

**GAME THEORY AND HUMAN CONFLICTS**

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**Econometric Research Program  
Research Memorandum No. 80  
March, 1966**

**The research described in this paper  
was supported by ONR Contract No.  
Nonr 1858(16).**

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## GAME THEORY AND HUMAN CONFLICTS

A review of Bruno de Finetti's outline of the impact of Game Theory on economic, social, and political problems.<sup>1</sup>

In his article "La teoria dei Giochi" (The Theory of Games) Bruno de Finetti tries to evaluate the implication game theory has for the solution of economic and social conflicts. As the article contains historical notes on the evolution of game theory and the social sciences, an exposition of game theory and problems encountered therein, and finally economic, social and political conclusions drawn from this analysis, I divided this review into three sections. Combined with this exposition are some side remarks and extensions of de Finetti's ideas.

### I. Historical Notes

Up to the last century apparently no specific contributions were made with regard to the resolution of conflicts arising in games or society. This is only in part true. Already in the sixteenth century a connection between

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<sup>1</sup>Bruno de Finetti, "La teoria dei giochi" in *Civiltà delle Macchine*, Vol. XI, No. 4, and No. 5, 1963. Bruno de Finetti, an Italian mathematician, made among others important contributions to the development of subjective probability theory. Of his many articles in this field I want to mention only: "Foresight: Its Logical Laws, Its Subjective Sources", (1937), English translation in Studies of Subjective Probability, Henry E. Kyburg, Howard E. Smokler (edts.), New York, 1964, pp. 93-158. Civiltà delle Macchine is an Italian bimonthly publication which covers subjects of cultural and scientific interest. The periodical is edited in Rome and sponsored by IRI (Istituto per la Ricostruzione Industriale).

gambling, probability theory, and the analysis of human behavior existed. This connection proved to be very useful in later times. Gambling can be regarded as the very starting point of probability theory and also of the analysis of optimal human behavior: Italian princes and other noblemen did not hesitate to take scientific advice and to consult the most eminent mathematicians of their times when trying to outdo others in gambling. So it happened that Girolamo Cardano first solved the problem of determining the resulting expectations in a probabilistic manner when casting three dice. Cardano himself was a passionate gambler and by gambling he got probably both inspiration and empirical verification of his probabilistic approaches.<sup>2</sup>

This problem was treated more extensively by Galileo Galilei in his "Sopra le scoperte dei dadi" and, interestingly enough, Galileo already touches in another context upon the question of arithmetic and geometric progression of utility in a controversy with Nozzolini. The question was whether 10 or 1,000 is a "better" estimate of a good really worth 100.<sup>3</sup>

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<sup>2</sup> Girolamo Cardano, "Liber de Ludo Aleae" (1525?, 1575?), Basel, 1663. Oystein Ore, "Cardano, the Gambling Scholar", Princeton, 1953. Samuel Wilks, "Sydney H. Gould, 'The Book on Games of Chances'", Princeton, 1961.

<sup>3</sup> Galileo Galilei, "Opere", Florence, 1898, "Sopra le scoperte dei dadi", 1613, 1623?, pp. 591-594. This piece is translated in F. N. David, "Games, Gods and Gambling", New York, 1962. The dispute with Nozzolini occurred in 1627 and is solved in a letter by Galilei. In part this letter is again discussed by David, p. 67.

A solution to optimal behavior in games was first formulated by Pascal. In 1654 the Pascal-Fermat correspondence stated, concerning the problem submitted by the Chevalier de Mere:<sup>4</sup> in a game points are accumulated and the first of the two gamblers who reaches  $n$  points wins. In which way should the gamblers now divide the stakes if they end the play with  $x$  points accumulated by the first gambler and  $y$  points by the second gambler? In this connection Pascal finally came to his theorem of the "Regle de Partis": optimal behavior in games was determined by choosing that pure strategy among those available which yields the highest mathematical expectation. This solution is known as Pascal solution.

From that time to Daniel Bernoulli's Petersburg Game, a full century passed. Through the concept of decreasing marginal utility of income Daniel Bernoulli succeeded in solving the paradox given by the favorable mathematical expectations in monetary terms and the disinterest of the public to take the chances offered by the Petersburg Game.<sup>5</sup> The discrepancy between monetary sums and the utility associated with them is the basis on which Bruno de Finetti will place his discussion on game theory.

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<sup>4</sup>See Leon Brunschvicg and Pierre Boutroux, "Oevres de Blaise Pascal", Paris, 1908, Vol. 3, pp. 375 ff, first letter on the 29th of July from Pascal to Fermat, second letter on the 9th of August from Pascal to Fermat, third letter on the 25th of September from Fermat to Pascal and fourth letter on the 27th of October from Pascal to Fermat, all written in 1654.

<sup>5</sup>Daniel Bernoulli, "Specimen Theoriae Novae de Mensura Sortis", in: Commentarii academiae scientiarum imperialis Petropolitanae, 1738.

Even before Daniel Bernoulli's explanation of the Petersburg Paradox another interesting result was presented. Monsieur de Waldegrave applied in 1712 the fundamental theorem of game theory, the minimax. Waldegrave's theorem is presented and illustrated in that letter discovered by G. T. Guilbaud and first discussed in 1959 at a colloquium of the C. N. R. S. in Paris.<sup>6</sup> But like some other potential developments which were not noticed until a later reappraisal, so also Waldegrave's solution to a particular game went unnoticed and so did Bernoulli's explanation of the Petersburg Paradox, at least as regards economic science.

In a letter of Raimond de Montmort to Nicolas Bernoulli the problem at issue was a game of cards, called Here. De Finetti gives a description of Here and the solution that was found by Waldegrave. The rules of the game may be omitted here. It suffices to say that the problem reduces to the situation where two individuals are faced with the decision of whether to hold the card they have or to exchange it. With what probability shall Paul exchange his card for another one when, at present, he holds a 7 and Paul is the first one to announce his option? And with what probability shall Pierre exchange his card, an 8, when he has to choose after Paul's option? With

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English version in D. Bernoulli, "Exposition of a New Theory of the Measurement of Risk", Econometrica, 1954, pp. 23-46. The most extensive treatment of the Petersburg Paradox and its economic implications is found in Karl Menger's article "Das Unsicherheitsmoment in der Wertlehre" Zeitschrift für Nationalökonomie, Vol. 5, 1934, pp. 459 ff.

<sup>6</sup>G. T. Guilbaud, "Faut-il jouer au plus fin", in: La Decison, C. N. R. S., Paris, 1961, pp. 171-182.

low points it is, in general, desirable to exchange the card; with high points, to hold on to it. The cards 7 and 8 represent just a marginal case. One had to compare the probabilities of success of Paul, while Pierre's probability was the residual left. When Paul has a 7 and Pierre has an 8, should they both hold (HH) or both exchange (XX), or Paul hold and Pierre exchange (HX) or vice versa (XH)? Montmort, according to de Finetti, analyzed the situation in 1710 and found the probabilities 0.51185 in the two cases HH and XX, 0.51367 for HX and 0.51294 for XH. This would mean that Paul has a slight advantage and also that the differences between the four possibilities are very small. Nevertheless, Montmort states, "it is impossible to say what strategy is best for each player." Paul would profit by "holding with 7" if he knew that Pierre followed the strategy of "holding with 8" and vice versa. Bruno de Finetti gives the following payoff configuration of the game, with slightly simplified values, but without altering the essence of the game:

Probabilities of success for Paul		Paul (first) having a 7:	
		holds	exchanges
Pierre (second) having an 8	holds	0.5	0.74
	exchanges	0.90	0.5

After two years of discussion, doubts, and play where the mathematicians were joined soon by more or less dedicated players, and wherein also the idea emerged that the players should decide "by chance" with a mechanism

like "head" and "tails" which was unknown to the counterplayer (mixed strategies), finally Waldegrave found the solution: there exists a procedure of chance both for Paul and for Pierre which eliminates the ambiguity of the problem, extracting a marble from a box which contains three white and five black ones and whereby Paul holds if it is a white marble (and similarly Pierre decides identically but the other way around). And indeed, the probability of success remains in this case (whether with the numbers given by de Finetti or those given by Waldegrave) unaltered by whatsoever behavior of the counterpart. In our case the probabilities of success are 0.65 for Paul and 0.35 for Pierre. If Paul chooses by this chance-apparatus, in fact, the probability of success is given by  $\frac{3}{8}$  of 0.50 and  $\frac{5}{8}$  of 0.74, in case Pierre holds. It is  $\frac{3}{8}$  of 0.9 and  $\frac{5}{8}$  of 0.5 in case Pierre exchanges, and in both cases we have a 0.65 combined probability of success. Paul always can secure himself this expected value in whatever way Pierre might act, even when Pierre himself applies a mixed strategy of whatever kind. When we apply the analysis to find out the best behavior for Pierre, we come to an identical result with altered interpretation. This is the Minimax solution, and Waldegrave correctly described its properties.<sup>7</sup>

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<sup>7</sup> The French text reads as follows: "Il s'ensuit de-la que le sort de Paul est au moins  $\frac{2831}{5525} + \frac{3}{5525.4}$  [= 0.5 12434], puisqu'il ne tient qu'a lui de prendre trois jettons blancs et cinq noirs, et si Paul tient une autre conduite, c'est ce qu'il espere rendre son sort encore meilleur" and "J'ai oublie de vous faire observer que Pierre a une voye pour borner le sort de Paul a  $\frac{2831}{5525} + \frac{1}{5525}$ , en faisant c = 5 et d = 3, ce que vous verres encore avec

Waldegrave himself is also the first who does not attribute too great an importance to his discovery. It seemed to him that this was a not too convincing rule for those who try to "jouer au plus fin" in the faint hope to improve their expectations. Furthermore, one recognizes from the passages cited that Waldegrave's approach was merely heuristic and did not state any general rule of behavior regarding games or situations of conflict. The solution Waldegrave arrived at was just a solution to that particular game without any claim that this solution could be applied to other games or all of them. But nevertheless, Waldegrave's achievement is very remarkable. Trembly (1802), Todhunter (1865), and Bertrand (1889) achieved similar results. Interesting in this regard are also some passages in H. Minkowski.<sup>8</sup>

Emil Borel gave a proof for the special case where the number of players was restricted to  $n = 3$ , later  $n = 5$  and  $n = 7$ . However, Borel failed to prove the general minimax theorem and doubted up to 1927 the possibility of a proof.<sup>9</sup>

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evidence . . . . On pourroit croire que... ce qui seroit une maxime constant pour l'un et l'autre Joueur. "

<sup>8</sup>H. Minkowski, Geometrie der Zahlen, 1895, p. 45 and H. Minkowski, Ueber die Theorie der einfachen Ungleichungen, Journal für die reine und angewandte Mathematik, Vol. 124, 1901.

<sup>9</sup>Emil Borel, La theorie du jeu et les equations integrales a noyau symetrique gauche, Comptes Rendus Academie des Sciences, Vol. 173, 1921, pp. 1304-1308. Proof for  $n = 3$ , Emil Borel, Sur les jeux ou interviennent l'hasard et l'habilité des joueurs, in Theorie des Probabilites, Paris, 1924, pp. 204-224.



Moreover, the proof for  $n = 3, 5, 7$  was given only for symmetric games. Borel introduced, however, the concepts of pure and mixed strategies.

In 1928 J. von Neumann proved the minimax theorem and related it to the theory of fixed points,<sup>10</sup> while J. Ville discovered the connection of the minimax theorem with the theory of convex sets.<sup>11</sup>

In the meantime marginal utility theory had found its introduction and application in economic science and wrought great changes in this field. The history of this process is known and omitted here, with the only remark that up to the game theoretical analysis of von Neumann-Morgenstern in 1944 no appropriate procedure to measure utility was developed. On the other side, utility theory was soon adopted to provide a basis for some theory of general equilibrium in a laissez faire-laissez passer environment. Challenged by theories of the collapse of such systems during the evolution of industrial societies one notes an interesting and gradual change in harmonious explanations: assertions that first were advanced even by their creators or discoverers as hypotheses slowly lost their hypothetical character and evolved into preconceptions or even "facts". This process was and

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<sup>10</sup> J. von Neumann, "Zur Theorie der Gesellschaftsspiele", Mathematische Annalen, 100, pp. 295 ff., 1928.

<sup>11</sup> J. Ville, "Sur la theorie generale des jeux ou intervient l'habilité des joueurs", Traite du calcul des probabilités et des ses applications, Applications des jeux de hasard, E. Borel and Collaborators, Paris, 1938, Vol. IV, fascicule 2, pp. 105-113.

is helped by at least three basic attitudes taken at face value by some writers on social sciences. Bruno de Finetti outlined all three of them:

1. The notion of certain aspects of optimality which are connected with the spontaneous equilibrium of certain economic (and social) models. This optimality connected with the equilibrium in those special models was readily extended as a necessary by-product of every equilibrium in more complicated problems. This first basic misconception in many cases reduced economic science merely to proving the existence or non-existence of equilibrium, leaving to the reader the "trivial" and "obvious" thought that this equilibrium also included optimality. And, moreover, the equilibrium proof, constructed, e. g. , for a two person and two goods economy, supposedly implied "ceteris paribus" the equilibrium for any market economy.

2. The notion of Pangloss, according to which we are living in the best of all possible worlds. This notion and the laissez faire-laissez passer attitude, which originated in Italy and through Paris found its way to England and Adam Smith, are the overall justifications found in many parts of economic and social theories. The necessary and useful corollary to this is the blissful acceptance of the given institutions, whatever they may be.

3. The Darwinian principle of natural selection, which assigns, if applied to economic and social sciences, positive values to any form of possible evolution (without immediate regard of the consequences implied

for individuals in this environment.

But nonetheless, after all the centenarian effort, economic and social reality seems to contradict this harmonious concept of the outcome of individual and collective human behavior. In this century alone social conflicts have twice ended in social disaster in the form of two world wars. Furthermore, there are the economic crises of the 1920's and the seemingly hopeless cause of underdeveloped regions which suggest rather an equilibrium of dis-equilibrium than anything else.

Against the Panglossian hypothesis and a world built on it by the exploitation of the marginal utility concept, the new results of game theory lend themselves to a far more realistic explanation of human behavior and all the intricate questions connected with it. Some of the possible implications of game theory are advanced by Bruno de Finetti.

## II. On Game Theory and When the Minimax Fails

In his exposition of game theory Bruno de Finetti explains the concepts of zero sum and non-zero sum games, two person, three person, and n-person games and, in connection with Waldegrave's result, the minimax theorem. Furthermore, pure and mixed strategies are explained.

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<sup>12</sup> J. von Neumann, O. Morgenstern, Theory of Games and Economic Behavior, Princeton, 1944.

The general strict determinateness of two person zero sum games

( $\Gamma$ )

$$\Gamma = (S_1, S_2; A)$$

where  $S_1$  and  $S_2$  are the set of pure strategies of players 1, 2 and  $A$  the payoff function is expressed in the extension to mixed strategies

$$\bar{\Gamma} = (\bar{S}_1, \bar{S}_2; E)$$

by the equality of the achievable values  $v_1$  and  $v_2$  of players 1 and 2, i. e. ,

$$\text{Max}_{X \in S_1} \text{Min}_{Y \in S_2} E(X, Y) = \text{Min}_{Y \in S_2} \text{Max}_{X \in S_1} E(X, Y)$$

The theorem is proved by showing that the possibility

$$v_1 < v_2$$

is excluded. The application of Brouwer's fixed point theorem gives the claimed result.<sup>13</sup>

This theorem can be extended to games involving more than two persons. Each zero sum game has thus a uniquely determined solution, or in pure strategies (games with saddle points) or in mixed strategies. Given two rational players, none of them can do better. De Finetti remarks that the minimax theorem is an essential and profound complement to the up to then professed pure maximization of mathematical expectations: the minimax is, says de Finetti, quite more important and applicable than first

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<sup>13</sup> J. von Neumann, O. Morgenstern, Theory of Games and Economic Behavior, op. cit., pp. 153 ff.

realized when it was applied in the 18th century by Waldegrave.

The capacity to "jouer au plus fin" beyond the minimax solution would imply not only that the other player(s) deviated from his minimax strategy, but in addition he must have a knowledge in what direction (and when) the other player will differ from the minimax solution - a highly improbable assumption which underlines the validity of the minimax theorem, says de Finetti.

In this sense the minimax solution is neither "offensive" nor "defensive" in its character. The possibility of a deviation from the minimax solution in a game, when one knows that the opponent himself will deviate and furthermore when one knows in what direction, is already stressed in in the "Theory of Games". But a necessary condition is that all this additional information is available to one of the players. To quote von Neumann and Morgenstern:<sup>14</sup> "It should be remembered, however, that our deductions ... are nevertheless cogent, i. e. , a theory of the offensive, in this sense, is not possible without essentially new ideas." Von Neumann and Morgenstern give an example where they discuss the offensive steps indicated in case the opponent deviates from optimal strategy in the case of poker:<sup>15</sup> "If the opponent 'bluffs' too much (little) for a certain hand  $z_0$ , then he can be punished by the following deviations from the good strategy: 'Bluffing' less (more) for hands weaker than  $z_0$ , and 'Bluffing' more (less)

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<sup>14</sup> J. von Neumann, O. Morgenstern, Theory of Games ..., op. cit. p.164.

<sup>15</sup> Ibid., p. 207.

for hands stronger than  $z_0$ . I. e. , by imitating his mistake for hands which are stronger than  $z_0$  and by doing the opposite for weaker ones." The mathematical proof is omitted here.<sup>16</sup>

Now de Finetti asks an important question which, as we will see, will give rise to a whole set of new problems: how does one take account of different risk attitudes of the players? De Finetti makes here the distinction between monetary values, which do not include different risk attitudes of the players, and utility units wherein all the values are adjusted according to the preferences of the players. In this, de Finetti starts from the point which Daniel Bernoulli found as solution to the Petersburg Problem. In the previous discussion the "certain sum" of money (e. g. , 1000 dollars) was equivalent to the mathematical expectation (of, e. g. , 50 per cent probability to win 2000 dollars). Now the two values might differ in utility terms, depending on the persons involved (attitude, wealth, etc. ). De Finetti does not enter into a more detailed elaboration of the controversy around utility and its measurability. In this regard a major breakthrough occurred again in the "Theory of Games" in 1944. During all the history of the theory of utility most economists agreed that it was difficult to apply this concept in economic theory in a useful way, as long as utility was not measurable. Intrinsically connected with this set of problems were also the questions of uncertainty and risk. The backdoor that led away from these problems,

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<sup>16</sup>Ibid. , p. 202 ff.

indifference curves as applied by Pareto, Edgeworth, Auspitz and Lieben, was only an elegant retreat. Von Neumann and Morgenstern succeeded in defining utility in a strict way and showed that utility could be determined in a quantitative way up to a linear transformation, i. e. , addition of constants and multiplication by real numbers. The problem of risk and utility is most evident in the actuarial science and the question of their measurability is here a fundamental one. The controversy in von Neumann-Morgenstern's theory centers around the question whether one can establish an equivalence between certain and uncertain events. In the field of insurance the assumption becomes a reality that there will always exist a certain amount of money which represents the lowest premium at which the insurance company is prepared to pay a claim after an event with a known probability distribution. If we accept that one can establish such an equivalence, then the measurability of utility follows immediately as shown by Karl Borch.<sup>17</sup>

Given the distinction between monetary sums and the utilities they incorporate and, furthermore, the measurability of utility by some deliberate scale one can now construct zero-sum games in utility terms which then

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<sup>17</sup> Karl Borch, "The Utility Concept Applied to the Theory of Insurance", The Astin Bulletin, Vol. 1, Part 5, 1961, pp. 245 ff.

Karl Borch, "Reciprocal Reinsurance Treaties Seen as Two-Person Cooperative Game", Upsala, 1961.

Karl Borch, "Forsikringens Okonomiske Teori", Norges Handelshoyskoles Saertrykk-Serie, 1, 1964.

Karl Borch, "A Note on Utility and Attitudes to Risk", Management Science, Vol. 9, No. 4, 1963, pp. 697 ff. , and others.

may be non-zero sum games in monetary values, given different attitudes of the players. Equivalently, zero-sum games in monetary units will then turn out to be non-zero sum games in utility units. Thus the distinction between monetary and utility terms which provides the basis for the explanation of the Petersburg Paradox results here in one of the main problems of the application of game theory, as we will see.

Starting from his original zero-sum game in monetary terms, de Finetti introduces a different attitude against risk by the two players, which leads to a non-zero sum game in utility terms. The numerical example de Finetti gives is the following zero-sum two-person game:

<u>Original Game (game value + 65)</u>		<u>Modified Game (game value = 0)</u>	
50	74	- 15	+ 9
90	50	+ 25	- 15

Both the original and the modified games are zero-sum games. The values of the payoff of the column player are in the modified game - 15, 9, 25, and of the row player just the opposite + 15, - 9, - 25, in monetary values.

Supposing a different risk attitude (character, wealth, etc. of the players) those values are transformed into different utility payoffs by discounting at different rates the identical monetary values:<sup>18</sup>

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<sup>18</sup>In the essay of de Finetti occurred a printing error, inserting twice the value 25 instead of 25 and 15 for player two.



Player One:

monetary values	- 15	+ 9	+ 25
utilities	- 17	+ 8	+ 19

Player Two:

monetary values	- 25	- 9	+ 15
utilities	- 36	- 11	+ 11

The discount factor of player two is greater and the adjustment to utilities yields the following non-zero sum game in utility terms:

	<u>First Player</u>	
	- 17, + 11	+ 8, - 11
<u>Second Player</u>	+ 19, - 36	- 17, + 11

Other examples of non-zero sum games are given by Bruno de Finetti, among others a quantification in form of a matrix of the conflict between Tosca and Scarpia, an example first constructed by Rapoport.<sup>19</sup>

In general terms, however, one can represent all the examples analyzed by de Finetti in the following payoff configuration:

		<u>Player Two</u>	
		1	2
<u>Player One</u>	1	a, a	c, b
	2	b, c	d, d

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<sup>19</sup> Anatol Rapoport, "The Use and Misuse of Game Theory", Scientific American, Vol. 202, No. 6, 1962, pp. 108-118.

where  $b > a \geq d > c$ .

Now comes the central part of de Finetti's essay. "The solution of these games", says de Finetti, "is much more complicated; if not for other reasons, then due to the fact that one has to distinguish now between more possible cases." It may be that both know the utilities of the other player, or each one might guess those utilities with a higher or lesser degree of certainty, or the first knows those values, but the second does not, etc. Each one might consider (in order to find the minimax strategies) the payoff in the own utilities or in terms of the utilities of the counter player as if it were a zero-sum game; or find and compare both solutions. But does there exist a unique solution without the introduction of new criteria? The answer of the minimax strategy applied by both fails now; one can make various considerations under various perspectives in relation to the mentioned subcases.

"It suffices to say that," says Bruno de Finetti, "in order to acquire an independence from the other player's decision, each player should adopt the minimax solution of the own utility-payoff (which is evident, if he wants to adopt that criterion as an ad hoc norm). But if at the same time he knew that the other player(s) follow(s) this norm (with a selection in accordance with the payoff in utilities of that player, and in addition he knew in what direction that player would therefore differ from his own minimax) each player would have the possibility of choosing a pure strategy (from the above mentioned two) which would be preferable to him. In so doing he would not

harm the opposite player, as he would increase his own utility leaving unchanged the utility of the other, which is not true in the case of the zero-sum game."

"But if both apply the same reasoning they are confronted with the unsolvable problem of finding a pure strategy solution (which does not exist for both players simultaneously) and indeed the main difficulty of non-zero sum games lies in the fact that the arising dilemma of the pure strategy solution cannot be overcome by the adoption of mixed strategies, as was the case in zero-sum games."

However, the best each one can achieve in the first example, whether by separate decisions or a coalitional one, is a negative game value and both would be better off not to play the game at all.

De Finetti points to the fact that optimal solutions in the Paretian sense do exist, if one admits a coalition of both players "against nature". But this again goes against the very principle on which the whole "Lausanne-world" rests, i. e. , directly against the concept of perfect competition.<sup>20</sup> Furthermore, there exists an infinite number of those optimal points and the difficulty arises when both players have to come to terms about which point to choose along the bargaining curve (or bargaining plane). Various economists have given more or less convincing criteria to determine the

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<sup>20</sup> Oskar Morgenstern, "Pareto Optimum and Economic Organization", Systeme und Methoden in den Wirtschafts- und Sozialwissenschaften, edited in honor of Erwin von Beckerath, Tübingen, 1964, pp. 573-586.

final solution; almost all sustain - with Edgeworth - that players will finally arrive at least at one of those points, but it is doubtful that such a solution will necessarily result. Important progress has been made in the field of bargaining theory in the last years and the research herein is in full progress (J. Nash, R. J. Aumann, M. Maschler, B. Peleg, R. Selten).

De Finetti affirms, "from the experience of man as social animal", that the bargaining curve (i. e. , a situation where everybody is better off) is not realizable "as one player, or both, may not be disposed to discuss an alternative, beneficial to both of them, out of the fear that the discussion might endanger the existent and tolerable status quo." On this problem two recent studies are worth mentioning. One by Karl Borch on the economics of uncertainty<sup>21</sup> and one by Clem Tisdell upon the Pareto-optimality of group behavior.<sup>22</sup>

When the argument is extended to three and n-person games, not very much changes in the nature of the problems. In the case of zero-sum games again the von Neumann-Morgenstern solution holds. The formation of coalitions and the criterion of dominance will again provide the desired solution.

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<sup>21</sup>Karl Borch, "The Economics of Uncertainty", Chapter 10, Western Management Science Institute, Working Paper No. 83, Los Angeles, 1965.

<sup>22</sup>Clem Tisdell, "Upon the Pareto Optimality of Group Behavior", Econometric Research Memorandum No. 76, Princeton, 1965.

The difference between "dominance" and "majorization" has to be kept in mind.<sup>23</sup> Any imputation which is dominated by some coalition of S players is not acceptable as a solution according to von Neumann and Morgenstern, as those S players could do better by realizing that coalition.<sup>24</sup>

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<sup>23</sup>The concept of majorization was developed in connection with strictly determined games: a row vector  $a_k$  (column vector  $b_h$ ) is said to majorize another row vector  $a_i$  (column vector  $b_j$ ) of a general matrix if and only if

$$a_k \geq a_i, \quad (b_h \geq b_j)$$

i. e. , each element of the row vector  $a_k$  (column vector  $b_h$ ) is greater than or equal to any corresponding element  $a_i$  ( $b_j$ ) with at least one element "greater than".

The game is strictly determined if and only if a row or a column majorizes all the others in the general matrix of strategies, i. e. ,

$$\begin{aligned} a_k &\geq a_i, & i = 1, \dots, k-1, k+1, \dots, n \\ b_h &\geq b_j, & j = 1, \dots, h-1, h+1, \dots, m \end{aligned}$$

In such a game the row (column) player is enabled to play just one pure strategy which is better than all his other possible alternatives and the counterplayer therefore knows what to expect.

The concept of "dominance" was introduced in order to choose from all the possible imputations those which are equally acceptable as solutions, while those eliminated by this criterion are not.

Specifically, an imputation  $y = y_1, y_2, \dots, y_n$  is said to dominate an imputation  $x = x_1, x_2, \dots, x_n$  if there exists a coalition of S players among the set of N players, such that

$$v(S) \geq \sum_{i \in S} y_i$$

and

$$y_i > x_i \quad \text{for all } i \in S,$$

i. e. , the  $y_i$  are strictly greater than the  $x_i$  for all players who participate in the coalition.

<sup>24</sup>J. von Neumann, O. Morgenstern, "Theory of Games...", op. cit. pp. 263 ff.

In case each one is better off when playing alone, we have an inessential game. Institutional factors might interfere and influence the formation of coalitions or prevent them at all. Some studies exist on the impact of those factors (ceremonials) on the extensive form of games and their solutions.<sup>25</sup>

But the field of games with imperfect coalitions takes us too far away in our discussion. In non-zero sum games the difficulties already encountered in the two person case are easily seen to arise in three and n-person games. The possible solutions are of a great variety. According to de Finetti, "under whatever circumstances the coalitions were formed and whatever their imputational rules, there always exists a group of players which, forming a coalition, may achieve a better result as they do together in the coalitions which they belong to at present (and therefore an imputation is possible wherein everybody of the new coalition partners is better off)." And, de Finetti continues, "We have expressed this situation of circularity, of disequilibrium, of instability, saying that it does not allow for one solution. The desire to reexamine under various aspects again and again this intricate problem is certainly justified, not only out of the dissatisfaction of the mathematician when being unable to find something acceptable as 'a solution' to an interesting problem, but also from the possibility of

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<sup>25</sup> Robert Reichardt, "Games with Imperfect Coalitions", Recent Advances in Game Theory, Princeton, 1961, pp. 177 ff, and "Toward Sociological Applications of the Game-theoretical Treatment of Coalitions", paper delivered at the Fifth Conference on Game Theory at Princeton University, April 5-7, 1965.

finding out something not immediately obvious or foreseeable (as occurred already earlier at Waldegrave), and from the desire to give some operational indication - who knows - within the practical field of games, or the economy, or the strategy of war, or politics, or other similar subjects."

In respect to the von Neumann-Morgenstern solution, de Finetti remarks that it gives not a single equilibrium point but a whole set of possible points; there does not exist an equilibrium point if considered individually, but it exists when considered as related reciprocally by particular connections based on "fear of blackmail and fear of fear of blackmail". Other approaches, according to de Finetti, tend to consider individual stable situations when there are restrictions imposed on the alteration of the status quo, e. g. , only one individual can change coalition each time the game is played. The Shapley value, on the other hand, is viewed as the application of the minimax not as norm for the players involved but for a third party called upon as umpire. To adopt, however, an arbitration scheme of this kind again presupposes a not hostile and not exaggerated egoistic attitude of the participants.

As just mentioned, the introduction of a few additional criteria will, however, lead to determinate solutions in the sense that unique payoff vectors will be selected as the equilibrium outcomes corresponding to the bargaining power of the players, their threat possibilities. These criteria were summarized by J. Harsanyi and the connection between Zeuthen's

Principle and games determined by risk dominance (Nash solution).<sup>26</sup>

These rationality postulates can be classified into postulates of rational behavior and postulates of rational expectations. The postulates of rational behavior comprise expected utility maximization by each player, individual efficiency in the sense that each player will choose an efficient individual strategy which is not dominated by any other strategy in a weak sense, mutual optimality, i. e. , in non-cooperative games the players will always use a strategy vector that is an equilibrium point and joint efficiency, where the dominance principle is now applied to joint strategies either in cooperative or non-cooperative games.

The postulates of rational expectations are the mutually expected rationality in the sense that each player expects equally rational behavior from his counter players, and the exclusion of irrelevant variables in the solution to the game.<sup>27</sup>

In particular, if player 1 acts under the above mentioned rationality postulates, Harsanyi showed that player 1 will make further concessions to player 2 only if

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<sup>26</sup> John C. Harsanyi, "Rationality Postulates for Bargaining Solutions in Cooperative and in Non-cooperative Games", Recent Advances in Game Theory, Princeton, 1961, pp. 233-247.

<sup>27</sup> John C. Harsanyi, "Rationality Postulates for Bargaining Solutions in Cooperative and in Non-cooperative Games", op. cit., pp. 237 ff.



$$p_{12} \leq \frac{u_1(A_1) - u_1(A_2)}{u_1(A_1) - u_1(C)} = q_1$$

where  $p_{12}$  is the subjective probability that player one assigns to the possibility that player two will refuse to go beyond his last offer,  $A_1, A_2$  are the last offers of players one and two,  $u_1$  or  $u_2$  are the utility functions of player one and player two,  $C$  denotes the conflict situation if the two parties do not reach an agreement. Similarly,  $q_2$  can be calculated.<sup>28</sup> Zeuthen's Principle then is that<sup>29</sup>

(a) player one will make a further concession if

$$q_1 < q_2$$

while player two will not.

(b) Both players will make concessions if

$$q_1 = q_2.$$

The postulates of rational behavior and rational expectations then lead to the Nash solution of non-zero sum games, which again is a determinate solution.

In particular, if  $u_1$  and  $u_2$  are the utility functions of players one and two in the von-Neumann-Morgenstern sense,  $c(S)$  a solution points in  $S$ , a compact, convex set which includes the origin, then Nash assumes<sup>30</sup>

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<sup>28</sup> John C. Harsanyi, "Approaches to the Bargaining Problem before and after the Theory of Games: A Critical Discussion of Zenthen's, Hick's, and Nash's Theories", Econometrica, 1956, pp. 144-157.

John C. Harsanyi, "On the Rationality Postulates Underlying the Theory of Cooperative Games", Journal of Conflict Resolution, 1961, pp. 179-196.

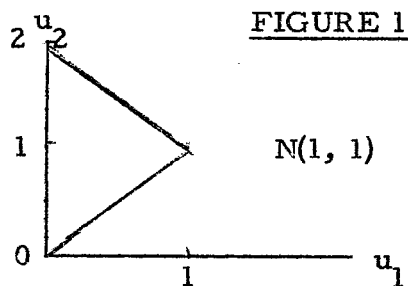
<sup>29</sup> Frederic Zeuthen, Problems of Monopoly and Economic Warfare, London, 1930.

<sup>30</sup> John F. Nash, "The Bargaining Problem", Econometrica, 1955, p. 159.

- 1) If  $\alpha \in S$  such that there exists another point  $\beta \in S$  with property  $u_1(\beta) > u_1(\alpha)$  and  $u_2(\beta) > u_2(\alpha)$  then  $\alpha \notin c(S)$ .
- 2) If the set  $T$  contains the set  $S$  and  $c(T)$  is in  $S$ , then  $c(T) = c(S)$ .
- 3) If  $S$  is symmetric and  $u_1$  and  $u_2$  display this, then  $c(S)$  is a point of the form  $(a, a)$ , i. e., on the diagonal  $u_1 = u_2$ .

The determinate solution is then given by the strategy pairs where  $u_1 \cdot u_2$  is maximized. By payoff-dominance solutions to cooperative games, two person constant sum games, non-cooperative games with solutions in the Nash sense can be found.<sup>31</sup>

Also, the Nash solution is easily acceptable in many cases. There exist however cases in which the Nash solution hardly appears to be "obvious" to both players. If an egoistic attitude prevails that point will never be achieved. An example is given by Borch:<sup>32</sup> consider the game where the possible alternatives the players can agree upon are represented by the triangle in Figure 1.



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<sup>31</sup>John F. Nash, "Non-Cooperative Games", Annals of Mathematics, 1951, pp. 286-295 and John F. Nash, "Two Person Cooperative Games", Econometrica, 1953, pp. 128-140.

<sup>32</sup>Karl Borch, "Economics of Uncertainty", Chapter 10, op. cit. pp. 14 f.

The Nash solution is given by the point  $N(1, 1)$  as one can easily see. The point  $N(1, 1)$  maximizes the product of the two utilities  $u_1$  and  $u_2$ . A closer inspection, however, reveals that the solution point might seem "unfair" to player 2: while through the cooperation of player 2 player 1 achieves the maximum of the possible outcomes, player 2 can by that cooperation only achieve half of his possible maximum of two units. By not cooperating at all he could prevent player 1 from getting any payoff at all. Especially in a competitive environment it is very unlikely that the Nash solution will be achieved at all in this game, as player 2 can always threaten with non-cooperation. Any other point in the triangle, however, is "sub-optimal" and by refusing cooperation at all, not even a Pareto optimal solution will be achieved.

There exist, however, non-zero sum games in which also the payoff-dominance postulate will not lead to a solution beyond the maximin payoffs in case of non-cooperation. I. e., in all those cases where a deadlock arises. An example is given by Luce and Raiffa, called "The Battle of Sexes"<sup>33</sup>, where in general we have the following payoff configuration:

		<u>Deadlock Situations</u>	
		<u>Player Two</u>	
<u>Player One</u>		1	2
		1	a, b
2	c, c	b, a	

with  $a \gg b > c$  or  $b \gg a > c$ .

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<sup>33</sup>R. D. Luce, H. Raiffa, "Games and Decisions", New York, 1957, pp. 90 ff.

A specific example to the second class will show the difference more clearly:

		<u>Player Two</u>	
		1	2
<u>Player One</u>	1	1, 2	0, 0
	2	0, 0	2, 1

In this conflict even communication between the two players is only of restricted value to both of them; by cooperation they will rule out the two pairs of strategies (1, 2) and (2, 1) as in these cases the payoff to both players is zero. Both can do better. But in addition to that the players have now to decide which of the two cooperative strategies (1, 1) or (2, 2) to choose, as in one case the first, in the other case the second player is better (worse) off. If the two players agree on a random device to decide which pair to use, e. g. , by tossing a coin (probability of each side 0.5), their expected payoffs will turn out to be 1.5 each or, depending on the random device they choose, anything between 1 and 2; and the sum of both payoffs will always be equal to three. But if during those negotiations about how to choose one of the admissible pairs of strategies a non-cooperative climate develops, each side insisting on the more advantageous strategy, even communication cannot solve the problem here to the satisfaction of both players. If both parties retire then and each one plays for himself, the expected payoff each player can secure for himself will be also under the rationality postulates mentioned only the maximin values: if they choose

their first strategy with probabilities  $\alpha$  and  $\beta$  respectively, their expected payoffs are

$$E_1(\alpha, \beta) = 2\alpha\beta + (1-\alpha)(1-\beta)$$

and

$$E_2(\alpha, \beta) = \alpha\beta + 2(1-\alpha)(1-\beta)$$

By choosing  $\alpha = 1/3$ , player one can secure himself an expected payoff of  $2/3$ , regardless of what player two might do.

$$E_1 = 1 - \alpha = 2/3$$

in this case, and player two can assure the same payoff to himself by playing strategy 1 with probability  $\beta = 2/3$ :

$$E_2 = 2 - 2\beta = 2/3.$$

Both values are less than the possible payoffs in a cooperational environment (1.5).

The basic difference to the previous cases (including Prisoner's Dilemma) is, that player one and player 2 can never obtain more than his maximin payoff. Games of this character are also called intractable.<sup>34</sup>

With regard to the question whether a cooperative or non-cooperative solution will be achieved by the (two) players, it will also be important whether the game is played only once or whether it is repeated, and how often

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<sup>34</sup> John C. Harsanyi, "Rationality Postulates for Bargaining Solutions in Cooperative and in Non-Cooperative Games", *op. cit.*, pp. 238 ff.

it is repeated between the two players. In a single event "bluffing" may seem to be a very efficient and profitable device. In a repetitive situation, however, through a process of learning, the two players find out that the realization of a cooperative solution will yield better results than the non-cooperative solution. One would expect therefore a convergence toward cooperation the more often non-zero sum games are played. Experiments in this regard were made and showed in effect a trend toward cooperation the larger the number of repetitions.<sup>35</sup> Decisive might also be whether there exist similar conflicts between the players in other games played simultaneously or whether those connections do not exist. Again, one would expect a more cooperative attitude when the number of various games played among them is larger. The question of whether a game is played once or more often might also affect the players' attitude toward acceptance of an arbitration scheme, a requisite to the achievement of the Shapley value in games.<sup>36</sup>

Finally, de Finetti introduces a further distinction between games and quarrels. In games only a finalistic attitude is operative and the foundation of behavior. In a game each player tries to maximize his own payoff, where the only restrictions are imposed by the possible strategies

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<sup>35</sup> L. Lave, "An Empirical Approach to the Prisoner's Dilemma", Quarterly Journal of Economics, 1962, pp. 42ff, and "Factors Affecting Cooperation in the Prisoner's Dilemma", Behavioral Science, 1965, pp. 26ff.

<sup>36</sup> L. S. Shapley, "A Value for N-Person Games", Annals of Mathematics Studies, 1953, pp. 307ff.

of the other player. In quarrels, on the contrary, behavior is determined by past events which serve as a pretext for present actions, i. e., quarrels are deterministic. This distinction of de Finetti does not mean that the latter form of behavior cannot be taken into account by a game-theoretical approach. The distinction arises from the notion that optimal behavior excludes any deterministic restraints which characterize the quarrels. The minimax in case of zero-sum games and the postulate of payoff domination in case of non-zero sum games determine the outcome of the game by selecting the set of strategies which satisfy those criteria. All solutions of this kind secure to the player at least the game value, or more in case the other player deviates from optimal behavior. One could easily take account of revengeful attitudes in the utility-payoff matrix insofar as they influence the utilities derived from different outcomes. The game still is determined by the finalistic attitude of the player, who tries to maximize his payoff. In quarrels this behavior gives way to reactions to past events. A quarrelsome attitude hurts the player who acts in this way. In this light many aspects of the present legal structure and social ceremonials are based, according to de Finetti, on a quarrel attitude and are of a deterministic character rather than oriented toward a behavior which would be optimal for societies. Though the distinction between quarrels and games will be irrelevant in zero-sum games (whatever preferences the players hold will be expressed in their utilities), in the case of non-zero sum games a quarrel attitude will give rise to non-cooperational solutions, while in absence of that attitude cooperation is likely to be achieved. Insofar the distinction will be relevant.

### III. Economic, Social and Political Implications

Based on many of the previous notions and the scepticism about the realizability of a determinate solution de Finetti then proceeds to the analysis of, not so much of what game theory says to us, but on what game theory makes us think. In this part de Finetti bases his thoughts mainly on Rapoport's article and M. Shubik's "Strategy and Market Structure".<sup>37</sup> The substance of the conclusions de Finetti comes to are the following:

1) That decisions based only on calculated self-interest can lead to non-optimal outcomes, or losses to both conflicting parties.

2) That the main emphasis has to be put on bringing about reciprocal contacts between the conflicting parties, to gain understanding of the adversary's view, his utility function so to speak, and, if possible, to cooperate even given the risk of a break of agreement by the other party.

These thoughts are extended in general to all possible conflicts in society. On the political scene, de Finetti affirms, the signs in 1963 were encouraging, and in support of this de Finetti cites Pope John XXIII, U-Thant, J. F. Kennedy, and others. We could, at least what regards statements - if not intentions, amend the list of citations by more recent ones. Apparently the benefits from cooperation in many conflicts outweigh in themselves all other possible gains from other strategies, i. e. , a rational conflict does not

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<sup>37</sup> Martin Shubik, "Strategy and Market Structure", New York, 1959.



exist in the first place. But even in games where a conflict of interest exists, as in the Prisoner's Dilemma, the concept of payoff-domination will lead "rational" players to a unique optimal solution in cooperative and (tractable) non-cooperative games. De Finetti wants to overcome maximin - deadlocks solely by cooperation between the players. However, also in the non-cooperational non-zero sum game rational players will come to a strategy, n-tuple, such that it constitutes an efficient joint strategy within the set of all (Pareto) equilibrium points. De Finetti dismisses the uniqueness of this outcome as "over-rationalization" by blackmail, fear of blackmail and fear of fear of blackmail. By bringing about cooperation, a solution will be found that dominates all other possible solutions, but again based on the same rationality postulates. The real dilemma, however, arises if one of the players does not adhere to the cooperational solution in trying to "exploit" the confidence of his counterplayers. By choosing then a best reply to the (known) strategy of the confident player, the first thinks to profit thereby assuming implicitly that the counterplayer does not realize that opportunity himself, which again violates the postulates of rational expectations. What occurs in this case is a payoff illusion, apparent in the Prisoner's Dilemma, which on a political-military basis again will have unexpected negative consequences.

Supposing an underlying scheme of gradual escalation of the conflict, a series of similar situations might occur and reoccur through time and can very easily be interconnected to a whole chainreaction of "egoistic" decisions,

where the additional element of the "quarrel" situation will come into play: because of similar past decisions one does not allow oneself to be "inconsistent", even given the realization that these same past decisions did not result in the warranted outcome. Results in experiments suggest a convergence toward cooperation. In a historical context, however, this quarrel attitude might persist, though it is irrational. In case the game is intractable, the non-cooperational solution will coincide then with the maximin strategies (deadlock situations). The maximin strategies will also occur as solutions to the game, if a completely hostile (then irrational) attitude between the players prevails, i. e. , if they set out not to maximize their own utilities but only to minimize those of their opponents. Similarly, if no information whatsoever is available between the players on their respective utilities, each player will be restricted to choose his strategies as if it were a zero (constant) sum game. All these considerations apply to political, as well as social and economic conflicts.

Bruno de Finetti especially emphasizes in this context that ideological differences in the social systems of today should not outweigh the common interest that lies in cooperation. In many ways the situation is comparable to religious conflicts and wars of the past. Similarly, ideological conflicts will settle down or be outdated by time. As religion, so ideologies should not be hostile as to interfere adversely in the conduct of human relations and as to endanger the very existence of the societies involved. In many ways,

those differences appear anyhow to be only based on concepts and reasonings of past times. Especially in cases of deadlock situations only cooperation will lead to strategies by the postulate of payoff-domination different and better than the maximin strategies. Often an arbitrational scheme will be called for, in case the parties cannot agree on a joint and efficient strategy during the bargaining process.

However, in each case, cooperation not only in attitude but also in deeds has to be achieved. We might conclude, as de Finetti did, with the Encyclical words:

"Populi omnes inter se fraterno more contemplantur, in  
iisque semper floreat semperque dominetur optatissima pax."

**DOCUMENT CONTROL DATA - R&D**

*(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified;*

1. ORIGINATING ACTIVITY (Corporate author)  Princeton University	2a. REPORT SECURITY CLASSIFICATION unclassified
	2b. GROUP

3. REPORT TITLE  
  
Game Theory and Human Conflicts

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)  
Research Memorandum No. 80, March, 1966

5. AUTHOR(S) (Last name, first name, initial)  
  
Heiss, Klaus-Peter

6. REPORT DATE March, 1966	7a. TOTAL NO. OF PAGES 35	7b. NO. OF REFS 37
-------------------------------	------------------------------	-----------------------

8a. CONTRACT OR GRANT NO. Nonr 1858(16)  b. PROJECT NO.  c. Task No. NR 047 086  d.	9a. ORIGINATOR'S REPORT NUMBER(S)  Research Memorandum No. 80
	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. AVAILABILITY/LIMITATION NOTICES  
  
releasable without limitations on dissemination

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Logistics and Mathematical Branch, Office of Naval Research Washington, D. C. 20360
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13. ABSTRACT

Review of Bruno de Finetti's article "La teoria dei Giochi" (1963) and comments on the history of game theory, the role of minimax theorem in zero sum games and non-zero sum games, its implications for the solution of social conflicts as seen by de Finetti.

14.

KEY WORDS

Game Theory History  
 Minimax Theorem  
 Zero Sum Games  
 Non-Zero sum Games  
 Cooperative Games  
 Non-Cooperative Games  
 Payoff-domination

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