ON METHODS AND PURPOSES
IN
ECONOMETRIC MODEL BUILDING

Camilo Dagum

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Princeton University
Econometric Research Program
92-A Nassau Street
Princeton, New Jersey

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1. Introduction

A model is a useful device in the building, modification, and acceptance of a scientific theory and moreover, it serves the pedagogical purpose of explanation and elucidation of complex ideas in simple, schematic form.

The characteristics and operational uses of every model are determined by the principles upon which the model is built and the objectives which it pursues.

Corrado Gini systematized a series of characteristics of these models, especially those referring to Economics, in two lectures delivered in Milan at the University Luigi Bocconi, in 1952 (Ref. 9). He pointed out that the main object of the model, like that of science itself, is to obtain an economy of thought. The models analyzed in his lectures referred mainly to economic phenomena,

> "yet, - the author says, - they are not prerogatives of Economics; on the contrary, models, in the past have been developed with special preference to other fields of knowledge. In fact, most of the sciences have turned to these models. In the realm of Physics, perhaps the best known example is that of the atom, which served as a basis for the investigations, discoveries and inventions which led to

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> the creation of the atomic bomb. Another familiar model is that of gas molecules, for which laws regarding gas distribution and pressure have been derived by means of the Calculus of Probability. Equally well known is the model of the solar system where the Copernican replaced the older, Ptolemaic model. Similarly, Newton's laws of universal gravity are based on a model which afterwards was replaced by a more comprehensive one expressed by Einstein's theory of relativity."

The construction of models in the statement of scientific knowledge, as Gini well points out, responds to the necessity of accomplishing an economy of thought. Models should sum up and abstract the permanent and outstanding characteristics of the phenomena under consideration and conceptually systematized, in order to obtain a reasonably complete and consistent body of facts. But models should not only meet the requirements of an economy of thought, in the sense of minimizing "inputs" of intellectual efforts in their attainment of a given degree of rigor; they should also have clarity and accuracy of exposition. These three qualities serve to measure the methodological and operational efficiency of any scientific theory or model. H. W. Brand (Ref. 3) has also dealt with this important subject of the fruitfulness of mathematical methods in economic theory but I think he misses the target. His argument and illustrations are hardly convincing. He develops his analysis on the surface of the argument without openly confronting it. The sterility of the effort, from which solid considerations could be expected, is due to the fact that the author takes some pre-conceptions as his starting point, and all his study tends deliberately to confirm them, but without success. Brand's argument which refers to mathematical statements in economic analysis does not add anything new to the oral statement, and can easily be compared with the clear and profound observation of an authority on the subject: Oskar Morgenstern (Ref. 14), who, after expressing that

> "eventually, a new calculus may have to be invented or discovered, as specifically suited to economic-social problems as the differential calculus was to classical mechanics," adds further:
>
> The new theory also shows clearly a feature that must be expected in any true mathematization: the mathematical theory must first yield the same insights which can be obtained from common sense. Thereafter it will give results which go far beyond common sense, results which common sense could never even guess at. When this happens, some of the new results may be translatable into ordinary language, but, for still others, this will become impossible. They will remain in mathematical symbolism. When that point has been passed, a higher state in the development of a science has been reached. It is only attainable by means of mathematics. Game theory has already entered this phase of development, although, for classical or neoclassical theory, no such instances appear to exist. This is another indication of what is involved in the successful use of mathematics."

It would be wise to quote another paragraph of the Morgenstern work:

> "if we were to ask today what the limitations of mathematics are in physics, both mathematicians and physicists would be baffled by the question, brush it aside as meaningless, and go on with their work. It could not be performed without mathematics being used to an ever increasing degree. The fact that this question is not asked is a sign of the maturity of physics and a consequence of the tremendous success mathematics had in developing that science - nay, indeed, in developing together with physics."

The extent to which a model can be called a fruitful model, as defined by the aforementioned qualities,

1. economy of thought
2. clarity of exposition
3. accuracy

depends on logical characteristics that every theory (as well as the models they outline) should have, namely: 1) Hypothesis, 2) Thesis. We shall turn to this subject later on, in paragraph 8.

2. Model Conceptions

We have already seen that the idea of the model was extensively developed in the field of physics. In the realm of the social sciences we must remember the sociologist, Max Weber, (Ref. 22) and his concept of ideal types. Weber says that,

> "the scientific method consists of the construction of types; it investigates and states all the connections of irrational sense, effectively conditioned, of the conduct that influences action, as 'shifts' of an extension of the same, 'constructed' as purely rational with regards to aims." He later adds that "the construction of a strictly rational action with regard to aims, serves Sociology in these cases - by virtue of its self-evident intelligibility and univocity, in the rational sense - as a type (ideal type) by means of which one can understand a real action influenced by irrationalities of all kinds (emotions and errors), as a shift of the extension expected of a rational action."

We must remember one of the first and most controversial models in Economics: the "homo economicus." This was a point at issue for those who only thought about the existence of an agreement between the model and reality, or held a value judgment concerning the behavior of individuals in economic activity. They ignored or set aside the systems of hypotheses which underlay the creation of the model and conditioned its practical use. This model was generalized by that of the "homo stochasticus," proposed by economists of the Cowles Commission, among whom J. Marschak was a prominent figure. The concept of "perfect competition," another classical model, proved to have a hypothesis too strong and too rigid for the real world. This model was afterwards integrated with the models of imperfect competition, oligopoly, duopoly, etc., which composed market morphology. The analysis of real people's behavior in economic activity, their power relationships, their strategies, the size of economic units and so on, led us to construct the more comprehensive and more empirically based theory of games (Ref. 21) and theory of domination (Ref. 16).

The late beginnings of a systematic and rigorous formulation of models in Economics, and the even later start in the Social Sciences, are explained by the mistaken methodological approach of economists in their use of Mathematics up until 1930. The formulation of models mainly involves the step from qualitative and descriptive analysis to quantitive and causal analysis with which mathematical statement of causality and dependence relationships is unavoidable because of the given degree of interaction between the phenomena considered.

Economists, encouraged by the successful results attained in Physics long before Adam Smith wrote his classic work, tried to assimilate the models and tools of analysis of Physics and to apply them toward the economic and social sciences in general. The unrestricted economic application of differential and integral calculus (methods derived in answer to the needs of the Physical Sciences) greatly circumscribed and often foredoomed the efforts of mathematically-minded economists.

It was from the thirties onwards that economic problems began to attract the interest and attention of mathematicians and, as a result, Economic Science started to contribute to the creation of new mathematical theorems and methods. Two names and dates marked the beginnings of the so-called Social Mathematics: John von Neumann and his minimax theorem, which was developed in 1928, and Abraham Wald with his contribution in 1934 to the conditions of existence and uniqueness of feasible solutions under several kinds of conditions (Ref. 14). The work of von Neumann and Wald stimulated the growth of a mathematical system geared to the needs of the science of economics. The growth of Social Mathematics, in turn, served to encourage a growing interest among mathematicians in the problems of economics. Out of this matrix came the pioneering efforts of von Neumann and Morgenstern which represented, for the first time, a systematization of a new mathematics built to fit the challenge of economics . Subsequent to von Neumann and Morgenstern's theory of Games came the theories of Information, Inventory, Mathematical Programming, in short, the body of Social Mathematics.

These recent contributions to Social Mathematics in no wise diminish the importance of the historic developments in Probability and Statistics, which constituted authentic contributions to the Social Mathematics, and which were in large part stimulated by those self-same problems of

Social Science. A partial list of the contribution to the theories of Probability and Statistics is replete with renowned figures as T. Bayes, A. de Moivre, J. Bernoulli, P.S. Laplace, A. Quetelet, G. Mendel, F. Galton, K. Pearson, A.A. Markov, C. Gini, R.A. Fisher. But we should observe here the dearth of mathematical, i.e., purely mathematical contributions to the development of Social Mathematics, concommitant to the contributions of Statistics and Probability, limited the application of Statistical and Probability techniques to the solution of socioeconomic problems of a mathematical nature.

3. Definitions

Most writers, whether they are philosophers, physicists, biologists, economists, sociologists, etc., agree upon the basic characteristics of models; viz., they serve as simplified representations of complex ideas. To cite a few of the many writers on this subject:

Corrado Gini (Ref. 9) thinks that a model is nothing but a simplified representation of the manner in which certain phenomena are constituted or developed. This concept is valid for all fields of learning. Moreover, we must emphasize here the explicit consideration of the structural aspect when one refers to the way in which certain phenomena are constituted - and of the functional and dynamic aspects - when one refers to the way in which observed phenomena develop.

Edmond Malinvaud (Ref. 10) defines a model as the formal representation of ideas and knowledge relating to a phenomenon. As in Gini's definition, this is valid for any field of learning.

José Ortega y Gasset (Ref. 15) states that Physics, after taking observations as his starting point, builds up ideal bodies and deduces their laws of motion, with the aim to furnish a unified scheme to which the bewildering multiformity of physical phenomena can be referred and thus ordered and reduced to a system.

Enders A. Robinson (Ref. 18) states that a model is a simplifed and idealized abstraction whose purpose is to approximate the behavior of a system. He stresses: (i) any body of theory deals with models and (ii) a model of necessity must always be a compromise between simplicity and reality.

José Luis Sanpedro (Ref. 19) defines a model as the simplified representation in mathematic symbols of a given set of economic relations.

Max Weber (Ref. 22) states that Sociology constructs concepts and tries to find rules for their occurrence.

4. Principles

In order to get a clear conception of the model and understand the process of its elaboration, we must start from Dilthey's (Ref. 8) fundamental assertion: "all science is empirical." Then, the philosopher adds, "but all experience has its original connection and its validity as determined by this connection in the conditions of our consciousness wherein it arises, in the whole of our nature."

Philosophy is excluded, since a part of it, the metaphysical objects, have no reality. Logic and mathematics also remain excluded from this conception since they are instrumental sciences that provide the method of logical process for the formation of knowledge. Nevertheless, it is worthwhile to remember that mathematics also has a good deal of empirical origin. Von Neumann discusses that theme in a lively paper (Ref. 20). He states:

> "it is undeniable that some of the best inspirations in mathematics - in those parts of it which are as pure mathematics as one can imagine - have come from the natural sciences (...) Geometry was the major part of ancient mathematics (...) There can be no doubt that its origin in antiquity was empirical and that it began as a discipline not unlike theoretical physics today. Apart from all other evidence, the very name "geometry" indicates this. Euclid's postulational treatment represents a great step from empiricism, but it is not at all simple to defend the position that this was the decisive and final step, producing an absolute separation."

As von Neumann states,

> "there are various important parts of modern mathematics in which the empirical origin is untraceable, or, if traceable, so remote that it is clear that the subject has undergone a complete metamorphosis since it was cut off from its empirical roots."

The empirical base that engenders scientific knowledge by means of logic, mathematics and statistics as instrumental sciences, involves knowledge of reality as a whole in the same way that philosophy aims to reach the unity of all our knowledge, or specialized cognition of reality that determines scientific disciplines, like Economics, History, Sociology, Physics, etc. The social scientist theorizes on the basis of empirical observation of relevant phenomena.

The empirical observations, experimental or non-experimental, play an outstanding role in the development of sciences. We can remember the most brilliant example, that of the Danish astronomer, Tycho de Brahe. After realizing that astronomical tables based on the Ptolemaic model were much too inaccurate, Tycho compiled astronomical observations that led him to abandon the Ptolemaic model but without accepting the Copernican one. These practical observations, carried over for more than twenty years and accomplished in pretelescope times, were left with Kepler in Prague, where he moved and met Kepler in 1599. On the basis of Tycho's observations, Kepler formulated his theories. His main contributions are oriented to explaining the form of which the phenomena are constituted. That is, Kepler is concerned with the structural aspect. The dynamic approach, i. e., the way that the phenomena develops, is done in this context, by Galileo Galiley. To that sense, Galileo complemented Kepler. So, the triplets: Tycho, the observer, Kepler and Galileo, the theorists, are at the very basis of the development of modern natural empirical sciences.

The corresponding social scientist in the history field came a century after. The Tycho of history was the French philosopher and critic, Pierre Bayle, regarded as a founder of eighteenth-century rationalism. His masterpiece "Dictionnaire Historique et Critique," first published in 1697, was an extraordinary source of historical information and criticism. Bayle's observations and criticism of historical life allowed Voltaire, the Kepler in history, to write his "Essai sur les moeurs et l'esprit des nations," a masterpiece of historical analysis of customs and spirit of human life. His work does not consider the outstanding features of wars and battles, political conspiracies, etc. without interconnecting them with the customs and spirit of peoples. The Galileo of history is Montesquieu who completed the Voltaire contributions, mainly in the dynamical context.

> "Montesquieu is the first to interpret historical phenomena dynamically. He conceives human life as made, in its ultimate reality, not of fixed patterns but of acting impulses, the monarchical form of government being the manifestation and result of 'honor' in action, the republican of 'virtue' in action. Honor and virtue are pure agents, when their impetus dwindles and dies, monarchy and republic decline and fall" (Ref. 15).

We gave two examples, one in natural sciences and another in social sciences, concerned with the empirical basis of scientific knowledge. Also, we must remember von Neumann's brilliant analysis about the empirical trace of some parts of mathematics. Economics is an empirical and, in general, non-experimental science. If we can manage to do experimental economics, we will be able to develop this science faster. Professor R. Selten of Frankfort am Main is oriented in this manner. He is the co-author of a book that deals with experimental economics and which is expected to be published in 1967. His students of Mathematical Economics play the role of investors, etc. This approach poses an extraordinary methodological problem that calls for a clarifying analysis. It assumes that economic students can imitate the "rational" behavior of subject of economic activity. On the other hand, it is a universally accepted principle that the human mind cannot imitate chance; Emile Borel convinced himself that he had proved this astounding theorem (Ref. 2).

Referring to economics as an empirical science, von Neumann and Morgenstern (Ref. 21) state:

> "The empirical background of economic science is definitely inadequate. Our knowledge of the relevant facts of economics is incomparably smaller than that commanded in physics at the time when the mathematization of that subject was achieved. Indeed, the decisive break which came in physics in the seventeenth century, specifically in the field of mechanics, was possibly only because of previous developments in astronomy. It was backed by several millennia of systematic, scientific, astronomical observation, culminating in an observer of unparalleled caliber, Tycho de Brahe. Nothing of this sort has occurred in economic science. It would have been absurd in physics to expect Kepler and Newton without Tycho, - there is no reason to hope for an easier development in economics."

5. The Stochastic Process

The development of the natural and social sciences has consisted of observations of real phenomena and subsequent construction of models to explain and interpret those observations.

In the progress of knowledge, we start with empirical observations (experimental or non-experimental) and later group them.

In the particular case of Economics, the subject under consideration, we are particularly interested in the behavior of individual subjects in economic activity; in their roles as consumers, producers, savers, investors, etc. Our observations will constitute the empirical base which will describe the relevant variables that serve to explain the phenomena under consideration, as well as those permanent or regular features which determine the behavior of such variables and their causal relationships.

Each observation constitutes a realization of a stochastic process, and is necessarily random and therefore precludes the possibility of prediction of outcome of individual cases. This randomness is explained by the variability of statistical observation from one case to the next, thus giving rise to random fluctuations which make prediction impossible in any given case.

To each stochastic process there corresponds a k-dimensional stochastic variable

$$\xi_t = (\xi_{1t}, \ldots, \xi_{kt}) \; ; \quad k \geq 1$$

which provide, for each realization of the stochastic process, a k empirical quantitative observation.

6. Statement of the Model

In the repeated realization of the stochastic process, we have a set of observations that constitutes a sort of empirical quantitative model, from which we can construct, accept or reject an economic theory in terms of its probability. In this set of observations, we notice a declining importance of random fluctuations in single observations giving place to statistical regularity, upon which basis an economic or econometric model can be mathematically constructed.

The process of analysis - deductive and inductive in its integrated form - leads us to the construction of a model that explains the behavior of the observed subjects in the activity of a system or sub-system; of a sector or a sub-sector of the economy.

Thus, for example, if we plan to study the behavior of subjects (in an economy) such as milk consumers in an area of unskilled workers, each observation of a household unit of a population of unskilled workers will constitute a realization of a stochastic process, which provides us with a set of statistical values, corresponding to the variables considered in a unit of time: quantities demanded, price, household income, etc. which will constitute a particular observation of the k-dimensional vector variable, ξ_t .

If we repeat the realization of the stochastic process a sufficient number of times to get a representative sample of the population of unskilled labor in the economic activity under consideration - milk consumption - we can proceed to analyze the statistical information obtained with the aim of deriving consistent regular relationships.

If we proceed in a similar way with producers and intermediaries we will be able to specify the relationships between variables which quantitatively reflect the behaviour of individuals in the economy, as consumers, producers and intermediaries. Each one of these specifications is expressed by means of an equation which is, in this particular case, a behavioral equation. These three equations provide an econometric model of the market.

Thus, if we start from the observed regularities, we can select the most relevant variables which explain the behavior of decision-units in each sector of the economy: consumers, producers and intermediaries.

We then proceed to classify the variables of economic theory according to their character: exogenous and lag variables - independent or explanatory - and dependent or explained endogenous variables. The immediate consequence of this classification is a symbolic formulation of the law of mathematical relationships postulated for the variables considered.

In our particular example, we begin with the following symbolic and simplified relations for each sector of the economy:

$$(1) \qquad \begin{aligned} & F_1 (D_t , p_t , u_{1t}) = 0 \\ & F_2 (S_t , p_{t-1} , u_{2t}) = 0 \\ & F_3 (D_{t-1} , S_t , p_t , p_{t-1} , u_{3t}) = 0 \end{aligned}$$

A further step in the model building implies the specification of each relation, i.e., to specify the mathematical correspondence between the variables of each equation. If we assume linear correspondence, the result is the following structural model:

$$(2) \qquad \begin{aligned} & D_t = \alpha_1 - \beta_1 p_t + u_{1t} \; ; \; \beta_1 > 0 \\ & S_t = \alpha_2 + \beta_2 p_{t-1} + u_{2t} \; ; \; \beta_2 > 0 \\ & p_t = p_{t-1} + \lambda (D_{t-1} - S_t) + u_{3t} \; ; \; \lambda > 0 \; . \end{aligned}$$

The additional specification of the signs of the coefficients (parameters β_1 , β_2 and λ) is derived from the empirical observations of decision making behavior in economic activity; i.e., the behavior of consumers, producers or intermediaries, respectively, as derived from the sample realization of the stochastic process.

In the standard and compact notation we have

$$(3) \qquad By_t + \Gamma z_t = u_t$$

where y_t is the vector of endogenous variables D_t, S_t and p_t ; z_t is the vector of predetermined variables, and u_t, the vector of errors terms. B and Γ are matrices of the structural parameters.

Matrix B is a third order matrix of the form:

$$(4) \qquad B = \begin{pmatrix} 1 & \beta_1 & 0 \\ 0 & 1 & \lambda \\ 0 & 0 & 1 \end{pmatrix} .$$

B, then, is a regular matrix (not singular) and it is also a triangular matrix since its elements are null on one side of the main diagonal. When maxtrix B is triangular it fulfills the necessary requirement of the recursive model. This is an especially important type of model which was developed and systematized largely as a result of the contributions of Herman Wold (Ref. 23 to 29).

Model (2) is structurally complete, since B is a matrix of rank three. Therefore, it is sequential as well as dynamic; it is a stochastic, multi-equational, linear model, and, since maxtrix B is triangular, it is recursive provided that the components of the stochastic vector are mutually independent. That is to say, that the requirement of a triangular matrix B , is necessary but notsufficient, since it is a stochastic model. If the model is deterministic, however, such a condition is necessary and sufficient.

7. Definition of Model

What we have stated above enables us to give a definition of an economic model, in conjunction with the definition of economic theory and assumption. In fact, if we start from observations and groups of observations obtained by a sample realization of stochastic processes, we notice the features of regularity which lead us to the statement of our system of assumptions. From these assumptions we derive the propositions which constitute the economic theory whose formal representation is given by the economic model.

The definition of each one of these concepts is given in the following paragraphs. (Ref. 5 to 7)

ASSUMPTION: The simplified and idealized expression of the essential and basic characteristics of the regularity observed in the behavior of subjects of economic activity, in a sample of realization of a given stochastic process.

ECONOMIC THEORY: A system of logically consistent propositions obtained from those assumptions.

ECONOMIC MODEL: The simplified and idealized mathematical formulation of economic theory.

8. Characteristics of an Economic Theory

The relevant characteristics of an economic theory are the following:

Characteristics of an Economic Theory
- logical
  - hypothesis
    - independence
    - mutually consistent
    - substance
  - thesis
    - logical consistency
    - substance
- empirical
  - generality
  - validity
- operational : operational viability

These characteristics refer to the behavior, in time (taking into account distributed lag patterns) and form (the motivations behind a decision), of the subjects of economic activity in the system or subsystem, sector or subsector, for which we are trying to formulate a theory.

Hypothesis: By hypothesis we mean the set of assumptions or initial propositions that deal with the behavior of the subjects of economic activity in relation to an assumed institutional order and technological structure. The substance of the hypothesis is an appraisal of the degree of specificity, rigor, and clarity of the assumptions. A hypothetic system should have the characteristics of internal consistency and independence.

Thesis: The thesis is the set of final propositions and conclusions concerning the behavior of the subjects of economic activity and deduced from the hypothesis. Its substance is an appraisal of the degree of specificity, rigor and clarity of the final propositions. The necessary condition for the formulation of a theory from a given hypothesis is logical consistency. In effect, given the system of assumptions, we will have an unequivocal statement of the truth or falsity of our conclusions.

Generality: The degree to which the assumptions of economic theory explain the real behavior of the subjects of economic activity in the time and space dimension.

Validity: The degree to which the conclusions or final propositions, deduced from the primary assumptions, explain the real behavior of the subjects of economic activity in the time and space dimension.

The qualifying adjective real and the concept of time-space (relatively) stress historical (concrete) characteristics, as opposed to theoretical (abstract) characteristics, of a living institutional system and a given technological structure incorporated in an economic context and within a defined period of time (in which the analysis of the behavior of the subjects of economic activity takes place).

A theory which is formulated by strict observation of logical rules need not be, nor should it be, in agreement with any determinate reality, time - space or otherwise. All that is necessary for the completion of a theory is the introduction of empirical characteristics. These, by definition, are measures of reality; respectively, they are measures of the reality of the hypothesis (generality) and measures of the reality of the thesis (validity).

Operational Viability: A theory which can be utilized towards the practical realization of its ends (decision-making, prediction, etc.,) is said to be operationally viable.

Let us now examine the interaction of the logical, empirical and operational characteristics of a theory.

The substance of the hypothesis and thesis maintain a high degree of correspondence. In effect, the more specific and concrete the hypothesis, the more specific and concrete is the resultant thesis.

By way of illustration, consider the simple market model presented in deterministic form:

$$
\begin{aligned}
D_t &= \alpha_1 - \beta_1 p_t \; ; \; \beta_1 > 0 \\
S_t &= \alpha_2 + \beta_2 p_{t-1} \; ; \; \beta_2 > 0 \qquad (5) \\
p_t &= p_{t-1} + \lambda \, (D_{t-1} - S_t) \; ; \; \lambda > 0 \, .
\end{aligned}
$$

The above model has the following basic specifications:

(a) It is operative only in a space with a decentralized economic system;

(b) Demand, within a given period, is a decreasing linear function of price in the same period;

(c) Supply, within a given period, is an increasing linear function of price in the preceding period;

(d) The increase in price in the period following any given period is an increasing linear function of anticipated excess demand as measured by the difference between the demand within the given period and the supply of the following period.

This set of facts, however, is not sufficient for the formulation of specific conclusions concerning the dynamic behavior of price p_t from period 0 in which price p_t takes on a value distinct from the equilibrium price p^*. In order to be able to formulate a conclusion we need an additional specification in respect to the parameter of λ. In effect, upon the resolution of the model, with respect to the price variable (which constitutes the logical-mathematical process that we applied to the set of hypotheses implied in [5]), we obtain the following equation:

$$(6) \qquad p_t = p^* + [1 - \lambda(\beta_1 + \beta_2)]^t (p_0 - p_t) \; ; \; p_0 \neq p^* .$$

Starting from equation (6), we can deduce the following three types of dynamic price behavior (which types constitute our thesis):

1) For $0 < \lambda < \frac{2}{\beta_1 + \beta_2}$, the dynamic behavior of price p_t approaches the former equilibrium price. Price behavior being convergent, i.e., damped;

2) For $\lambda > \frac{2}{\beta_1 + \beta_2}$, the price behavior is divergent, i.e., explosive;

3) $\lambda = \frac{2}{\beta_1 + \beta_2}$, the behavior is of a regularly oscillating form, that form being $p_{2t} = p_0$; $p_{2t+1} = 2p^* - p_0$.

Up to this point we have dealt exclusively with the logical characteristics of the theory under consideration. If our assumptions are consistent with reality, then we can say that price behavior in the time-space dimension is dependent upon the value of the parameter λ, compared with β_1 and β_2. For each of the previously demonstrated alternatives it is consistent with reality to say that price p_t either converges, diverges, or maintains regular oscillations with respect to equilibrium price p^*.

As examples of theories which satisfy the prerequisites of the logical characteristics of hypothesis and thesis we can cite the theories of perfect competition and the comparative cost theory of international trade. But these theories do not possess the other prerequisites of generality and validity. This is especially true in the case of under-developed economics. The postulates of these theories are excessively strong, and by this measure are able to add substance to the hypothesis and facilitate the reaching of incontestable conclusions in the purely theoretical aspect of economic policy. This is all by way of saying that these theories lack a firm basis in the real world, and precisely because of this, they have no practical utility.

The realization of this fact coupled with the nonconformist spirit of the scientist have led to the formulation of theories which possess a greater degree of validity and generality. As examples of scientific developments along these lines, we can cite the work of E. H. Chamberlain and Joan Robinson, which resulted respectively in the theories of monopoly competition and imperfect competition, which theories have given us a more complete picture of market morphology. In addition to representing significant contributions to the development of social mathematics, the previously cited work of Morgenstern and von Neumann also contain a high degree of the logical and empirical characteristics which we have introduced in our evaluation of the characteristics of theory.

In general, there exists a certain incompatibility between the concepts of the substance and generality of an hypothesis. In effect, to the degree that we increase the specification of the hypothesis, we gain substance and lose generality. Thus, ceteris paribus, increasing substance implies decreasing representativeness of the real world, although it should be added that this process imparts a greater operational fecundity to the hypothesis. Turning to the market model which we have previously analyzed, we see that linear specification adds substance to the hypothesis but also represents a loss in generality, provided that the stochastic process that generates the empirical observations is not linear. This linearization of the model allows us to determine, once equilibrium is lost, whether the dynamic behavior of the price converges upon the same or different level, or whether it behaves in an explosive or oscillating pattern. But, to reiterate, this greater operational fecundity which is implicit in the additional specification and which, in turn, implies the linearization of the model, leads to an increasing disparity between the hypothesis and the real behavior of the producers, consumers, and intermediaries, in the time-space dimension. In short, what we have done is to reduce the generality of the theory, a reduction which has resulted in a loss of validity in our conclusions.

In order to have practical utility, an economic theory, however general or valid it may be, needs to be operationally viable. The application of the inductive-deductive reasoning process to quantitative empirical observations, generated by a determinate stochastic process, can result in the specification of an equation-system without operational viability. Such is generally the case of the maximum likelihood method of R. A. Fisher when we try to apply it to interdependent, multiequational linear models, with at least one overidentifiable equation. This method presents us with a system of non-linear equations in unknown parameters, a problem for which we have not fully accomplished either mathematical or numerical methods for solution. Another example, with the reservation that this time our model lacks both generality and validity, is the Walrasian system of general economic equilibrium; the fact that the number of equations and unknowns are equal is not sufficient information for us to say that the model is determinate; the contribution of A. Wald in 1934 established the conditions for the existence and uniqueness of solutions.

9. Basic Prerequisites for Theories Representative of Reality

In attempting to scientifically analyze and explain a given economic phenomenon, one theory is said to be more firmly based in reality than another if it represents a simultaneous advancement of the substance and generality of the hypothesis and, provided that it works within a logical system, if it is adequate for the formulation of conclusions with a high degree of validity.

The prerequisites for the formulation of rigorous and representative theories are as follows:

1. A humanistic background (with respect to the economist);
2. A mathematical and statistical background (again, in reference to the economist);
3. The development of social mathematics, that is to say, mathematics necessary for the solution of the problems posed by the science of economics;
4. The development of a statistical method applicable to the social sciences;
5. Perfection of electronic computers;
6. Further developments of methods of numerical calculus;
7. Rich and accurate statistical series.

The simultaneous advancement of the substance and generality of a hypothetic system requires a clear understanding of the motivations of decision-making: social, religious, political, and, most importantly for our purposes, economic. Such an advancement would give the economist an understanding of the place of Economics in the context of the Social Sciences, emphasizing its distinguishing features. The realization of this goal will be a direct result of the economist's acquisition of a humanistic background. If we hope to effect the simultaneous advancement of the logical and empirical characteristics of an economic theory we must keep in mind the concept of economic relativity in a four dimensional (time-space) system.

This process will in turn require concomitant developments in the statistical and mathematical background of economics. The empirical base of economic science and the interpretation of statistical observations, as a realization of stochastic processes, demonstrate that it is necessary for the economist to have a mathematical and statistical background. Such a background would enable the economist to make a more rational judgment as to which variables are relevant in the formulation of an econometric model.

In order to effectively use the model we need:

1. The mathematical transformation of the model (using the deductive process) toward the end of reaching conclusions or final propositions;

2. The conditions of validity and uniqueness of said propositions;

3. Feasible statistical methods and their properties.

This implies a reliance upon a mathematical and statistical method, or its development. Such is the case of the previously cited contribution of A. Wald, J. von Neumann and O. Morgenstern's Theory of Games (to which we can also apply the previous commentary concerning the humanistic, mathematical, and statistical background of the economist), the methods of parameter estimation in multiequational models with overidentifiable equations, the methods of parameter estimation with errors in the variables, etc. Specifically, in attempting to apply the method of least squares to multiequational interdependent linear models, it is necessary that the model be exactly identifiable if we hope to produce estimations with acceptable statistical properties. On the other hand, the application of the method of maximum likelihood requires a solution of a system of nonlinear equations generated by the Jacobian transformation. Out of the limitations of these two methods came the discovery of various new methods of parameter estimation which represented a significant contribution to econometric theory.

The further development of electronic computers and of methods of numerical calculation will lead to the achievement of the desired degree of disaggregation of variables and sectors and to a more accurate specification of equations.

The results of these developments will be an increase in: 1) the substance and generality of a given hypothetic system; 2) thesis validity; and 3) operational viability.

Finally, in order to effect the most fruitful combination of the logical, empirical and operational characteristics of a theory, we need a great deal of accurate statistical information covering a sufficient period of time. Given this, we will be able to formulate theories of a high degree of generality. On this very important subject, Oskar Morgenstern has written an incisive work (Ref. 13) distinguished by a great number of penetrating observations. From that book, we cite the following passage:

"Econometric theory has, instead, considered another possibility: discrepancies noted between the model and the data entering into equations may appear to be due to influence of specific factors not considered in the equations, to 'hidden variables' rather than to errors and inaccuracies of the data. This, then, poses a theoretical problem of accounting for the other factors rather than one of 'mere' correction of the basic information. The two approaches are necessarily inseparable from each other; but they need to be further characterized in their mutual relation to each other.

The differences between 'true' and 'observed' variables is not necessarily that of overlooked economic forces showing up as alleged random disturbances. The differences may be far more elementary and lie in the data themselves on the basis of which the econometric theory was presumably established. The word 'elementary' must in this connection be used in both its meanings: as simple from a technical point of view and as fundamental from the point of view of theory construction."

Edmond Malinvaud stands in substantial agreement with the thinking of Morgenstern's book; in the epilogue to his Statistical Methods of Econometrics (Ref. 11) he says:

"Finally, we must never forget that our progress in understanding economic laws depends strictly on the quality and abundance of statistical data. Nothing can take the place of the painstaking work of objective observation of the facts. All improvements in methodology would be in vain if they had to be applied to mediocre data."

The interaction of statistical information with the other basic prerequisites for the formulation of theories with a high degree of generality and validity is emphasized in the following statement of Professor Morgenstern (Ref. 13):

"A simultaneous development has to occur in the theory, in its application, and in observation. The first is affected by the latter; it proves its value if it remains unaffected when either more points of observation are brought to bear upon it or when the existing measurements are refined and sharpened. It can also happen that a theory is in no perceptible way influenced by any additional and/or finer measurements. This will be the case when it is not related to the facts at all or only in such a hazy way that it deserves no consideration whatsoever as an empirical theory. It is likely that parts of accepted economics fall into this category. For example, no amount of improved observations of a modern economy will have any bearing upon the Walrasian system which, using the inadequate conceptual-mathematical notion of maximization, describes only a hypothetical case of economic organization, far removed from reality however coarsely or finely described."

10. Practical Utility of Models

The empirical base of economic science leads to an analysis of whether or not there exists a significant level of agreement between theory and reality. In order to bring about an operational use for the model, i.e., a use which is aimed at prediction and decision-making, we need to complement this empirical base by the study of the structural permanence of the equations of the model. If we find that our theory is an acceptable approximation of reality then we can apply it towards practical ends. In this connection, J. Marschak said, "knowledge is useful if it helps to make the best decisions" (Ref. 12).

We accept, reject, or modify a theory on the basis of an econometric analysis of the relation of its logical to its empirical characteristics. In order to ascertain the generality of our hypothesis and the validity of our thesis we calculate the differences between estimated and observed values and then decide whether or not the difference is significant. If it is, we reject the theory. If it is not we accept the theory and account the differences to random fluctuations.

Once it has passed the test of analysis, the theory can be utilized towards the realization of the following goals:

1. Explanation
2. Prediction
3. Decision

This realization opens to us one of the most stimulating areas of social research; more specifically, it allows us to weigh the costs and benefits, social and economic, of different policies and to apply our decision to the real world.

In order to utilize the theory in decision-making we must: 1) reclassify our variables into the categories of controllable and non-controllable, 2) purposefully determine those controllable exogenous variables which will act as instrumental variables, and 3) determine those endogenous variables which will be the objectives of the model.

At the same time the study of the degree of structural permanence of model-defining equations will be of great importance, both in decision-making and prediction models. The decision-making models, in particular, will take into account the degree of structural permanence and, according to the objectives of economic policy, whether to change or maintain a given structure. By structural permanence of an equation we mean that that particular equation will continue to explain its corresponding sector of economic activity even after technological, behavioral and institutional changes have invalidated other equations of the model. In this case we will be able to make solidly founded predictions but decisions which attempt structural changes will have a high economic, social and political cost as their only alternative to failure.

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13. ABSTRACT

This paper is concerned with the empirical foundations of the natural and social sciences, namely, the formulation and testing of theories based on logically objective analyses of empirical observations. In many instances, both in the experimental and non-experimental sciences, this involves the use of logical empiricism in a stochastic context. The manner in which a scientist begins to construct a model is described, and some relevant definitions are put forward. Illustrations are drawn from several fields, and in particular economics, in which the empirical observations are analyzed as a sample realization of a stochastic process. In this fashion the concepts of stochastic economics are introduced, together with an integrated system of definitions to cover the basic assumptions, the economic theory relevant under these assumptions, and the model into which both these are incorporated. The logical, empirical and operational characteristics of economic theories are next considered, and certain basic requirements are put forward for simultaneous advancement of the generality and validity of these theories, in order to increase their explanatory power. The paper concludes with a short analysis of the practical utility of models for purposes of explanation, prediction, and decision making.

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