial conditions. The period of the solution of the homogeneous part of (2.4), $2\pi/\theta$, is referred to below as the autoregressive period of the scheme. Since $b < 1$, the first term in the solution approaches zero as t increases so that asymptotically

$$(2.5') \qquad x_t = \sum_{j=0}^{\infty} \xi_j \epsilon_{t-j+1} .$$

It is the periodicity of the sequence $\{x_t\}$ as determined by this last expression which is considered for illustrative purposes.

Measures of Periodicity

Consider now the problem of determining the period of an observed time series $\{x_t\}$. At least four ways of measuring the period of a series are available. These include

(1) the mean-distance between peaks (troughs),

(2) the mean-distance between upcrosses (downcrosses),

(3) the correlogram period, and

(4) the spectrum period.

In general, the expected values of (1) - (4) differ from each other. Hence, one's findings may depend critically on the way in which periodicity is measured. Since the empirical results of this study are presented in the form of estimates of the spectrum, it is of interest to compare the spectrum with the other three,

more traditional, methods of characterizing a time series.[4]

(1) Mean-distance between peaks. The expected mean-distance between peaks, where a peak is said to occur at time t if $x_{t-1} \leq x_t \geq x_{t+1}$, can be determined in the following way. Let p denote the probability that $x_t$ is a relative maximum (peak), i.e.,

$$p = \Pr\{\lambda_t \leq 0, \quad \lambda_{t+1} \geq 0\}$$

where

$$\lambda_t = x_{t-1} - x_t$$
$$\lambda_{t+1} = x_t - x_{t+1} .$$

Then in a series of N observations, one would expect to find $Np = n$ peaks. The mean-distance between peaks is thus $N/n = 1/p$, i.e., the inverse of the probability that $x_t$ is a peak. For a normal series $p = \frac{\cos^{-1} \tau}{2\pi}$ where $\tau$ is the correlation between $\lambda_t$ and $\lambda_{t+1}$. In terms of the autocorrelation coefficients of the original series, the mean-distance between peaks is given by

$$(2.6) \qquad P_1 = 2\pi/\cos^{-1}\left[\frac{-1 + 2\rho(1) - \rho(2)}{2(1 - \rho(1))}\right]$$

where $\rho(s) = \gamma(s)/\gamma(0)$ is the correlation between $x_t$ and $x_{t+s}$. For the autoregressive process (2.4), the mean-distance between peaks is given by

$$(2.6') \qquad P_1 = 2\pi/\cos^{-1}\left[\frac{b^2 - (1 + a)^2}{2(1 + a + b)}\right].$$

(2) Mean-distance between upcrosses. An upcross is said to have taken place between t-1 and t provided $x_{t-1} \leq \mu \leq x_t$, where $\mu$ is the mean of

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[4] The following discussion of the mean-distance between peaks, mean-distance between upcrosses, and the correlogram period is based on Kendall [15] to which the reader is referred for a more detailed discussion and derivation.

the series. By an argument analogous to that given for the mean-distance between peaks, the expected mean-distance between upcrosses is

$$(2.7) \qquad P_2 = 2\pi/\cos^{-1} \rho(1) .$$

For the autoregressive process this may be written as

$$(2.7') \qquad P_2 = 2\pi/\cos^{-1} \left[\frac{-a}{1+b}\right] .$$

It is of interest to note that the mean-distance between upcrosses depends only on the first-order autocorrelation of the series, whereas the mean-distance between peaks depends on the first- and second-order autocorrelation coefficients of the series. Both of these measures are based on the probability of occurrence of a given event (peak or upcross) so that the stochastic nature of the series is explictly taken into consideration. However, in these definitions no subsidiary constraints such as conditions which have the effect of reducing "ripple" have been imposed.[5] Although this limits to some extent the applicability of these two measures to economic time series, they are suggestive and lead to interesting comparisons with the correlogram and spectrum periods.

(3) Correlogram period. A third measure of the periodicity of a series can be derived from the correlogram. In general, the correlogram period is defined as the mean-distance between troughs (peaks) or downcrosses (upcrosses) in the sequence of serial correlation coefficients. For the

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[5] An additional constraint on the mean-distance between peaks which has the effect of reducing ripple, namely, $x_t \geq x_{t+h}$, has been discussed by Dodd [8].

autoregressive process (2.4), the theoretical values of the serial correlation coefficients are given by

$$(2.8) \qquad \rho(s) + a\,\rho(s-1) + b\,\rho(s-2) = 0 \qquad (s \geq 1);$$

that is, the serial correlation coefficients are generated by the homogeneous part of the difference equation. It follows that the correlogram oscillates with a periodicity which is identical to the autoregressive period, namely,

$$(2.9) \qquad P_3 = 2\pi/\cos^{-1}\left[\frac{-a}{2\sqrt{b}}\right] .$$

The fact that the correlogram period is equal to the period of the solution of the homogeneous part of the difference equation probably accounts for the intuitive appeal of this measure.

(4) Spectrum period. The fourth measure of the period of a series which is considered here is the spectrum period. The spectrum period is defined as the inverse of the frequency at which the power spectrum exhibits a relative peak (provided one exists). The power spectrum of the autoregressive process (2.4) is given by[6]

$$(2.10) \qquad f_x(\omega) = \left|1 + a\,e^{-i\omega} + b\,e^{-i2\omega}\right|^{-2} f_\epsilon(\omega) \qquad (0 \leq \omega \leq \pi)$$

or, equivalently, by

$$(2.11) \qquad f_x(\omega) = \frac{\sigma_\epsilon^2}{2\pi(1 + a^2 + b^2 + 2a(1 + b)\cos\omega + 2b\cos 2\omega)} .$$

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[6] The method by which this expression is obtained is described in Granger and Hatanaka [11, pp. 35-37].

The theoretical spectrum of the autoregressive with $a = -1.1$, $b = 0.5$, and $\sigma_\epsilon^2 = \pi$ is shown in Figure 1. This power spectrum exhibits a relative peak at $\omega = \cos^{-1}\left[\frac{-a(1+b)}{4b}\right]$ so that the spectrum period of the autoregressive is[7]

$$(2.12) \qquad P_4 = 2\pi/\cos^{-1}\left[\frac{-a(1+b)}{4b}\right] .$$

The interpretation of the spectrum period is relatively straightforward. It is simply the inverse of the center frequency of that band of frequencies which makes the largest contribution to the variance of the series.

In general, each of these measures of periodicity is different. The extent to which these measures diverge from one another depends, of course, on the exact nature of the generating process. With reference to the autoregressive process, all four of these measures yield the same result if $b = 1$, in which case the process contains a deterministic component since the first term in the general solution (2.5) does not damp out. With both a and b equal to zero, $\{x_t\}$ is simply a random series. The mean-distance between peaks is three units of time and the mean-distance between upcrosses is four units of time, both of which are well-known results. The correlogram does not oscillate in this case since $\rho(s) = 0$, $(s > 0)$ and the power spectrum is flat so that neither the correlogram period nor the spectrum period is defined in this case. The values of the period as measured by each of these methods are set out in Table I for different values of a and b of the autoregressive process. From this table it is obvious that the period which is contained in a time series depends critically on the way in which periodicity is defined and measured.

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[7] The expression for $\omega$ is obtained by setting the derivative of $f_x(\omega)$ given in (2.11) equal to zero and solving for $\omega$.

TABLE I

Theoretical Periodicity of Series Generated by

$$x_t + a\,x_{t-1} + b\,x_{t-2} = \epsilon_t$$

| a<br>b | -1.5<br>0.9 | -1.7<br>0.8 | -1.615<br>0.7 | -1.52<br>0.6 | -1.1<br>0.5 |
|---|---|---|---|---|---|
| $P_1$ | 7.90 | 8.69 | 7.36 | 6.43 | 4.96 |
| $P_2$ | 9.51 | 18.76 | 19.79 | 19.79 | 8.40 |
| $P_3$ | 9.53 | 19.85 | 23.73 | 32.31 | 9.25 |
| $P_4$ | 9.56 | 21.16 | 31.79 | $\infty$ | 10.46 |

A tacit assumption underlying this discussion of measures of periodicity is that the series under consideration consists of a sequence of fluctuations which can be characterized by an average periodicity. This does not mean that each successive fluctuation must be exactly the same length, but that the dispersion about the average is not so great that the average is meaningless. Provided this assumption is satisfied, the empirical implementation of each of the measures of periodicity is straightforward. With respect to the mean-distance between peaks (upcrosses), an obvious procedure is to mark off the peaks (upcrosses) and then determine the average distance between them. For the correlogram and spectrum, the theoretical serial correlation coefficients could be replaced by their estimates.[8]

[8] This is not the usual procedure which is used to estimate the spectrum. Spectrum estimation is discussed in Section III and the Appendix of this paper.

An economic time series, however, is not likely to satisfy such an assumption for it implies that the series contains a single period. It is much more reasonable to assume that economic time series are composed of several fluctuations, each of which can be characterized by an average periodicity. Indeed, the long-swing hypothesis is specifically concerned with the existence of a fluctuation which is longer in duration than, and superimposed upon, the ordinary business cycle. With a series of superimposed variations, the problem of decomposing the series into meaningful components immediately arises. If, for example, periodicity is measured by the mean-distance between peaks, it is necessary to establish a criterion that will enable the investigator to distinguish between business-cycle peaks and long-swing peaks in the time series under consideration. One method that has been used in this connection, low-pass filtering, will now be considered.

Filtering Operations.

In most earlier studies of the long-swing hypothesis, an indirect method has been used to distinguish between long swings and business cycles in the series of observations. The usual procedure involves applying a low-pass filter to the original series in order to reduce or eliminate short-run fluctuations.[9] The filtered series is then analyzed as if it were identical to the original series but with the high-frequency components removed. A low-pass filter which has often been used in this connection is the simple moving

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[9] For example, Kuznets [16] has used a low-order moving average filter to eliminate short-run fluctuations from the series. Various studies of the long-swing hypothesis in which some filtering technique has been used are described by Adelman [5].

average of length 2m+1:

$$(2.13) \qquad y_t = \sum_{k=-m}^{m} x_{t+k}/(2m+1)$$

where $\{y_t\}$ denotes the filtered series and $\{x_t\}$ denotes the original series. The effect of the filtering operation described by (2.13) on each of the four measures of periodicity enumerated above will now be examined and illustrated with reference to the autoregressive process (2.4).

Each of the measures of periodicity depends on the serial correlation coefficients of the series which is being analyzed. The general form of the autocovariance function of the filtered series obtained from an original series by (2.13) is

$$(2.14) \qquad \gamma^y(s) = \sum_{k=-m}^{m} (2m + 1 - |k|)\, \gamma^x(k+s)$$

where $\gamma^y(s)$ and $\gamma^x(s)$ denote, respectively, the autocovariance function of the filtered and original series. Given the autocovariance of the original series, the filtered autocovariance and autocorrelation functions can easily be obtained. The effect of the filtering operation on the periodicity of the series is not, however, immediately apparent. A general argument in terms of the spectrum is given below, but for the other three measures of periodicity the effect of the filtering is merely suggested by referring to numerical examples. For expository purposes it is assumed that the original series is generated by the autoregressive scheme (2.4) and the filtered series is obtained from (2.13). The period of the original and filtered series for various values of the autoregressive coefficients a and b and different values of m, where m determines

the length of the filter, are set out in Table II.[10]

TABLE II

Period of Original and Filtered Autoregressive Series for Different Values of a, b, and m

| $P_j$ | m \ a = / b = | -1.5 0.9 | -1.7 0.8 | -1.615 0.7 | -1.52 0.6 | -1.1 0.5 |
|---|---|---|---|---|---|---|
| $P_1$ | 0 | 7.9 | 8.7 | 7.4 | 6.4 | 5.0 |
| | 2 | 9.6 | 15.3 | 14.6 | 13.6 | 8.6 |
| | 3 | 10.0 | 17.0 | 16.6 | 15.8 | 8.8 |
| | 4 | 9.9 | 18.3 | 18.2 | 17.4 | 8.4 |
| $P_2$ | 0 | 9.5 | 18.8 | 19.6 | 19.8 | 8.4 |
| | 2 | 10.7 | 22.3 | 25.6 | 27.8 | 14.1 |
| | 3 | 12.4 | 24.2 | 28.5 | 31.6 | 17.7 |
| | 4 | 16.7 | 26.5 | 31.7 | 35.5 | 20.5 |
| $P_3$ | | 9.5 | 19.9 | 23.7 | 32.3 | 9.3 |
| $P_4$ | 0 | 9.6 | 21.2 | 31.8 | $\infty$ | 10.5 |
| | 2 | 9.6 | 22.2 | 36.4 | $\infty$ | 26.7 |
| | 3 | 9.8 | 22.2 | 44.4 | $\infty$ | $\infty$ |
| | 4 | $\infty$ | 23.6 | 80.0 | $\infty$ | $\infty$ |

This table suggests that the mean-distance between peaks, $P_1$, and the mean-distance between upcrosses, $P_2$, are both increased by the simple moving average filter. For example, with a = -1.7 and b = 0.8, the theoretical mean-distance between peaks in the original series is 8.7 and the mean-distance

[10] The values for a and b in this table are the same as those used in Table I. When m = 0, the length of the filter is 2m+1 = 1 so that the filtered and original series are identical.

between upcrosses is 18.8 units of time. The expected mean-distance between peaks in the five-item moving average of this series (m = 2) is 15.3 while the mean-distance between upcrosses is 22.3 units of time. For higher-order moving averages (m = 3, 4) the period of the filtered series is somewhat longer in this example. It is apparent from this table that the period of the filtered series is greater than the period of the original series, at least for a certain range of values of a, b, and m. This indicates that in general it is not valid to conclude that the period which is determined by marking off peaks or upcrosses in the filtered series came about as a result of a fluctuation with the same period in the original series.

The correlogram period is not subject to this difficulty in the case of the autoregressive process. The reason for this is that the autocorrelation coefficients of the original series are generated by (2.7) which has as its solution[11]

$$(2.15) \qquad \rho(k) = D^k \frac{\sin(k\theta + \psi)}{\sin \psi}$$

where $D = \sqrt{b}$

$$\theta = \cos^{-1}(-a/2D)$$

$$\tan \psi = \frac{1 + b}{1 - b} \tan \theta.$$

According to (2.14) the autocorrelation coefficients of the filtered series are linear combinations of the autocorrelation coefficients of the original series. Since the autocorrelation coefficients of the original series all have the same

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[11] Cf. Kendall [15, p. 26].

period and damping factor, it follows that the period of oscillation of the correlogram of the filtered series is the same as that of the original series which in turn is equal to the autoregressive period of the scheme.[12] For this reason only one number is given in Table II for the correlogram period.

The spectrum period exhibits the same tendency as the mean-distance between peaks when a moving average filter is applied to a series. In this case a general argument is much simpler to construct than in the preceding cases. The power spectrum of the filtered series, $f_y(\omega)$, is related to the power spectrum of the original series by

$$(2.16) \qquad f_y(\omega) = G(\omega)\, f_x(\omega) \qquad (-\pi \leq \omega \leq \pi)$$

where $G(\omega)$, the gain of the $(2m + 1)$-item moving average, is

$$(2.17) \qquad G(\omega) = \left| \sum_{s=-m}^{m} e^{i\omega s}/(2m + 1) \right|^2 .$$

The gain of a five-item moving average is shown in Figure 2.a. It is apparent that the low frequencies are passed by the filter and the high frequencies are rejected.

Suppose that the spectrum of the original series, $f_x(\omega)$, exhibits a peak at $\hat{\omega}$, i.e.,

$$(2.18) \qquad \left. \frac{df_x(\omega)}{d\omega} \right|_{\hat{\omega}} = 0 .$$

The corresponding peak in the filtered series occurs at $\bar{\omega}$, where $\bar{\omega}$ is determined by solving

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[12] Cf. Allen [6, pp. 129-131].

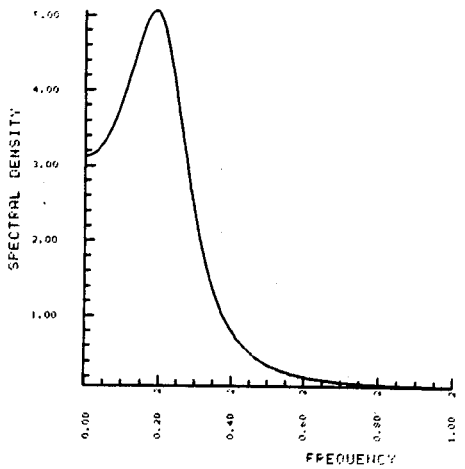
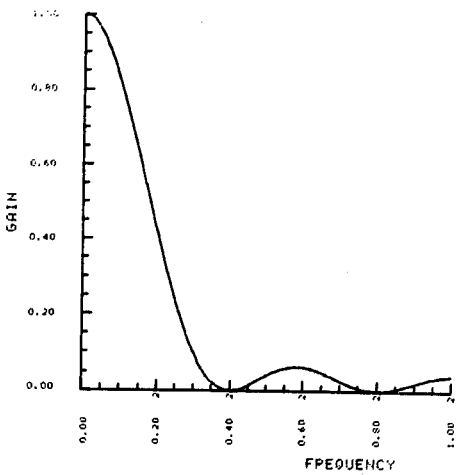



Figure 1. Theoretical Spectrum of $x(t) - 1.1x(t-1) + 0.5\,x(t-2) = \epsilon(t)$.



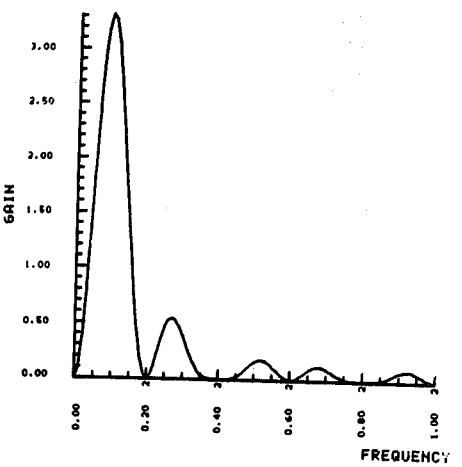
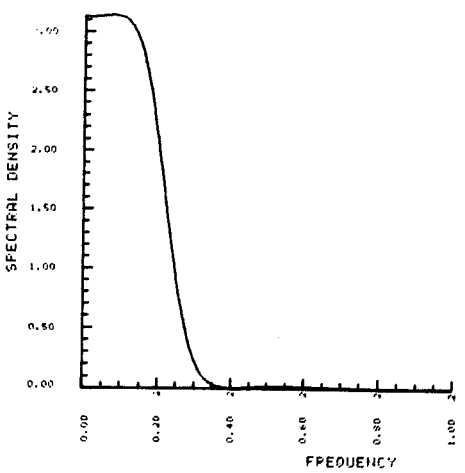




Figure 2.a Gain of an Unweighted Five-Item Moving Average

Figure 2.b Gain of Decadal Difference of Quinquennial Average



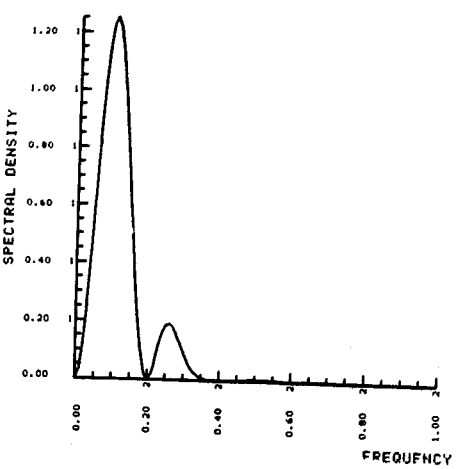




Figure 3.a Theoretical Spectrum of Quinquennial Average of Autoregressive

Figure 3.b Theoretical Spectrum of Decadal Difference of Quinquennial Average of Autoregressive

$$(2.19) \qquad \frac{df_y(\omega)}{d\omega} = f_x(\omega)\frac{d\,G(\omega)}{d\omega} + G(\omega)\frac{df_x(\omega)}{d\omega} = 0$$

or, equivalently, by solving

$$(2.20) \qquad \frac{df_x(\omega)}{d\omega} = -f_x(\omega)\frac{d\,\ln G(\omega)}{d\omega} \quad .$$

Since $G(\omega)$ is a positive but decreasing function of $\omega$ for $\omega < 2\pi/5$, it follows that $\frac{d\,\ln G(\omega)}{d\omega} < 0$ for $0 \leq \omega \leq 2\pi/5$. Therefore the right-hand side of (2.20) is positive for $\omega < 2\pi/5$. In order for $\hat{\omega}$ to afford $f_x(\omega)$ a true local maximum, $\frac{df_x(\omega)}{d\omega}$ must be positive for $\omega < \hat{\omega}$ and negative for $\omega > \hat{\omega}$. A comparison of (2.18) and (2.20) indicates that $\bar{\omega} < \hat{\omega}$ provided $\hat{\omega} < 2\pi/5$. If the original series contains an important component of periodicity greater than five units of time per cycle, the filtered series will contain an important component of duration longer than that of the original series.[13] The theoretical spectrum of a five-item moving average of the series generated by the autoregressive process (2.4) with $a = -1.1$ and $b = 0.5$ is shown in Figure 3.a.

In addition to the simple moving average, a second filtering operation is sometimes applied to the filtered series. For example, Kuznets [16], after smoothing the original series with a quinquennial average, analyzes decadal rates of change. This second filtering operation,

$$(2.21) \qquad z_t = y_{t+5} - y_{t-5} \ ,$$

yields a series the spectrum of which is related to that of the smoothed series by

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[13] In general, a $(2m+1)$-item moving average "shifts" peaks in the spectrum which are located below $1/(2m+1)$ years per cycle to still lower frequencies. It should be noted that this has nothing to do with aliasing which is described by Blackman and Tukey [7, pp. 31-33] and which is explored by Taubman [13] in connection with the long-swing hypothesis.

$$(2.22) \qquad f_z(\omega) = 2(1 - \cos 10\omega)\, f_y(\omega) .$$

The combined operations of smoothing and taking decadal differences produces a series the spectrum of which is related to that of the original series by

$$(2.23) \qquad f_z(\omega) = 2(1 - \cos 10\omega)(1 + 2\sum_{k=1}^{2} \cos \omega k)^2 f_x(\omega)/25 .$$

The gain of these combined operations is shown in Figure 2.b. In Figure 3.b the effect of applying these two operations to the series generated by the autoregressive scheme (2.4) with a = -1.1 and b = 0.5 is shown. It is clear once again that inferences about the original series based on estimates obtained from a filtered series can be quite misleading.

The fact that the correlogram period of the autoregressive process (2.4) is invariant with respect to the filtering operation described by (2.13), while the spectrum period is changed, may appear to be rather surprising. There is, however, a rather simple explanation for this. The spectrum of the original series, $f_x(\omega)$, is the Fourier-transform of the autocovariance function of $\{x_t\}$, i.e.,

$$(2.2) \qquad f_x(\omega) = \sum_{s=-\infty}^{\infty} \gamma^x(s) \cos \omega s .$$

From (2.15) it follows that the autocovariance function of the autoregressive process is

$$(2.24) \qquad \gamma^x(s) = A_x D^s \sin(s\theta + \psi_x)$$

where $A_x = \text{var } x/\sin \psi_x$ and D and $\psi_x$ are determined as above (Equation (2.15)). Substituting this expression for the autocovariance function into (2.2) and

simplifying yields

$$(2.25) \qquad f_x(\omega) = A_x \cos \psi_x K(\omega) + A_x \sin \psi_x L(\omega)$$

where

$$K(\omega) = \sum_{s=-\infty}^{\infty} D^s \sin \theta s \cos \omega s$$

$$L(\omega) = \sum_{s=-\infty}^{\infty} D^s \cos \theta s \cos \omega s \ .$$

It was shown above that the autocovariance function of the filtered series differs from that of the original series only in its amplitude and phase so that it may be written as

$$(2.26) \qquad \gamma^y(s) = A_y D^s \sin(\theta s + \psi_y)$$

where $A_y = \text{var } y/\sin \psi_y$. Transforming $\gamma^y(s)$ yields

$$(2.27) \qquad f_y(\omega) = A_y \cos \psi_y K(\omega) + A_y \sin \psi_y L(\omega)$$

for the spectrum of the filtered series.

A comparison of (2.25) and (2.27) indicates immediately why the spectrum period is not invariant with respect to the filtering operation. The spectrum period of the original series is obtained by solving

$$(2.28) \qquad \frac{df_x(\omega)}{d\omega} = A_x \cos \psi_x K'(\omega) + A_x \sin \psi_x L'(\omega) = 0$$

or

$$(2.28') \qquad K'(\omega) = - \tan \psi_x L'(\omega)$$

for $\bar{\omega}$. The spectrum period of the filtered series is found by solving

$$(2.29) \qquad \frac{df_y(\omega)}{d\omega} = A_y \cos \psi_y K'(\omega) + A_y \sin \psi_y L'(\omega) = 0$$

or

$$(2.29') \qquad K'(\omega) = - \tan \psi_y L'(\omega)$$

for $\hat{\omega}$. Since the phase of the filtered autocovariance function will, in general, differ from that of the original series, i.e., $\psi_y \neq \psi_x$, it follows that $\bar{\omega} \neq \hat{\omega}$.

Summary

This discussion of the three measures of periodicity which are not invariant with respect to filtering operations, namely, the mean-distance between peaks, the mean-distance between upcrosses, and the spectrum period, suggests an important point in connection with the long-swing hypothesis. Specifically, a major cycle with a periodicity of between eight and eleven years in the original series may appear as a long swing with a periodicity of between fifteen and twenty-five years in the filtered series. This point is strikingly illustrated by the numerical results set out in Table II. Since several earlier studies of the long-swing hypothesis have used filters that are identical or similar to those described above, the results of these studies must be interpreted with extreme caution. In order to avoid the possibility of drawing misleading inferences from estimates obtained from a filtered series, it is necessary to consider the effect of the filter.

One of the distinct advantages of the spectral-analytic approach to time-series analysis is that this sort of adjustment problem can easily be handled. In those cases in which a filter is used, the estimates of the

spectrum can be adjusted for the effect of the filter in a relatively straightforward way.[14] In many cases, however, it is not necessary to process the series in order to eliminate the short cycles before proceeding with the estimation. This is because the power spectrum provides a decomposition of the variance of the series over the entire frequency axis. This means that with the spectral analytic approach a direct comparison of the power contained in different frequency bands is possible. Thus, the relative importance of the long swing can be compared with that of the major and minor business cycle.

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[14] For a discussion of situations in which it might be advisable to filter the series before proceeding with the estimation, see Blackman and Tukey [7, pp. 39-43].

## III. SPECTRAL ESTIMATION AND THE LONG-SWING HYPOTHESIS

In this section the empirical results of this study, presented in the form of estimates of the spectral density functions of a number of macroeconomic time series, are described. The particular form of the long-swing hypothesis with which this study is concerned is the growth-rate variant. This choice was made, in part, in view of the stationarity assumption of spectral analysis. While the absolute level of most economic variables cannot possibly be considered to be generated by a stationary stochastic process because of the dominant trend in mean, the sequence of growth rates is somewhat less questionable.[15]

Each of the series analyzed was first transformed by computing relative rates of growth according to

$$(3.1) \qquad y(t) = [x(t+1) - x(t)]/x(t)$$

where $\{x(t)\}$ denotes the original series and $\{y(t)\}$ denotes the series of growth rates. The power spectra of the growth-rate series were then estimated by Fourier-transforming weighted estimates of the autocovariance function. These normalized spectral estimates, referred to as spectral densities, are of the form

$$\frac{\pi \hat{f}(\omega)}{\hat{\gamma}(0)} \qquad (0 \leq \omega \leq \pi)$$

where $\hat{f}(\omega)$ is an estimate of $f(\omega)$ as given by (2.2) and $\hat{\gamma}(0)$ is an estimate of the variance of the series. This method of normalization is such that the

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[15] Adelman [5] has experimented with residuals from a log-linear trend. Although this transformation might be expected to eliminate the trend in the mean, the trace of the residual series indicates that the variance of the residuals is by no means stationary. The non-stationarity of the growth rate series is less conspicuous, although perhaps no less real.

theoretical spectral density of a random series is 0.5 at all frequencies. This provides a convenient standard with which to compare the estimated spectra. The actual formulas used in the estimation as well as some of the sampling properties of the estimates are discussed in the Appendix.

The spectral estimates obtained from a finite realization of a process can be interpreted heuristically as estimates of the average power contributed by a band of frequencies in the interval $\omega \pm \delta\omega$ $(0 \leq \omega \leq \pi)$. In this study the spectrum is estimated at the frequencies centered on $\omega_j = j\pi/100$ $(j = 0, 1, \ldots, 100)$. Obviously only $\pi/2\delta\omega$ of these estimates are (almost) independent, namely, those separated by a distance $2\delta\omega$, which is twice the bandwidth of the estimate. Thus the frequency axis can be divided into $\pi/2\delta\omega$ (almost) disjoint frequency bands. For the estimation procedure used here the width of each band is approximately $2\delta\omega = 4\pi/T$ radians per unit of time and the number of independent estimates is approximately $T/4$, where T is the truncation point (or number of lags) used in the estimation. During the course of the investigation, spectral densities were estimated with T = 10, 20, 30, 40, 60, and 80. As the number of independent estimates of the spectrum is increased, a sharper resolution of the frequency axis is possible, but this is achieved only at the expense of an increase in the variance of the estimate. It was found that T = 20 provides an estimate which describes reasonably well the main features of the spectra of the series dealt with here, so only the twenty-lag estimates are discussed in some detail.

The resolution of the frequency axis which corresponds to the twenty-lag estimate is shown in Table III. Although only ten points at which the spectral density is estimated are (almost) independent, the function has been

TABLE III

Resolution (in years per cycle) of the Frequency Axis for a Twenty-Lag Estimate of the Spectral Density

| Frequency Point | Lower Bound | Period | Upper Bound |
|---|---|---|---|
| 0 | 20.00 | $\infty$ | $\infty$ |
| 10 | 10.00 | 20.00 | $\infty$ |
| 20 | 6.67 | 10.00 | 20.00 |
| 30 | 5.00 | 6.67 | 10.00 |
| 40 | 4.00 | 5.00 | 6.67 |
| 50 | 3.33 | 4.00 | 5.00 |
| 60 | 2.86 | 3.33 | 4.00 |
| 70 | 2.50 | 2.86 | 3.33 |
| 80 | 2.22 | 2.50 | 2.86 |
| 90 | 2.00 | 2.22 | 2.50 |
| 100 | 2.00 | 2.00 | 2.22 |

plotted in the following diagrams at the frequency points $\omega_j = j\pi/100$ ($j = 0, 1, \ldots, 100$) by interpolation from the estimating equation. The periodicity of the fluctuation of the estimate centered on the $j^{th}$ frequency point together with its upper and lower limits as determined by the bandwidth of the estimate are shown in this table in terms of years per cycle. It should be emphasized that the values given in Table III are only approximate. This is discussed in more detail in the Appendix.

The frequency band which corresponds to the Kuznets cycle or long swing is centered on the tenth frequency point, i.e., $10\pi/100$ radians per year

or 20 years per cycle. The long-swing hypothesis can be interpreted as stating that the variance-contribution of this band of frequencies is significantly greater than that of neighboring frequency bands. This intuitive statement of the hypothesis suggests that its rejection be based on the absence of a local peak in the spectrum near this long-swing frequency. For the use of the estimated spectrum as a descriptive statistic, this statement of the hypothesis seems to be adequate. However, a more precise formulation of the hypothesis in terms of conventional tests of significance is possible. The (100 - 2 $\alpha$) percent confidence band for normally distributed independent random variables, referred to as white noise, can be determined from

$$\Pr\left\{\chi^2_{1-\alpha}(\ell) \leq \frac{\ell \hat{f}(\omega)}{f(\omega)} \leq \chi^2_{\alpha}(\ell)\right\} = 1 - 2\alpha$$

where $\ell$, the equivalent degrees of freedom of each estimate, is determined by dividing the number of observations used in the estimation of the spectrum by m/4 (i.e., $\ell = 4n/m$). These confidence limits provide a method for testing the hypothesis that the underlying process is random. Specifically, an estimate which lies outside the (100 - 2 $\alpha$) percent confidence limits is said to be significantly different from white noise at that level. The 90 percent and 95 percent confidence limits for the spectral estimates from a sequence of independent random variables of the same length as the series discussed below (n = 86) is shown in Table IV for different values of T.[16]

The general features of the spectral density functions estimated with T = 20, 30, and 40 are set out in Table V. For each truncation point

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[16] Since the series of industrial production and pig iron production cover the period 1860-1961, the confidence limits for these series are slightly different from those given in the table.

TABLE IV

Approximate Confidence Limits for Spectral Estimates from (Gaussian) White Noise

| Degrees of Freedom | 90 percent limits | | 95 percent limits | |
|---|---|---|---|---|
| 8 (T = 40, n = 86) | .171 | .969 | .136 | 1.001 |
| 11 (T = 30, n = 86) | .208 | .894 | .172 | .996 |
| 17 (T = 20, n = 86) | .255 | .812 | .222 | .888 |

used in the estimation, the location of the relative peaks in terms of years per cycle (ypc) and the estimated peak value are given. For expository purposes, the relative peaks are grouped in the table according to their location: 15 - ∞ years per cycle, 9 - 15 ypc, 5 - 9 ypc, 3 - 5 ypc, and 2 - 3 ypc. The first number in each column indicates the center of the band in which the spectrum exhibits a relative peak. Where no number is shown, the spectrum does not exhibit a relative peak in the band. The peak value of the spectrum is shown directly below the location figure. A single (double) underscore denotes peaks which are significantly different from the spectrum of white noise at the 90 percent (95 percent) confidence level.

For example, the spectrum of the Gross National Product series estimated with twenty lags exhibits local peaks in the bands centered on 11.8, 5.6, and 3.4 years per cycle. These fluctuations have relative amplitudes of .48, .78, and .77 respectively, none of which lies outside the upper 90 percent

TABLE V

General Features of the Spectral Estimates[17]

| Series (Variance) | Number of Lags | | Location and Value of Spectral Peaks | | | | |
|---|---|---|---|---|---|---|---|
| | | | 15-∞ ypc | 9-15 ypc | 5-9 ypc | 3-5 ypc | 2-3 ypc |
| 1. Gross National Product (.0047) | 20 | duration | | 11.8 | 5.6 | 3.4 | |
| | | amplitude | | .48 | .78 | .77 | |
| | 30 | duration | | 11.1 | 5.6 | 3.4 | 2.4 |
| | | amplitude | | .52 | .90 | .94 | .41 |
| | 40 | duration | 20.0 | 10.5 | 5.7 | 3.4 | 2.5 |
| | | amplitude | .47 | .56 | .97 | 1.08 | .46 |
| 2. Net National Product (.0069) | 20 | duration | | 11.1 | 5.6 | 3.3 | 2.4 |
| | | amplitude | | .42 | .73 | .87 | .44 |
| | 30 | duration | | 10.5 | 5.6 | 3.3 | 2.5 |
| | | amplitude | | .46 | .87 | 1.03 | .53 |
| | 40 | duration | 18.2 | 10.0 | 5.7 | 3.4 | 2.5 |
| | | amplitude | .38 | .51 | .97 | 1.17 | .62 |
| 3. Industrial Production (.0142) | 20 | duration | | | 7.1 | 3.4 | |
| | | amplitude | | | .82 | .83 | |
| | 30 | duration | | 12.5 | 6.7 | 3.4 | 2.8 |
| | | amplitude | | .59 | .94 | 1.05 | .66 |
| | 40 | duration | | 13.3 | 6.5 | 3.4 | 2.8 |
| | | amplitude | | .63 | 1.04 | 1.26 | .74 |

[17] The sources of the various series are given at the end of the table. The value in parenthesis following the description of the series is an estimate of the variance. Amplitudes with a single (double) underscore are peaks in the spectrum which are significantly different from white noise at the 90 percent (95 percent) confidence level.

| Series (Variance) | Number of Lags | Location and Value of Spectral Peaks | | | | |
|---|---|---|---|---|---|---|
| | | 15-∞ ypc | 9-15 ypc | 5-9 ypc | 3-5 ypc | 2-3 ypc |
| 4. Pig Iron Production (.0574) | 20 duration | | | 6.2 | 3.4 | 2.8 |
| | 20 amplitude | | | .60 | .96 | .69 |
| | 30 duration | | 13.3 | 6.2 | 3.4 | 2.7 |
| | 30 amplitude | | .44 | .72 | 1.26 | .83 |
| | 40 duration | | 14.3 | 6.2 | 3.4 | 2.7 |
| | 40 amplitude | | .48 | .82 | 1.56 | .94 |
| 5. Flow of Goods to Consumers (.0025) | 20 duration | 25.0 | | 5.7 | 3.6 | 2.5 |
| | 20 amplitude | .60 | | .54 | .52 | .62 |
| | 30 duration | 22.2 | 9.5 | 5.7 | 3.6 | 2.5 |
| | 30 amplitude | .68 | .43 | .60 | .62 | .80 |
| | 40 duration | 22.2 | 9.5 | 5.9 | 3.4 | 2.5 |
| | 40 amplitude | .77 | .46 | .65 | .67 | .93 |
| 6. Consumers' Durables (.0260) | 20 duration | | 10.5 | 5.6 | 3.3 | 2.3 |
| | 20 amplitude | | .72 | .81 | .84 | .36 |
| | 30 duration | | 10.5 | 5.6 | 3.3 | 2.4 |
| | 30 amplitude | | .90 | .94 | 1.05 | .40 |
| | 40 duration | 28.6 | 11.1 | 5.7 | 3.4 | 2.4 |
| | 40 amplitude | .26 | 1.05 | 1.02 | 1.12 | .43 |
| 7. Consumers' Semi-Durables (.0048) | 20 duration | 16.7 | | 5.1 | 3.2 | 2.4 |
| | 20 amplitude | .55 | | .56 | .53 | .65 |
| | 30 duration | 16.7 | | 5.0 | 3.3 | 2.5 |
| | 30 amplitude | .66 | | .65 | .62 | .75 |
| | 40 duration | 16.7 | 6.9 | 5.0 | 3.3 | 2.5 |
| | 40 amplitude | .74 | .42 | .72 | .70 | .82 |

| Series (Variance) | Number of Lags | Location and Value of Spectral Peaks | | | | |
|---|---|---|---|---|---|---|
| | | 15-∞ ypc | 9-15 ypc | 5-9 ypc | 3.5 ypc | 2-3 ypc |
| 8. Gross Capital Formation (.0574) | 20 duration | | | 5.7 | 3.2 | 2.2 |
| | 20 amplitude | | | 1.04 | .63 | .38 |
| | 30 duration | | 11.1 | 5.4 | 3.2 | 2.2 |
| | 30 amplitude | | .69 | 1.23 | .71 | .41 |
| | 40 duration | | 11.1 | 5.6 | 3.1 | 2.2 |
| | 40 amplitude | | .73 | 1.33 | .77 | .42 |
| 9. Gross Producers' Durables (.0412) | 20 duration | | | 7.1 | 3.4 | 2.1 |
| | 20 amplitude | | | 1.13 | .87 | .18 |
| | 30 duration | 22.2 | | 7.7 | 3.4 | 2.2 |
| | 30 amplitude | .31 | | 1.24 | 1.16 | .18 |
| | 40 duration | 22.2 | | 8.0 | 3.4 | 2.2 |
| | 40 amplitude | .36 | | 1.32 | 1.42 | .19 |
| 10. Gross Nonfarm Residential Construction (.1440) | 20 duration | | 11.8 | | 3.4 | 2.3 |
| | 20 amplitude | | .98 | | .43 | .46 |
| | 30 duration | | 11.8 | 5.6 | 3.5 | 2.4 |
| | 30 amplitude | | 1.14 | .61 | .46 | .48 |
| | 40 duration | | 12.5 | 5.3 | 3.5 | 2.4 |
| | 40 amplitude | | 1.25 | .63 | .51 | .54 |
| 11. Inventory Investment (6.068) | 20 duration | 20.0 | | | 4.1 | 2.3 |
| | 20 amplitude | .65 | | | .89 | .66 |
| | 30 duration | 20.0 | 10.0 | | 4.2 | 2.3 |
| | 30 amplitude | .78 | .47 | | 1.07 | .70 |
| | 40 duration | 20.0 | 9.5 | | 4.2 | 2.4 |
| | 40 amplitude | .92 | .53 | | 1.25 | .76 |

| Series (Variance) | Number of Lags | Location and Value of Spectral Peaks | | | | |
|---|---|---|---|---|---|---|
| | | 15-∞ ypc | 9-15 ypc | 5-9 ypc | 3-5 ypc | 2-3 ypc |
| 12. GNP per Worker (.0051) | 20 duration | | 10.5 | 5.4 | 3.4 | |
| | 20 amplitude | | .55 | .85 | .70 | |
| | 30 duration | | 10.5 | 5.6 | 3.4 | 2.4 |
| | 30 amplitude | | .52 | .91 | .96 | .42 |
| | 40 duration | 20.0 | 10.5 | 5.7 | 3.4 | 2.5 |
| | 40 amplitude | .50 | .56 | .98 | 1.10 | .47 |
| 13. GNP per Capita (.0046) | 20 duration | | 10.5 | 5.4 | 3.4 | |
| | 20 amplitude | | .55 | .85 | .70 | |
| | 30 duration | | 10.5 | 5.6 | 3.4 | 2.4 |
| | 30 amplitude | | .67 | 1.01 | .85 | .43 |
| | 40 duration | 22.2 | 10.5 | 5.7 | 3.4 | 2.4 |
| | 40 amplitude | .34 | .76 | 1.09 | .97 | .40 |
| 14. Population (.00003) | 20 duration | ∞ | | | 3.5 | 2.3 |
| | 20 amplitude | 4.60 | | | .10 | .03 |
| | 30 duration | ∞ | 10.5 | 4.9 | 3.5 | 2.3 |
| | 30 amplitude | 5.96 | .73 | .08 | .12 | .03 |
| | 40 duration | ∞ | 10.0 | 4.9 | 3.5 | 2.3 |
| | 40 amplitude | 6.87 | .82 | .09 | .13 | .03 |

Source: The data used in this study, with the exception of the Index of Industrial Production and Pig Iron Production, are the annual estimates underlying the series published in Kuznets [16].[18] Where more than one estimate was available, Kuznets' Variant III was used. The series are expressed in constant (1929) dollars and cover the period 1869-1955. The Index of Industrial Production with 1929 as the base year was constructed by splicing three series: Frickey's Index of Production for Manufacture [10, p. 54] covering the period 1860-1914; Fabricants' Index of Output of Manufacturing Industries [9, p. 44] covering the period 1915-1918; and the Federal Reserve Board's Index of Industrial Production [21 and 22] covering the period 1919-1960. The Pig Iron Production series was taken from [23] for the years 1860-1945 and from [24] for the years 1946-1961.

[18] Permission by S. Kuznets and the National Bureau of Economic Research to use these series is gratefully acknowledged.

white-noise confidence limit given in Table IV. This summary presentation facilitates a comparison of spectral estimates which differ from one another in the focussing power of the spectral window used in their estimation. The amount by which the focussing power of the window differs for different truncation points can be inferred from Figure A.1 in the Appendix.

The results of the estimation are shown in graphical form in Figures 4 -17 for the truncation point T = 20. As suggested above, the twenty-lag estimates adequately reflect the general features of the thirty- and forty-lag estimates, so only the twenty-lag estimates are shown in graphical form. In addition to the estimated spectrum the trace of the relative rate of growth of each series is also shown. These are included as a basis for making some judgment about the stationarity of the series from which the spectrum is estimated.

A comparison of the spectra which relate to national income and production with the spectra of the consumption and investment components of national product reveals several interesting points. The spectra of the income and production series (1-4 in Table III) shown in Figures 4-7 exhibit major peaks in the ranges [5.6 - 7.1] and [3.3 - 3.4] years per cycle (ypc).[19] The latter peak, which corresponds to the well-known forty-month cycle, is the more prominent of the two, at least in the Net National Product and Pig Iron Production series. Relatively weak peaks emerge in the intervals [10.0 - 14.3]

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[19] The notation $[p_1 - p_2]$ is used to denote the location of the relative peaks in the set of series under consideration. The value of $p_1$ is the shortest duration at which the spectrum exhibits a relative peak over all the estimates of all the series, and $p_2$ is the longest duration over all the estimates of all the series in the group. The grouping of the relative peaks in this way is to a certain extent arbitrary, but the overall picture is relatively clear.

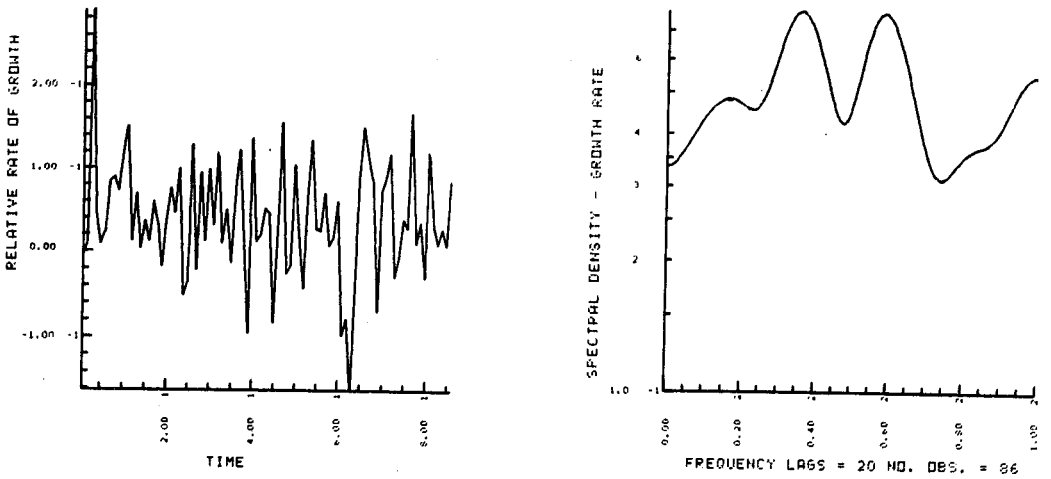


Figure 4. Gross National Product, 1869 - 1955..

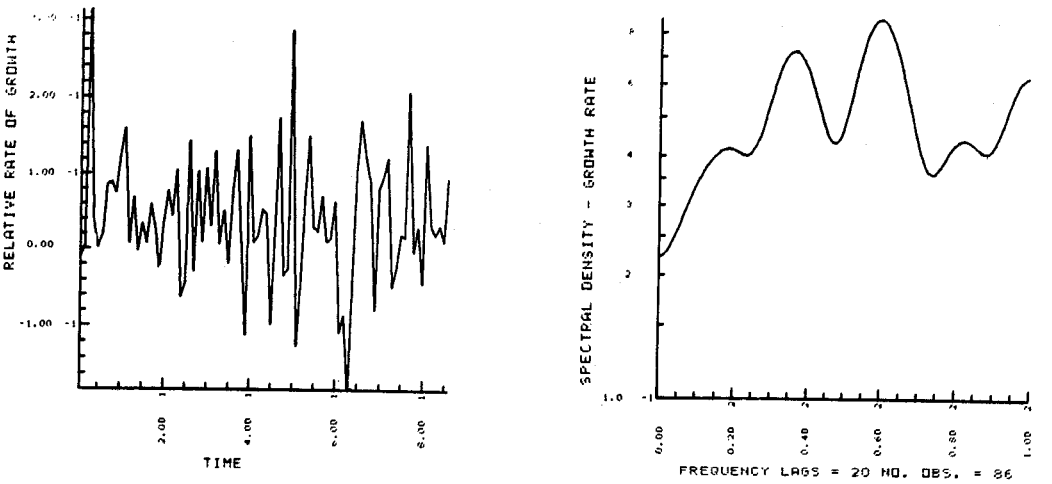


Figure 5. Net National Product, 1869 - 1955.

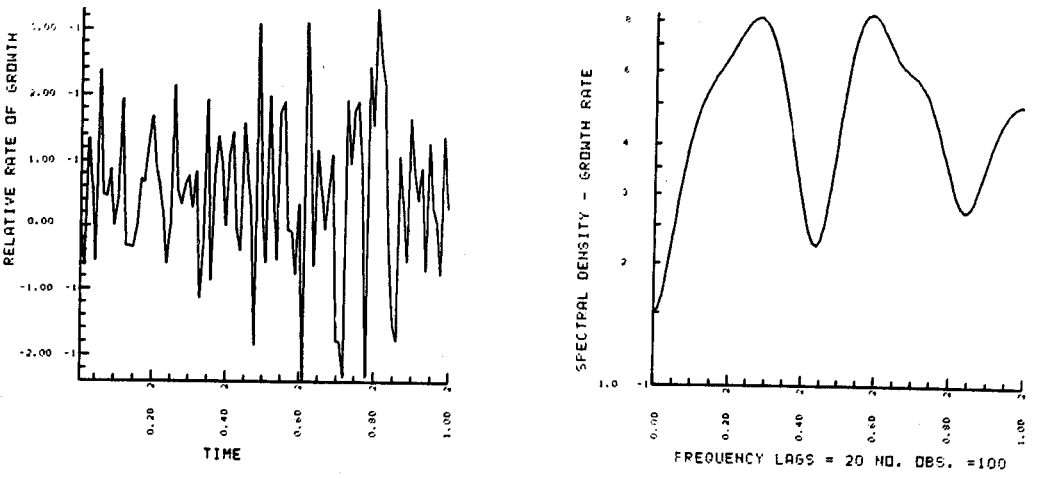


Figure 6. Index of Industrial Production, 1860 - 1960.

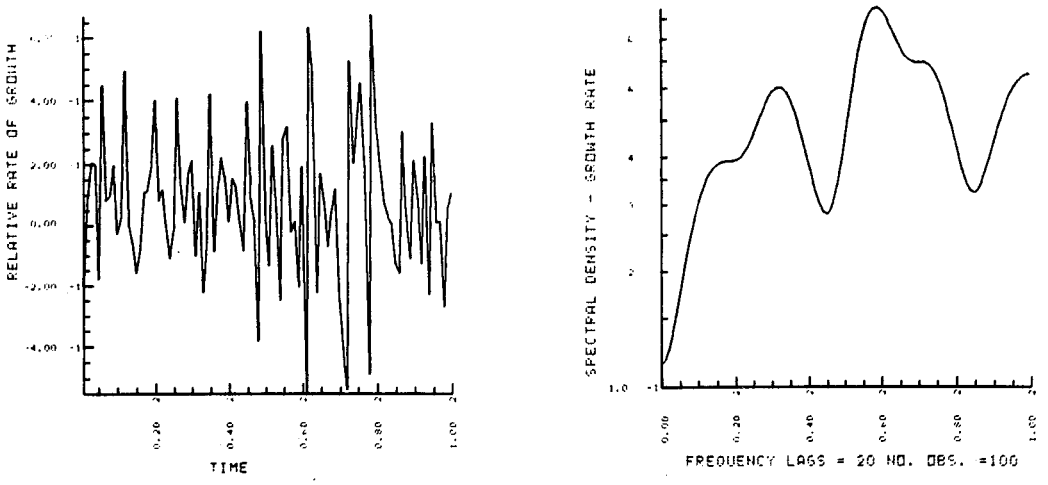


Figure 7. Pig Iron Production, 1860 - 1961.

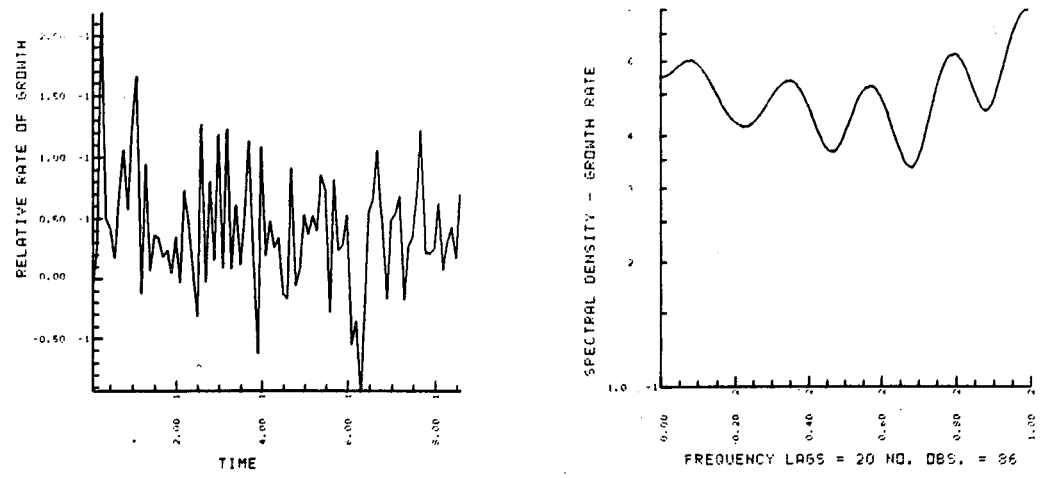


Figure 8. Flow of Goods to Consumers, 1869 - 1955.

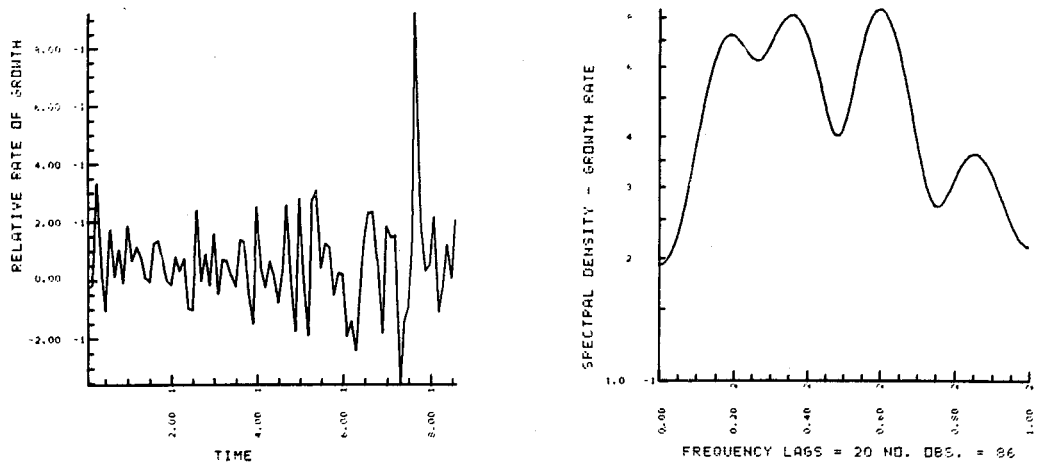


Figure 9. Consumers' Durables, 1869 - 1955.

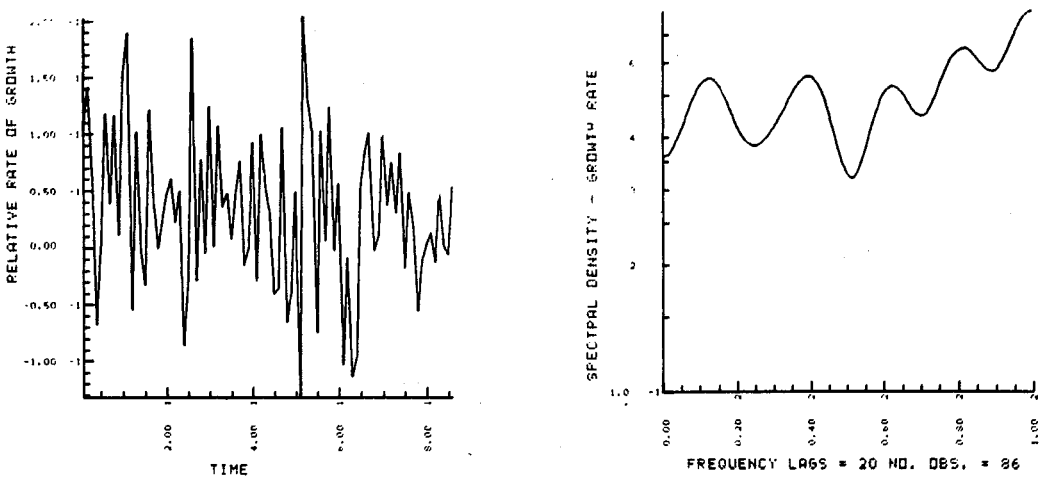


Figure 10. Consumers' Semi-durables, 1869 - 1955.

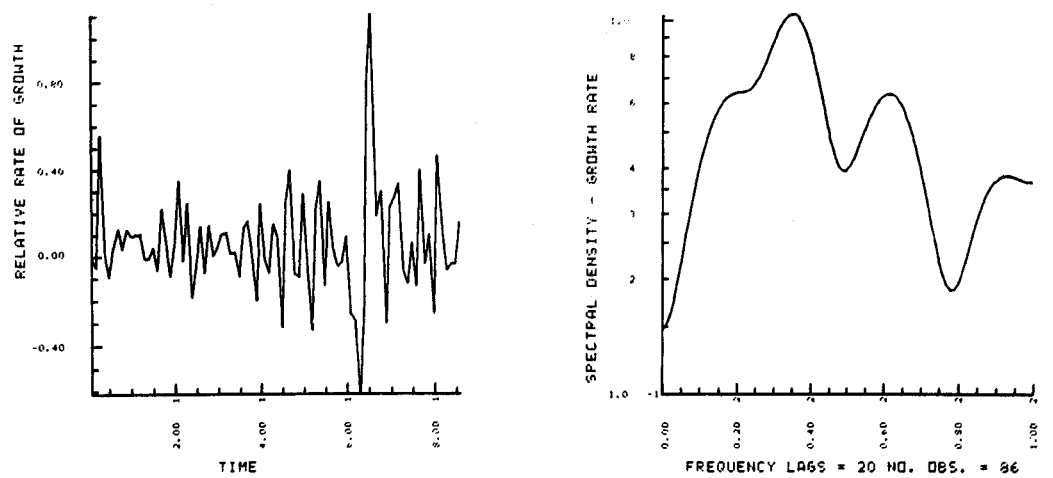


Figure 11. Gross Capital Formation, 1869 - 1955.

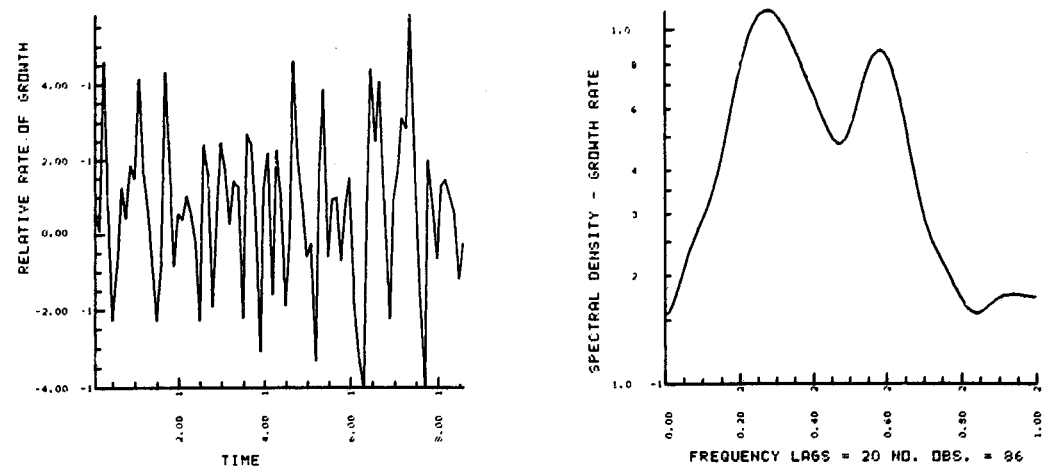


Figure 12. Gross Producers' Durables, 1869 - 1955.

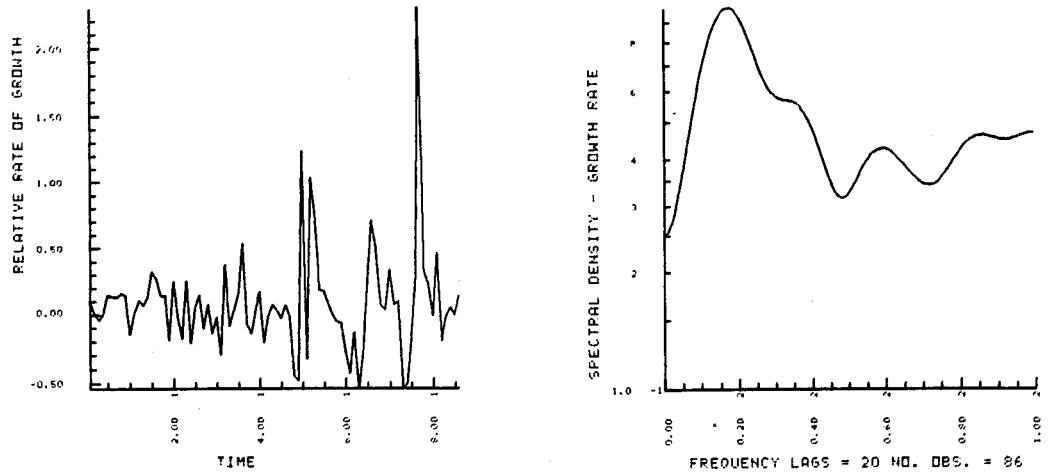


Figure 13 Gross Nonfarm Residential Construction, 1869-1955

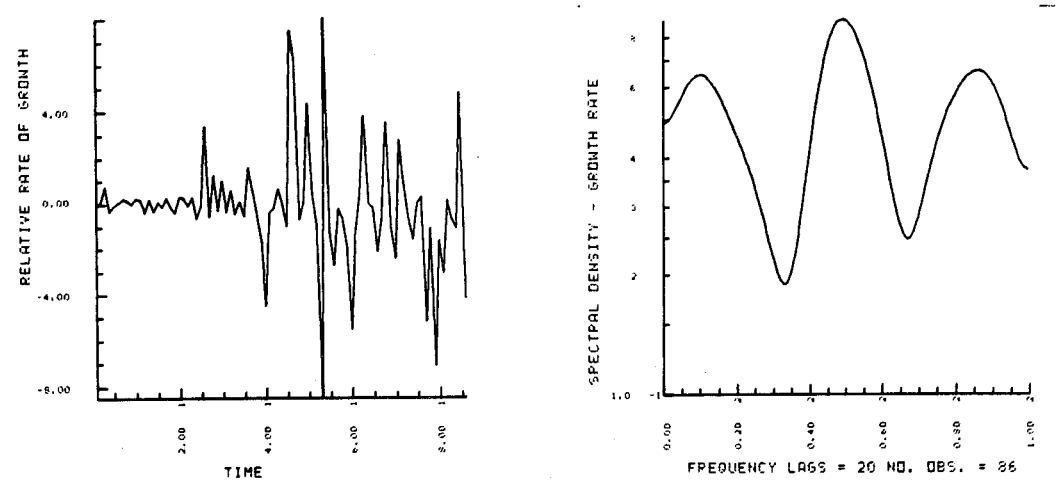


Figure 14. Inventory Investment, 1869 - 1955.

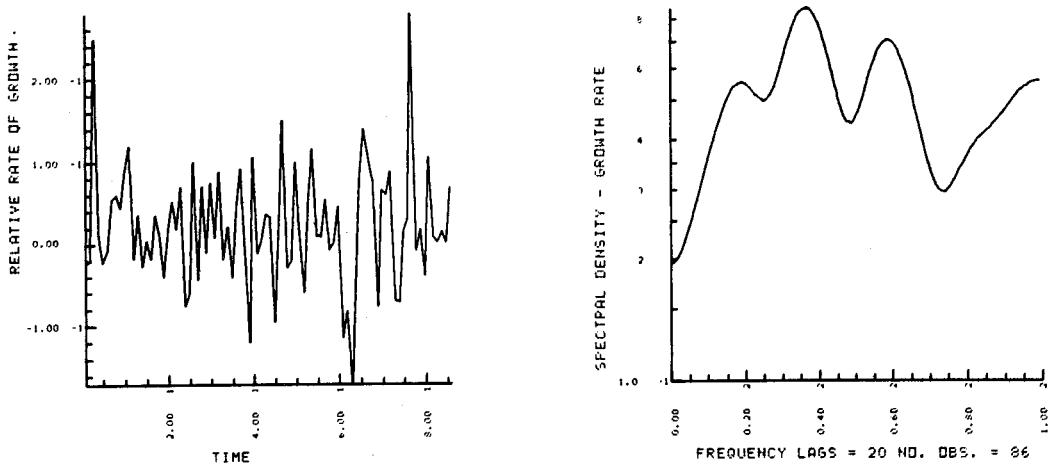


Figure 15. Gross National Product Per Worker, 1869 - 1955.

and [2.4 - 2.8] ypc. Only when the truncation point is increased to 40 does anything remotely resembling a long swing emerge in these series; and then only in the GNP and NNP series. The long swing seems to be entirely absent from the two production series.

The estimates derived from the series relating to aggregate consumption (5 - 7 in Table III), shown in Figures 8 - 10, are interesting in several respects. The spectra of the total consumption and consumers' semi-durables series are very weak (i.e., not statistically different from the spectrum of white noise) but do exhibit a relative peak in the [16.7 - 25.0] ypc range. The spectrum of the consumers' durables series is much like that of the income series in that relatively strong peaks emerge in the ranges [5.6 - 5.7] and [3.3 - 3.4] ypc. The major difference is that the major cycle of periodicity [10.5 - 11.1] ypc is much more pronounced in the consumers' durables series than in the income series. All this agrees reasonably well with the accepted notions about the volatility of the various components of consumption expenditure. The curious thing about these series is the emergence of a weak long-swing peak in the total consumption and consumers' semi-durables series when there is no corresponding concentration of power in this neighborhood in the income series.

The spectral estimates of the investment series (8 - 11 in Table III) shown in Figures 11 - 14 are interesting in several respects. The gross investment series, although similar to the production series (Figures 6 - 7) in terms of the location of the spectral peaks, is more strongly influenced by a [5.4 - 5.7] year fluctuation than are the production series. The gross producers' durables series exhibits important [7.1 - 8.0] and 3.4 ypc peaks

and a very weak long-swing peak centered on 22.2 ypc. The gross nonfarm residential construction series exhibits the highly publicized long building cycle, although the periodicity of [11.8 - 12.5] ypc is somewhat shorter than that of previous estimates. It is very interesting to note that this frequency band contributes more than twice as much as any other to the variance of the series. The inventory investment series is interesting for two reasons. The periodicity of the business-cycle component emerges as 4.1 - 4.2 ypc, almost .5 ypc longer than in the other series. This is very curious in that a considerable amount of the explanation of business cycles, at least recently, has been centered around the inventory adjustment process.[20] The other interesting property of the inventory series is that, like the consumption series, it exhibits a long-swing peak.

The estimated spectra for GNP per Worker and GNP per Capita (Figures 15 - 16) closely resemble the spectrum of the GNP series. This is not particularly surprising in view of the fact that the spectrum of total population (Figure 17) closely resembles that of the GNP series except for the considerable amount of power concentrated near the zero frequency. As can be seen from the trace of the relative rate of growth of population, this concentration of power is probably due to the downward trend in mean.

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[20] This unexpected result may very well be explained by the inadequacy of the inventory investment series, especially during the earlier years of the series.

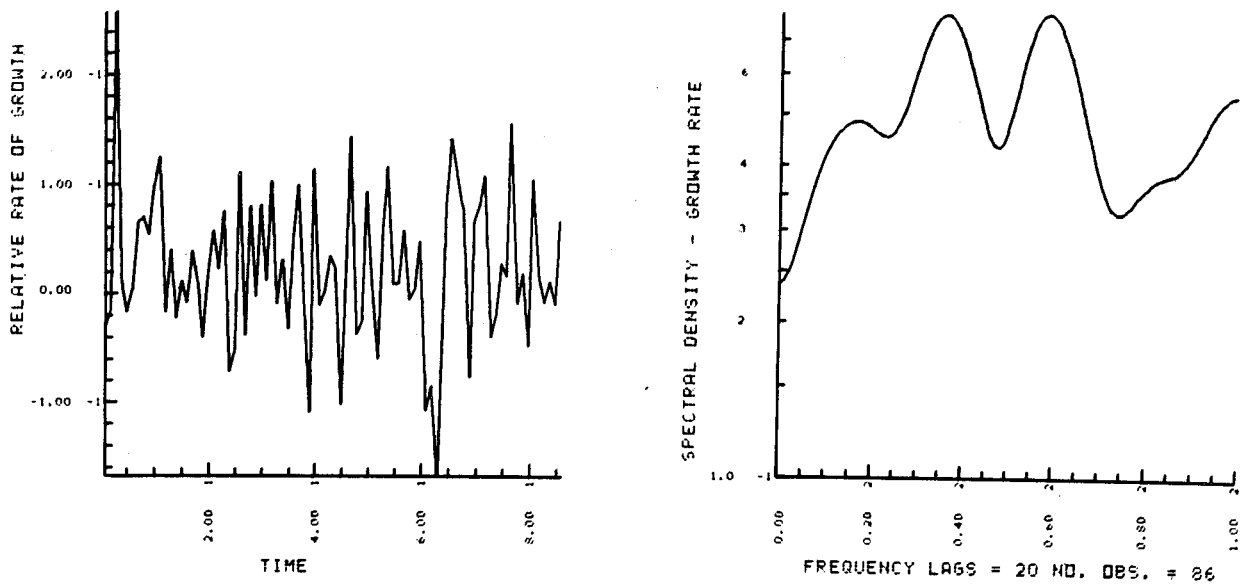


Figure 16. Gross National Product Per Capita, 1869 - 1955.

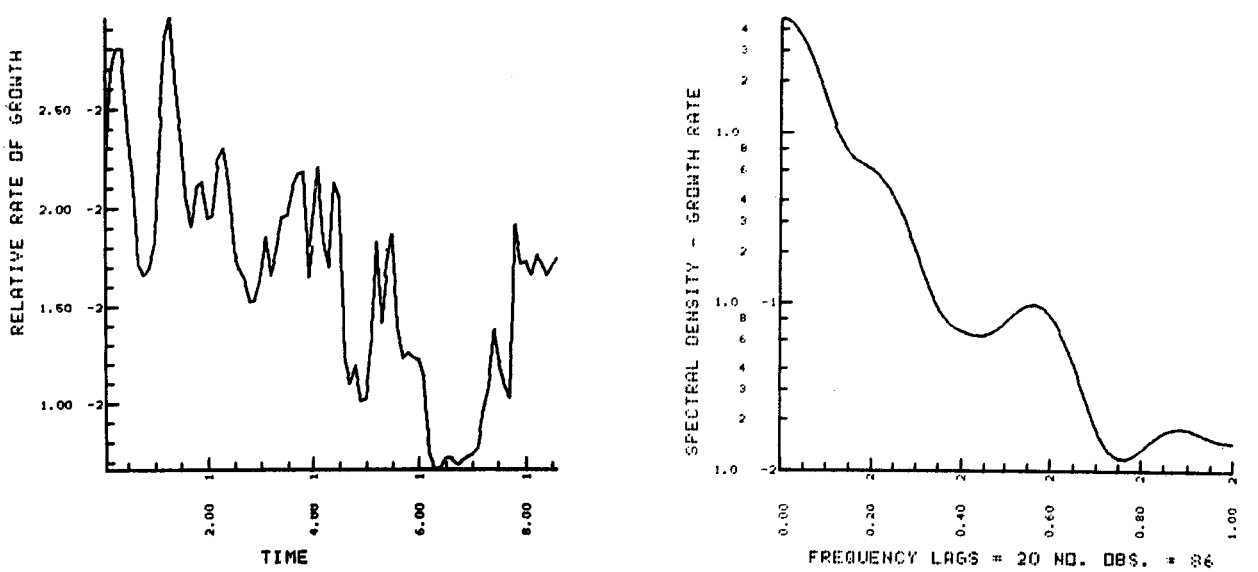


Figure 17. Total Population, 1869 - 1955.

## IV. CONCLUSION

In this paper the growth-rate variant of the long-swing hypothesis has been explored. In order to determine the relative importance of long swings in the relative rate of growth, spectral densities of a number of macro-economic variables were estimated. These estimates, while not providing a definitive answer to the question of the existence of the Kuznets cycle, do nothing to dispel the skepticism which has been voiced in connection with the long-swing hypothesis. The spectral peaks which do emerge in the long-swing frequency band are in most cases extremely weak; in no case are they statistically significant. The fact that neither the construction series nor the population series exhibits a peak in the long-swing frequency band is particularly discouraging with respect to the long-swing hypothesis. This is especially true in view of the fact that the theory of the long cycle relies heavily on the long adjustment process inherent in the construction sector or on the push (pull) of long swings in the rate of growth of population.

The impact of major and minor fluctuations in economic activity is evident from the estimated spectra. It does not seem possible to dismiss these fluctuations as purely random events in the sense that the spectral peaks arise from sampling variability. With respect to the long swing, however, the observation that the usual filtering methods can "shift" a major-cycle peak into a long-swing peak tends to cast considerable doubt on the existence of long swings in the rate of growth which are distinctly different from and independent of the major cycle.

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# APPENDIX[1]

## DESCRIPTION OF THE ESTIMATION PROCEDURE

The methods used in this study to estimate the spectral density function and some of the properties of the estimation technique are described in this appendix. The problem which is considered is that of estimating from a finite sample $\{x(t);\ t = 1, 2, \ldots, n\}$ the true spectral density of the process which is defined by

$$(A.1) \qquad f(\omega) = \frac{1}{2\pi}\left[1 + 2\sum_{s=1}^{\infty}\rho(s)\cos\omega s\right] \qquad (0 \leq \omega \leq \pi)$$

where $\rho(s)$ is the autocorrelation function of the process. The estimates discussed in the text were derived from the estimating equation

$$(A.2) \qquad \hat{f}_n(\omega_j) = \frac{1}{2\pi}\left[1 + 2\sum_{s=1}^{T} k(s)\, r(s)\cos\omega_j s\right]$$

$$(\omega_j = j\pi/100;\quad j = 0, 1, \ldots, 100)$$

where

$$(A.3) \qquad r(s) = \frac{\sum_{t=1}^{n-s}(x(t) - \bar{x})(x(t+s) - \bar{x})}{\sum_{t=1}^{n}(x(t) - \bar{x})^2} \qquad (s = 1, 2, \ldots, m)$$

$$(A.4) \qquad \bar{x} = \frac{1}{n}\sum_{t=1}^{n} x(t)$$

$$(A.5) \qquad k(s) = \begin{cases} 1 - 6(s/T)^2(1 - s/T) & 0 \leq s \leq T/2 \\ 2(1 - s/T)^3 & T/2 \leq s \leq T. \end{cases}$$

[1] This appendix is based primarily on Jenkins [13].

An approximate description of the meaning of the estimates obtained from (A.2) is facilitated by referring to the large-sample properties of these estimates.

Consider first the expected value of the estimate $\hat{f}_n(\omega)$ as the number of observations in the sample increases without limit. This may be shown to be

$$\text{(A.6)} \qquad \lim_{n\to\infty} E[\hat{f}_n(\omega)] = \int_0^{\pi} f(y)\, K(\omega, y)\, dy$$

where

$$\text{(A.7)} \qquad K(\omega, y) = \frac{1}{2}\{\mu(\omega - y) + \mu(\omega + y)\}$$

$$\text{(A.8)} \qquad \mu(y) = \frac{1}{\pi}\left[1 + 2\sum_{s=1}^{T} k(s) \cos ys\right].$$

The expected value of the point estimate $\hat{f}_n(\omega)$ is seen from (A.6) to be a weighted average of the (continuous) spectral density function over the frequency domain. The way in which the true spectrum is averaged depends on the spectral window $K(\omega, y)$. The spectral window centered on $\omega = 0$ which corresponds to the weights given by (A.5) is shown in Figure A.1 for T = 20, 30, 40. It can be seen that the spectral window emphasizes the frequencies near $\omega$ and suppresses frequencies distant from the frequency on which it is centered. This means that if the spectrum is not changing rapidly near $\omega$, $\hat{f}_n(\omega)$ provides an asymptotically unbiased estimate of $f(\omega)$.

Although equations (A. 7 - 8) describe the spectral window perfectly, it has been found convenient to characterize the window by its bandwidth. The definition of bandwidth used here is that of Jenkins [13], viz., half the base width of the rectangle with the same area as the spectral window and which gives rise to the same variance as the given window. For the window used here

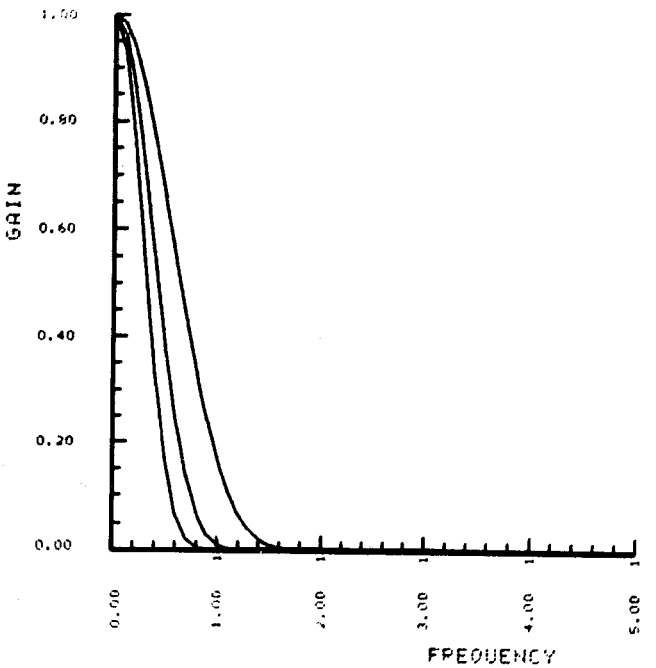


Figure A.1. Normalized Parzen Window, m = 20, 30, 40.

the bandwidth is $\frac{2\pi}{T}$ radians per unit of time so that the estimate $\hat{f}_n(\omega)$ can be thought of as an estimate of the mass in the interval $(\omega \pm \frac{2\pi}{T})$. This is the sense in which the resolution of the frequency axis presented in Table III of the text is to be interpreted. It might be noted that the number of independent (almost non-overlapping) estimates is equal to $\frac{T}{4}$. Only estimates which are separated by a distance of $\frac{4\pi}{T}$ radians are almost independent.

This discussion of the spectral window indicates that the resolution of the frequency axis is directly related to the truncation point T of the estimate. By increasing T a sharper resolution of the frequency axis is possible. However, the variance of the estimates increases with T since

$$\text{(A.9)} \qquad \text{Var } \hat{f}_n(\omega) \simeq \frac{2\pi}{n} f^2(\omega) \int_0^{\pi} K^2(\omega, y)\, dy = \frac{T}{2n} f^2(\omega).$$

This indicates the conflict between resolution or bias and variance of the estimate which arises in the estimation of the spectral density. Since no definite compromise has as yet been reached on this point, the usual procedure is to estimate the spectrum with several values for the truncation point T.