ON DETERMINISTIC AND STOCHASTIC STRUCTURES

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1. Observed Structure, Objective Structure and Philosophy of Action

The concepts of structure, development and change are authentic themes of our time that called first upon the attention of the economist and more recently that of the sociologist.

The fact that these concepts have received more attention is mainly due to the non-conformist verification of the economic, political and social dramatic reality of the underdeveloped and developing countries joined by a desire for change, all which implies a triple process of analysis and decision:

a) A quantitative and causal analysis of the operant forces (variables) in the social reality (knowledge of the economic structure).

b) Formulation of the objectives to be achieved, bearing in mind the quantities of human and natural resources, in agreement with a consistent social philosophy (objective structure).

c) Formulation of a philosophy of action, which will comprise the decisions made in the time interval between observation and attainment of the objective structure. It must also include an explicit estimate of the economic and social costs due to the process of change and a comparison with alternative decisions. Among possible philosophies of action, those which should be

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considered are the philosophies which stress a will for change. This desire for change is characteristic of the non-conformist people (progressives) who are not committed to the status-quo. The philosophies to be considered should also stress the social and economic costs of maintaining the existing structure, which are characteristic of traditional societies.

2. Structure and Model

The concepts of structure and structural change already have a rigorous meaning among econometricians and mathematical economists; i.e., those economic analysts who combine theory and measure. On the other hand, literary economists generally distort these concepts with vague ideas, prejudices and sometimes even ideological implications.

F. Perroux [Ref. 21 and 23], T. C. Koopmans [Ref. 12] and J. Marschak [Ref. 17] were the first to give a clear and precise formulation of this subject. Later, A. Marchal [Ref. 16] contributed a valuable analysis and systematization.

The concept of structure is strictly related to the concept of model. The latter was treated by the author in previous papers [Ref. 6,7,8 and 9], which specified how, in economics, a system of assumptions, theory and models derive from a process of analysis based on empirical observations. It is assumed that each empirical observation associated with the behavior of the economic decision-makers is a sampling from a stochastic process. This stochastic process is the economic structure corresponding to the system, subsystem, sector, subsector, etc., submitted to analysis. This structure is the unknown that will be estimated from a sample of T observations.

Once the estimation has been done we have a specific economic structure, denoted by g_T. This economic structure is an element of the set of all possible structures, denoted by $\mathcal{G}_T$, that might explain the system, subsystem, sector, subsector, etc., of the economic activity

The set of structures $\mathcal{G}_T$, defines a model.

3. Marginal Indeterminism and Stochastic Economics

The description of structure and model made above, is concerned with stochastic models and stochastic structures. It is usual in econometrics to distinguish between deterministic and stochastic structures and also between deterministic and stochastic models. This unnecessary and superfluous distinction has its roots in the historic development of quantitative economic analysis.

Initially this analysis was strictly deterministic and had as its main purpose the determination of the exact value of the vector of the jointly determined variables as a function of the predetermined ones. The limitations of this approach led to the introduction of the stochastic component in the structural relations. The stochastic component explains probabilistically the differences between the observed and estimated values due to causes such as errors of specification in the model equations, errors of measurement, exclusion of explanatory variables, et cetera.

The introduction of the stochastic component was made only to explain what was at that time considered a residual or marginal error in the structural relations. Therefore any stochastic model constructed in that way will be called, following J. Neyman's terminology [Ref. 20], marginally indeterministic.

The present stage of the development of quantitative economic analysis is characterized by the full introduction of stochastic processes into economic analysis. According to the Jerzy Neyman terminology [Ref. 20] into his study on the indeterminism in sciences, this stage would be called dynamic indeterminism.

Dynamic-deterministic economic analysis has achieved a high level of development during the two last decades but the main interest has recently been shifted to dynamic-stochastic analysis.

4. Elements of a Model

Any model and structure are composed of some or all of the following elements:

Equations
- behavioral
- technological
- institutional
- identities

Variables
- endogenous or jointly determined (explained by the model or structure). E.g.: $y_t = (y_{1t}, \ldots, y_{Gt})$;
- predetermined (explanatory variables); include exogenous and lagged endogenous. E.g.: $z_t = (z_{1t}, \ldots, z_{Kt})$;
- stochastic $u_t = (u_{1t}, \ldots, u_{Gt})$.

Parameters
- $B = (\beta_{g,j})$; $g,j=1,\ldots,G$, $|B| \neq 0$; matrix of the coefficients of the jointly determined variables. It is regular, i.e. non-singular, of order $G \times G$.
- $\Gamma = (\gamma_{g,h})$; $g=1,\ldots,G$; $h=1,\ldots,K$; matrix of the coefficients of the predetermined variables, of order $G \times K$.
- $A = (B,\Gamma) = (\alpha_{g,i})$; $g=1,\ldots,G$; $i=1,\ldots,G+K$.
- $\Lambda = (\sigma_{g,j})$; $g,j=1,\ldots,G$; variance-covariance matrix of the stochastic variable.
- ψ other structural parameters associated with the stochastic vector (autocorrelation coefficients, etc.).

Using the preceding matrix notation, any general linear stochastic model can be set out compactly, as follows:

(1) $$By_t + \Gamma z_t = u_t$$

The model is complete if B is a regular matrix and it includes H. Wold's recursive type models as well as the Cowles Commission's interdependent models. To be a recursive model, B must be a triangular matrix and Λ a diagonal variance-covariance matrix. If (1) satisfies the necessary and sufficient condition for identifiability, then it is a set of G linearly independent equations, each of which explains the behavior of the particular sector (or subsector, etc.) of the economy corresponding to that equation.

The deterministic formulation equivalent to the linear stochastic model (1) is :[1]

(2) $$By_t + \Gamma z_t = 0 \ .$$

Determinism in the sciences has been extensively studied by different disciplines, mainly philosophy, with emphasis on dynamic analysis and on its relevance to forecasting the future through knowledge of the past.

In accordance with these observations, Russell [Ref. 24] has written that a system is said to be deterministic when given certain data, $e_1, e_2, \ldots, e_n$, at times, $t_1, t_2, \ldots, t_n$, respectively, concerning this system, and if Y_t is the state of the system at any time t , there is a functional relation of the form,

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[1] Using the modern notation and in a more generalized form, i.e., without explicit specification of the mathematical law of correspondence among the variables, we have

$$F: z_t \to y_t$$

For a stochastic model as (1), then

$$F: (z_t, u_t) \to y_t \ .$$

(3) $$Y_t = F(e_1,t_1;\ e_2,t_2;\ldots;e_n,t_n;t)$$

The system will be deterministic throughout a given period if t, in the above formula may be any time within that period, though outside that period the formula may be no longer true. If the universe, as a whole, is such a system, determinism is true of the universe; if not, it is not so.

H. Feigl [Ref. 10] stated that a deterministic model is characterized by its ability to predict, in a strict and precise way, individual events or some of their aspects.[2]

5. An Example of Stochastic and Determinist Structures and Models

We can illustrate the above concepts of deterministic and stochastic model (and structure) with some examples.

Example 1. The following multiplier-accelerator Keynesian type model is deterministic.

(4) $$C_t = \alpha + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \beta_3 Y_{t-3}$$

$$I_t = v_1(Y_{t-1}-Y_{t-2}) + v_2(Y_{t-2}-Y_{t-3})$$

$$Y_t = C_t + I_t + A_t$$

---

[2] As example of a less restrictive formulation of determinism we can consider the existence of laws of least action and a finite limit v of propagation forces (economics, physics, etc.). According to it, a model is deterministic if given the laws of least action and a finite limit v of propagation forces, then it is possible to predict exactly the state Y_t for every point (e_1,t_1) in the space-time dimension.

Given the laws of least action, a finite limit v of propagation forces and the coordinates (e,t) for all $t < t_1$, and for all points e of the space, such that the distance $|e_1-e| < v(t_1-t)$, then if it is possible to predict exactly the state Y_t for every point (e_1,t_1) in the space-time dimension, the model is said to be deterministic.

where A_t is autonomous investment, considered as an exogenous variable, $y_t = (C_t, I_t, Y_t)$ is the vector of the endogenous variables and α represents autonomous consumption.

If we assume as a real case that:

$$
(5) \quad
\begin{aligned}
C_t &= 140 + 0.50\, Y_{t-1} + 0.15\, Y_{t-2} + 0.10\, Y_{t-3} \\
I_t &= 1.50\, (Y_{t-1} - Y_{t-2}) + 0.60\, (Y_{t-2} - Y_{t-3}) \\
Y_t &= C_t + I_t + A_t
\end{aligned}
$$

then we have a deterministic structure; i.e., there is a functional relation from the predetermined variables to the jointly determined variables.

The deterministic structure (5) is an element g_T of the set $\mathcal{G}_T$ of all the possible deterministic structures corresponding to (4); i.e., $g_T \in \mathcal{G}_T$.

Example 2. A stochastic formulation of (4) is:

$$
(6) \quad
\begin{aligned}
C_t &= \alpha + \beta_1\, Y_{t-1} + \beta_2\, Y_{t-2} + \beta_3\, Y_{t-3} + u_{1t} \\
I_t &= v_1(Y_{t-1} - Y_{t-2}) + v_2(Y_{t-2} - Y_{t-3}) + u_{2t} \\
Y_t &= C_t + I_t + A_t
\end{aligned}
$$

A particular stochastic structure in the set of all possible stochastic structures in (6) is the following:

$$
(7) \quad \begin{aligned}
C_t &= 140 + 0.50\, Y_{t-1} + 0.15\, Y_{t-2} + 0.10\, Y_{t-3} + u_{1t} \\
I_t &= 1.50\, (Y_{t-1} - Y_{t-2}) + 0.60\, (Y_{t-2} - Y_{t-3}) + u_{2t} \\
Y_t &= C_t + I_t + A_t
\end{aligned}
$$

$$
f(u_{1t}, u_{2t}) = \frac{1}{2.16\, \Pi \sqrt{1-0.35}} \exp\left[-\frac{1}{1.30}\left(\frac{u_{1t}^2}{0.81} - \frac{1.18}{1.08} u_{1t}\, u_{2t} + \frac{u_{2t}^2}{1.44}\right)\right]
$$

$$
E(u_{i,t}, u_{j,t+\tau}) = 0 \qquad i,j=1,2; \quad \tau \neq 0
$$

The system of equations (7) defines a stochastic structure when the values of the component of the vector of predetermined variables, uniquely determined the conditional probability distribution of the vector of jointly determined variables.

As in every stochastic structure, (7) is specified by the numerical values associated with the structural parameters including those of the stochastic vector and the probability distribution of the latter.

6. Common Characteristics of a Set of Structures

A set of structures $\mathcal{G}_T$ defines a model when it satisfies the following four characteristics:

(i) All the structures have the same number of equations;

(ii) The structures have the same variables (jointly determined, predetermined and stochastics);

(iii) The structures have identical variables in each equation; i.e., the same a priori restrictions;

(iv) Each equation must have the same mathematical law of correspondence (functional or stochastic) among its variables.

C. Christ [Ref. 2] considers only the first three properties. The inclusion of the fourth limits the generality of the model and makes the concepts of structure and model more useful, particularly in reference to a quantitative interpretation of structural changes through statistical inference.

For example, if an estimate obtained for β_1 in (5) is significantly different from 0.50, for a new period $t_1 \leq t \leq t_2$, then the consumption function in (5) is said to have another structure.

The conclusion achieved is immediate and precise, since we are comparing two structural relations having the same mathematical law of correspondence F (linear in this case) and identical variables.

7. Structural Change and Its Causes

Once the structural change has been tested the next step is to find the causes that generated it.

In (5), a variation in β can be caused by different motives, such as change in the income distribution, change in the consumer tastes, change in savings decisions, et cetera. It is not possible to evaluate structural change by comparing equations with different mathematical laws of correspondence. If this occurs, then the structures to which the equations belong, are associated with different models. Thus, for example, the following equations belong to different models.

$$C_t = \alpha + \beta_0 Y_t + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \dots + \beta_h Y_{t-h} + \dots \tag{8}$$

$$C_t = \alpha + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} \quad . \tag{9}$$

They cannot be produced by the same process. In such a case, the analysis will consist of determining, probabilistically, which of the two specifications explains better the behavior of its respective economic sector or allows a better prediction when the equation is used for prediction purposes. In this last sense, it is convenient to recall the Janus Coefficient [A. Gadd and H. Wold, Ref. 11] that has the following properties:

(i) It measures the accuracy of the prediction;

(ii) It gives a criterion for determining whether a structural change has occurred between the observed and predicted period.

8. Stochastic and Deterministic Postulates

For a precise distinction between deterministic and stochastic structures and models, it will be necessary to introduce the concepts of condition space, sample space and probability space [Ref. 15 and 19]. These concepts will allow us to introduce the deterministic and stochastic postulates.

A sample space R is the set of all possible outcomes of a stochastic process, under a given set of conditions. Each possible outcome is called a sample point or elementary event e and a subset of sample points e is called an event E, then $e \epsilon E \subset R$. The number of sample points may be finite or infinite.

A condition space $\mathcal{C}$ is the set of all possible causes c that can produce an event E.

A product space or R and $\mathcal{C}$, denoted by $R \times \mathcal{C}$, is the set of all possible pairs (E, C). A subset of the form $E \times C = \{(e,c): e \epsilon E, c \epsilon C\}$ defines a rectangle in $R \times \mathcal{C}$.

In econometrics, a sample space R is the set of all possible values for the jointly determined variable $y_t = (y_{1t}, \ldots, y_{Gt})$ and, The above three axioms imply that a probability $P(\circ)$ on a σ-algebra $\mathcal{B}$ of events of a sample space $\mathcal{R}$ is a mapping of $\mathcal{B}$ into the real and closed interval $[0,1]$.

We now state the following postulates:

Stochastic Postulate: For every set of conditions $c \in \mathcal{C}$, there exists a probability measure $P(\cdot|c)$ on the measurable space $(R, \mathcal{B})$. If $E \in \mathcal{B}$ then $P(E|c)$ is the probability that E occurs given that condition c occurs.

Deterministic Postulate: For every $c \in \mathcal{C}$, the probability measure $P(\cdot|c)$ is a Dirac measure; i.e. there exists $e \in R$ such that $P(e|c) = 1$.

The stochastic postulate and stochastic causality can be regarded as corresponding concepts as can the deterministic postulate and deterministic causality. Further, deterministic causality is a limit case of stochastic causality[3].

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[3] Two historical statements on deterministic causality that pervade all of classical physics are due respectively to Descartes and Laplace. They are also two outstanding representatives of rationalism that characterized the "enlightenment" period.

René Descartes, in his Discours de la Méthode, states: "The long chains of simple and easy reasonings by means of which geometers are accustomed to reach the conclusions of their most difficult demonstrations, had led me to imagine that all things, to the knowledge of which man is competent, are mutually connected in the same way, and that there is nothing so far removed from us as to be beyond our reach, or so hidden that we cannot discover it, provided only we abstain from accepting the false for the true, and always preserve in our thoughts the order necessary for the deduction of one truth from another".

P. S. Laplace, in his Théorie Analytique des Probabilités, states: "An intelligence knowing, at a given instant of time, all forces acting in nature, as well as the momentary positions of all things of which the universe consists, would be able to comprehend the motions of the largest bodies of the world and those of the smallest atoms in one single formula, provided it were sufficiently powerful to subject all data to analysis; to it, nothing would be uncertain, both future and past would be present before its eyes".

9. Parameter Space

To the structural parameters of the matrices B , Γ and Λ in (1), there is associated a point in the parameters space Ω . If $\omega \in \Omega$ and N is the number of structural parameters, then $\Omega \subset R_N$ where R_N is the N-dimensional Euclidean space. Then, ω is a point in the parameter space Ω , which itself is a subspace of R_N.

Parameter Space: The set of all possible values that can be assumed by the structural parameters of a model defines a parameter space, denoted by Ω .

For example, in the following deterministic model of the Keynesian type,

$$(10) \qquad C_t = \alpha + \beta Y_t \; ; \quad \alpha > 0 \qquad 0 < \beta < 1$$

$$Y_t = C_t + I_t$$

the parameter space is:

$$\Omega = \{ \alpha, \beta \; : \; \alpha > 0 \; , \quad 0 < \beta < 1 \}$$

Its corresponding graph is indicated in Figure 1 .

In the following market stochastic model,

$$(11) \qquad \begin{aligned} D_t &= \alpha_1 - \beta_1 p_t + u_{1t} \; ; \quad \alpha_1 > 0 \qquad \beta_1 > 0 \\ S_t &= \alpha_2 + \beta_2 p_{t-1} + u_{2t} \; ; \qquad \beta_2 > 0 \\ p_t &= p_{t-1} + \lambda (D_{t-1} - S_t) + u_{3t} \; ; \qquad \lambda > 0 \end{aligned}$$

$$\Lambda = \begin{pmatrix} \sigma_1^2 & 0 & 0 \\ 0 & \sigma_2^2 & 0 \\ 0 & 0 & \sigma_3^2 \end{pmatrix}$$

$$E(u_{i,t} \, u_{j,t+\tau}) = 0 \; ; \; i,j = 1,2,3, \quad \tau \neq 0 \; .$$

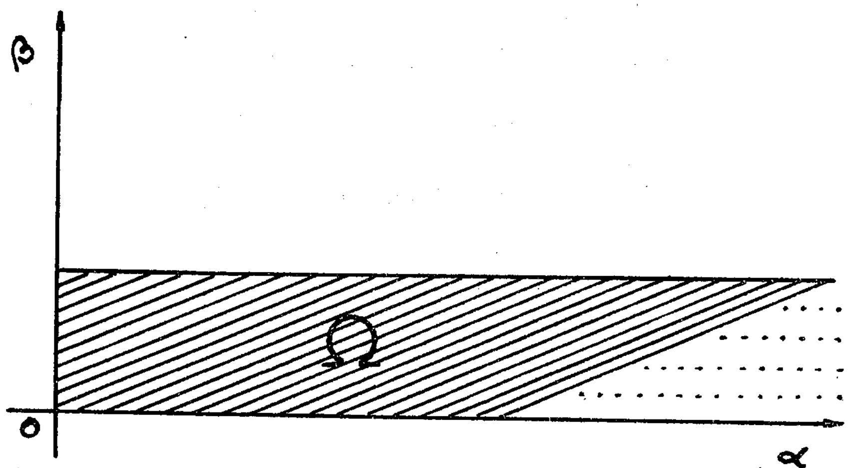

Fig. 1: Parameter space corresponding to model (10).

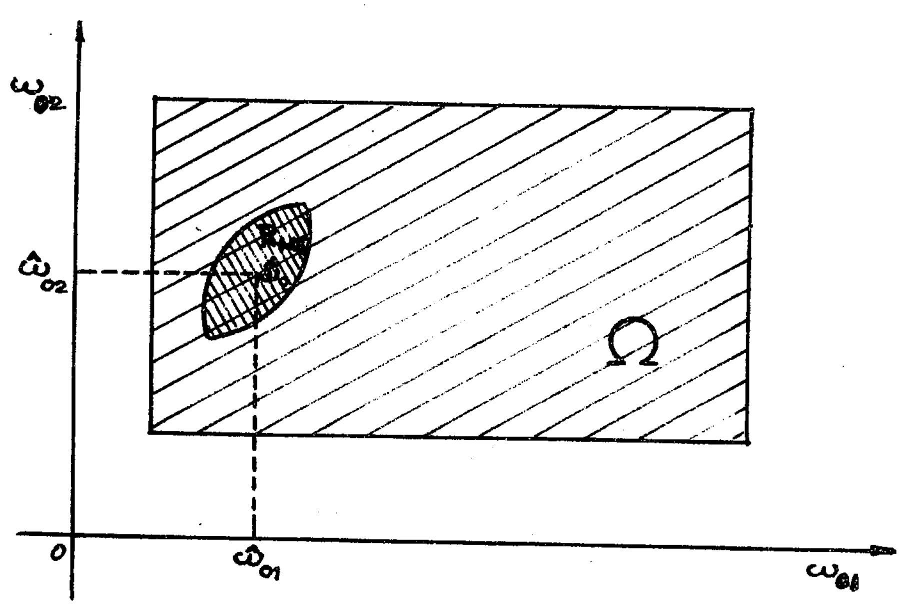

Fig. 2: Confidence region $R_{1-\alpha}$ (ellipse) and Parameter space Ω for N=2.

the parameter space is eight-dimensional, $\Omega \subset R_8$. Any point of the Euclidean space R_8 to which correspond real values for α_2 and positive values for the remaining structural parameters defines a point ω in the parameter space Ω .

10. Definition of Structure

Bearing in mind the deterministic and stochastic postulates and the concept of parameter space, it is possible to formulate a precise definition of economic model and economic structure.

Deterministic Model. A set of autonomous relations defines a deterministic model when the following specifications are known:

(i) a mathematical law F of functional correspondence among the jointly determined and predetermined variables;

(ii) a sample space R ;

(iii) a condition space $\mathcal{C}$;

(iv) a parameter space Ω .

Stochastic Model. A set of autonomous relations defines a stochastic model when the following specifications are known:

(i) a mathematical law F of stochastic correspondence among the jointly determined, predetermined and stochastic variables;

(ii) a probability space $(R, \mathcal{B}, P)$;

(iii) a condition space $\mathcal{C}$;

(iv) a parameter space Ω .

Considering that a model is a set of structures, we can then give the following definition.

Stochastic Structure. A set of autonomous relations defines a stochastic structure when the following specifications are known:

(i) a mathematical law F of stochastic correspondence among the variables in each equation or autonomous relation;

(ii) a probability space $(R, \mathcal{B}, P)$;

(iii) a condition space $\mathcal{C}$;

(iv) a parameter point $\omega_o = (\omega_{o1}, \ldots, \omega_{oN})$ in Ω .

The definition of stochastic structure has now a simple intuitive interpretation. In effect, given the mathematical law F and given the point $\omega_o \epsilon \Omega$ (i.e., the numerical values of the structural parameters), the set of conditions $C \epsilon \mathcal{C}$ (i.e., specific values to the predetermined variables) uniquely determine the conditional probability distribution of the jointly determined variables.

In reference to the stochastic model (1), we have symbolically:

$$(12) \qquad P(y_t \epsilon E_y | z_t) = |B| P(u_t \epsilon E_u)$$

The conditional mathematical expectation of the vector of the jointly determined variables under the reduced form, defines an eo ipso predictor [Ref. 25]. That is

$$(13) \qquad E(y_t | z_t) = -B^{-1} \Gamma z_t \quad .$$

The definition of deterministic structure is obvious.

11. Definition of Structural Change

Given the stochastic structure corresponding to the parameter value $\omega_o = (\omega_{o1}, \ldots, \omega_{oN})$ in Ω , a significance level α and a sample realization of a stochastic structure of size T , then there exists a point estimate

$\hat{\omega}_o$ for ω_o and a random ellipsoid $R_{1-\alpha}$ in Ω, centered at $\hat{\omega}_o = (\hat{\omega}_{o1}, \ldots, \hat{\omega}_{oN})$ and corresponding to a level of significance α such that

$$P(\omega_o \epsilon R_{1-\alpha}) = 1-\alpha$$

The ellipsoid $R_{1-\alpha}$ is a $100(1-\alpha)\%$ confidence region for ω_o and $1-\alpha$ is the confidence coefficient. Further, the ellipsoid $R_{1-\alpha}$ is an estimator region for ω_o such that the probability is $1-\alpha$ that the ellipsoid contains the structural parameter ω_o.

In a similar manner we can set up an estimator region for any subset of the parameters in the vector ω_o. This is of particular importance for multiequational models of the recursive type because of the possibility of obtaining an estimator region for the parameters of each equation. For $N = 2$, $\omega_o = (\omega_{o1}, \omega_{o2})$ and the confidence region is an ellipse (Figure 2). For a single ω_o, say ω_{o1}, the estimator confidence interval is, of course, a one-dimensional region estimator interval. We can now give the following definition.

Structural Change. A structural change has occurred when a structure for the period $t_o \leq t \leq t_1$, $t_1 > t_o$, is estimated to be the point $\hat{\omega}_o \epsilon \Omega$ in the N-dimensional space, i.e.,

$$(14) \quad \hat{\omega}_o = (\hat{\omega}_{o1}, \ldots, \hat{\omega}_{oi}, \ldots, \hat{\omega}_{oN}) = (\hat{\omega}_{oi}) \; ; i=1,\ldots,N$$

and, when for the period $t_s \leq t \leq t_2$ $(t_s > t_o \; ; t_2 > t_1)$, the structure has been estimated to be

$$(15) \quad \hat{\omega}_1 = (\hat{\omega}_{11}, \ldots, \hat{\omega}_{1i}, \ldots, \hat{\omega}_{1N}) = (\hat{\omega}_{oi}); \; i=1,\ldots,N$$

and when the difference $\hat{\omega}_1 - \hat{\omega}_0$ does not admit the null hypothesis.

The new vector $\hat{\omega}_1$ belongs to Ω . If the structural change has been observed in the parameters of only one equation or a proper subset of equations belonging to that structure, then the remaining relations are said to have permanence or structural autonomy; i.e., in reference to the technological, institutional, or behavioral causes that produced the change[4].

12. Economic Development and Structural Change

The concepts of structure and structural change are useful for defining and programming economic development.

One of the most widely accepted definitions of development is due to F. Perroux [Ref. 22] and states: Development is a regular and permanent growth of Gross National Product brought about by a combination of psychological and social changes in the population.

This definition emphasizes that in order for there to be economic development there must be changes in the behavioral equations corresponding to psychological and social aspects of the population. These behavioral changes will be followed by institutional and technological changes. These must be such as to increase economic efficiency and social welfare; i.e., there must be greater productivity of resources,an equitable income distribution and larger investment in the social and economic infrastructure.

[4] Gregory C. Chow [Ref. 1] has proposed a testing method for the null hypothesis of the difference between two points in the parameter space Ω when we are presented with two linear uni-equational structures belonging to a given model with normal distribution in the disturbances. It is possible to extend this result to non-linear multi-equational structures, using Transvariation Theory [Ref. 3 and 5].

The realization of development requires as a necessary though not sufficient condition, that the new structure differs significantly from the old one; i.e., the new estimated vector $\hat{\omega}_1 \in \Omega$ must show a statistically significant difference from the original vector $\hat{\omega}_o$.

Abstract

The concept of a modern society as opposed to a traditional (backward) society and the idea of structural change underline nearly all up to date economic policies of development. The notion of structural change is at stake first of all in the current polemic on causes of inflation between the Latin American school of inflation (structuralist school) and the more orthodox monetarist school. We can trace it also in the theoretical foundations of any economic policy. Nevertheless, the concept of structure has not a clearcut statement outside econometrics. The purpose of this paper is to develop a quantitative and operational concept of structure and, as a consequence, a quantitative and operational concept of structural change. With that aim the deterministic and stochastic postulates are introduced in terms of the notion of probability space. These concepts together with the notion of parameter space bring out definitions of stochastic and deterministic structures and structural changes. These elements allow a direct interpretation and measure of economic development in terms of a notion of structural change derived from: (a) an observed structure, (b) the formulation of an objective structure and (c) a given decision model (philosophy of action).

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The concept of a modern society as opposed to a traditional (backward) society and the idea of structural change underline nearly all up to date economic policies of development. The notion of structural change is at stake first of all in the current polemic on causes of inflation between the Latin American school of inflation (structuralist school) the the more orthodox monetarist school. We can trace it also in the theoretical foundations of any economic policy. Nevertheless, the concept of structure has not a clearcut statement outside econometrics. The purpose of this paper is to develop a quantitative and operational concept of structure and, as a consequence, a quantitative and operational concept of structural change. With that aim the deterministic and stochastic postulates are introduced in terms of the notion of probability space. These concepts together with the notion of parameter space bring out definitions of stochastic and deterministic structures and structural changes. These elements allow a direct interpretation and measure of economic development in terms of a notion of structural change derived from: (a) an observed structure, (b) the formulation of an objective structure and (c) a given decision model (philosophy of action).

14. KEY WORDS	LINK A		LINK B		LINK C	
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