FLUCTUATIONS IN THE AUSTRALIAN ECONOMY: VIEWED BY FREQUENCY TECHNIQUES

S. Peter Burley

Econometric Research Program
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Econometric Research Program
Princeton University
207 Dickinson Hall
Princeton, New Jersey

ABSTRACT

The Australian economy has long been characterized by a fluctuating growth pattern concerning which any precise knowledge is rather limited. The analysis of this phenomenon has been complicated by the fact that a large sector of the economy produces for export, and so might be expected to import fluctuations from other countries.

This study is concerned to develop and use some new frequency techniques of time series analysis to describe, and make inductive inferences concerning, fluctuations in the growth rates of Australian economic time series. Particular attention is paid to the situation since World War II, when there has been a fairly regular "stop-go" growth cycle in many variables, and when export prices, especially wool prices, have fluctuated strongly.

As had been found previously for other economies, the methods of spectral analysis and complex demodulation provided a concise description of most cyclical phenomena of interest. In this study a related technique of data analysis is suggested, in the form of a consideration of the sequence of relative phases of the demodulates of different economic time series. It is shown that these are closely related to the coherence and phase of the cross-spectrum, and that in practice they are highly stable and interesting statistics to use in describing dynamic economic processes.

It was seen that the sequence of relative phases of demodulates provides a systematic alternative to the highly subjective and arbitrary graphical techniques popularized by the National Bureau of Economic Research as a way of analyzing business cycles. Further this type of data analysis provided some new and useful insights into the cyclical dynamics of the Australian economy, such as the fact that the post-war business cycle

appears to be unrelated to wool price fluctuations, which in turn appear to be connected with activity fluctuations in the major industrial countries of the Northern Hemisphere. Also it appears that labor market variables lag at least some production indicators, and that the Phillips curve provides a good description of the relationship between demand for labor and the rate of change of average earnings over the cycle.

Further it is shown that the behavior of the relative phases of demodulates over time, and over a range of frequencies, provides useful information about the stability and time structure of the hypothetical lagged relationships proposed by economic dynamics. This is illustrated by a discussion of attempts to estimate the time lag of the Australian accelerator function. An endeavor is also made in this context to render explicit, in the shape of an uncertainty principle, the limitations inherent in statistical attempts to estimate the dynamic structure of non-stationary economic processes.

PREFACE

Problems associated with fluctuating growth are very much at the forefront of present research efforts in Australian economics. However, despite the great attention directed at these questions, our understanding of them is far from complete. No doubt this is due in a large measure to the fact that there is so little empirical knowledge to guide research in this field. One reason for this may be the restricted coverage and limited reliability of the data available. More important however seems to be the fact that until recently there has been a lack of really suitable statistical tools to derive, from the available economic time series, information which is immediately related to the theoretical concepts of economic dynamics.

The present thesis is undertaken with the objective of helping to develop and use techniques permitting a better statistical description of the physiology of fluctuations in the growth of the Australian economy by taking advantage of some new frequency-spectrum methods of time series analysis. Although this study is the first approach of its kind in Australian economics, the reader will note its relation to the recent work of Morgenstern, Tukey, Godfrey, Granger, Hatanaka and others who pioneered the use of spectral analysis in econometrics. To all these individuals I stand in debt for the paths they made, and to Professors Godfrey and Tukey especially for the helpfulness they showed initiating me into their techniques of applied spectral analysis.

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Secretariat and the Australian Commonwealth Bureau of Census and Statistics.

All research work for this thesis was done while I was a member of the Econometric Research Program of Princeton University. I am particularly grateful to its director, Professor Oskar Morgenstern, for his general encouragement throughout, and for serving on my thesis committee together with Professor Godfrey. The committee's many helpful suggestions and continual interest have benefited this study enormously in both form and content.

The computational parts of the work were carried out on the IBM 7094 computer at the Princeton University Computer Center. These computer facilities were supported in part by National Science Foundation Grant NSF GP579 and, for the present project, by a grant from the Textile Research Institute. Finally I must record my thanks to the Ford Foundation for Fellowship aid in the year 1966-7 and to the International Wool Secretariat for their support of a research grant through the Textile Research Institute in the year 1967-8.

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CHAPTER ONE

INTRODUCTORY REMARKS

I. Orientation of the Study

This thesis describes an attempt to use some recently developed frequency domain techniques of time series analysis to describe fluctuations in the Australian economy. The statistical background of the computational methods used was summed up some years ago as a result of the earlier work by Tukey[1] and Bartlett[2] in laying the foundations of empirical frequency spectrum analysis. No attempt will be made here to review, or significantly extend, this theory. The aim is rather to complement endeavors to develop and apply Fourier techniques to econometrics, as has been undertaken notably by Morgenstern[3], Granger[4], and Godfrey[5]. Accordingly statistical theory

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[1] R. B. Blackman and J. W. Tukey, The Measurement of Power Spectra, (Dover, New York, 1959).

[2] M. S. Bartlett, Stochastic Processes, (Cambridge University Press, Cambridge, 1955).

[3] Oskar Morgenstern, "A New Look at Economic Time Series Analysis", Money, Growth and Methodology and Other Essays in Economics in Honor of Johan Akerman, Hugo Hegeland Ed., (C.W.K. Gleerup, Lund, Sweden, 1961). The bibliography of this essay summarizes the literature leading up to the modern use of Fourier methods in econometrics.

[4] C. W. Granger in association with M. Hatanaka, Spectral Analysis of Economic Time Series, (Princeton University Press, Princeton, N. J., 1964).

[5] M. D. Godfrey, "The Statistical Analysis of Stochastic Processes in Economics", Kyklos, February 1967, pp. 373-386.

will receive only a heuristic treatment, with the emphasis being placed on an effective characterization of the properties of Australian economic time series and the derivation of numerical measures related to the analytic aspirations of economists interested in the fluctuations found in such series.

It should be emphasized at the outset that these newly developed techniques of spectral analysis are in principle entirely different from both the charting methods developed by the National Bureau of Economic Research and the (lagged) regression equation techniques originally associated with the names of Tinbergen and the Cowles Commission and more recently with the Brookings Institution. However, features of both these approaches will also be discussed in the hope of contributing towards a more integrative view of Australian dynamic econometrics as well as viewing it from a new perspective. It should also be observed that the spectral approach represents a considerable technical advance on the simpler periodogram methods whose earlier use in economics has been admirably described by Davis.[6]

As the use of spectral methods in economics is still rather novel, this study will be somewhat exploratory from the point of view of technique. Thus, early in the thesis, it will be necessary to consider the ways in which spectral models provide a language in which economic assumptions may be stated about the observed Australian time series. Then it will be possible to go on to select the more realistic and statistically tractable of these assumptions to characterize the way in which the data can be considered to have been generated. Consequently the approach will be descriptive and preliminary compared with the more usual statistical and

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[6] H. T. Davis, The Analysis of Economic Time Series, (Principia Press, Bloomington, Indiana, 1941). I am grateful to Professor Godfrey for drawing my attention to the potentialities of this source, especially its discussion of the moving periodogram.

econometric procedures of estimating coefficients and making predictions from parameterized models. The interest here will be rather in discovering what light spectral methods can shed on the general nature of the fundamental relationships of economic dynamics. Others have pointed out the primitive state of scientific knowledge of this subject, especially as it applied to the Australian economy, is one of the reasons for the dearth of attempts at Australian econometric model building. Thus one long-time worker in this field, in reviewing the work to date, observed:[7]

> ... dynamic economic theory that must provide the framework for forecasting models is not in a very good state.

In order to set this analysis in context, it seems appropriate to begin by outlining some peculiarities of the Australian economy and the general nature of the economic time series describing it, as these will form the subject matter of this study.

II. The Australian Economy

The Australian economy differs from that of most other advanced nations in having a relatively large primary sector, which sells most of its output (largely wool) in overseas markets. The primary sector, is thus directly subject to the notorious instabilities of international commodity markets as well as the almost equally variable character of the Australian climate. However the secondary and tertiary sectors of the economy lead a rather different existence in half a dozen large urban communities sheltered by tariffs and import controls from many changes in the external environment.

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[7]D. J. Smyth, "How Well Do Australian Economists Forecast?", The Economic Record, June 1966, pp. 293-311.

These characteristics make it necessary for econometricians to adopt different approaches to the farm and non-farm sectors of the economy. Thus even the traditional aggregate consumption function does not fit well with post-war Australian data, largely because of wide fluctuations in wool growers' incomes which have not been reflected in consumption expenditure.[8] Similarly, students of the Australian accelerator function have had to disaggregate farm and non-farm stocks because of the very different realities they seem to represent.[9] Hence, in an empirical study of fluctuations in the Australian economy such as this, it will be wise to begin by avoiding a specious aggregation of primary and other sectors.

The oldest major industry in Australia and the one which until recently has been held to determine its entire condition is of course the wool industry. As one Australian observer has put it:[10]

> Exponents of the conventional wisdom in Australia have long held that Australia rides on the sheep's back -- a claim that, at least until recently, has gone virtually unchallenged. From time to time, of course, someone has pointed out that the sheep provided a rather uneven and occasionally uncomfortable ride and has suggested we should seek some alternative conveyance of a more stable and dependable kind.

However, despite the traditional importance of its primary sector, Australia now counts as an industrialized economy, dominated by its secondary and tertiary sectors. This is largely the result of changes which took place

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[8] Jan Kmenta, "An Econometric Model of Australia, 1948-61", Australian Economic Papers, December 1966, pp. 131-164.

[9] D. J. Smyth, "The Inventory and Fixed Capital Accelerators", The Economic Record, August 1960, pp. 414-418.

[10] J. Lewis, "The Present Position of Wool in Australia", The Australian Quarterly, December 1959, pp. 1-20.

during the recent war and post-war eras. Again to quote a local writer:[11]

> In 1937/38 the net value of all primary production exceeded that of factories by about nine per cent. In the subsequent period the relationship has been quite reversed. Now the manufacturing contribution stands about eighty per cent higher than the primary; there can no longer be doubt about the status of Australia as an industrial society.

In Chapter Two of this dissertation, we shall discuss the cyclical dynamics of this recent acceleration in industrial growth. In Chapter Three, we shall consider the fluctuations in the longer established Australian wool industry. Then in Chapter Four, we shall consider attempts to pass from the description of cyclical processes to making inductive inferences concerning all fluctuations using a probability model. Finally, in Chapter Five, we shall sum up the major findings and lessons of this largely exploratory attempt to use some new statistical techniques to shed light on a variety of economic and econometric questions concerning Australian economic fluctuations. In the remainder of this chapter, we will discuss the properties of the raw materials of our study, Australian economic data.

III. Data Considerations

Australian economists have often complained of the limited coverage of the available figures compared with those of most other advanced countries.[12] However, as Appendix I of this thesis indicates in some detail, there does exist a vast collection of local statistics of interest to economists and which are available from various official sources in Australia. Doubtless to say, many more would be desirable,

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[11] Lachlan McGregor, "The Development of Manufacturing in Australia", Readings in Australian Economics, N. Drohan and J. Day, Eds., (Cassell Ltd., Melbourne, Australia, 1965), pp. 300-316.

[12] D. J. Smyth, "How Well Do Australian Economists Forecast?", The Economic Record, June 1966, pp. 293-311.

but much of the existing stock still awaits efficient use. It might be more profitable to remember that practically all the data used by economists are not collected with their analytical ends in view, but rather as a by-product of certain administrative procedures. Hence quite generally economic data have only a loose relationship with the "realities" envisaged by economic theory, being masked by both conceptual differences and (often considerable) measurement errors.[13]

The aim of this dissertation is to indicate some ways in which modern methods of data analysis can be applied to the imperfect figures available to obtain some information about the dynamic behavior of the Australian economy. The aim will be to set up suitable averaging procedures which will automatically capture the variations and covariations of interest but cancel out errors and other irrelevant factors in the masses of figures that are available. All efforts will be made to use statistical procedures which are as efficient as possible and hence the limits of our ability to obtain information will depend on the properties of the data used. One very important limit already mentioned is the accuracy of the data. No statistical procedures can extract information that simply is not there; and even if the information is present, but in a form masked by large random errors, then a large number of suitable replications will have to be averaged over to cancel out their effects. Also in the time series case, there is a new dimension to the information available, and sought from, the data, namely time. The time structure of the sample will thus be a limiting factor of its ability to provide a description of the system under consideration. For example five consecutive monthly figures will not serve to

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[13] Oskar Morgenstern, On the Accuracy of Economic Observations, (Princeton University Press, Princeton, N. J., 2nd ed., 1963).

identify a seasonal pattern. Indeed, in a statistical situation, twelve consecutive monthly figures would also leave much unsaid since it is only a sample of one so far as the seasonal effect is concerned. One of the objectives of modern time series analysis, and of this dissertation, is to consider what types of dynamic structures can be identified from various forms of time series data. But first we may note that official figures are not usually published in a form directly useful for spectral methods of analysis, and so it will be necessary to consider suitable ways of transforming them into a more useable form.

A. The Use of Proportional Changes in Variables

A methodological departure which this study makes from the more usual practice of econometrics is its consideration of proportional increases in published economic data over time, rather than the quantities themselves. The economic rationality of such a procedure was indicated long ago by Marshall in what seems to be a fundamental methodological observation:[14]

> There is more than a superficial connection between the advance made by the applications of the differential calculus to physics at the end of the eighteenth century and the beginning of the nineteenth, and the rise of the theory of evolution. In sociology as well as in biology we are learning to watch the accumulated effects of forces which, though weak at first, get greater strength from the growth of their own effects, ... This conclusion will remain valid even if further investigation confirms the suggestion, made by some Mendelians, that gradual changes in the race are originated by large differences of individuals from the prevailing type. For economics is a study of mankind, of particular nations, of particular social strata; and it is only indirectly concerned with the lives of men of exceptional genius or exceptional wickedness and violence.

[14] Alfred Marshall, Principles of Economics, (Macmillan and Co. Ltd., 8th ed., reprinted 1949), p. 694.

Later work notably by Cassel, Von Neumann, Harrod, Domar, and Joan Robinson has brought to a high state of development the notion of an economy growing exponentially as the result of impersonal economic laws. These models undoubtedly capture more in realism than the earlier stationary and static models, and although they abstract from some of the richer dynamics of observable economic data and say nothing about their statistical character, they suggest the appropriate background for an investigation of these further features.

Thus it is suggested that an acceptable statistical model for real world data might consider them as being generated by a multivariate stochastic process in the <u>proportional changes</u> of variables per unit time interval: these are the measures which should tend to show statistical stability, not the original variables themselves. Specifically if one is concerned with the fluctuations of a cumulatively growing economic process, it will ordinarily be found that the proportional fluctuations show more stability over time than the absolute fluctuations. Hence, on taking logarithms, one obtains a series which looks uniformly irregular about a linear upward trend (since on a log scale equal differences measure equal proportional movements, and a linear trend represents a constant growth rate). So, if we then take first differences, this trend is removed and we obtain a series homogeneous in the sense that one part looks very much like another, and hence it is an appropriate variable to form averages over. We may note in passing that provided the percentage changes between successive values of the original series are small, then the first difference of logarithms series corresponds approximately to the sequence of incremental growth rates per unit time.

B. Stationarity of Processes

In order to undertake a systematic statistical analysis, it is necessary to refine somewhat further the assumptions concerning the processes under study. Thus, for time averages to provide reasonable estimates of a stochastic process, it is necessary that the random mechanism governing the process should be well behaved over the time span of the average. We will be particularly concerned with linear interaction terms between series which (hopefully) are time-invariant.

Such a notion is implicit in the principles of Galilean relativity which underlie the relationships of the observables of classical physics. The reasonableness of invoking similar invariance properties for the relationships between proportional changes in economic variables in a growth situation will depend on many things staying constant. In an overall sense, as Marshall pointed out concerning some related problems, it is necessary to have an epoch where:[15]

> ....the methods of production and trade change but little; and above all where the character of man himself is a constant quantity. For in such a case by far the most important conditions of production and consumption, of exchange and distribution will remain of the same quality, and in the same general relations to one another though they are all increasing in volume.

Clearly this precludes any unusual interference with the normal workings of the economic relationships under consideration, such as would be the result of wars or other major changes in economic organization. Hence there is need for care and *a priori* knowledge in the selection of variables and historical periods to be used as statistical samples so as to ensure that the data are free from the influence of factors in which there is no

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[15] Alfred Marshall, *op. cit.*, p. 306.

current interest. In this study, particular attention will be paid to some fairly simple cyclical phenomena which it is felt may satisfy these conditions better than most economic mechanisms.

It is often useful to assume that all the data series being analyzed can be considered as being generated by a random process in which only the relative timing of matching events (and not their absolute historical time) is significant, then the lagged moments and lagged cross moments can be estimated consistently by averaging down a single realization of the data series. Under these conditions, the process is said to be "stationary".

Since spectral and cross spectral analysis is crucially concerned with functions of second order lagged moments, it will suffice to assume that these moments have the required stability over time -- a condition on the process known as "second order stationarity". This condition is never likely to hold exactly for economic data collected over time, but provided it holds approximately, reasonable estimates can be obtained for the average over the time period of the spectral functions of interest. Although it is rarely stated explicitly, it should be remembered that a similar idea of stationarity is also necessary for the validity of the more usual time domain techniques of econometrics, in order to ensure that it is reasonable to think in terms of a convergence of the time average estimates on some fixed ensemble parameters.[16]

The more sophisticated of these techniques appear to be derived from some of the earlier work of Yule who also noted that the fluctuations

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[16] A. S. Goldberger, Econometric Theory, (John Wiley and Sons, New York, 1964), pp. 142-155. Actually what is required is that the process be "ergodic" which in general is a stricter condition than second order stationarity. However as Birkoff, Von Neumann and others have shown, a process that is stationary to any order (and this is true of a second order stationary normal process) is also ergodic provided it satisfies the conditions of metric transitivity, see e.g. Emanuel Parzen, Stochastic Processes, (Holden-Day, San Francisco, 1962), pp. 72-76.

observable in economic time series appeared to be similar to those generated by an "autoregressive process"[17]:

$$x(t) = \sum_{j=1}^{k} a(j) \cdot x(t-j) + n(t) \tag{1.1}$$

where n(t) is a random error term.

The corresponding time path for x(t), starting at time t=0, is given by the solution to Equation 1.1.

$$\begin{aligned} x(t) &= \sum_{j=0}^{k} c(j) \cdot z(j)^{t} + \sum_{j=0}^{\infty} b(j) \cdot n(t-j) \\ &= \qquad x(1,t) \quad + \quad x(2,t) \end{aligned} \tag{1.2}$$

Thus x(t) can be expressed as the sum of a deterministic function of time x(1,t), and a non-deterministic moving average of noise terms x(2,t). If the roots z(j) of the characteristic equation:

$$z^{0} - \sum_{j=1}^{k} a(j) \cdot z^{-j} = 0 \tag{1.3}$$

are complex roots and with modulus equal to one, then the x(1,t) component can be represented by the deterministic "signal":

$$x(1,t) = \sum_{j=0}^{k} d(j) \cdot \cos(\omega(j) \cdot t + \theta(j)) \tag{1.4}$$

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[17] G. U. Yule, "On a Method of Investigating Periodicities in Disturbed Series", Transactions of the Royal Society (Series A), April 1927, pp. 226-298.

Further work, notably by von Neumann, Wold, and Cramér, to be discussed later, indicates that every process stationary to second order can be decomposed into two noncorrelated components:

$$x(t) = x(1,t) + x(2,t) \tag{1.5}$$

where x(1,t) is of the form shown in Equation 1.4. Empirical spectral analysis, however, is theoretically concerned with the non-deterministic term:

$$x(2,t) = \sum_{j=0}^{\infty} b(j)\cdot n(t-j) \tag{1.6}$$

From the point of view of the empirical description of finite samples all stationary series can be represented this way. However the coefficients b(j) of Equation 1.6 can be such that the behavior of x(2,t) is rather similar over a finite sequence to that of the series x(1,t) shown in Equation 1.4. In such "signal-like" cases, the language of deterministic processes (e.g. "cycles") can be a useful shorthand and will be used extensively in this thesis. It will be recollected that stationary deterministic components are possible results of the difference and differential equation models of economic dynamics cast in non-stochastic linear forms.[18]

At the other extreme, the coefficients b(j) might be of the Kronecker delta form, so that the behavior of x(2,t) can be similar to that of a random noise series, n(t). In such "noise-like" cases, the concepts of classical statistics (e.g. "degrees of freedom") can be useful. Such notions are normal in econometric studies which commonly employ classical

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[18] P. A. Samuelson, "Interactions Between the Multiplier Analysis and the Principle of Acceleration", The Review of Economic Statistics, May 1959, pp. 75-78.

estimation techniques which depend for their full validity on the data having a "noise-like" character.

In practice of course, we would expect the properties of economic time series to lie somewhere between these two characterizations. The question as to which, if any, of them is particularly appropriate to describe the business cycle fluctuations in the Australian economy over the periods in question seems to be an empirical consideration and one which this thesis is especially concerned to investigate.

Making the preliminary assumption that many aspects of the post-war Australian economic growth processes can be considered to satisfy the second order stationarity conditions, we now turn to a study of some of them from the point of view of spectral analysis.

CHAPTER TWO

LEADING AND LAGGING BUSINESS CYCLE FLUCTUATIONS

I. Post-war Economic Time Series

As has already been observed, the Australian economy, especially in its secondary and tertiary sectors, has grown rapidly since World War II. This growth however was not continuous, but took place in a rhythmic "stop-go" pattern; and it is with such "growth cycles" rather than the overall average growth itself that we will be concerned.

Recently there has been an interest in approaching this subject by an application to Australian statistics of some well-known methods devised by the National Bureau of Economic Research (N.B.E.R.) for analyzing the U. S. business cycle.[1] Specifically peaks and troughs are found for individual economic time series showing the business cycle, either in the raw data or in short moving averages of it. From the conspectus of these, the peaks and troughs of a "general reference cycle" are determined. Finally a distinction is made between "leading", "roughly coincident", and "lagging" series according to the typical relationship between cycles of individual series and "the general reference cycle". One of the strongest findings of this type of analysis in Australia is that both registered unemployment and unfilled vacancies are roughly coincident with the general reference cycle, (though it has been suggested that female vacancies may show

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[1] J. S. Mallyon, "Statistical Indicators of the Australian Trade Cycle", Australian Economic Papers, June 1966, pp. 1-20; and A. M. Waterman, "The Timing of Economic Fluctuations in Australia: January 1948 to December 1964", Australian Economic Papers, June 1967, pp. 77-102.

a slight lead of about one month). This finding seems to be in accordance with a popular custom of using either of these series (or the difference between them) as a measure of excess demand in Keynesian style analyses of macro fluctuations.[2]

The aim of this chapter is to re-examine this whole area, using the methods of spectral and cross-spectral analysis to achieve a better characterization and estimation of the properties of the business cycle in different areas of the Australian economy. It should be observed at the outset that it is a contention of this thesis that the four to five year Australian business cycle is a phenomenon of the "business" sectors of the economy. Thus it is not a feature of the primary sector, including the all-important wool industry which has its own economically significant fluctuations and which will be treated in Chapter Three. It may be noted that this point is similar to that made by the N.B.E.R. concerning the extent of business cycle fluctuations in the U. S. economy.

The idea of different economic time series, exhibiting more or less regular fluctuations which differ between series only in their typical timing with respect to one another, is accomodated by the model of joint stationarity described in Chapter One. Hence, in order to study the relationships between two such series, it seems appropriate, as a first approach, to focus on their lagged auto-correlation and cross-correlation functions since these statistics are assumed to be stable over time. Provided this model is merely a reasonably good approximation to the facts, it is possible to use the theorems of pseudo-spectral and pseudo-cross-spectral analysis to show that ordinary spectral procedures will give a very neat description of

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[2] P. H. Karmel, "The Australian Economy, April 1957", <u>The Economic Record</u>, April 1957, pp. 1-22.

the average joint properties of the series over the time period studied.[3] Apart from its statistical elegance, a spectral description seems particularly appropriate in a discussion of business cycle phenomena, where the basic heuristic notions can be expressed more simply and intuitively in the frequency domain than in real time.

The sample period to be considered in this chapter begins in 1948. By this time, most war-time controls were dismantled and so the insitutional structure of the economy became stabilized, a consideration that is important from the point of view of stationarity. Also a considerable amount of data becomes available in a reasonably reliable and compatible form from about this time. As the interest here centers on a business cycle that has an average period of about fifty-four months, a longer time period might seem to have been more satisfactory from the point of view of having a reasonable number of sample cycles. However even with this limited data it has been possible to derive some useful descriptive measures.

In order to indicate the meaning of spectral and cross-spectral measures, and the way in which they are formed, a detailed example of the analysis is now given. We will consider the joint evolution over time of the monthly growth rates of unemployment, x(t), and unfilled vacancies, y(t), (with the mean growth rate subtracted out in each case to facilitate later analysis). As has already been mentioned, unemployment and unfilled vacancies data have been at the basis of most previous discussions of Australian economic fluctuations, and hence they constitute a convenient starting point for this particular study.

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[3] M. Hatanaka and M. Suzuki, "A Theory of the Pseudospectrum and its Application to Non-stationary Dynamic Econometric Models", Essays in Mathematical Economics in Honor of Oskar Morgenstern, Martin Shubik, Ed., (Princeton University Press, Princeton, N. J., 1967), pp. 443-466.

Using the second order stationarity assumption, it is possible to construct meaningful estimates of a sequence of values of the serial correlation and cross-correlation functions in the low mean square error form advocated by Parzen:[4]

$$R_{xy}(s) = \frac{\sum_{t=1}^{n-s} x(t)\cdot y(t+s)}{\left(\sum_{t=1}^{n} x(t)^2 \cdot \sum_{t=1}^{n} y(t)^2\right)^{1/2}} \qquad (2.1)$$

where n is the number of months for which values of $x(t)$ and $y(t)$ are available, and

s takes the values $0,1,\ldots m$ months.

Such values for $R_{xx}(s)$, $R_{yy}(s)$, $R_{xy}(s)$, and $R_{yx}(s)$ are consistent though biased estimators of the lagged correlation functions at unit intervals up to some maximum lag m. Much more has been said and remains to be said concerning the statistical properties of these estimates.[5] But for the present purposes this writer merely wishes to draw attention to the desirability of having m ---

(a) small compared with n: so as to reduce the bias inherent in having only n-s terms in the numerator, and also so as to ensure "stability" in the sense of forming the m^{th} lag estimates by averaging over a reasonable number of pairs of terms, yet

(b) large compared with the decay time of the true second moment properties of the generating process: so as to capture, whole, and without serious bias, independent instances of the underlying linear economic interactions.

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[4] Emmanuel Parzen, "Mathematical Considerations in the Estimation of Spectra", Technometrics, May 1961, pp. 167-190.

[5] S. K. Zaremba, "Quartic Statistics in Spectral Analysis", Advanced Seminar on Spectral Analysis of Time Series, B. Harris, Ed., (John Wiley and Sons, Inc., New York, 1967), pp. 47-79.

These two criteria together point to the need for long, comparatively irregular series, devoid of fixed constant terms, trends, or periodicities which never die away, (this is another reason for taking growth rate series and subtracting out estimates of the deterministic non-zero means). It may be noted for future reference that the functions $R_{xx}(s)$ etc. may also be interpreted as lagged covariance functions of the original growth rate series normalized on zero mean and unit standard deviation.

Measures of lagged correlation similar to those above have long been used in econometrics to indicate leading and lagging interactions.[6] However as Tukey has pointed out, the interpretation of correlograms and related functions is a very tricky business.[7] We shall discuss some of these difficulties later on in Chapters Three and Four. Also it should be borne in mind that, in econometrics in general, and certainly in business cycle analysis, the interest is in the lead-lag relationshisp of certain components of the time series, rather than all components taken together. This amounts to saying that in many problems, especially those of interest here, there are relevant and irrelevant fluctuations in the data and corresponding serial correlation functions. This suggests a need to characterize them separately so as to be able to discriminate between them.

II. Frequency Decomposition of Second Order Moments

There is a long-established approach, originally associated with the name of W. M. Persons and later developed and popularized by the Harvard Economic Barometer and the N. B. E. R., which considers certain

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[6] Gerhard Tintner, Econometrics, (John Wiley and Sons, Inc., New York, 1952), especially Chapter 10.

[7] J. W. Tukey, "An Introduction to the Calculations of Numerical Spectrum Analysis", Advanced Seminar on Spectral Analysis of Time Series, B. Harris, Ed., (John Wiley and Sons, Inc., New York, 1967), pp. 25-46.

economic time series as linear combinations of trend, cyclical, seasonal, and irregular effects, each governed by different laws.[8] The cyclical component, being of particular relevance from the point of view of stabilization theory, is the one that is especially sought in studies like this. It should be noted that, although the U. S. cyclical component is often referred to as a "forty month cycle", experienced students of it, such as Gordon, have pointed out that:[9]

> Some cycles are mild and others are severe; some last two or three years and others eight or ten years.

This indicates a need to adopt a broad view of the "four to five year Australian growth cycle" although at sight it appears to have been comparatively regular over the period studied.[10] As a first approximation, it was decided here to pay particular attention to fluctuations in the frequency range 1/81 cycles per month to 2/81 cycles per month: an interval which is symmetrical about the point 1/4.5 cycles per year, and which is disjoint from the very low frequency "trend" components and the higher frequency "seasonal" components from whose effects we wish to abstract.

As is well known, a systematic way of discriminating between the effects of different frequency components in the true lagged correlation functions of a process is to take their Fourier transforms.[11] Thus if

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[8] A. F. Burns and W. C. Mitchell, Measuring Business Cycles, (National Bureau of Economic Research Inc., New York, 1946).

[9] R. A. Gordon, Business Fluctuations, (Harper and Row, New York, 2nd ed., 1961), p. 250.

[10] E. A. Boehm, "The Australian Economy, February 1964", The Economic Record, March 1964, pp. 1-32.

[11] For a very readable introduction to this method of analysis see Peter Whittle, "Some Recent Contributions to the Theory of Stationary Processes", which is Appendix 2 of Herman Wold, A Study in the Analysis of Stationary Time Series, (Almqvist and Wiksell, Stockholm, 2nd ed., 1954)., pp. 167-228.

$\underline{R}_{xx}(s)$, $\underline{R}_{yy}(s)$, and $\underline{R}_{yx}(s)$ are the theoretical lagged correlation functions of a discrete process, then we can take Fourier transforms of the form:

$$\underline{f}_{xx}(\omega) = \sum_{s=-\infty}^{\infty} \underline{R}_{xx}(s) \cdot \exp(i\omega s) \qquad (2.2)$$

Hence, from the theory of the inverse Fourier transform, we know that the corresponding lagged correlation function has the "spectral representation":

$$\underline{R}_{xx}(s) = (1/2\Pi) \cdot \int_{-\Pi}^{\Pi} \underline{f}_{xx}(\omega) \cdot \exp(-i\omega s)d\omega \qquad (2.3)$$

By considering Equation 2.3 for the case s=0, it can be seen that $\underline{f}_{xx}(\omega)$ and $\underline{f}_{yy}(\omega)$ have the intuitive meaning that, if the infinite series x(t) and y(t) are regarded as representing some combinations of fluctuations, then $\underline{f}_{xx}(\omega)$ and $\underline{f}_{yy}(\omega)$ are the theoretical probability density functions of the (normalized) fluctuation variance among the different frequency components ω. Similarly it can be seen that spectral analysis can be interpreted as an analysis of variance classified in terms of cells distributed over intervals of the frequency domain.[12] This indicates that spectral analysis is a useful procedure to use in cases where frequencies constitute a significant classification of the variance, i.e. where changes in conditions would change some frequency components (e.g. the cycle) while leaving others alone (e.g. the seasonal). Actually we have indicated a rather more general result, that the overall second moment of the ensemble of x(t) can be obtained by adding together the mean squares of each of the frequency components.

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[12] J. W. Tukey, "Discussion Emphasizing the Connection between Analysis of Variance and Spectrum Analysis", Technometrics, May 1961, pp. 191-219.

It will be remembered that the mean square is equal to the variance plus the square of the mean, and so the second moment component within a frequency cell can be considered in two extreme senses according to whether it would stay the same or change if another realization were taken under "similar" conditions, i.e. from the same statistical ensemble. The statistician is peculiarly interested in discussing examples of the second case, where the mean square frequency components can be considered as variances, which vary between realizations and have only expected values in common. However initially we shall not give prominence to this viewpoint (because we shall be discussing some fairly regular fluctuations over a comparatively short span where the degrees of freedom are limited). We shall consider the second moments in the business cycle frequency cells as being "mean-like", such as would be obtained if the economy were run over and over again under exactly the same conditions. It may be recollected that this would be appropriate for analyzing a signal-like component of the data such as is shown in Equation 1.4 of Chapter One. This rather trivializes the statistical meaning of our model but it seems to be a worthwhile endeavor when the historical properties of the components of the data sample in that frequency cell are held to be of interest. Further, when this has been done, we shall then be prepared to go on to consider a more general, statistical notion of a replication of the data, which is appropriate to considering the problems of predicting the cyclical phenomena likely to arise in future. Then we shall pay more attention to the significance of degrees of freedom and sampling variances.

Actually, Tukey and Bartlett, and more recently Parzen, have argued that, when we only have an estimate $R_{xx}(s)$ of $\underline{R}_{xx}(s)$ which is necessarily truncated at some maximum lag m, the finite Fourier transform does not give a very useful frequency decomposition. However they have shown that it is possible to get a more desirable type of frequency discrimination by transforming not the estimated correlation functions but specially weighted versions of them.

It should be noted that the estimated auto-correlation functions $R_{xx}(s)$ and $R_{yy}(s)$ are even functions of s, as also are the weighting functions $\lambda(s)$ proposed by Parzen.[13] Hence the desired transforms or "estimated spectra" as they are called, will be real valued functions of the form:

$$\begin{aligned} f_{xx}(\omega) &= \sum_{s=-m}^{m} R_{xx}(s)\cdot \lambda(s)\cdot \cos \omega s \\ &= R_{xx}(0) + 2 \sum_{s=1}^{m} R_{xx}(s)\cdot \lambda(s)\cdot \cos \omega s \end{aligned} \tag{2.4}$$

The use of Parzen weights, in the range -m, m of the original correlation function and effectively of zero weights outside that range, corresponds, in the frequency domain, to smoothing a raw version of the estimated spectrum by convolving it with a "Parzen window" (which is the Fourier Transform of the sequence $\lambda(s)$). This has the effect of mixing into the estimate, at any given value of ω, contributions from other values, particularly neighboring values. If we do not use some such window, but still have only a truncated estimate of the correlation function (as we always will in practice), there will be even greater contributions from more distant values of ω. This so-called "leakage" would be especially inconvenient

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[13] Emmanuel Parzen, loc. cit., Also we shall discuss this matter more fully in Section IV of this chapter.

for our present purposes, so a Parzen (or similar) window is highly desirable. Figures 2.1 and 2.2 show the raw Fourier and Parzen windows for the case in which m equals 60 months, on the frequency scale 0 - 1/12 cycles per month.

The fundamental fact is that, with merely a finite number of data points, we can only hope to estimate a finite number of values of the lagged correlation function. Hence we can only approximate its Fourier transform by a finite order cosine polynomial. However, by a choice of a suitable window, we can ensure that the polynomial is fitted in such a way as to reflect more or less independently the average values of the spectrum in certain local frequency ranges. Thus the estimated spectrum has effectively been smoothed by a weighted moving average operating in the frequency domain and having only very small weights outside a finite "bandwidth" somehow defined.

As has been mentioned, the aims of this project made a Parzen window particularly desirable, so as to capture (free from trend and seasonal leakage) the "cyclical component" which we have suggested corresponds largely to fluctuations in the range 1/81 to 2/81 cycles per month (c.p.m.). Hence initially a Parzen window was selected having a "rectangular bandwidth" of 1/81 c.p.m.[14] This means that the estimate "at" 1/54 c.p.m. will give a very strong emphasis to fluctuations in the band 1/54 ± .5/81 c.p.m., (giving particularly strong weights to frequencies near the center of the band and weak and declining weights to frequencies away from it). Actually

[14] This corresponds to using Parzen weights on the lagged correlation functions truncated at m=108 months. In view of the seemingly regular and undamped character of our cycle over a period as short as this, we will expect bias in our spectral estimates, but bandwidth considerations already mentioned, suggested it would not be appropriate to consider longer lags. It should also be noted that there are other notions of "the bandwidth", e.g., "the bandwidth of independent estimates", which in this case would be 1/54 c.p.m. See Emmanuel Parzen, "The Role of Spectral Analysis in Time Series Analysis", Technical Report No. 2, (Department of Statistics, Stanford University, July 1965).

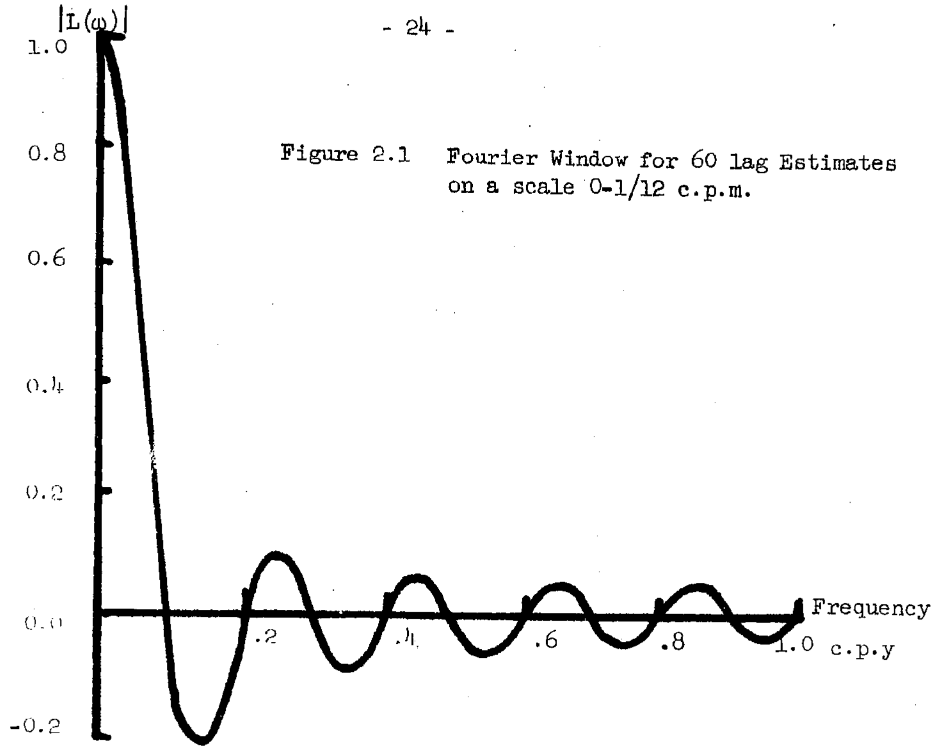


Figure 2.1 Fourier Window for 60 lag Estimates on a scale 0-1/12 c.p.m.

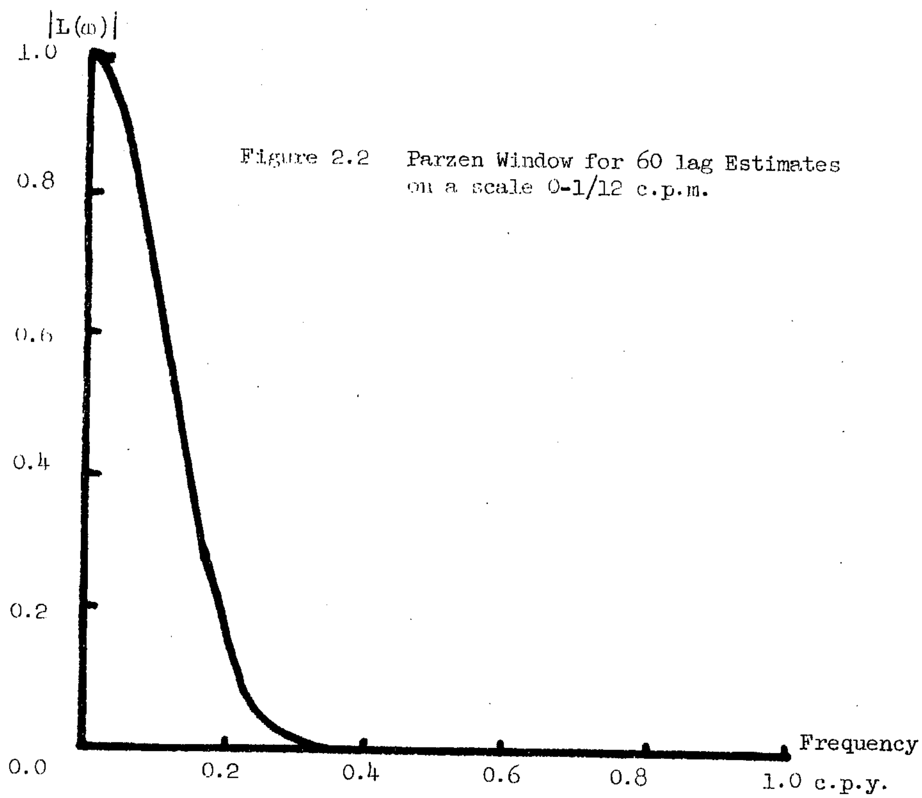


Figure 2.2 Parzen Window for 60 lag Estimates on a scale 0-1/12 c.p.m.

Figure 2.3 Spectrum of Monthly Vacancies 1948-66 (108 lag Parzen Estimates on a scale 0 - 1/2 c.p.m.)

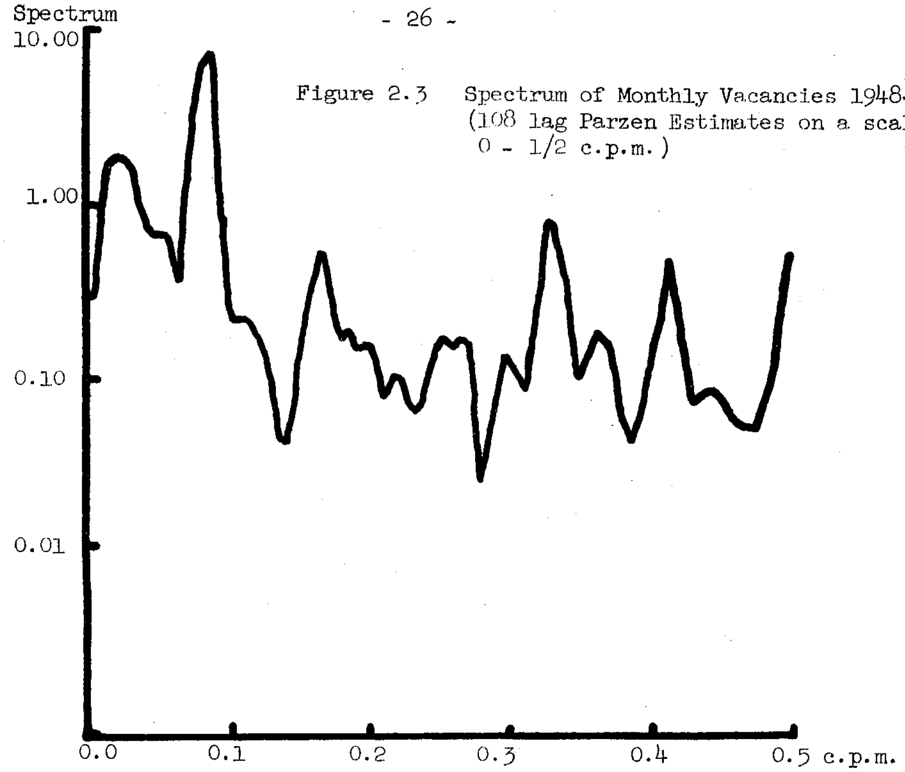
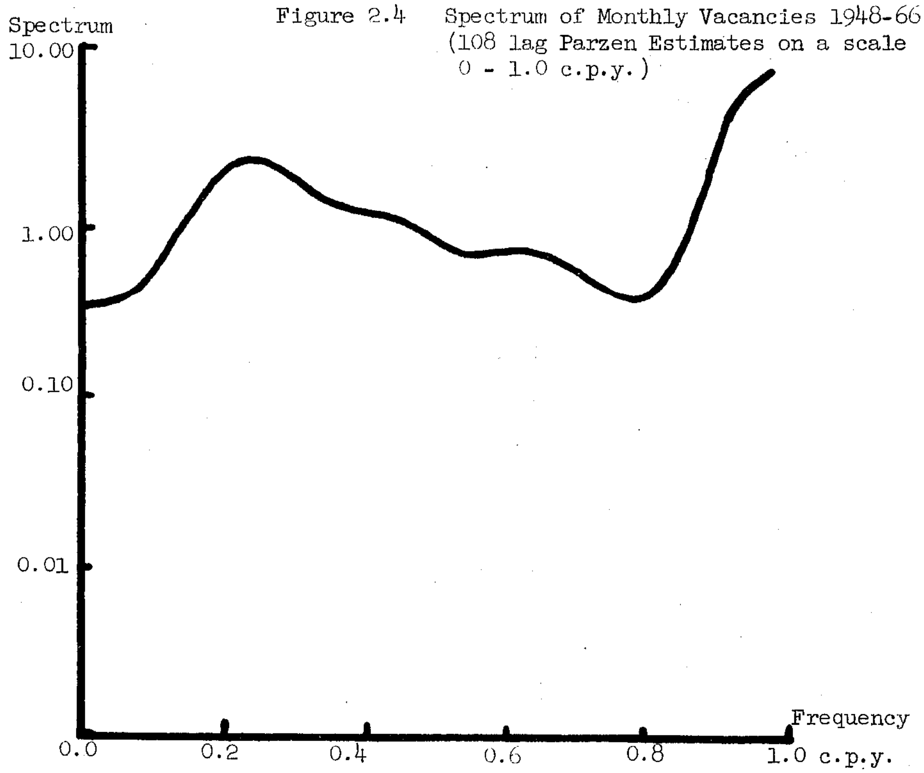


Figure 2.4 Spectrum of Monthly Vacancies 1948-66 (108 lag Parzen Estimates on a scale 0 - 1.0 c.p.y.)



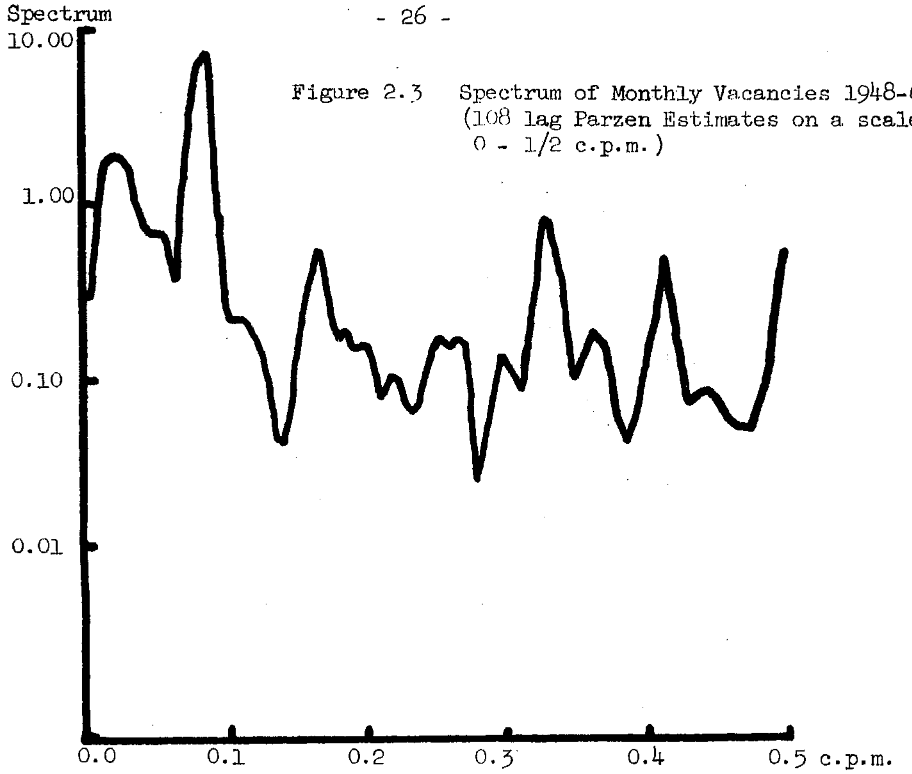


Figure 2.3 Spectrum of Monthly Vacancies 1948-66 (108 lag Parzen Estimates on a scale 0 - 1/2 c.p.m.)

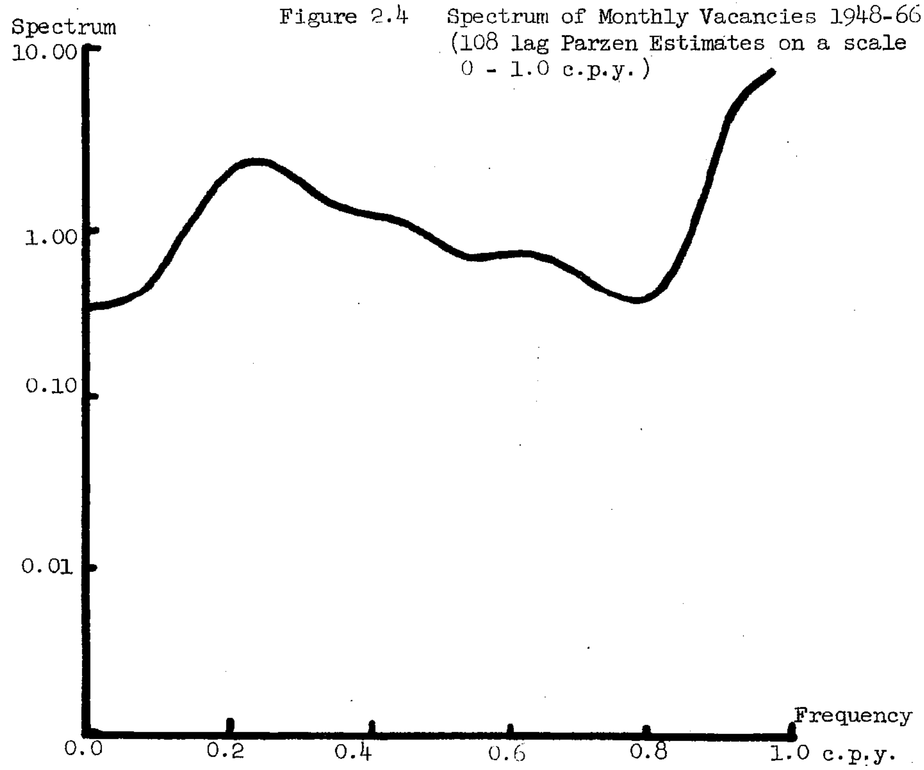


Figure 2.4 Spectrum of Monthly Vacancies 1948-66 (108 lag Parzen Estimates on a scale 0 - 1.0 c.p.y.)

Similar to the estimated spectrum, there is an estimated cross-spectrum, formed by transforming truncated and weighted estimates of the lagged cross-correlation function. However this latter function, unlike the auto-correlation function, is not even in s, and hence its Fourier transform is complex valued with the form:

$$
\begin{aligned}
f_{yx}(\omega) &= \sum_{s=-m}^{m} R_{yx}(s) \cdot \lambda(s) \cdot \exp(i\,\omega s) \\
&= 1/2 \sum_{s=-m}^{m} (R_{yx}(s) + R_{yx}(-s)) \cdot \lambda(s) \cdot \exp(i\,\omega s) \\
&+ 1/2 \sum_{s=-m}^{m} (R_{yx}(s) - R_{yx}(-s)) \cdot \lambda(s) \cdot \exp(i\,\omega s) \\
&= R_{yx}(0) + \sum_{s=1}^{m} (R_{yx}(s) + R_{xy}(s)) \cdot \lambda(s) \cdot \cos \omega s \\
&+ i \left(\sum_{s=1}^{m} (R_{yx}(s) - R_{xy}(s)) \cdot \lambda(s) \cdot \sin \omega s \right) \qquad (2.5)
\end{aligned}
$$

Analogous to the interpretation of the spectrum as an analysis of variance, it can be seen by taking the inverse transform of the cross-spectrum that it may be considered as representing an analysis of the covariance $R_{yx}(0)$. The cyclical cell of this decomposition, especially its mean-like interpretation will be an important focus in what follows. For notational convenience we sometimes write:

$$
\begin{aligned}
f_{yx}(\omega) &= c_{yx}(\omega) + i \cdot q_{yx}(\omega) \\
&= |f_{yx}(\omega)| \cdot \exp(i\phi(\omega)) \qquad (2.6)
\end{aligned}
$$

where $c_{yx}(\omega)$ is known as the cospectrum,

$q_{yx}(\omega)$ is known as the quadspectrum, and

$\phi(\omega) = \tan^{-1}(q_{yx}(\omega)/c_{yx}(\omega))$ is called the relative phase.

Once the laborious work of computing the spectral and cross-spectral estimates of the time series is completed, it is relatively easy to calculate some useful statistics describing their joint behavior in the frequency domain. Thus analogous to the definition of a regression coefficient of x on y (in normalized form) as $R_{yx}(0)/R_{xx}(0)$ for all variations in the sample, there is a similar statistic for fluctuations near each frequency, known as the "transfer function":

$$\begin{aligned} L_{yx}(\omega) &= f_{yx}(\omega)/f_{yy}(\omega) \\ &= |f_{yx}(\omega)|\cdot\exp(i\phi(\omega))/f_{yy}(\omega) \\ &= G_{yx}(\omega)\cdot\exp(i\phi(\omega)) \end{aligned} \tag{2.7}$$

This function will be seen to express the fact that, if we consider the normalized series as being made up of frequency components $k_x(\omega)\cdot\exp(i\omega t)$ and $k_y(\omega)\cdot\exp(i\omega t)$, then these components are related by the open loop regression equation:

$$\begin{aligned} k_x(\omega)\cdot\exp(i\omega t) &= G_{yx}(\omega)\cdot\exp(i\phi(\omega))k_y(\omega)\cdot\exp(i\omega t) + k_n(t,\omega) \\ &= G_{yx}(\omega)\cdot k_y(\omega)\cdot\exp(i\omega t + i\phi(\omega)) + k_n(t,\omega) \end{aligned} \tag{2.8}$$

where n(t) is an error term, uncorrelated with y(t).

As will be shown in Section IV of this chapter, the whole foregoing discussion can be summarized neatly by considering that the jointly stationary variables corresponding to the normalized x and y series can be represented as a sum of frequency components grouped in independent bands by the estimation window. Squaring and averaging these sinusoidal terms over time and over a window gives estimates of their contribution to that region of the spectrum (Equation 2.4), while regressing these terms, one on the other, over the same time and frequency range gives an estimate of the transfer function between the related (coherent) components of the two series in that time and frequency region (Equation 2.8). To complete this interpretation in terms similar to those of standard econometric regression analysis, it is necessary to say something about the percentage of variation explained by the regression. This can be measured by a statistic known as the "coherence" which corresponds to the square of the correlation coefficient between the normalized series in regard to those parts of them which can be described, over the span of the sample, by sinusoidal terms with frequencies intersecting the window centered at ω. Thus coherence is given by:

$$C_{yx}(\omega) = |f_{yx}(\omega)|^2/(f_{xx}(\omega)\cdot f_{yy}(\omega)) \tag{2.9}$$

Such a statistic seems a useful tool for business cycle analysis as it provides a means of determining if two series are in fact strongly correlated at the frequencies of interest, and hence whether it makes sense to attempt to determine their relative timing (e.g. via the relative phase of the coherent components).

It should be noted that the estimates of both the coherence and the phase are unaffected by the direction of the causality in Equation 2.8, (apart of course from the sign of the phase). Thus these are correlation rather than causal measures, and hence abstract from a host of controversial problems connected with causality in econometrics.[16] In this respect they conform with the phenomenological perspective of the N.B.E.R. measures.

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[16] See e.g. Herman Wold, Econometric Model Building (North-Holland, Amsterdam, 1964), especially Chapter 1.

The N.B.E.R. methods were described long ago as "measurement without theory",[17] however the standpoint of its critics (interdependent regression equation models in the time domain) has since appeared to be rather less than the last word on the subject.[18] It is not proposed here to contribute to this exceedingly complex and unresolved controversy, but instead to proceed with a comparatively simple descriptive approach which seems to yield useful information about some simple regularities of interest, and which in any case seems to be a necessary preliminary to the more ambitious "whole economy" models of the business cycle. This does not mean that we are unaware of the problems connected with the assumptions of alteris paribus and equation identification, but that, for the moment, we will be especially concerned with some simple relationships where these conditions may be less crucial. Hence our present analytic aspirations are merely concerned with factoring out some other influences which also tend to bias estimates of cyclical relationships of interest, (we have in mind not only the important low frequency trends, but also the higher frequency seasonal and short run effects).

In the present analysis of the relation between the growth rates of unemployment and unfilled vacancies, we can begin by examining the estimates of the coherence and the transfer function which are shown in Figures 2.5 and 2.6 for the frequency range 0 to 1/12 c.p.m.

It can be shown that the estimates of coherence, like the analogous R^2 statistics of conventional econometrics, are constrained to the range

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[17] T. C. Koopmans, "Measurement Without Theory", The Review of Economics and Statistics, August 1947, pp. 161-72.

[18] See e.g. the above reference by Wold for a discussion of this question and a bibliography of some of the extensive literature on this subject.

Figure 2.5 Coherence of Vacancies and Unemployed (108 lag Parzen Estimates)

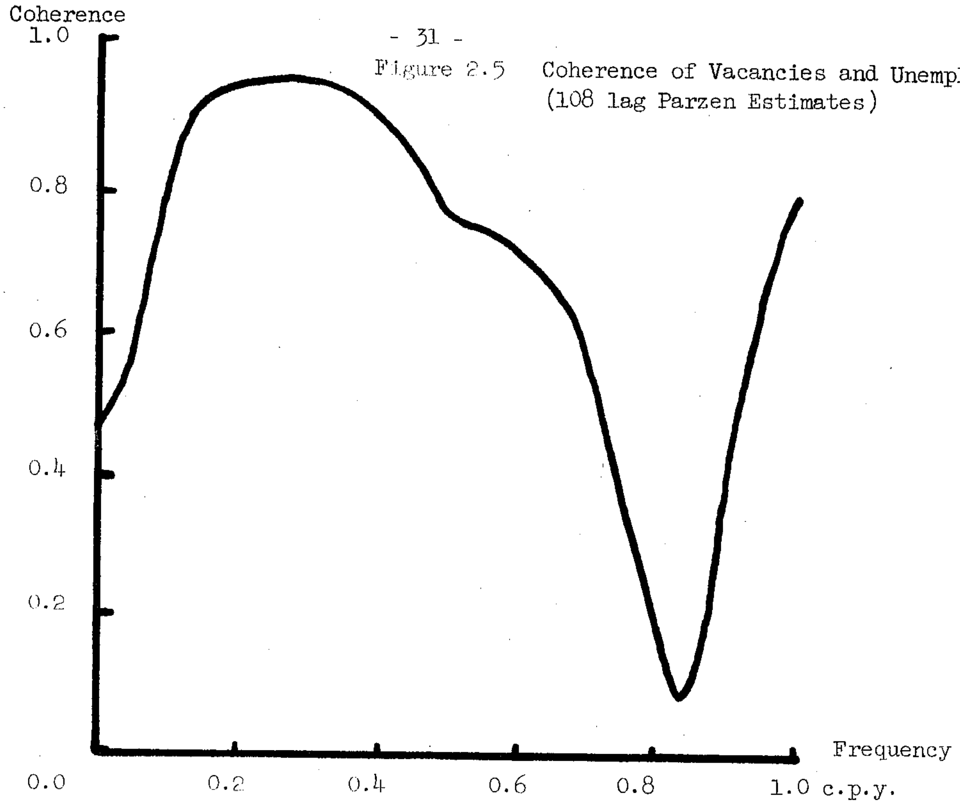
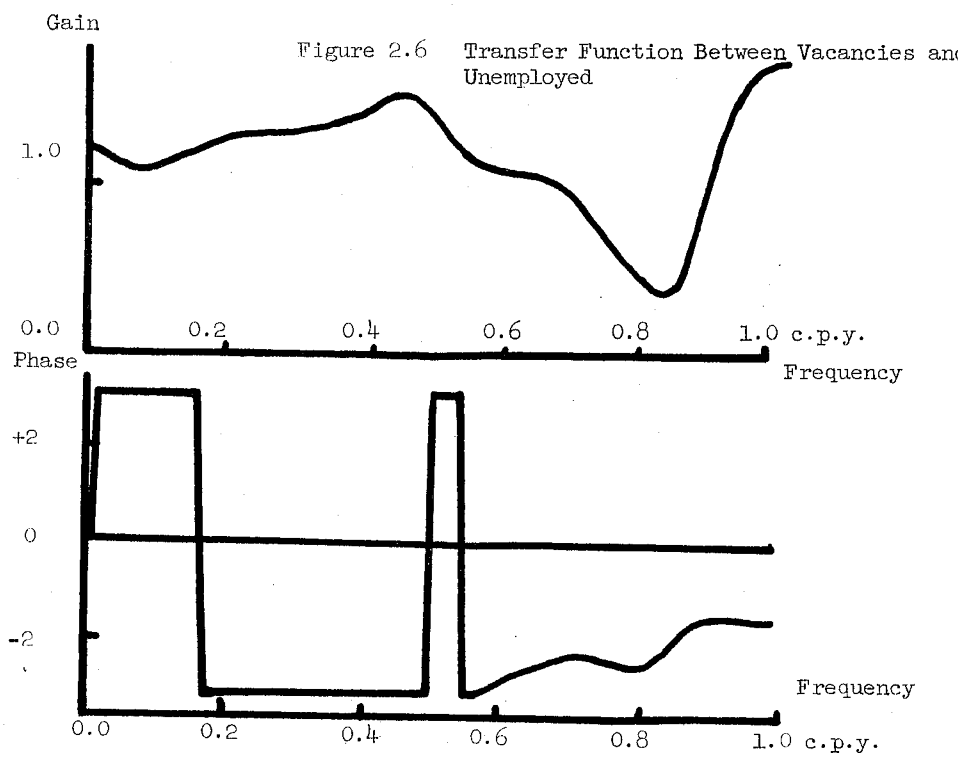


Figure 2.6 Transfer Function Between Vacancies and Unemployed



0 to 1, and may be biased upward in the small sample case.[19] However no detailed analysis of statistical inference in terms of a probability model has been attempted for cases like the present one because we are more fundamentally concerned simply to describe the relative phasing of some semi-regular fluctuations, and in such a case the number of degrees of freedom is low and rather unimportant. We might also allude to the vast complications presented by the likelihood of the processes being either non-normal or non-stationary or having properties which are not stable over the estimation window. So the estimates derived here (like the N.B.E.R. measures) are offered initially as the descriptive statistics of a historical process rather than the sampling statistics from a probability ensemble. However by using the language of spectral analysis we are facilitating a later raising of our analytic ambitions to representing our analysis in terms of a probability model. In fact one of the aims of this study is to add to empirical knowledge of what constitutes "high" and "low" coherence by accumulating experience concerning what we can expect to obtain from various types of economic relationships. In a similar vein, we will also be concerned with the robustness of typical results to variations in the estimation procedures. To begin with, we may note that Hatanaka, in his study of U.S. business cycle indicators, while pointing to the need for more empirical work to increase background knowledge in this area, took the value 0.5 as the dividing line between high and low coherence.[20] Some such

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[19] For k degrees of freedom this bias is $(2/k)(1 - \text{true coherence})$, or $2/k$ when the true coherence is zero, (i.e. 0.25 in the present case of 8 degrees of freedom for independent estimates). For a discussion of these and other subtleties associated with coherence estimation see, L. J. Tick, "Estimation of Coherency", Advanced Seminar on Spectral Analysis, B. Harris, Ed., (John Wiley and Sons, New York, 1967).

[20] M. Hatanaka, "A Spectral Analysis of Business Cycle Indicators: Lead Lag in Terms of All Time Points", Research Memorandum No. 53, (Econometric Research Program, Princeton University, February 1963), p. 43 and p. 54.

feeling for coherence, however crude, would seem to represent a significant addition to the measures sought by N.B.E.R. procedures, especially as the fundamental idea of a "business cycle" is that it is a more or less regular fluctuation which is shared by many important economic variables.

In the particular case under discussion, the coherence between the growth rates of unemployment and unfilled vacancies at the business cycle frequencies turned out to 0.95 which is very high by any standard, while the phase difference was Π. This indicates that the cyclical components of the two series are mirror images of one another as well as being strongly related. It may be observed that this result is similar to Mallyon's, and it seems theoretically simple enough, since both series are proxies for the supply-demand situation in the labor market. Hence it suffices to take just one of these series (unfilled vacancies) as an indicator of the cyclical state of the labor market and, because of the economy-wide importance of this market, such an indicator will also constitute a convenient business cycle reference series. It may also be observed that a cross-spectral analysis of unfilled vacancies and a zig-zag line connecting the peaks and troughs of a "general reference cycle" found by N.B.E.R. methods showed a coherence of 0.85 and a phase difference of zero at the business cycle frequencies.[21] This result is also in accordance with Mallyon's more visual examination and perhaps indicates a further rationale for using unfilled vacancies as a reference series, at least initially.

III. Cross-spectral Comparison of Cyclical Indicators

Having chosen a reference indicator, we can use it to make comparisons with other series by the methods of two-variable cross-spectral

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[21] In this particular calculation the peaks and troughs of the (triangular) general reference cycle were the same as those given by Mallyon, op. cit. except that the series was extended to include a peak in June 1965, and the mid point of the long slump October 1956 - December 1958 was chosen as the local trough.

analysis. Thus, provided the coherence between unfilled vacancies and the other series is acceptable, it is profitable to go on to consider the relative phase lag between the two series at the frequencies of interest. This is directly translatable into a time lag rather similar to that studied by the N.B.E.R. if such is desired. An average phase lag of ϕ in the neighborhood of 1/54 c.p.m., for example, corresponds to an average time lag of $(\phi/2\Pi)\cdot 54$ months for those frequency components. However, the idea of a fixed phase lag over the cyclic band often seems intuitively more reasonable in an economic sense than a fixed time lag. It is also a better statistical notion here to consider the phase as being reasonably constant over the estimation window. It should also be noted that the above measure of time lag is not estimated on exactly the same basis as the N.B.E.R. measure in that information about the relative timing of the fluctuations of interest is obtained from all data points over the cycle, rather than just those at the peaks and troughs. However, it seems a far more systematic procedure and obviates the large amount of mental stress to say nothing of arbitrariness involved in attempts to determine, by mere inspection of irregular data series, the precise months in which business cycle turning points occurred.[22]

One further difference between the analysis given here and Mallyon's N.B.E.R. approach is more apparent than real. Mallyon examines the data in logarithmic form, while here first differences of logs are studied. But it should be noted that, from the point of view of the relative phases of narrow band components disjoint from zero frequency, there is no difference, since first differencing changes the phase of both series equally.

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[22] Oskar Morgenstern, On the Accuracy of Economic Observations, (Princeton University Press, Princeton, N.J., 2nd ed. 1963), pp. 58-61.

The cross-spectral relationship of most available economic time series with vacancies is not as close or as simple as that of unemployment. This may be seen by referring to Table 2.1 where we show the cross-spectral

Table 2.1 Cross-spectral Estimates with Vacancies near 1/54 c.p.m.

Series	Coherence	Phase
A. Labor Market Variables		
Unemployed	.95	3.1
Employed	.65	0.2
Net Immigration	.65	0.1
B. Wage and Price Indicators		
Security Prices	.80	0.0
Consumer Prices	.60	2.0
Wholesale Prices	.55	1.5
Average Earnings	.50	1.5
Money Supply	.45	-0.1
C. Production Indicators		
Refrigerator Production	.80	-0.8
Brick Production	.65	-0.5
Stove Production	.45	-0.2
Cement Production	.35	-0.6
Steel Production	.35	-2.5
Timber Production	.30	0.2
Coal Production	.20	1.8
Electricity Production	.10	0.0
Gas Production	.01	2.0
D. Sales Figures		
New Automobile Registrations	.80	-0.3
Volume of Retail Sales	.75	0.0
E. International Transactions		
Imports	.80	0.3
Exchange Reserves	.60	-1.5
Wool Price	.10	0.2
Exports	.00	0.3

phase and coherence with respect to the reference cycle of a number of economic variables ranked by their coherences within some economically meaningful groupings. Some of these results appear to give an interesting reflection of the operation of economic tendencies pertinent to business cycle studies and hence will warrant an attempt at theoretical analysis. However, before aspiring to make any elaborate inferences from the face value of these measures, it seems appropriate to give some consideration to their statistical meaning and robustness, as is indicated in the next section.

IV. Direct Methods of Frequency Analysis

It is comforting to note that, in all the foregoing cases, wherever the coherence was reasonable (e.g. greater than 0.5), the cross-spectral estimates of the cyclic phase difference between the pairs of series was not particularly sensitive to the width of the window used. Specifically the phase was not noticeably affected by increasing or decreasing the band-width by as much as 50 per cent. This indicates that the measurement procedures have a certain robustness and that for some reason or other (perhaps due to the prominence of effects near the center of our window[23]) we can say that from the point of view of estimation, the cyclical processes seem to be well characterized by our frequency domain models. One might still wonder about the time stability of the cyclical processes under study, especially with regard to the phases. This has hitherto merely been assumed. Hence it seems worth considering strategies for an empirical investigation of this kind of stability.

We may begin by observing that the foregoing analysis was in terms of a statistical description of the cyclical part of the processes

[23] This interpretation is supported by the peaking of the spectra and cross-spectra near 1/54 c.p.m. as the estimation window is narrowed. In fact even the periodograms of most of the series of interest had local spikes there.

under study obtained by the now almost classical "spectral" procedures. These are based on the idea of forming a frequency decomposition of previously obtained second order time domain statistics which were computed by taking averages over the entire time span of data, on the basis of the stationarity assumption. Recently however it has been suggested that it may sometimes be more useful to invert the order of the operations of time averaging and frequency decomposition.[24] For studies which like the present one focus on particular narrow frequency bands, these methods seem to be computationally simpler than the corresponding spectral calculations and they have the added advantage of permitting a generalization of the analysis to include a consideration of certain slowly changing forms of non-stationarity.

The rationale of these procedures hinges on the fact that from the spectral representation of the serial correlation functions of the discrete time variates of a stationary bivariate process (shown in Equation 2.3) Cramér has deduced a related representation for the variates themselves:

$$\begin{matrix} x \\ y \end{matrix}(t) = \int_{-\Pi}^{\Pi} \exp(i\,\omega t)\cdot k_{\begin{smallmatrix} x \\ y \end{smallmatrix}}(\omega)d\omega \qquad (2.10)$$

These equalities are to be understood as holding as limits in mean square. Actually any (infinite time) realization of the non-deterministic stationary time domain variates is neither strictly periodic nor convergent on zero, and hence it does not have any obvious meaning as a direct Fourier series or as a Fourier integral. However a widening span of a realization may be

[24] C. Bingham, M. D. Godfrey and J. W. Tukey, "Modern Techniques of Power Spectrum Estimation", IEEE Transactions on Audio and Electroacoustics, June 1967, pp. 56-66.

approximated, as close as we like in a mean square sense, by an increasing number of Fourier series terms, converging in the limits of an infinite time span and infinitesimal frequency spacing to an integral of the form shown in Equation 2.10, thus implying a realization of the random variable $k(\omega)$.[25]

Although the Cramér representation is fundamental to the approach of this dissertation, its precise mathematical meaning and relationship with the spectral representation present some mathematical subtleties beyond the scope of this data analytic study. In any case they have already been more ably discussed elsewhere.[26] We merely present the following highly heuristic account to indicate the meaning of the computations which will be made later.

If "E" represents the expectation operator and $x(t)$ and $y(t)$ come from a stationary bivariate process (and as before, are understood to be normalized on zero mean and unit variance), then the true values of the serial correlation coefficients can be written in a form which is independent of the absolute time t, viz.:

$$\underline{R}_{yx}(s) = E[y(t)\cdot x^*(t+s)]$$

---

[25] H. Cramer, "On Harmonic Analysis in Certain Functional Spaces", Arkiv for Mathematik, Astronomi o. Fysik, March 1942, pp. 1-7. We should note that Equation 2.10 refers to the average weighting of different frequencies in the process, (not in a realization such as we will be discussing). Thus $k(\omega)$ is a random variable in ω, though it may also include a number of fixed Dirac delta function spikes corresponding to any deterministic components.

[26] A fundamental article is H. Cramer, "On the Theory of Stationary Random Processes", Annals of Mathematics, January 1940, pp. 215-230. A compact summary and listing of the associated literature is given by P. Whittle, "Some Recent Contributions to the Theory of Stationary Random Processes", which is Appendix 2 of Herman Wold, Stationary Time Series, (Almqvist and Wiksell, Stockholm, 2nd ed., 1954). Another treatment has been given by M. S. Bartlett, Stochastic Processes, (Cambridge University Press, Cambridge, 1955), pp. 186-195.

where * indicates the complex conjugate. The use of complex algebra here clearly represents no loss in generality, but will result in a great gain in simplicity. Hence using Equation 2.10, we may write:

$$\underline{R}_{yx}(s) = E[\int_{-\Pi}^{\Pi} \exp(i\omega_1 t)\cdot k_y(\omega_1)d\omega_1 \cdot \int_{-\Pi}^{\Pi} \exp(-i\omega_2(t+s)\cdot k_x^*(\omega_2)d\omega_2]$$

$$= E[\int_{-\Pi}^{\Pi}\int_{-\Pi}^{\Pi} \exp(it(\omega_1-\omega_2)-i\omega_2 s)\cdot k_y(\omega_1)\cdot k_x^*(\omega_2)d\omega_1 d\omega_2]$$

which is time independent in the general case if, and only if:

$$E[k_y(\omega_1)\cdot k_x^*(\omega_2)] = 0 \quad \text{for } \omega_1 \neq \omega_2 \; . \qquad (2.11)$$

Hence for a stationary process we can rewrite the variables of integration ω_1 and ω_2 as a single dummy variable ω, giving:

$$\underline{R}_{yx}(s) = \int_{-\Pi}^{\Pi} \exp(-i\omega s)\cdot E[k_y(\omega)\cdot k_x^*(\omega)]d\omega$$

which, by comparison with the spectral representation $\underline{R}_{yx}(s)$ in Equation 2.3, together with the result of Equation 2.11, gives:

$$E[k_y(\omega_1)\cdot k_x^*(\omega_2)] = \begin{cases} f_{yx}(\omega), & \omega_1 = \omega_2 \\ 0, & \omega_1 \neq \omega_2 \end{cases} \qquad (2.12)$$

Similarly it can be shown that:

$$E[k_{\substack{x\\y}}(\omega_1)\cdot k^*_{\substack{x\\y}}(\omega_2)] = \begin{cases} f_{\substack{xx\\yy}}(\omega) , & \omega_1 = \omega_2 \\ 0 , & \omega_1 \neq \omega_2 \end{cases} \qquad (2.13)$$

Thus it may be seen that $k_y(\omega)$ and $k_x(\omega)$, and hence $\exp(i\,\omega t)\cdot k_y(\omega)$ and $\exp(i\,\omega t)\cdot k_x(\omega)$ are orthogonal processes in ω, and that y(t) and x(t) are sums of all such uncorrelated components. It may also be noted that if y(t) and x(t) are stationary, then so also are each of the frequency components of $k_y(\omega)$, $k_x(\omega)$, $f_{yy}(\omega)$, $f_{xx}(\omega)$, and $f_{yx}(\omega)$, which accordingly might be estimated by averaging over successive time spans. However, as noted earlier in this chapter, given a restricted span of a function in the time domain, we can only estimate broad-band averages of its true continuous representation in the frequency domain. Still, if the characteristics of the process under study are fairly stable over such bands, then the corresponding estimates will provide a good description of them. Thus the art of forming "good" estimates (i.e. those with low relative errors) is to take averages of quantities which tend to be stable over the frequency and time intervals chosen. Hence it is suggested that a reasonable way to estimate the properties of a stationary process near some frequency ω would be first to divide each of the data series y(t) and x(t) into a number of independent time spans, then within each of these, $k_y(\omega)$ and $k_x(\omega)$ might be "estimated" by the complex Fourier coefficients at ω. These of course would be very crude "estimates", as even for noise-like data each of the real and imaginary parts would correspond to only about one degree of freedom. However, by averaging <u>quadratic functions of these quantities</u> across the independent time spans in the sense of Equations 2.12 and 2.13, it seems possible to obtain broad-band estimates of $f_{yy}(\omega)$, $f_{xx}(\omega)$, and $f_{yx}(\omega)$ with a reasonable number of degrees of freedom. This presupposes of course that the correlation structure of the process is such that the successive "estimates" of $k_y(\omega_1)\cdot k_x^*(\omega_2)$ etc. that we average over are in fact independent. It may be remembered that a similar requirement was mentioned earlier for the workability of more conventional spectral procedures: that the decay time of the true serial correlation functions of the non-deterministic generating process be less than the interval over which separate "independent" estimates of them are formed.[27]

This new estimation procedure suggests that one might for example consider the stability (or otherwise) of a moving average of such "estimates" of $k_y(\omega_1)\cdot k_x^*(\omega_2)$ etc. over time as evidence of the reasonableness of the assumption that the process is stationary. As has been pointed out earlier, the frequency windows corresponding to raw Fourier coefficients are rather unsuitable, in that the sidelobes are fairly large and consequently permit considerable leakage into the estimates at ω of effects due to quite distant frequencies. Consequently one might consider taking some suitably weighted frequency transform akin to that given by the Parzen window.

One convenient practical way to compute broad-band estimates of say $k_y(\omega)$, centered on some frequency ω' and evaluated over successive

---

[27] This notion of what constitutes independent replications and the way in which averaging them across leads to consistent estimates seems rather more fundamental than the idea that you need a certain minimum number of observations or "cycles" in the data before you can use spectral methods. Actually there is no periodic sequence of "cycles" in a non-deterministic process -- if such were present, the true correlogram would never damp out and so there would be no possibility of making averages of independent things over time to ensure consistent spectral estimates. Similarly we may note that if a cycle only damps out slowly in the correlogram one would need to form estimates up to a large number of lags to avoid biased estimates, and this of course imposes a limit on our ability to control sampling variance by averaging over separate things. This situation is a manifestation of the common trade-off that exists in statistics between variance and bias. However if we are merely interested in a description of what actually happened in the data rather than what might have happened (e.g. what was the actual average of the relative phasing of two fluctuations resembling signals at the cyclic frequencies), then these considerations become rather less important.

time spans, is via the methods of "complex demodulation". Here the time series $y(t)$ is multiplied by a complex exponential function of time $\exp(-i\omega' t)$ and the resulting product series is smoothed by some moving average process, giving the following series of time-dependent and weighted "Fourier coefficients":

$$y(t,\omega') = \sum_{s=-m}^{m} w(s)\cdot y(t+s)\cdot \exp(-i\omega'(t+s)) \qquad (2.14)$$

One may obtain a good physical idea of what is going on here by noting that this is a digital counterpart of the well-known electronic procedure of heterodyning an input signal $y(t)$ with a local oscillation $\exp(-i\omega' t)$ and then low pass filtering the mixed signal, thus extracting a low frequency component which corresponds to the complex envelope of the ω' band of the input signal. For example, if the input signal is a single sinusoidal component with a fixed frequency ω'' and a very slowly varying complex amplitude $K(t)\cdot\exp(i\Theta(t))$, (i.e. it does not change significantly over the range $-m,m$), then we have:

$$y(t) = K(t)\cdot\exp(i\Theta(t))\cdot\exp(i\omega'' t) \qquad (2.15)$$

which according to Equation 2.14 has as demodulates at frequency ω' :

$$\begin{aligned} y(t,\omega) &= \sum_{s=-m}^{m} w(s)\cdot K(t)\cdot\exp(i\Theta(t))\cdot\exp(i(\omega''-\omega')(t+s)) \\ &= \sum_{s=-m}^{m} w(s)\cdot\exp(i(\omega''-\omega')s)\cdot K(t)\cdot\exp(i\Theta(t))\cdot\exp(i(\omega''-\omega')t) \\ &= W(\omega''-\omega')\cdot K(t)\cdot\exp(i\Theta(t))\cdot\exp(i(\omega''-\omega')t) \end{aligned}$$

If the sequence $w(s)$, $s=-m\ldots m$, is chosen in such a way that $W(\omega)$ is unity near the origin and zero elsewhere, then:

$$y(t,\omega') = \begin{cases} K(t)\cdot\exp(i\Theta(t))\cdot\exp(i(\omega''-\omega')t), & \omega' \doteq \omega'' \\ 0, & \text{otherwise} \end{cases} \qquad (2.16)$$

which corresponds to a low frequency image of the apparent complex amplitude of the signal at frequency ω' . Thus if ω'' equals ω' , the demodulate has the form $K(t)\cdot\exp(i\Theta(t))$, while if ω'' is only approximately equal to ω' then the demodulate has the form $K(t)\cdot\exp(i(\omega''-\omega')t + i\Theta(t))$.

Considering now a stationary but non-deterministic process, we shall find it convenient to work with its Cramér representation as an infinite sum of complex exponential terms, which for a discrete variate is:

$$y(t) = \int_{-\Pi}^{\Pi} \exp(i\omega'' t)\cdot k_y(\omega'')d\omega'' \quad . \qquad (2.17)$$

This has the demodulates:

$$\begin{aligned} y(t,\omega') &= \sum_{s=-m}^{m} w(s) \int_{-\Pi}^{\Pi} \exp(i\omega''(t+s))\cdot k_y(\omega'')d\omega''\cdot\exp(-i\omega'(t+s)) \\ &= \int_{-\Pi}^{\Pi} \sum_{s=-m}^{m} w(s)\cdot\exp(i(\omega''-\omega')s)\cdot\exp(i(\omega''-\omega')t)\cdot k_y(\omega'')d\omega'' \\ &= \int_{-\Pi}^{\Pi} W(\omega''-\omega')\cdot\exp(i(\omega''-\omega')t)\cdot k_y(\omega'')d\omega'' \end{aligned}$$

Again if the time domain series $w(s)$ is chosen so that $W(\omega)$ is close to unity in the narrow band $\pm$ d and zero elsewhere, then:

$$y(t,\omega') = \int_{\omega'-d}^{\omega'+d} W(\omega''-\omega')\cdot\exp(i(\omega''-\omega')t)\cdot k_y(\omega'')d\omega'' \qquad (2.18)$$

which again is a low frequency image of the average component of the original series in the band near frequency ω'. We may note that, if $k_y(\omega)$ has the Dirac delta function form $K\cdot\exp(i\theta)\cdot\delta(\omega-\omega'')$ of Equation 2.15, then this result becomes identical with that of Equation 2.16. In fact a very similar result can be seen to hold if there is a "narrow" peak in the true spectrum near the demodulating frequency ω corresponding to something like a phase coherent signal (here "narrow" means that the peak is within the bandwidth $\omega' + d$ of the filter $W(\omega-\omega')$). Here the phase of the demodulates is reasonably stable and has a good physical interpretation. This becomes less true as we go to the more noise-like case of a smoother spectrum, where the phase becomes a highly random variable. However we shall argue that even in this case the relative phase of the demodulates of two different series is a stable and intuitively meaningful quantity, provided the coherence is reasonably high.

More generally we may note that, for a non-deterministic process with a smooth spectrum and randomized phasing near ω', the demodulates can be used to form estimates of the spectrum at ω' by observing that the variance of the demodulates may be written as:

$$E(y(t,\omega')\cdot y^*(t,\omega')) = E[\int_{-\Pi}^{\Pi} W(\omega'''-\omega')\cdot\exp(i(\omega'''-\omega')t)\cdot k_y(\omega''')d\omega'''\cdot$$

$$\int_{-\Pi}^{\Pi} W^*(\omega''-\omega')\cdot\exp(-i(\omega''-\omega')t)\cdot k_y^*(\omega'')d\omega'']$$

$$= \int_{-\Pi}^{\Pi} W(\omega'''-\omega')\cdot W^*(\omega''-\omega')\cdot\exp(i(\omega'''-\omega''))\cdot E[k_y(\omega''')\cdot k_y^*(\omega'')]d\omega'''d\omega''$$

$$= \int_{-\Pi}^{\Pi} |W(\omega-\omega')|^2 \cdot f_{yy}(\omega)d\omega \quad \text{(From Equation 2.13)}$$

$$= \int_{\omega'-d}^{\omega'+d} |W(\omega-\omega\ )|^2 \cdot f_{yy}(\omega)d\omega \quad \text{(for a suitable choice of window)} \qquad (2.19)$$

$$\doteqdot \hat{f}_{yy}(\omega) \qquad (2.19a)$$

(provided the spectrum is fairly constant over the (normalized) window).

This suggests that the spectrum of y(t) near ω' might be estimated by:

$$\hat{f}_{yy}(\omega') \doteqdot \frac{1}{2p+1} \sum_{t=-p}^{p} |y(t,\omega')|^2 \qquad (2.20)$$

Similarly from the estimated covariance of the complex demodulates of y(t) and x(t) we might get an estimate of the cross spectrum of the form:

$$\hat{f}_{yx}(\omega') \doteqdot \frac{1}{2p+1} \sum_{t=-p}^{p} y(t,\omega') \cdot x^*(t,\omega')$$

$$\doteqdot \frac{1}{2p+1} \sum_{t=-p}^{p} |y(t,\omega')| \cdot |x(t,\omega')| \cdot \exp[i(\theta_y(t,\omega')-\theta_x(t,\omega'))]$$

$$\doteqdot \frac{1}{2p+1} \sum_{t=-p}^{p} |y(t,\omega')| \cdot |x(t,\omega')| \cdot \exp(i\tilde{\phi}_{yx}(t,\omega')) \qquad (2.20a)$$

where $\tilde{\phi}_{yx}(t,\omega')$ represents the instantaneous difference between the phases $\theta_y(t,\omega')$ and $\theta_x(t,\omega')$ of the demodulates. This variable will be fundamental to this thesis. From these spectral and cross-spectral estimates it is possible to form estimates of the basic linear relationships between y(t) and x(t) as indicated by Equations 2.6 through 2.9. Thus from Equations 2.7, 2.20, and 2.20a an estimate of the transfer function is given by:

$$\hat{L}_{yx}(\omega') \doteq \frac{\sum_{t=-p}^{p} |y(t,\omega')| \cdot |x(t,\omega')| \cdot \exp[i(\Theta_y(t,\omega')-\Theta_x(t,\omega'))]}{\sum_{t=-p}^{p} |y(t,\omega')|^2} \qquad (2.21)$$

Similarly from 2.9, 2.20, and 2.20a the coherence can be estimated by:

$$\hat{C}_{yx}(\omega') \doteq \frac{|f_{yx}(\omega')|^2}{f_{xx}(\omega') \cdot f_{yy}(\omega')}$$

$$\doteq \frac{\sum_t \sum_{t'} |y(t,\omega')| \cdot |y(t',\omega')| \cdot |x(t,\omega')| \cdot |x(t',\omega')| \cdot \exp(i(\tilde{\phi}_{yx}(t,\omega')-\tilde{\phi}_{yx}(t',\omega')))}{\sum_t |x(t,\omega')|^2 \cdot \sum_{t'} |y(t',\omega')|^2}$$

$$\doteq \frac{\sum_t \sum_{t'} |y(t,\omega')| \cdot |y(t',\omega')| \cdot |x(t,\omega')| \cdot |x(t',\omega')| \cdot \cos(\tilde{\phi}_{yx}(t,\omega')-\tilde{\phi}_{yx}(t',\omega'))}{\sum_t |x(t,\omega')|^2 \cdot \sum_{t'} |y(t',\omega')|^2} \qquad (2.22)$$

(the imaginary parts must cancel, since the numerator is real from its original definition)

= a maximum ≤ 1 (with respect to the function $\tilde{\phi}_{yx}(t,\omega')$ if $\tilde{\phi}_{yx}(t,\omega')$ equals $\tilde{\phi}_{yx}(t',\omega')$ for all t, t'; further this maximum = 1 when $|y(t,\omega')| / |x(t,\omega')|$ equals $G_{yx}(\omega')$, with $G_{yx}(\omega')$ also independent of time). Similarly the estimated coherence between the two series will be high inasmuch as these conditions are approximated (i.e. that the relative phase and relative amplitude of the two demodulates stays close to constant). These conditions can readily be determined by inspecting for example plots of $\tilde{\phi}_{yx}(t,\omega')$ against time. We will focus particularly on this variable and note that a necessary condition for a high coherence is that $\tilde{\phi}_{yx}(t,\omega')$ be a constant over time, or almost so.

As far as an overall estimate of the phase itself goes, we have from Equation 2.6 that:

$$\tan\hat{\phi} = \frac{(\hat{f}_{yx}(\omega'))}{(\hat{f}_{yx}(\omega'))}$$

$$\therefore \frac{\sin\hat{\phi}_{yx}(\omega')}{\cos\hat{\phi}_{yx}(\omega')} \doteq \frac{\sum_t |y(t,\omega')|\cdot|x(t,\omega')|\cdot\sin\tilde{\phi}_{yx}(t,\omega')}{\sum_t |y(t,\omega')|\cdot|x(t,\omega')|\cdot\cos\tilde{\phi}_{yx}(t,\omega')} \qquad (2.23)$$

$$\doteq \frac{\sum_t \sin\tilde{\phi}_{yx}(t,\omega')}{\sum_t \cos\tilde{\phi}_{yx}(t,\omega')}$$

(if $\tilde{\phi}_{yx}(t,\omega')$ is fairly constant, which it will have to be if it is to be really worth calculating)

This suggests that a crude preliminary estimate of $\phi_{yx}(\omega')$ might be given by:

$$\hat{\hat{\phi}}_{yx}(\omega') = \frac{1}{2p+1}\sum_{t=-p}^{p}\tilde{\phi}_{yx}(t,\omega')$$

which seems reasonable again provided $\phi_{yx}(t,\omega')$ is fairly constant (as it will be in all cases where the coherence is high enough to make the phase calculation really worthwhile).

Hence it appears that time plots of the instantaneous phases and relative phases of demodulates make a useful low information starting point for empirical work in dynamic economics. Specifically, attention should be paid to both the variance and average value of the phase difference between the demodulates of pairs of series, as evidence of the existence and nature of any stable linear relation between them over the time period studied. Such an analysis has some obvious similarities with the aspirations of studies using N.B.E.R. methods but has the added advantage of being rather more systematic.

In fact it seems to the writer that the general theory of cyclical processes envisaged by practitioners of the N.B.E.R. procedures is analogous to the notion that a series y(t) and the corresponding reference series x(t) have instantaneous spectra with cyclical peaks whose properties might be understood in terms of their resemblance to the components:

$$K_x(t)\cdot\exp(i\,\omega t + i\Theta_x(t))$$

and

$$K_y(t)\cdot\exp(i\,\omega t + i\Theta_y(t))$$

where $\Theta_x(t)$ and $\Theta_y(t)$ may vary considerably over time but <u>only in such a way that</u>:

$$\Theta_x(t) - \Theta_y(t) = \phi_{xy} \doteqdot \text{constant} \tag{2.24}$$

If all K(t) and $\Theta(t)$ are band limited by some natural filter inherent in the generating process which cuts off at a lower frequency than some feasible estimation filter, we should be able to examine the reasonableness of the fundamental requirement 2.24 of this model by an inspection of the relative phase of the demodulates in the cyclical region of the spectrum.

The above discussion might also be compared with the intuitive exposition of spectral and cross-spectral estimates in terms of linear regression theory given in Section II of this chapter.[28] More

[28] See p. 28.

specifically we might return to an empirical study of the cyclical correlation between the growth rates of unfilled vacancies and unemployed (again removing the mean at the beginning to simplify later analysis).

These two series were demodulated according to Equation 2.14, using ω' equal to $2\Pi/54$ radians per month, and a simple moving average series w(s) equal to $1/(2m+1)$ over the range $\pm$ m where m equals 27. This results in demodulates which in fact are just complex Fourier coefficients and whose phase angles θ and relative phase ϕ are shown in Figure 2.7.

As may be verified by an inspection of this figure, the long run phase difference between these demodulates is almost exactly constant and equals Π radians. This indicates a high phase coherence between the series at the cyclical frequencies, and an average phase difference of half a cycle; a result that is in conformity with the cross-spectral finding shown in Table 2.1.

In a similar way, we may note from Figure 2.8 that the phase difference between unfilled vacancies and wool prices has no stable average value over time, and hence no apparent phase coherence. Again this agrees with the cross-spectral result of Table 2.1. We may note in passing that the phase of exports followed the same path as that of wool prices. In fact a comprehensive comparison of Table 2.1 and the results of complex demodulation indicates a very close agreement between the results of the two different ways of determining the average coherence and phasing of the series over the time period in question. An added advantage of going by the demodulate route however is, that by considering different time spans, one can obtain fairly direct information about the time structure of the frequency domain characteristics of the process, such as changes in the average cyclical frequency or changes in the average relative phasing of the cyclical components of different series.

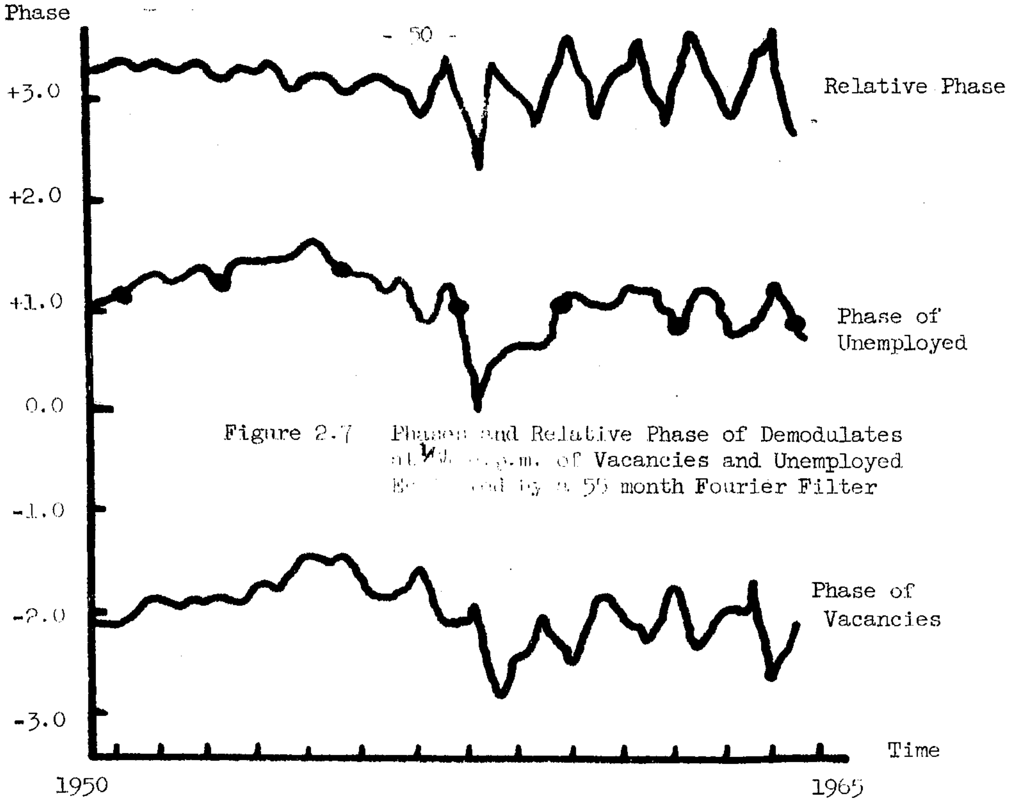

Figure 2.7 Phases and Relative Phase of Demodulates at 1/54 c.p.m. of Vacancies and Unemployed Estimated by a 55 month Fourier Filter

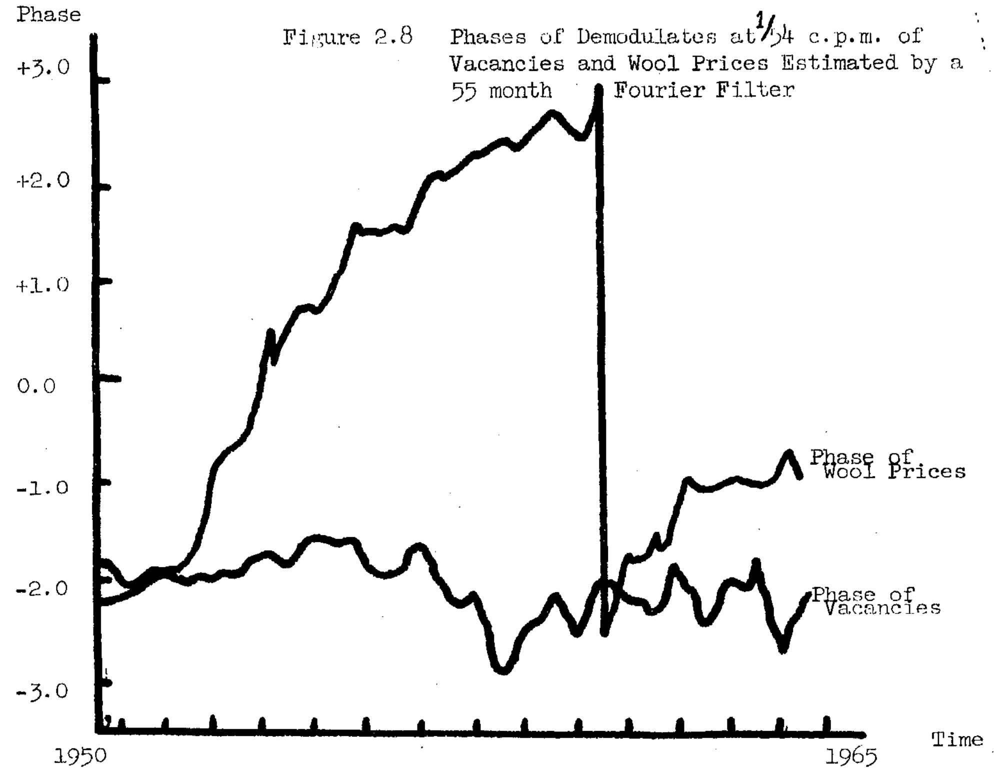

Figure 2.8 Phases of Demodulates at 1/54 c.p.m. of Vacancies and Wool Prices Estimated by a 55 month Fourier Filter

In considering the time structure of statistics based on the demodulates, one should bear in mind that the demodulates are by definition band limited, and hence successive values are only independent of one another if they are spaced far enough apart in time. In the previous calculations we were concerned to design a low pass filter which definitely discriminated between frequencies near zero and those near 1/54 c.p.m. In fact we chose a filter whose first lobe cut off at 1/55 c.p.m., and hence we know from the previously cited sampling theorem[29] that all the information sought from the demodulates is contained in values some 27 months apart. This fact is represented by the widely spaced large dots in Figure 2.7. In the limiting case of noise-like data the corresponding spectral estimates might be considered to be based on about one degree of freedom each. Hence these are a sufficient number of points to consider in attempting to ascertain the time structure of the cyclical components of the process.

However, an inspection of the actual demodulates already mentioned indicates that they plainly do include higher frequency components, and in particular a 1/12 c.p.m. fluctuation. This must be due to leakage of corresponding parts of the heterodyned signal through the side lobes of the rather crude low pass filter used. We may note that the frequency window $W(\omega)$ corresponding to the normalized moving average weighting scheme:

$$w(s) = \begin{cases} \frac{1}{2m+1}, & s = -m \ldots m \\ 0, & \text{otherwise} \end{cases} \qquad (2.25)$$

[29] See Appendix II.

has the form:

$$W(\omega) = \frac{\sin(\omega\cdot(2m+1)/2)}{\sin(\omega/2)\cdot(2m+1)/2}$$

which is shown in Figure 2.9 for the frequency range 0, 1/12 and where we choose 2m+1 equal to 60 for representational clarity.

In some cases, such as immigration, average earnings, brick production, and exports, the leakage appeared to be reasonably strong compared with the desired low frequency component passed by the main lobe of the low pass filter. This indicates the comparatively low importance of the variability in the band near the demodulation frequency in these series, and the possibility of serious bias due to leakage from other frequencies when using this type of window.

A better window, whose main lobe cuts out at the same point, can be achieved by using the moving average weights:

$$w(s) = \begin{cases} (2m+1-|s|)(2m+1)^2, & s = -2m\ldots 2m \\ 0, & \text{otherwise} \end{cases} \qquad (2.26)$$

This has the Fourier transform:

$$W(\omega) = \left(\frac{\sin(\omega\,(2m+1)/2)}{\sin(\omega/2)\cdot(2m+1)/2}\right)^2$$

Again taking 2m+1 equal to 60 and using the frequency scale 0, 1/12 this window is shown in Figure 2.10. It will be noted that it has smaller side lobes than those shown in Figure 2.9. This new filter is often described as a "Bartlett window" and in fact can be realized by the computationally

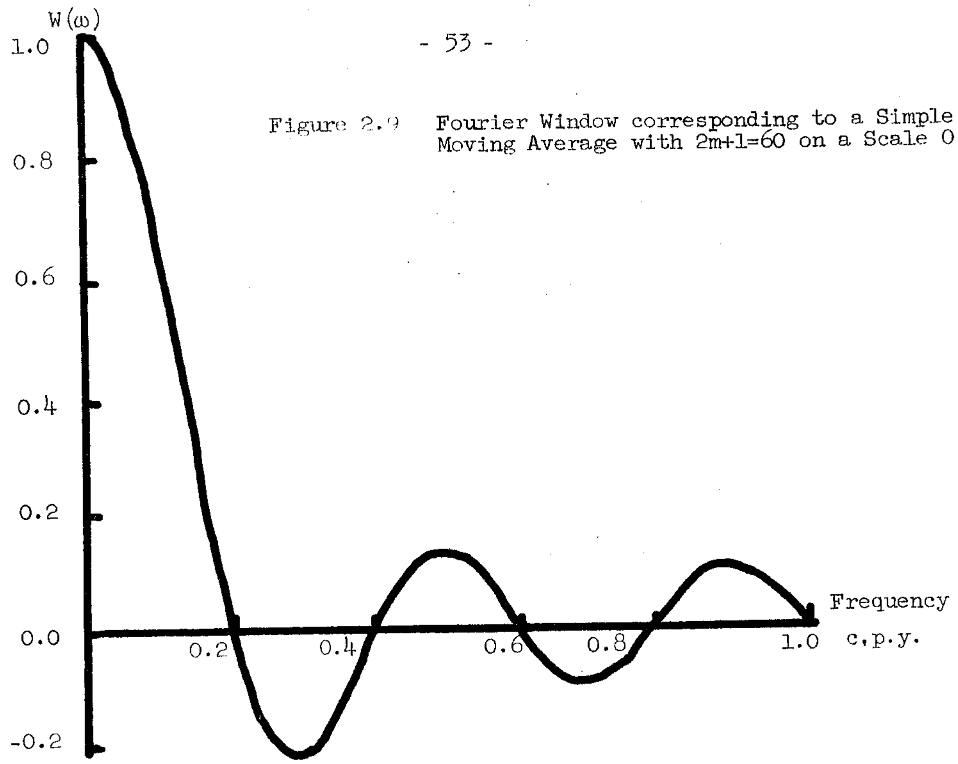

Figure 2.9 Fourier Window corresponding to a Simple Moving Average with 2m+1=60 on a Scale 0 - 1/12

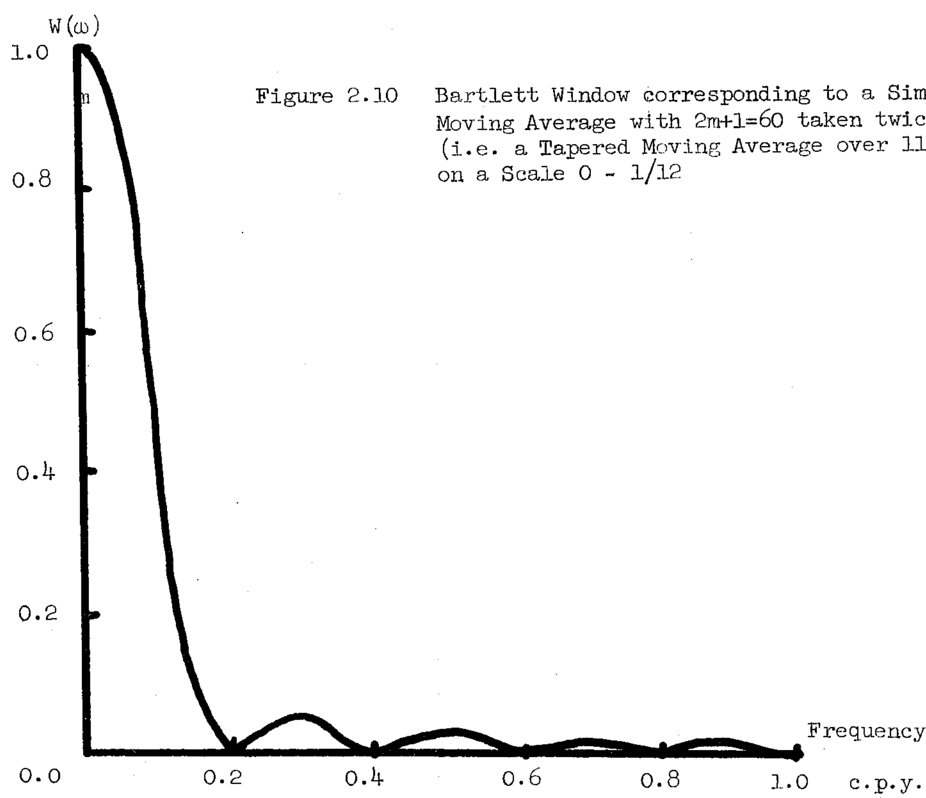

Figure 2.10 Bartlett Window corresponding to a Simple Moving Average with 2m+1=60 taken twice over, (i.e. a Tapered Moving Average over 119 terms), on a Scale 0 - 1/12

simple operation of taking a simple constant weight moving average of the type shown in Equation 2.25 twice over[30], thus:

$$\frac{1}{(2m+1)^2} \sum_{s=-m}^{m} \sum_{s=-m}^{m} y(s) = \frac{1}{(2m+1)^2} \sum_{s=-2m}^{2m} (2m+1-|s|)\cdot y(s)$$

From the point of view of this thesis, the Bartlett window has the added advantage that, in using the resulting demodulates to compute the spectrum and cross-spectrum as in Equation 2.19, the corresponding spectral estimation window is:

$$|W(\omega)|^2 = \left(\left(\frac{\sin(\omega.(2m+1)/2)}{\sin(\omega/2)\cdot(2m+1)/2}\right)^2\right)^2$$

$$= \left(\frac{\sin(\omega\cdot(2m+1)/2)}{\sin(\omega/2)\cdot(2m+1)/2}\right)^4$$

which in fact corresponds to the "Parzen window" used in our previous calculations of the spectrum and cross-spectrum from the serial correlation functions. Figure 2.11 indicates such a Parzen window with 2m+1 and the frequency scale chosen as above to facilitate comparisons.

One big disadvantage of demodulating with the Bartlett window however is that it uses a longer moving average sequence and hence considerably limits the time span covered by the demodulates derived from a given span of data. Thus if the data are given at the discrete points t=1,2...T; then using the Fourier weights of Equation 2.25 wigh m equals 27 one obtains demodulates at the time points t=28,29...T-27. Using the Bartlett weights of Equation 2.26, the demodulates are constrained to the range t=56,57...T-54

[30] C. Bingham, M. D. Godfrey and J. W. Tukey, op. cit.

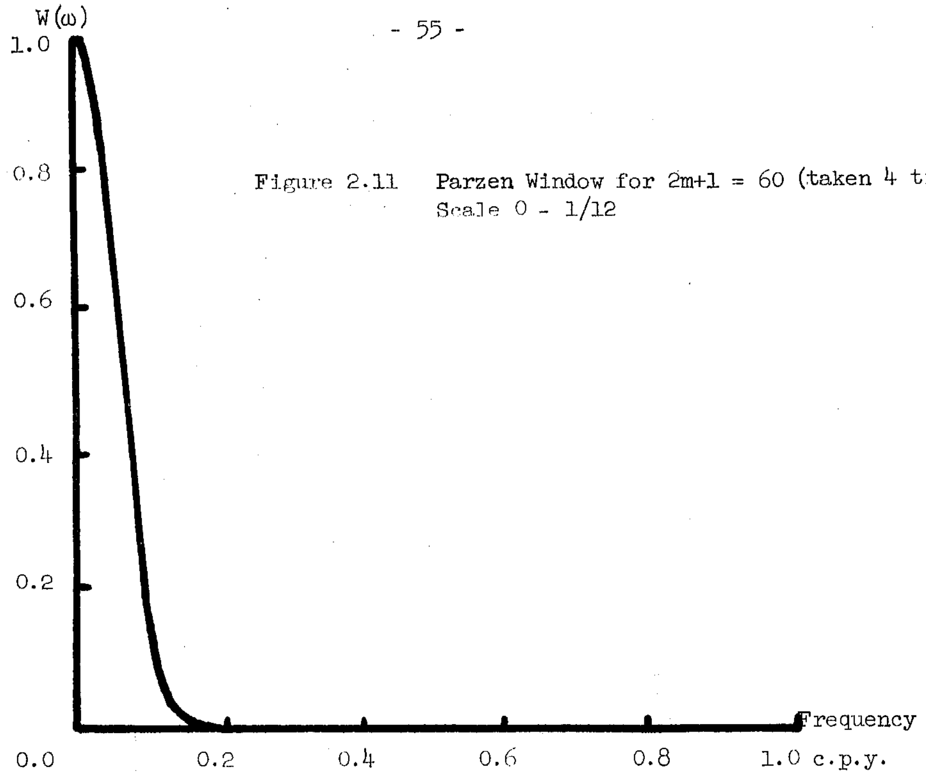

Figure 2.11 Parzen Window for 2m+1 = 60 (taken 4 times) on a Scale 0 - 1/12

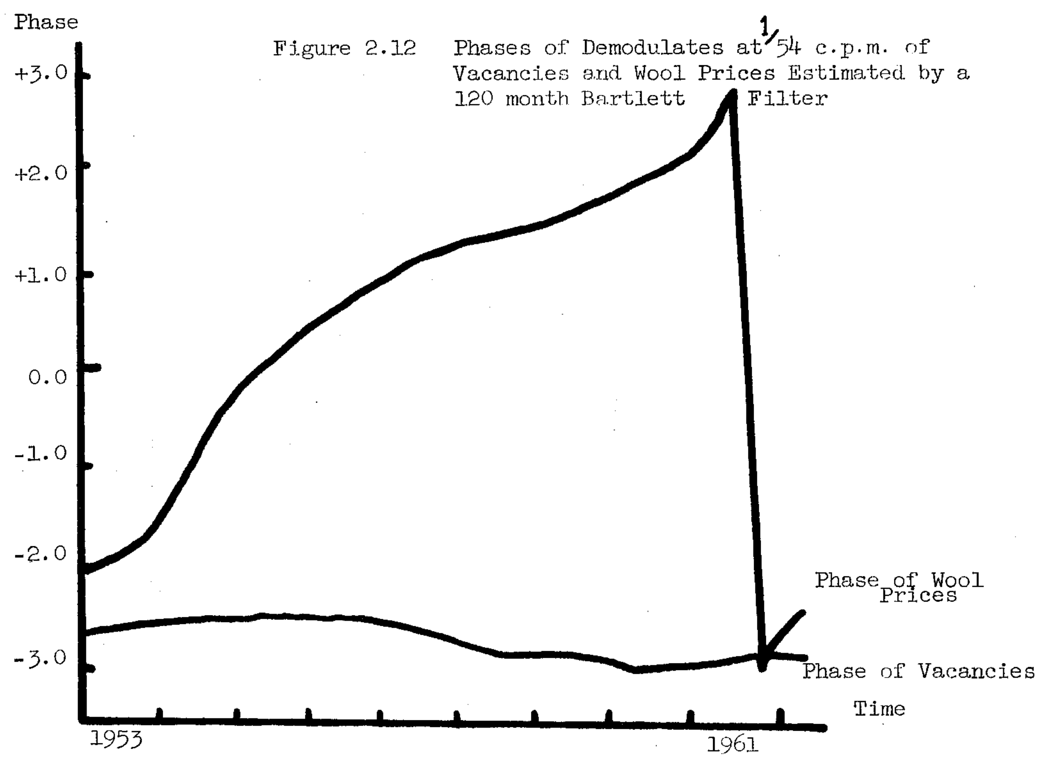

Figure 2.12 Phases of Demodulates at 1/54 c.p.m. of Vacancies and Wool Prices Estimated by a 120 month Bartlett Filter

and in our case where T is only 228 this is a serious restriction. But because of its other advantages this is the procedure we shall use henceforth in this chapter. To indicate the more effective smoothing properties of the Bartlett window, we show in Figure 2.12 the results obtained from its use to determine the phase of the demodulates of vacancies and wool prices. These can be compared with the earlier more irregular results of Figure 2.8.

V. Conclusions

Having thus indicated the general statistical meaning of the results shown in Table 2.1 and shown how we can study their stability over time in terms of the relative phase of demodulates, it is now appropriate to consider in some detail their economic interpretations. Some of these results appear to indicate the operation of a few rather interesting and consistent economic tendencies as will now be seen by considering them in the theoretically natural groupings of Table 2.1. Their robustness is indicated by Figures 2.13 through 2.18 showing the relative phase of their demodulates with respect to that of the reference series using the Bartlett window discussed above.

A. Labor Market Variables

As has been observed, registered unemployment and unfilled vacancies have a very high coherence (0.95), and are exactly out of phase at the cyclical frequencies (see Figure 2.13). Furthermore this has a more or less intuitive interpretation in terms of the inverse character of the excess demand and supply functions in the labor market. In a similar way it can be noted that the cyclical components of employment and unfilled vacancies are fairly closely related (coherence = 0.65), and are more or less in phase, though on average over time employment seems to lag vacancies by about two months. This indicates perhaps a general tendency towards short adjustment delays between a change in the number of workers sought and the actual change in the number of workers employed.

It was considered worth attempting to refine this analysis somewhat by considering the connection between employment and unemployment. This was studied in the form of an examination of the discouraged worker hypothesis of Okun's law, which suggests that the labor force participation rate is related to the fraction unemployed by a coefficient between zero and minus one. However despite the fact that the predicted relative phase of Π was obtained, the coherence was only 0.4 at the cyclical frequencies, indicating perhaps that this relationship is not very significant as a cyclical phenomenon in Australia.

Rather more interesting seemed to be the coherence of 0.65 and average lag of about one month of immigration behind unfilled vacancies (see Figure 2.13). This might suggest a re-examination of Kmenta's recent suggestion that the post-war fluctuations in immigration have been exogenous to the economy.[31] Other demographic variables, such as the birth rate, death rate, and marriage rate, have had a negligible coherence with the cyclical component of vacancies. Nor, incidentally, were these series closely related to one another (not even births and marriages).

B. Wage and Price Indicators

Average earnings and the aggregate wholesale price index for domestically produced goods shows a cyclical coherence of greater than 0.5 and an average phase lag of close to $\Pi/2$ with unfilled vacancies (see Figure 2.14). This might be considered as being consistent with the

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[31] Jan Kmenta, "An Econometric Model of Australia, 1948-61", Australian Economic Papers, December 1966, pp. 131-164.

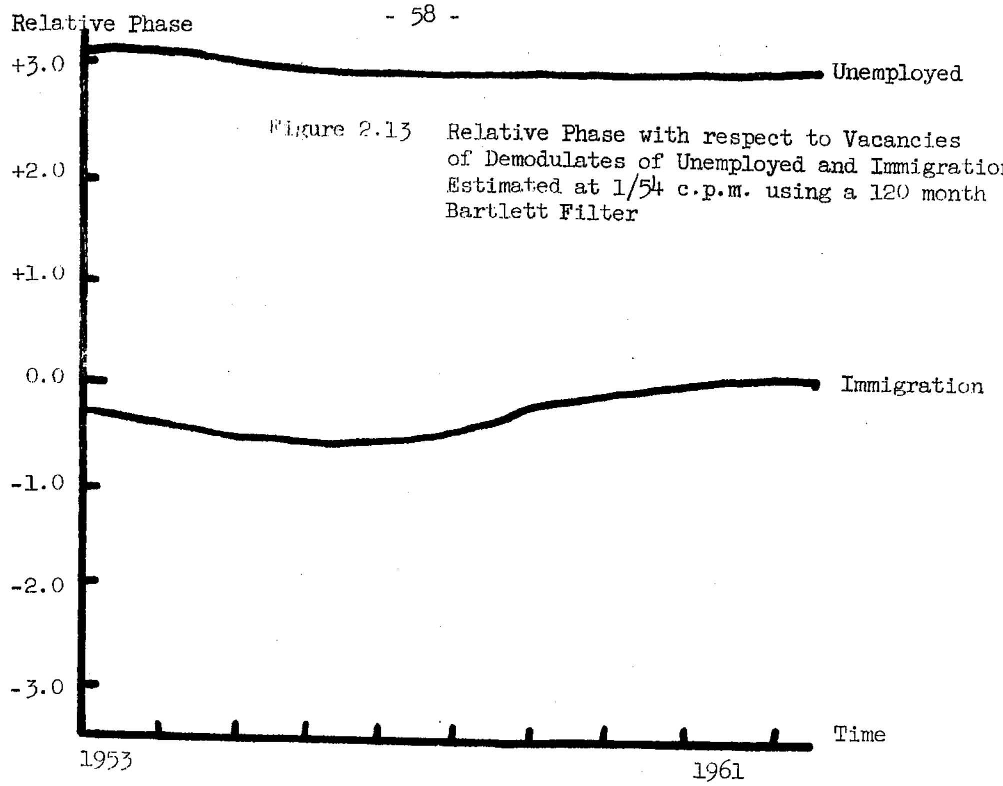

Figure 2.13 Relative Phase with respect to Vacancies of Demodulates of Unemployed and Immigration Estimated at 1/54 c.p.m. using a 120 month Bartlett Filter

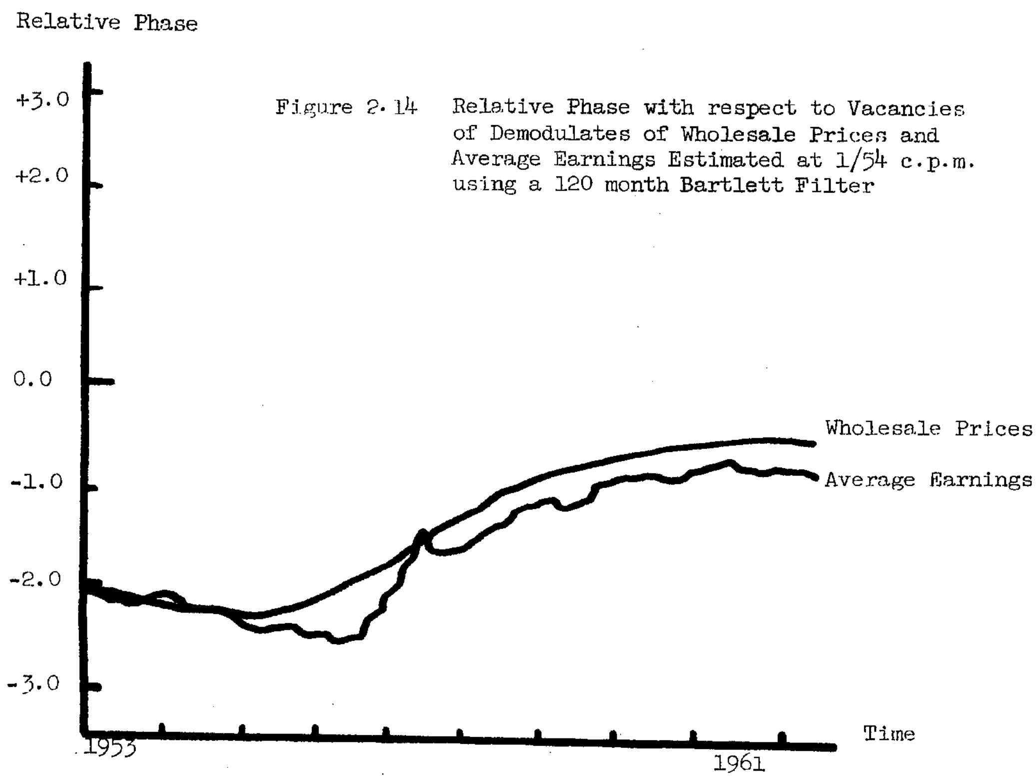

Figure 2. 14 Relative Phase with respect to Vacancies of Demodulates of Wholesale Prices and Average Earnings Estimated at 1/54 c.p.m. using a 120 month Bartlett Filter

Walrasian market adjustment function, currently popularized by Phillips curve studies. These suggest that the "proportionate rate of change of the money wage rate depends upon the 'rate' of unemployment".[32] It is only necessary to remember that the earnings or price series has, by definition, a phase lag of $\Pi/2$ on the corresponding proportionate rate of change series; which is held by the hypothesis to move with our proxy variable for excess demand (e.g. an inverse measure of unemployment, or as in this case, unfilled vacancies). This model then explains what has perhaps seemed a little startling to some observers: that the cyclical upswing of earnings and prices continues even after the peak of the boom in real activity has passed.[33]

Consumer prices also showed a reasonable coherence with the reference series but had a larger phase lag ($\Pi/2 + 0.4$). This might be interpreted as an additional time lag of about one quarter of consumer prices behind costs (earnings and wholesale prices). Such a suggestion is consistent with the analysis given by Boehm[34] and is in fact borne out by a coherence of 0.9 and a phase lag of 0.3 of consumer prices with respect to domestic wholesale prices (see Figure 2.15).

It is also revealing to observe that none of these earnings or price indicators has a significant coherence with the money supply: an observation which is consonant with the well-known variability in the velocity of circulation which has occurred in Australia in the post-war era.

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[32] W. A. Phillips, "The Relation Between Unemployment and the Rate of Change of Money Rates in the United Kingdom, 1861-1957", Economica, November 1958, pp. 283-299.

[33] P. H. Karmel, "The Australian Economy, March 1961", The Economic Record, March 1961, pp. 1-24.

[34] E. A. Boehm, "The Australian Economy, February 1964", The Economic Record, March 1964, pp. 1-32.

Share prices have a high coherence (0.8) with the reference series but a zero phase lag (see Figure 2.16). The idea of a close correlation between these series is broadly consistent with existing beliefs, though the phase lag contradicts a common notion that Australian share market fluctuations lead cycles in real activity.[35]

C. Production Indicators

The basic materials and fuel production series studied (gas, coal electricity, cement, timber and steel) had a negligible coherence with the reference series. This presumably is the basis of the difficulty Australian economists have in trying to interpret the diversity of movement of the available raw material production indices over the business cycle.[36] The only possible exception noted to this rule for basic materials was the clay brick production series, which had a coherence of 0.65 and a time lead of about five months (see Figure 2.17). This may be due to its close relation with the housing market which has been held to be a leading indicator.[37] The production of refrigerators shows an even higher cyclical coherence (0.8) and a time lead of about seven months over the reference series. Stove production also shows an overall average time lead, of about two months, but the coherence is only 0.45 (see Figure 2.17).

Thus what scant information there is suggests that Australian labor market indicators lag production, at least in the housing and consumer durables markets. A similar finding was made by Hatanaka for the

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[35] P. H. Karmel, "The Australian Economy, March 1961", The Economic Record, March 1961, pp. 1-24.

[36] See P. H. Karmel, ibid.

[37] B. L. Johns, "The Australian Economy, August 1964", The Economic Record, September 1964, pp. 325-362.

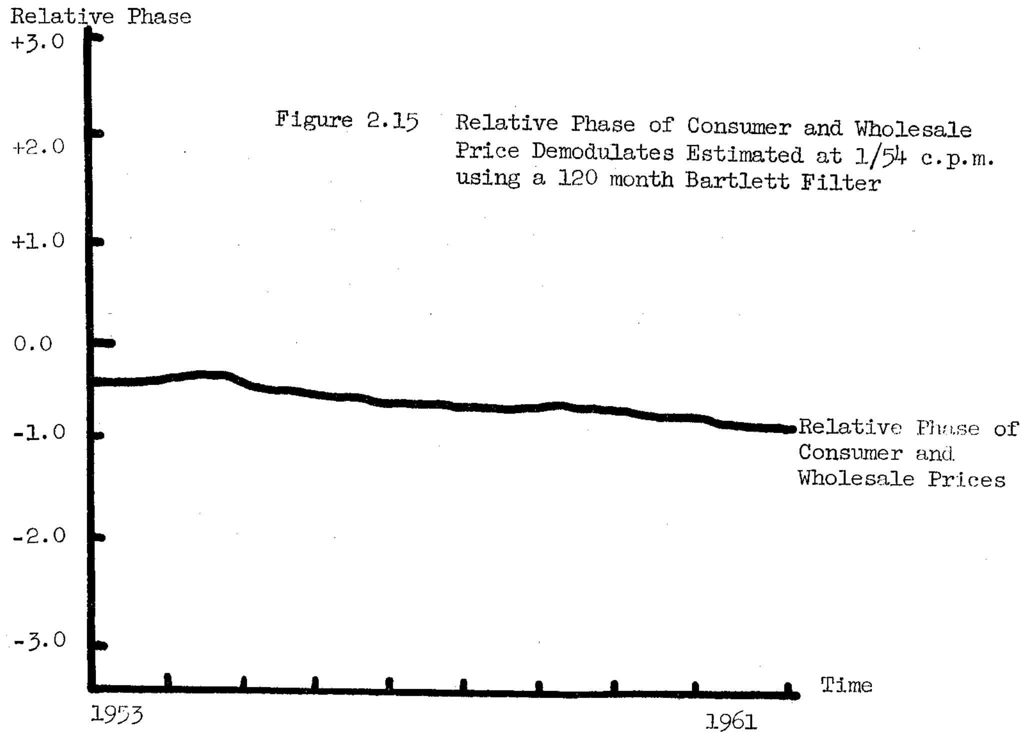

Figure 2.15 Relative Phase of Consumer and Wholesale Price Demodulates Estimated at 1/54 c.p.m. using a 120 month Bartlett Filter

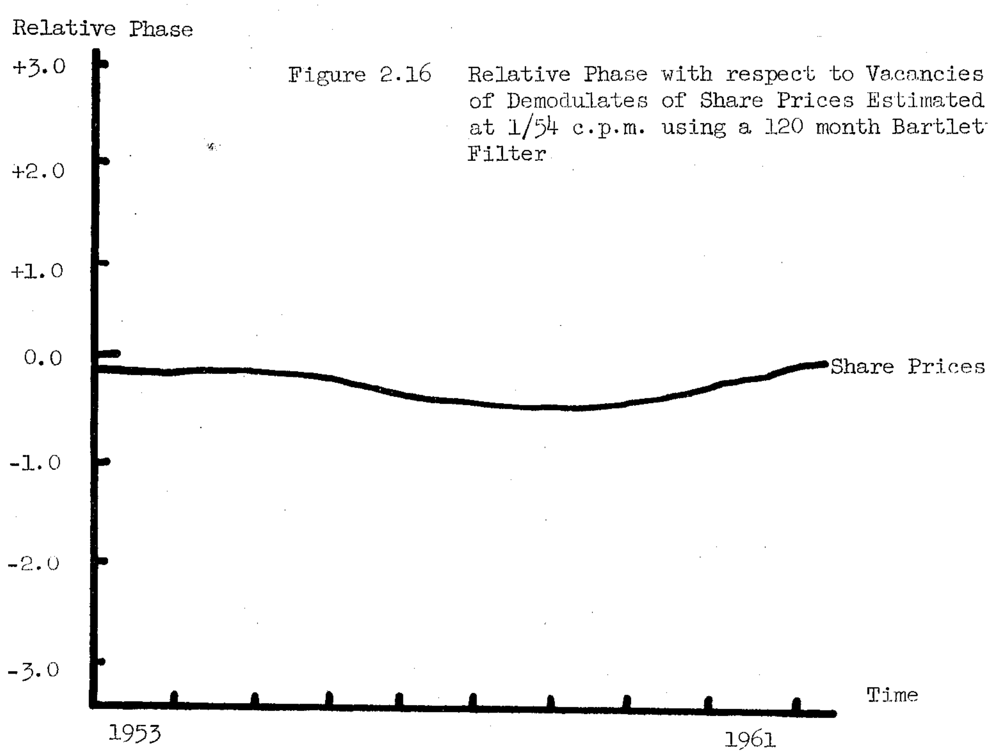

Figure 2.16 Relative Phase with respect to Vacancies of Demodulates of Share Prices Estimated at 1/54 c.p.m. using a 120 month Bartlett Filter

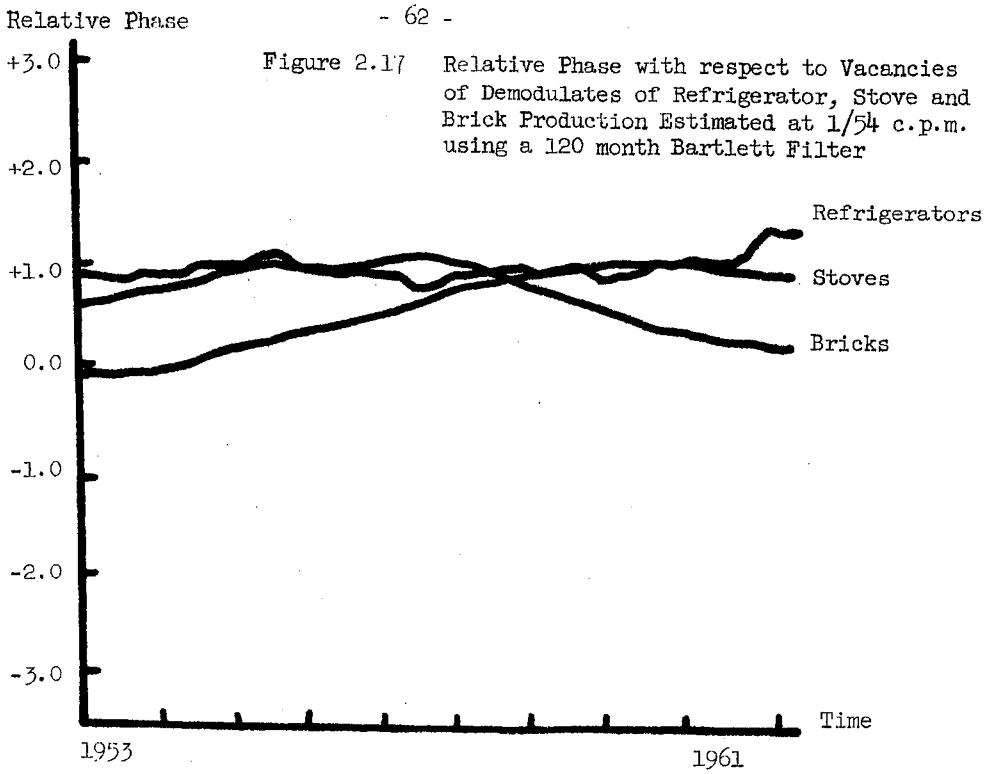

Figure 2.17 Relative Phase with respect to Vacancies of Demodulates of Refrigerator, Stove and Brick Production Estimated at 1/54 c.p.m. using a 120 month Bartlett Filter

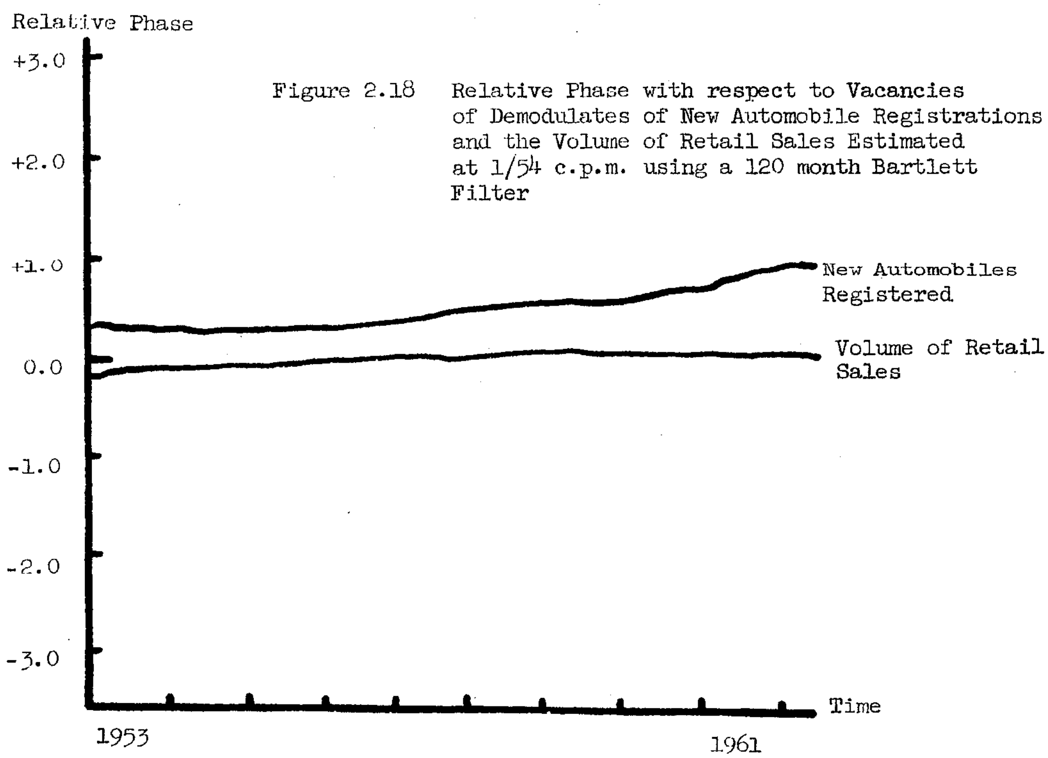

Figure 2.18 Relative Phase with respect to Vacancies of Demodulates of New Automobile Registrations and the Volume of Retail Sales Estimated at 1/54 c.p.m. using a 120 month Bartlett Filter

United States on comparing the factory employment and industrial production indices: a result which he found paradoxical on technological grounds, considering that labor is an input and production an output.[38] However this seems to overlook the distinction between the employment and utilization of labor, and the micro-economic consideration that the costs to a firm of changing its work force can justify a slow response in attempting to adjust employment to the level of output. As Paish observed, concerning the post-war British business cycle:[39]

> It is a matter of common observation that when the rise in output is checked, employers tend to retain, at least for some time, a good deal of labour, although it is no longer fully employed, so that some of the margin of unused capacity created is in the form of the under-utilization of labour which is recorded as employed. Conversely, when demand recovers, the first capacity to be re-employed tends to be this unused margin of capacity in the existing labour force. Thus fluctuations in the proportion of labour employed may be expected to lag behind changes in the proportion of total capacity in use.

Hence the idea of using labor market indicators as a proxy for the general cyclical state of economic activity seems to be questionable. But it should be borne in mind that our observation of the lag of Australian vacancies behind production has been based on only the few fragmentary production series that are available. Unfortunately there is no official aggregate production index published in Australia so it is not possible to investigate this matter more directly. However it should be possible to get a more complete picture of the situation by considering also the sales side, which we will now discuss.

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[38] C. W. Granger in association with M. Hatanaka, Spectral Analysis of Economic Time Series, (Princeton University Press, Princeton, N. J., 1964), p. 240.

[39] F. Paish, Studies in an Inflationary Economy: The United Kingdom 1948-1961, (Macmillan, London, 1962), p. 318.

D. Sales Figures

The monthly number of new automobiles registered was observed to have a coherence of 0.80 and a lead of about one quarter over unfilled vacancies: a finding which is akin to that made by Johns[40] (see Figure 2.18).

Quarterly figures representing the value of aggregate retail sales (exclusive of motor vehicles) have been published by the Commonwealth Statistician since 1950; but even before this, the Retail Traders Association of New South Wales published a quarterly index of the value of retail trade in Sydney (Australia's largest city, population 2,500,000). From these two series and a consumer price index (see Appendix I), an index of the volume of retail sales was constructed for the period 1948-66 and compared with unfilled vacancies. A high coherence of 0.75 was found, but with sales moving in phase with vacancies (see Figure 2.18). Perhaps the main lesson to draw from these varied results and those in Section C is that aggregation can clearly mask a variety of different effects and so statements about the relative phasing of the cyclical components of highly aggregated series have little obvious meaning.

E. International Transactions

One rather interesting result obtained in this study was the negligible coherence of the cyclical component of the reference series with exports: suggesting that the four to five year post-war growth cycle is more or less endogenous to the Australian economy. The export fluctuations at these and neighboring frequencies (down to 1/2 cycles per year) correspond to wool price fluctuations as is indicated by a coherence of about 0.8 and zero phase difference between the series. It may be noted that the low

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[40] B. L. Johns, op. cit.

average (broad band) coherence between either of these series and the reference series corresponds to a general upward trend in the phase of their cyclical demodulates as is shown for example in Figures 2.8 and 2.12. This upward slope, of $2\Pi/120$ radians per month, may be interpreted as being due to a semi-regular fluctuation at a neighboring frequency ω'' which according to Equation 2.16 is given by:[41]

$$2\Pi(\omega''/2\Pi - 1/54) = 2\Pi/120$$

Thus there appears to be an export (price) cycle of about 1/38 c.p.m. which of course is a very different thing from our reference cycle, at least with regard to the linear theory used in this thesis. This of course cannot be the last word on the much controverted subject of the "dependence of the Australian economy" on its export markets.[42] The dynamics of these markets (specifically the wool market) will be discussed in detail in the next chapter. All that is asserted at this stage is that the recent four to five year Australian growth cycle does not appear to have been imported directly. The value of merchandise imports on the other hand has a high coherence (0.80) and an average time lag of about one quarter behind the cyclical component of vacancies.

Mallyon lays considerable emphasis on the lead he noted of the gold and foreign exchange reserves over the reference cycle.[43] In this cross-spectral study, a lead was also found, though it was rather larger

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[41] A similar property for demodulates near the seasonal frequencies of economic time series has been reported by M. D. Godfrey, "An Exploratory Study of the Bi-Spectrum of Economic Time Series", Applied Statistics, January 1965, pp. 48-69.

[42] For conflicting views on this subject see for example: Sir Douglas Copland, "The Australian Economy, A New Look", The Economic Record, August 1957, pp. 141-152; and H. W. Arndt, "The Australian Economy, September 1957", The Economic Record, December 1957, pp. 285-310.

[43] J. S. Mallyon, "Statistical Indicators of the Australian Trade Cycle", Australian Economic Papers, June 1966, pp. 1-20.

(approximately $\Pi/2$) and corresponding to a coherence of about 0.60. This might perhaps be interpreted as the effect of a cumulation (i.e. integration) of payments for imports, which as we have seen are more or less coincident with the reference series. It should be remembered that integration corresponds to a phase change of $\Pi/2$. This explanation would assume of course that exports and capital flows are either relatively small, or not coherent with the cyclical component of the reference series.

CHAPTER THREE

WOOL MARKET FLUCTUATIONS

I. Australia and the International Wool Market

As the foregoing discussion indicates, there does not appear to be a close connection between the "cycle" in most sectors of the Australian economy and fluctuations in the Australian export sector, particularly the wool industry. Bearing in mind the international character of the wool market, it seems worth examining the possibility that the Australian wool industry fluctuations may be more closely related to the variation in activity of the wool consuming industries of other economies, than to anything which happens in various markets in Australia. In particular we might consider the plausability of the idea that the fluctuations in the wool markets of various countries are sufficiently similar to warrant establishing a uniform international approach to wool market fluctuations.

Nowadays, some 75 per cent of Australian sheep are of the merino breed, and in the past the percentage has been even higher.[1] Further the Australian wool industry produces a half to two-thirds of the total world's supply of merino wool, and so its role in the world market is extremely important. The Wool Handbook explains:[2]

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[1] Commonwealth of Australia, Bureau of Agricultural Economics, Statistical Handbook of the Sheep and Wool Industry (3rd ed., Canberra, Australia, 1961), p. 12.

[2] E. M. Pohl, "Grading and Production of Wool", which is Chapter 8 of Werner von Bergen (ed.), Wool Handbook, Vol. 1 (John Wiley and Sons, New York, 1963), p. 584.

> Merino wools are the most valuable wools produced in the world. Approximately 30 per cent of the world's wool production in 1958 came under the merino classification. Of this, from 65 to 70 per cent was produced in Australia according to figures available.
>
> The merino wools are noted for their softness, fineness, strength, and elasticity and are especially desired for their superior spinning and felting properties. For spinning the finest woolen and worsted yarns, merino wools are an absolute necessity. Flannels and knit goods of high quality, suiting and dress goods of fine texture, face-finished fabrics such as broadcloths, billiard cloths, doeskins, meltens and various uniform cloths are dependent on merino stock. No other wool would give the required appearance, handle, finish, and character that distinguishes each of these fabrics, and in many cases no other wool could be spun to the required fine yarn sizes.

As in Chapter Two of this dissertation, we shall especially be concerned with fairly regular short-run fluctuations and their explanation. We might begin by noting that one author has undertaken an examination of some correlograms describing the sheep population of New South Wales, the major wool-producing state. He argues that these show that there exists an "eighteen year cycle and an autocorrelated four year cycle" in the Australian wool industry.[3] However, in view of the development of empirical time series analysis, and especially spectral techniques, subsequent to this earlier analysis, it seems worth using these new methods to reconsider the existence and nature of the alleged cycles.

II. Fluctuations in Australian Sheep Numbers

Following Rutherford's approach, we consider the New South Wales sheep population at annual intervals over the period 1861-1940. We compute

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[3] R. S. Rutherford, "Fluctuations in the Sheep Population of New South Wales, 1860-1940", The Economic Record, June 1948, pp. 56-71.

the serial correlation coefficients of these data y(t) out to the fortieth lag, using the original data rather than Rutherford's deviations from fifteen year and five year moving averages. An inspection of this correlogram, Figure 2.1, does not suggest any obvious cycles, nor indeed does the corresponding spectrum, Figure 3.2, which in fact has merely the typical spectral shape for an economic variable.[4] Hence it seems appropriate to consider whether Rutherford's modifications of the data had anything to do with the cycles he observed.

It is clear that subtracting a 2m+1 year moving average from a series y(t) gives a series:

$$x(t) = y(t) - \frac{1}{2m+1} \sum_{s=-m}^{m} y(t+s) \tag{3.1}$$

So if the original series has the Cramér representation:

$$y(t) = \int_{-\Pi}^{\Pi} \exp(i\omega t) \cdot k_y(\omega) d\omega \tag{3.2}$$

then from Equation 3.1 we see that the modified series has the representation:

$$x(t) = \int_{-\Pi}^{\Pi} \exp(i\omega t) . L(\omega) \cdot k_y(\omega) d\omega \tag{3.3}$$

where:

$$L(\omega) = 1 - \frac{1}{2m+1} \sum_{s=-m}^{m} \exp(i\omega s) \tag{3.4}$$

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[4] C. W. Granger in association with M. Hatanaka, op. cit., pp. 58-9.

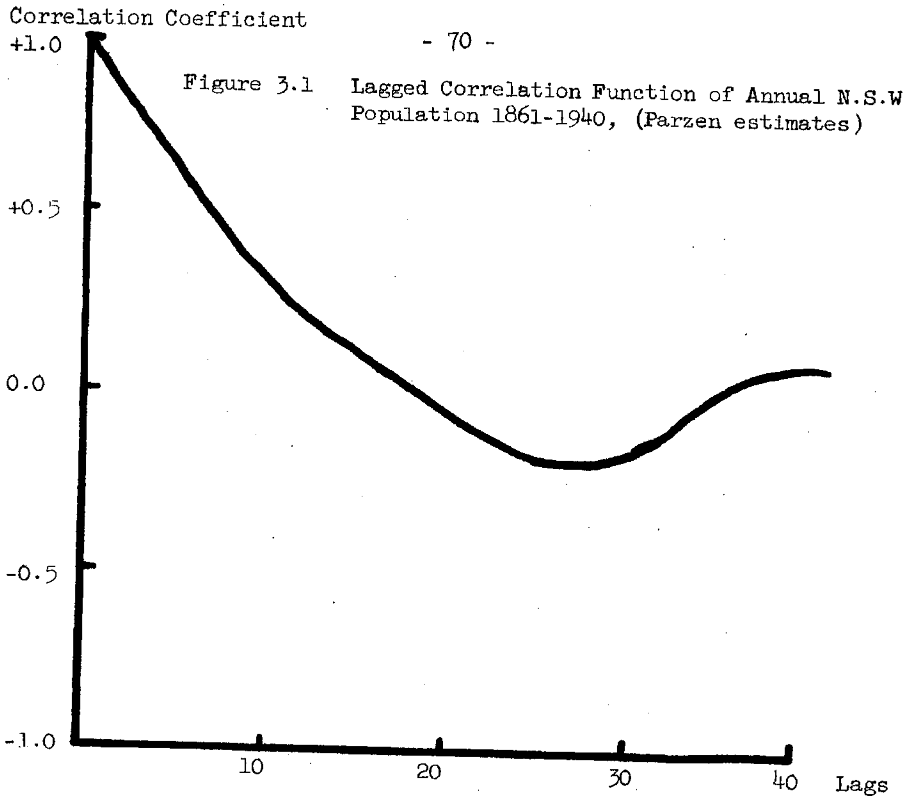

Figure 3.1 Lagged Correlation Function of Annual N.S.W. Sheep Population 1861-1940, (Parzen estimates)

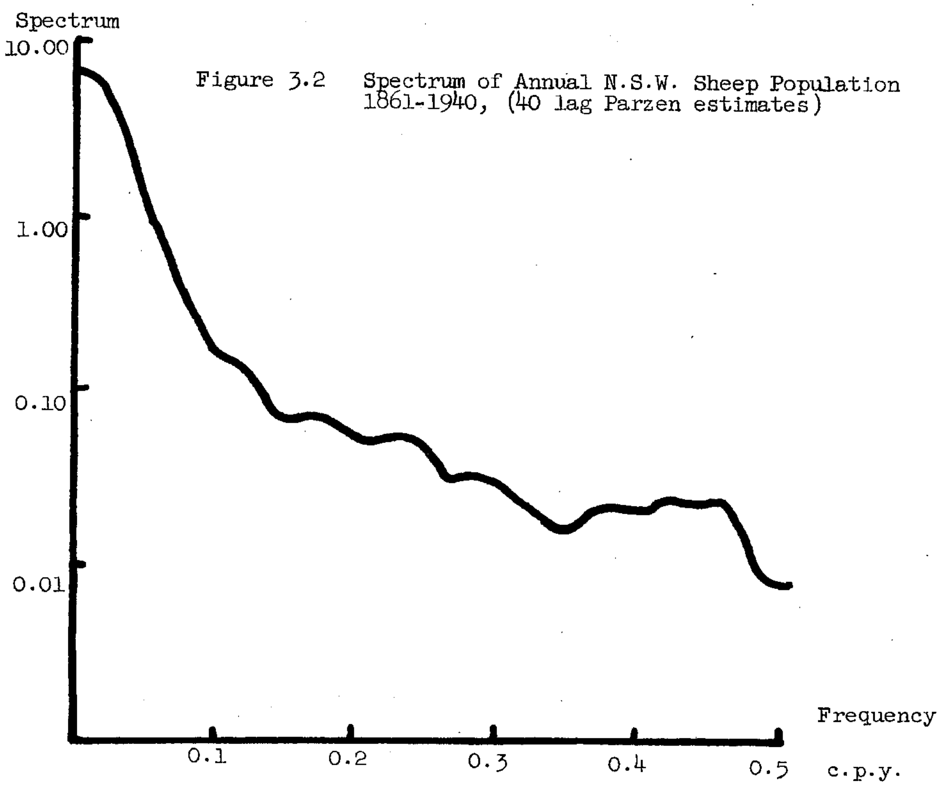

Figure 3.2 Spectrum of Annual N.S.W. Sheep Population 1861-1940, (40 lag Parzen estimates)

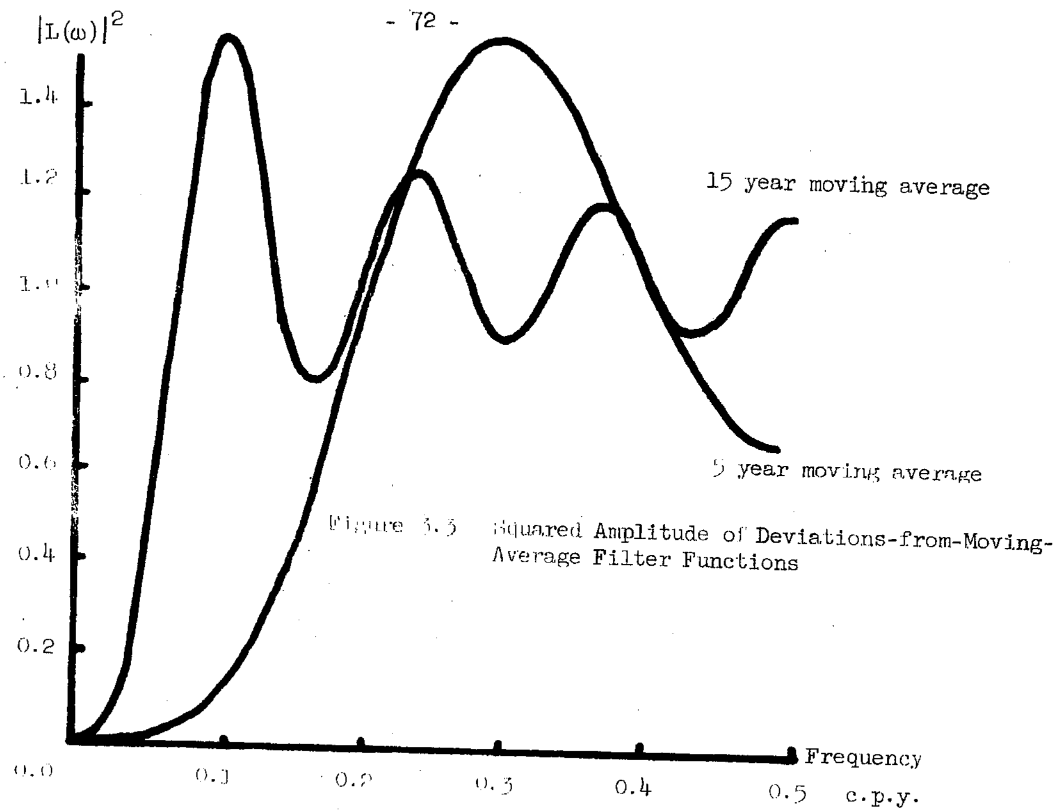

Figure 3.3 Squared Amplitude of Deviations-from-Moving-Average Filter Functions

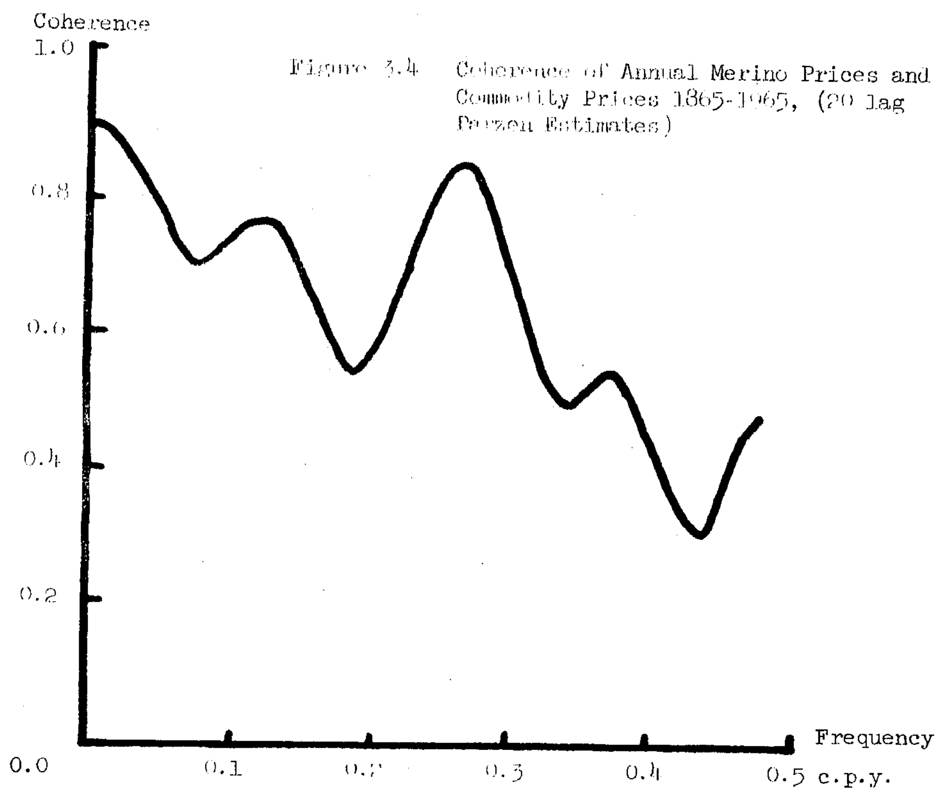

Figure 3.4 Coherence of Annual Merino Prices and Commodity Prices 1865-1965, (20 lag Parzen Estimates)

especially now that more comprehensive tools and data are available. Fortunately in the case of the wool market, we have reasonably good data going back a century or so and hence one might hope really to take advantage of the techniques discussed in the previous chapter.

III. Fluctuations in the Growth of Wool Market Variables

The arguments adduced in Chapter One for studying series based on the first differences of logarithms of variables, rather than the variables themselves, seem to be particularly pertinent when it comes to studying the biological growth of the sheep population of Australia over the past century. Series representing a transformation of this kind on the annually reported sheep populations for all six Australian states, taken separately, and in aggregate, were analyzed in this study. In all cases, the spectra were rather similar to those expected from random noises. Thus in no case was there evidence of anything that might be characterized as a sequence of even vaguely systematic growth cycles. Nor incidentally did the cross-spectra between the growth rate series for different states ever show a notable coherence at any particular frequency. A similar lack of evidence for any growth cycles was indicated by the spectrum of the first difference of logarithms of annual Australian wool production, also there was no notable coherence at any frequency between this series and a similarly transformed merino wool price index.

Thus there do not appear to be any regular flucatuations on the production side of the Australian wool market, nor does there appear to exist any systematic connection between fluctuations in wool prices and in the sheep population or wool production at any frequencies.

However both the spectra and the coherence (Figure 3.4) of the growth rate series (normalized as before) of the annual Sauerbeck indices for merino prices and general commodity prices over the past century[5] have local peaks in the neighborhood of a three to four year cycle. The corresponding gain is approximately equal to one. It should be pointed out however that the standard deviation of the original merino price index is about twice that of the commodity price index, and so it would be necessary to multiply this gain by a factor of two to obtain the transfer function between the unnormalized commodity prices and merino prices (in growth rate form as usual). Thus we observe that the three to four year fluctuations in the merino prices, besides having been closely correlated with corresponding fluctuations in annual commodity prices over the last century, have about twice their amplitude; and as the phase measurement indicates, they are on average almost coincident with it, with a small average lead of about 0.5 radians or 3 months.

As has been observed in the last chapter, fluctuations in the rate of growth of prices tend to move more or less in phase with the corresponding fluctuations in the level of excess demand for the goods in question, so the above results might seem to reflect the effect on wool prices and commodity prices of the well known Kitchin cycle in inventories and industrial activity. This is often characterized as a two to four year cycle and as we saw in the last chapter some such cycle appears to have been prominent in the post-war prices for Australian wool. Such cycles in the wool and cotton textile industries are often mentioned in the literature,[6] and hence it seems worth going on to investigate their nature and behavior over time by the methods of complex demodulation.

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[5] A. Sauerbeck, "Prices of Commodities and Precious Metals", Journal of the Royal Statistical Society, September 1886, pp. 581-643 and continued thereafter in the same source by Sauerbeck and subsequently by the Editor of The Statist.

[6] W. H. Miernyk and M. Zymelman, Inventories in the Textile Cycle, (U.S. Department of Commerce, Business and Defence Services Administration, Washington, D. C., 1961).

In Figures 3.5 through 3.8 are shown the amplitudes and phases of the demodulates at a 41 month cycle of the merino price index and the commodity price index over the period 1880-1950, using the low pass Parzen filter shown in Figure 3.9. Actually the data range from 1866 to 1964 but the filter consumes a number of end terms from such series. It will be observed that these results confirm the above cross spectral findings in the sense that the wool price fluctuations are consistently about double the commodity price fluctuations. Also as the relative phases of the demodulates shown in Figure 3.10 indicate, they tend to lead them slightly, by about 0.5 radians on average. Further this phase lead is highly stable, rather remarkably so for an economic relationship over such a long period of time (during which it corresponds to a time lead fluctuating between the limits of about zero and six months).

Apart from a similar upward trend in the amplitudes of both demodulates, one also may observe related dips which, it will be noted, correspond to concurrent phase discontinuities. These presumably represent flows of power out of the cyclical band at times corresponding to the impacts of the great depressions of the 1890's and the 1930's and of the two world wars.

It was found possible to confirm the above results, and refine them somewhat, with the aid of more precise and detailed data available over the period 1893-1966. Thus Blau[7] has published an end of quarter series for the London price of merino 66's wool (on a clean basis) for the period up to 1946, which we then extended by making use of current sources.[8] These figures

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[7]Gerda Blau, "Wool in the World Economy", Journal of the Royal Statistical Society, September 1946, pp. 179-235.

[8]As is indicated below, the end of quarter series are particularly useful in wool market studies. For the sources of these and other data used in this chapter, see Appendix I.

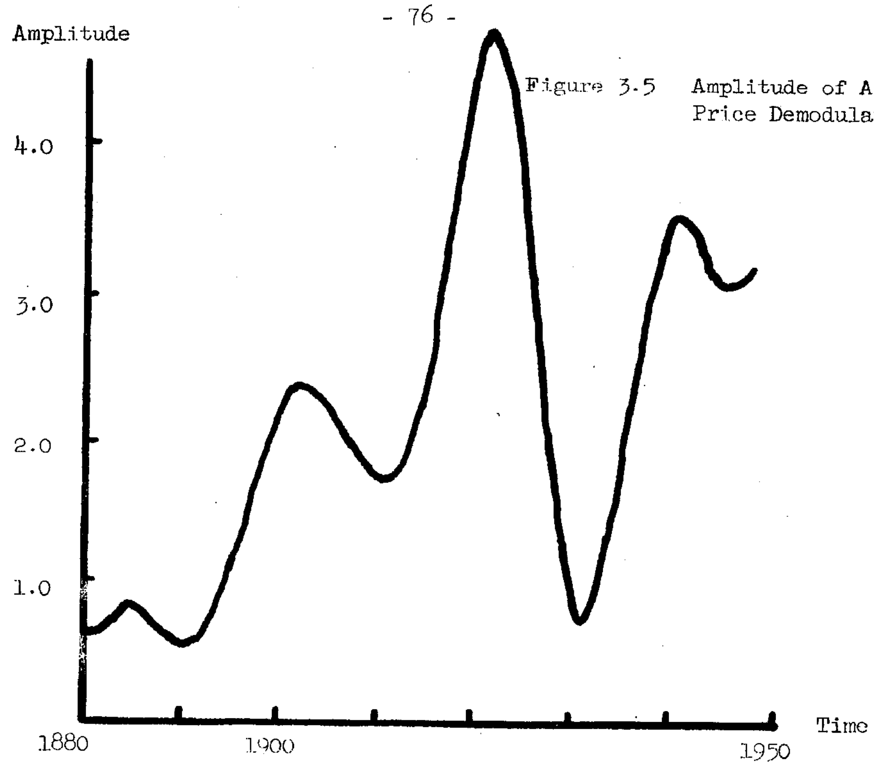


Figure 3.5 Amplitude of Annual Merino Price Demodulates

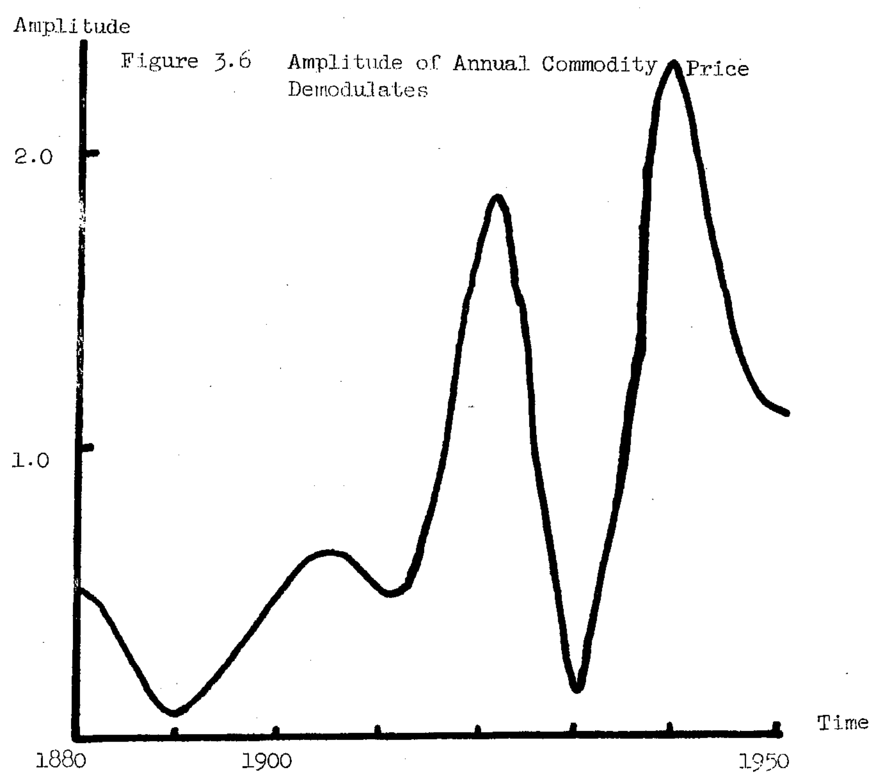


Figure 3.6 Amplitude of Annual Commodity Price Demodulates

Figure 3.7 Phase of Annual Merino Price Demodulates

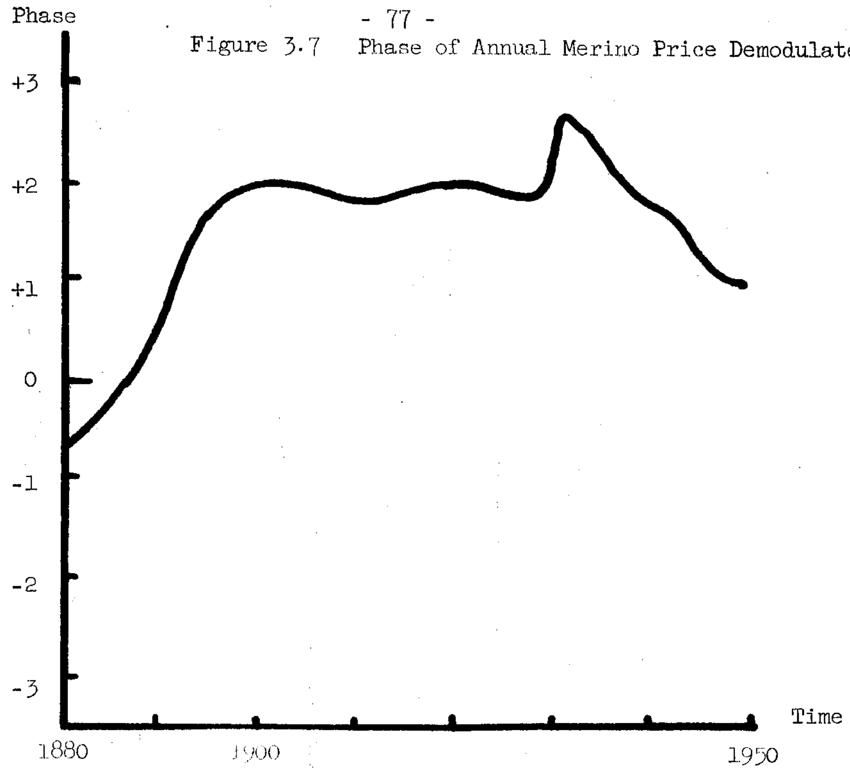
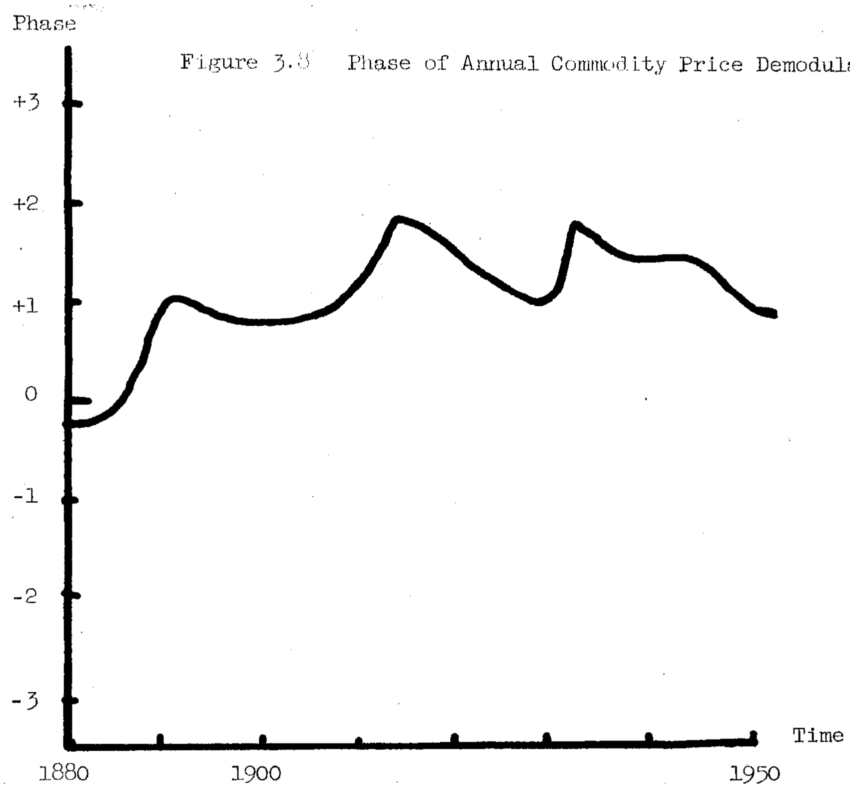


Figure 3.8 Phase of Annual Commodity Price Demodulates



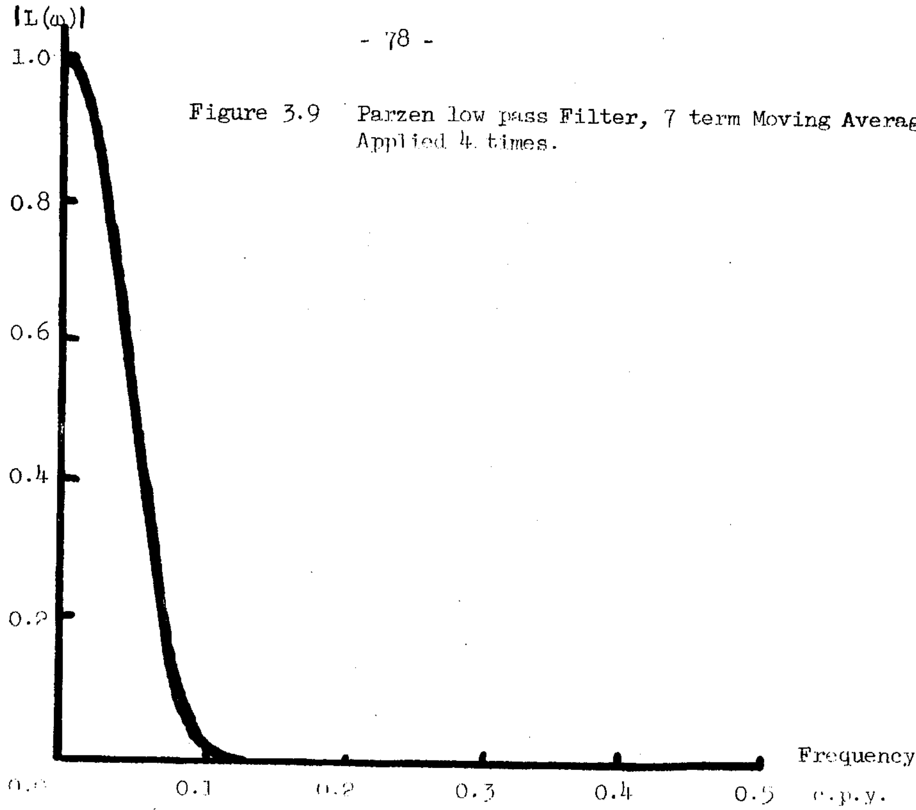


Figure 3.9 Parzen low pass Filter, 7 term Moving Average Applied 4 times.

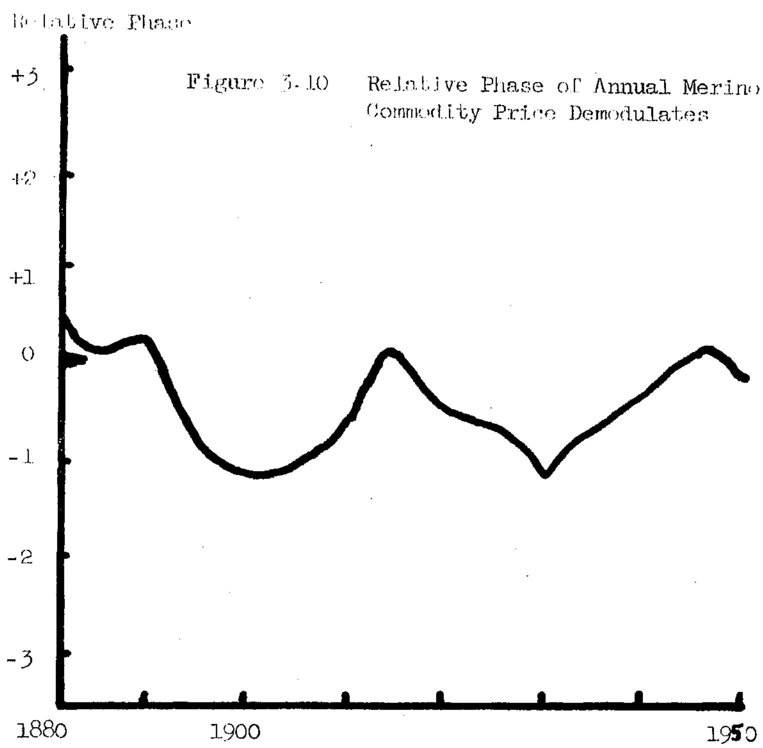


Figure 3.10 Relative Phase of Annual Merino Price and Commodity Price Demodulates

and the corresponding Sauerbeck commodity price figures were demodulated at the same frequency as that used above and with a similar window, and the corresponding phases and phase differences are shown in Figures 3.11 through 3.13 for the period 1905 to 1953. It will be seen that the results are similar to those previously obtained, the only observable difference being the sharper resolution of the dislocation in commodity markets beginning in 1929. Also the spectra and cross spectra of these new series peak in the two to four year range in substantially the same way as those obtained previously. See, for example, the spectrum of merino prices in Figure 3.14 (but note that in this case the abscissa covers the frequency range 0 to 1/12 c.p.m., instead of the 0 to 1/24 c.p.m. of Figure 3.4).

With this background in mind, we might now return to the analysis of the post-war fluctuations in Australian wool prices. But first we should note that nowadays over ninety-five per cent of the wool produced in Australia is sold at Australian auctions. Thus London has been displaced as the greatest wool-marketing center of the world by Sydney, Australia, which handles some twenty-five per cent of the total Australian clip.[9] This means that the basic international price for merino wool is now determined at the Australian auctions, which open each year in late August or early September and run through to the following June. These auctions are highly organized so as to ensure that most days of the week from September to June there will be a fairly continuous market for wool in at least two of the different selling centers.[10]

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[9] G. F. Rainnie, "Raw Materials and Markets" which is Chapter 1 of G. F. Rainnie (ed.), The Woolen and Worsted Industry: An Economic Analysis, (Clarendon Press, Oxford, 1965), p. 9.

[10] For a description of the Australian wool auction system see: Alan Barnard, "A Century and a Half of Wool Marketing", which is Chapter 31 of Alan Barnard (ed.), The Simple Fleece (Melbourne University Press, Melbourne, 1962), p. 475.

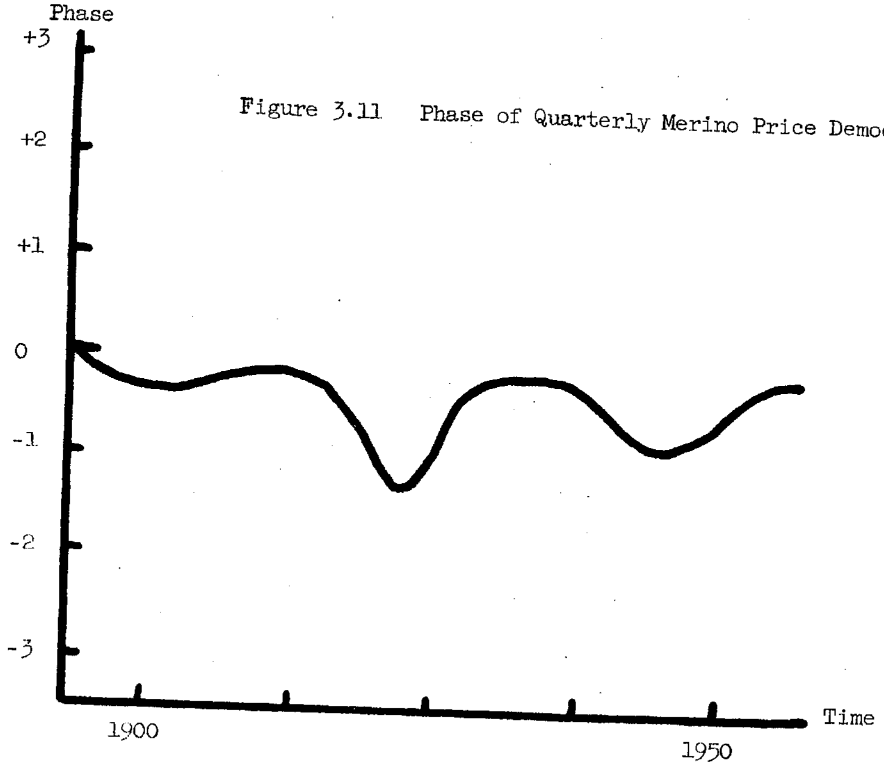


Figure 3.11 Phase of Quarterly Merino Price Demodulates

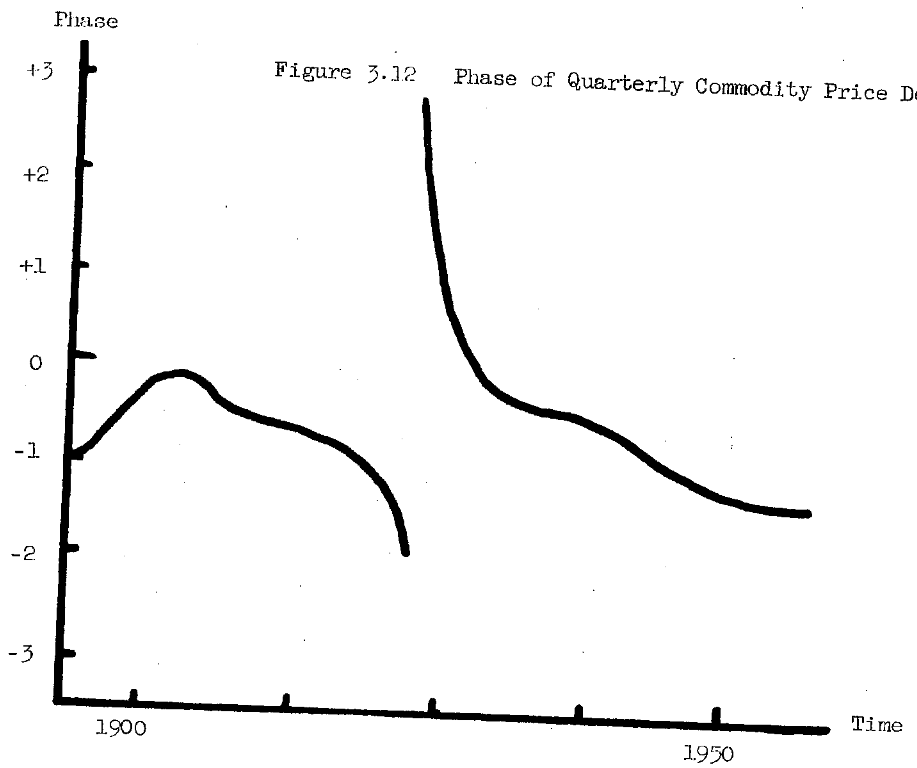


Figure 3.12 Phase of Quarterly Commodity Price Demodulates

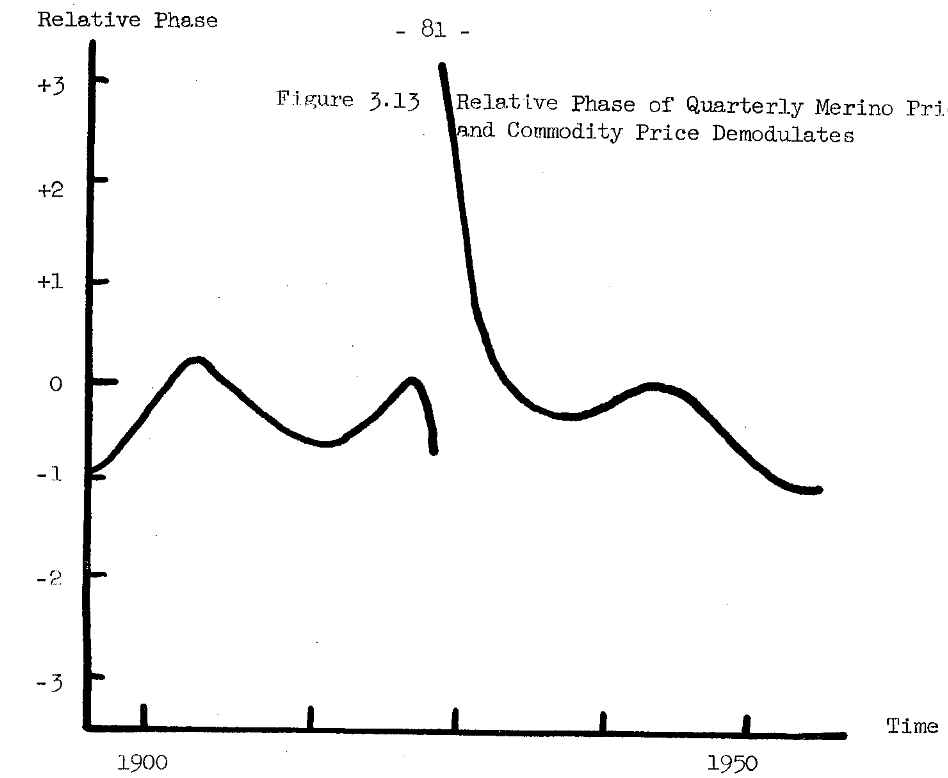


Figure 3.13 Relative Phase of Quarterly Merino Price and Commodity Price Demodulates

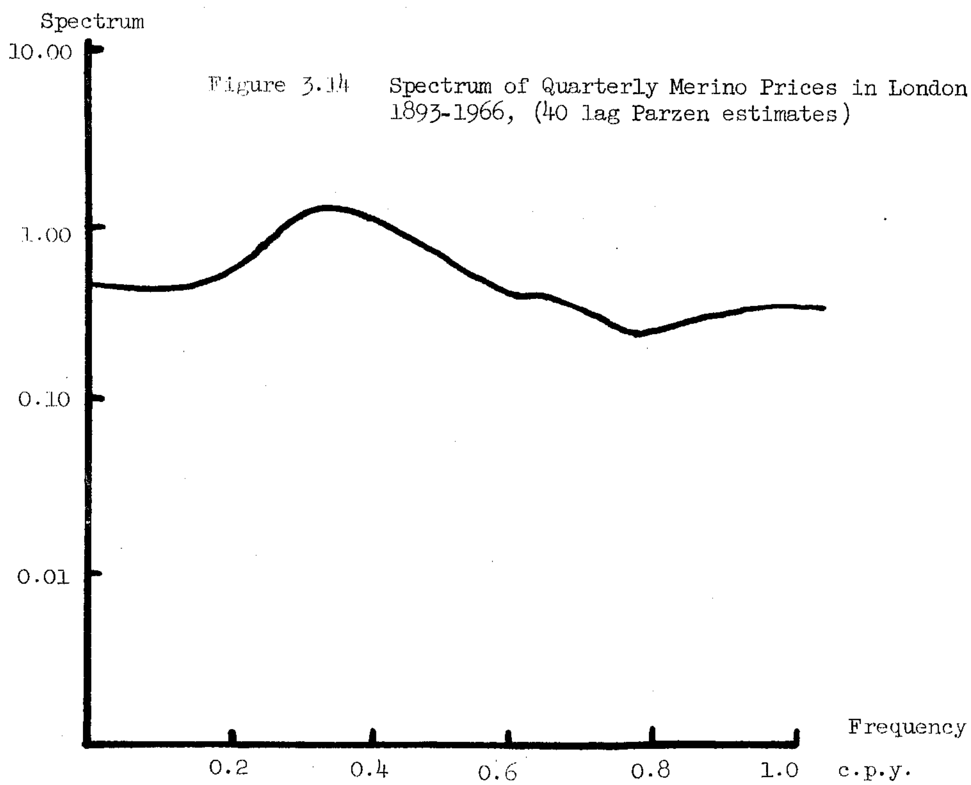


Figure 3.14 Spectrum of Quarterly Merino Prices in London 1893-1966, (40 lag Parzen estimates)

A corollary of this is that there are no effective values for these wool prices during the months of July and August, and the common practice of determining nominal values for these months by linear interpolation seems most inappropriate for use in frequency methods of statistical analysis, especially as far as short-run fluctuations are concerned. The procedure adopted here is to use averages over each of the equally spaced months of March, June, September, and December, and then transform them as before, i.e. to rates of growth of end-of-quarter data. This means that the highest frequency resolvable would be 1/6 c.p.m. instead of the 1/2 c.p.m. which monthly data would permit, but at least our procedure will not introduce spurious annual effects or otherwise bias the results the way interpolated monthly data would. In any case, as we are not here interested in very high frequencies, we shall only be looking at the results on the convenient scale 0 to 1/12 c.p.m.

Using such time series for the Australian wool export price index, and the prices of Australian merino 64's wool on a clean basis in Boston (in bond), and in London for the period 1947-66, one obtains spectral and cross-spectral functions of the form shown in Figures 3.15 through 3.19.[11] These indicate the extremely high dynamic solidarity of international wool prices, especially for fluctuations in the two to four year range. We may observe that this is the same frequency range as was found important for longer spans of data, as is shown for example by the spectrum of London merino prices 1893-1966 seen in Figure 3.14. Further, as the coherence and phase diagrams in Figures 3.20 and 3.21 show, such a solidarity appears to be shared not only by wool in different markets, but also, though to a

[11]Data from earlier years were not used as, prior to September 1947, wool prices were controlled by the Allied Governments so as to insulate the wool industry against war time changes in demand.

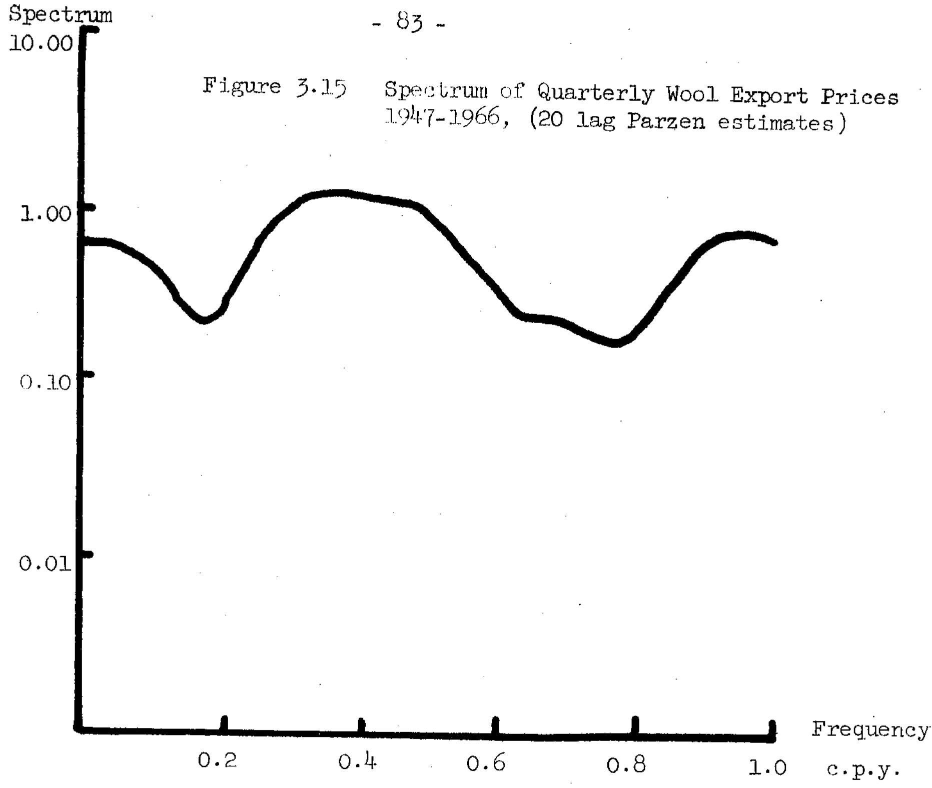


Figure 3.15 Spectrum of Quarterly Wool Export Prices 1947-1966, (20 lag Parzen estimates)

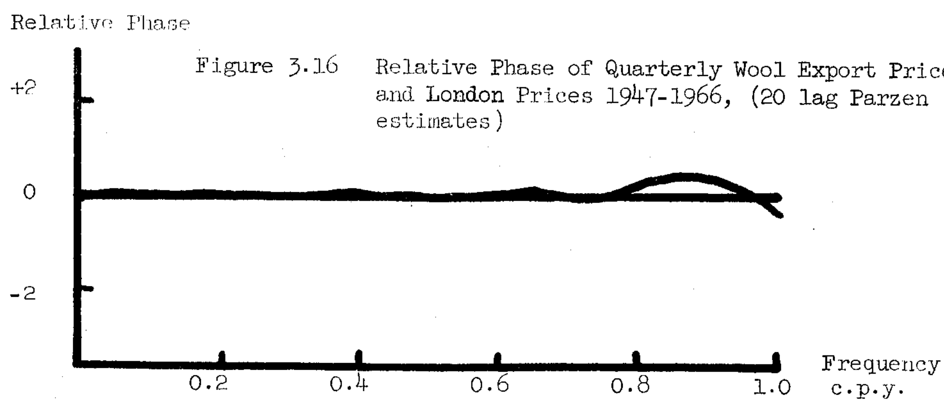


Figure 3.16 Relative Phase of Quarterly Wool Export Prices and London Prices 1947-1966, (20 lag Parzen estimates)

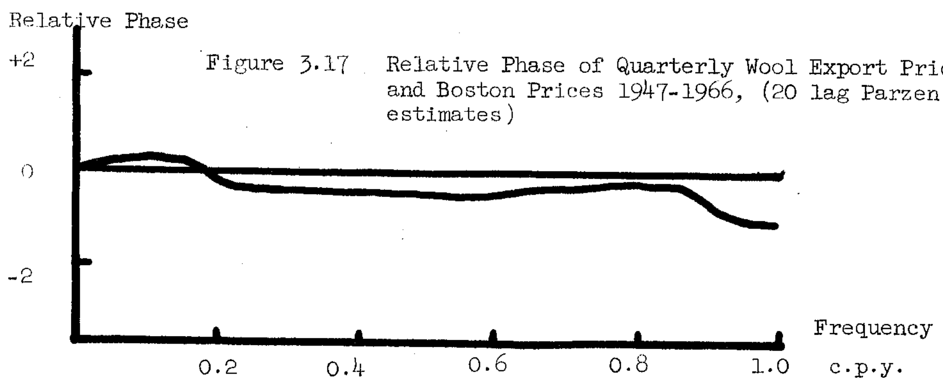


Figure 3.17 Relative Phase of Quarterly Wool Export Prices and Boston Prices 1947-1966, (20 lag Parzen estimates)

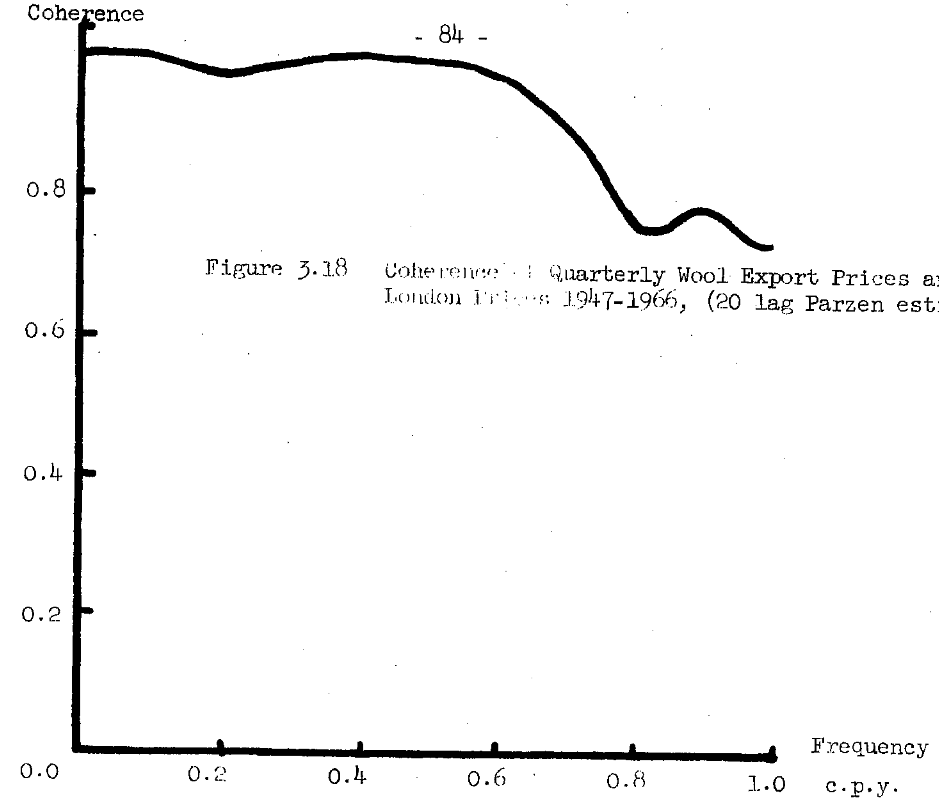


Figure 3.18 Coherence of Quarterly Wool Export Prices and London Prices 1947-1966, (20 lag Parzen estimates)

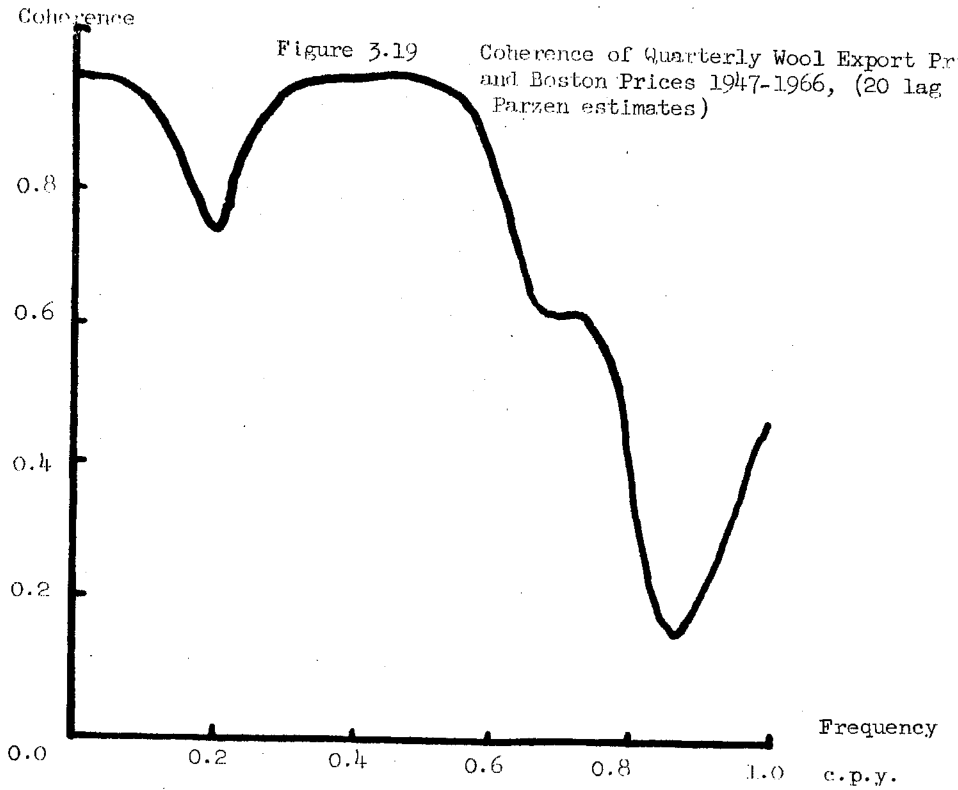


Figure 3.19 Coherence of Quarterly Wool Export Prices and Boston Prices 1947-1966, (20 lag Parzen estimates)

lesser extent, by Australian wool prices and international commodity prices generally in the post-war era. The last results fit in interestingly with the reasonably high coherence and small phase leads found earlier for similar relations over the much longer time periods 1893-1966 and 1865-1965.

As the foregoing analysis attaches a special importance to two to four year cycles in the post-war period, it was decided to demodulate the abovementioned post-war series at a three year cycle using a twenty quarter moving average applied twice (Bartlett window) as a low pass filter and note the time stability of their phases and phase differences as before. As the spectral peaks and high coherences lead us to expect, these are extremely stable for the various international wool prices, but somewhat less so for the phase of the commodity price index and for the difference between this phase and that of the Australian wool export price index. See, e.g. Figures 3.22 and 3.23.

IV. Conclusions

To sum up the above results concerning Australian wool price fluctuations, it seems that they are far more closely related to wool and commodity prices internationally (and probably associated with inventory and activity fluctuations in the major industrial countries) than to any supply side conditions in Australia. Thus the frequency domain approach serves to identify the semi-regular two to four year fluctuations in wool prices mainly with the international demand side of the market. This result seems to be fully consistent with the expected low price elasticities of supply and demand for wool, according to which wool prices would be strongly influenced by fluctuations in aggregate demand. In fact the exceptionally low price elasticities of demand and supply for wool in comparison with

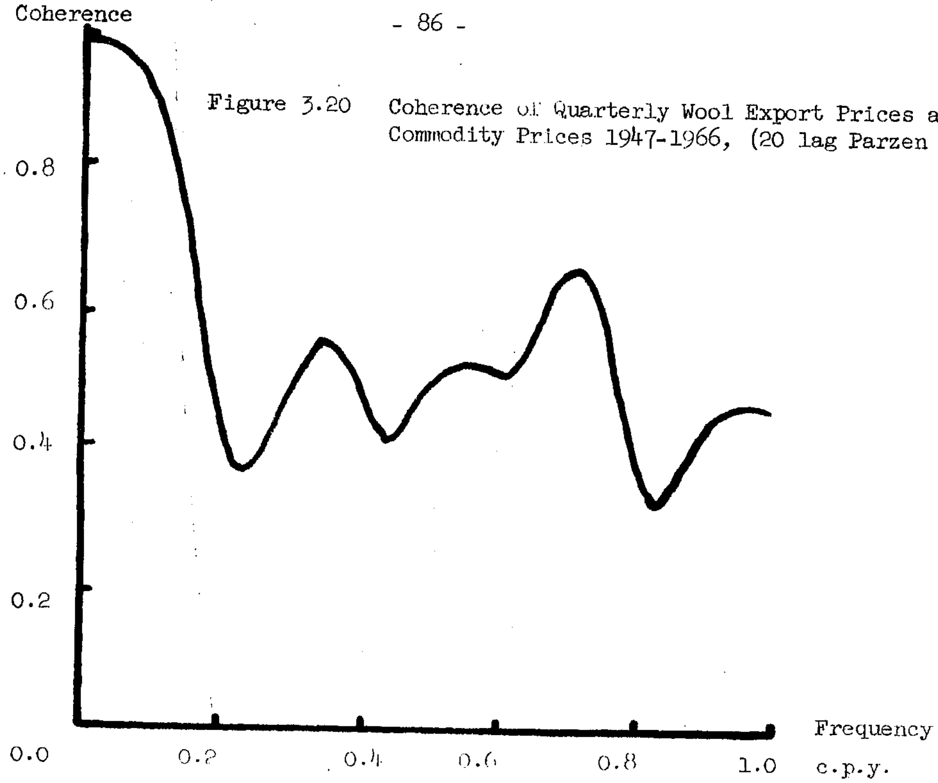


Figure 3.20 Coherence of Quarterly Wool Export Prices and Commodity Prices 1947-1966, (20 lag Parzen estimates)

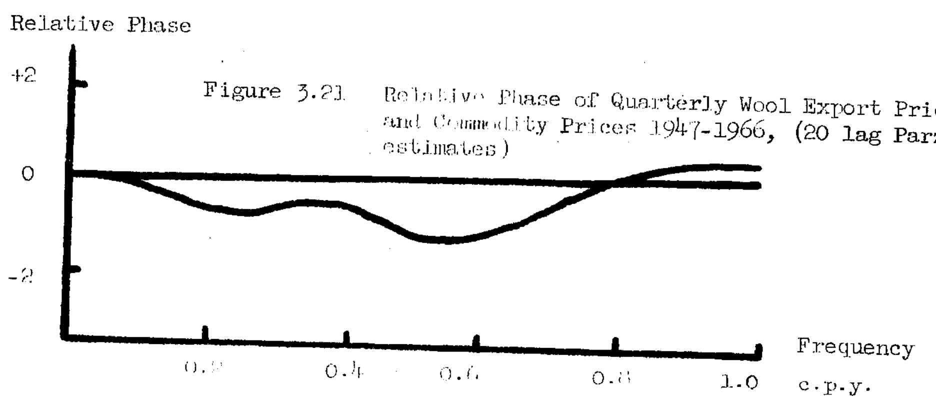


Figure 3.21 Relative Phase of Quarterly Wool Export Prices and Commodity Prices 1947-1966, (20 lag Parzen estimates)

Figure 3.22 Relative Phase of Wool Export Price Demodulates and London Merino Price Demodulates whose actual Phase is also shown

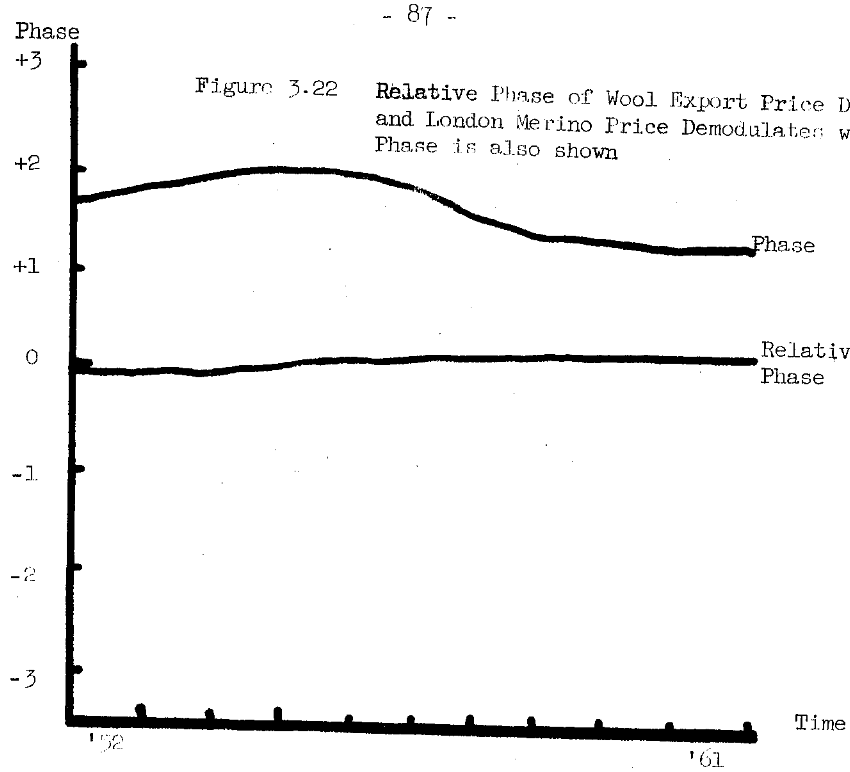
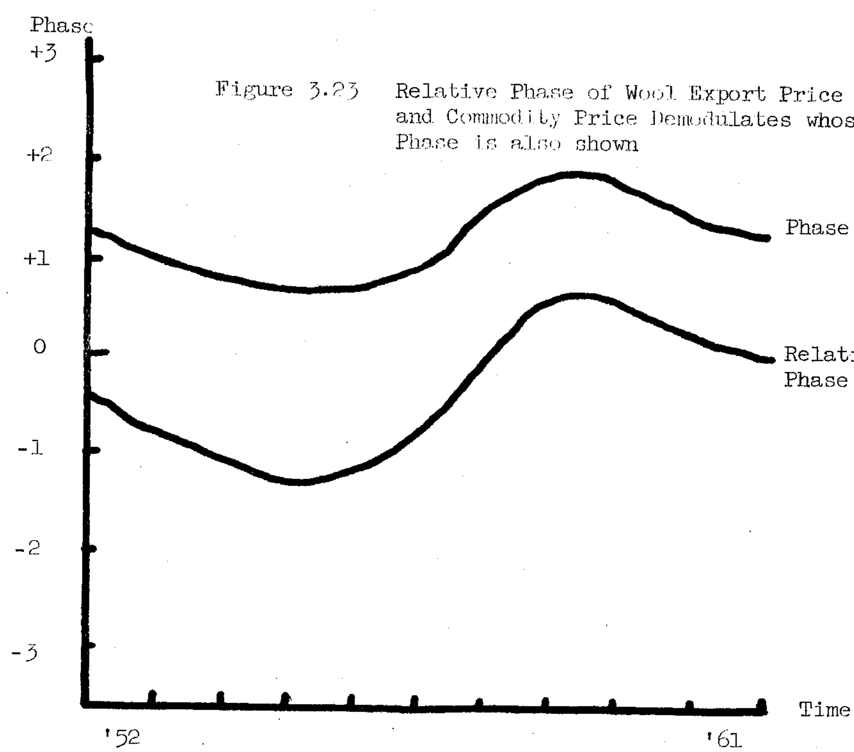


Figure 3.23 Relative Phase of Wool Export Price Demodulates and Commodity Price Demodulates whose actual Phase is also shown



other commodities are indicated in our analysis by the much higher amplitude, and slight phase lead, of wool price fluctuations over corresponding fluctuations in commodity prices generally. However to go further would require a detailed testing of economic hypotheses specified in terms of an invariant time domain behavioral structure. This would involve a more comprehensive statistical methodology than we have indicated to date. So in the next chapter we go on to discuss the question of raising our analytic aspirations in this way.

CHAPTER FOUR

MORE ELABORATE MODELS OF INTERACTING FLUCTUATIONS

I. The Time Domain Approach

So far this thesis has concentrated on models of the form given in Equation 2.8, namely:

$$k_x(\omega)\cdot\exp(i\omega t) = G_{yx}(\omega)\cdot\exp(i\phi(\omega))\cdot k_y(\omega)\cdot\exp(i\omega t) + k_n(t,\omega)$$

where ω is taken as being in the neighborhood of some semi-regular cycle. It has been proposed that information about the relative timing of fluctuations at cyclical frequencies can efficiently be obtained by forming broad band estimates of the corresponding values of $\phi(\omega)$. This, it is felt, represents a considerable improvement on the N.B.E.R. procedures designed to obtain information of this type.

In recent decades however the whole approach based on focussing on particular cyclical fluctuations has been largely supplanted by dynamic regression equation models which aim at describing the interactions at all frequencies present in the discrete data. Normally however the data are in the form of annual observations, so the interest is presumably in frequencies between zero and 1/2 cycles per year. This change seems to have been due in part to the development of Yule's autoregressive model by Tinbergen and others from the 1930's on,[1] and in part to the well-known failure of the

[1] A comprehensive statistical discussion of the asymptotic properties of the models commonly used nowadays has been given by H. B. Mann and A. Wald, "On the Statistical Treatment of Linear Stochastic Difference Equations", Econometrica, July-October 1943, pp. 173-220.

Harvard Economic Barometer to predict the major downturn of 1929. So, in recent years, econometrics has been largely concerned with the estimation of the coefficients of linear invariant relationships between economic variables by regression methods in real time. Thus interest is in equations of the general form:

$$x(t) = \sum_{s} a(s) \cdot y(t+s) + n(t) \qquad (4.1)$$

where $n(t)$ is an error term uncorrelated with $y(t+s)$ and s is non-positive (for causal relations). The actual form of the relationship seems typically to be suggested by a rather unclear combination of economic theory and statistical fitting considerations (e.g. high values of the R^2 and t statistics).[2] As an example of this type of procedure we might refer to the fixed capital adjustment relation proposed by Smyth for the Australian economy. He argues that R^2 computations indicate that, instead of a relation of the form:

$$I(t) = a(Y(t) - Y(t-1)) + n(t) \qquad (4.2)$$

which seems appropriate for the U. S. economy, the Australian case is better described by:[3]

$$I(t) = a(Y(t-1) - Y(t-2)) + n(t) \qquad (4.3)$$

---

[2] It is surprising that such a confused methodology should exist today when this matter was very clearly analyzed nearly thirty years ago by Tinbergen and Keynes who, despite their other differences, were in agreement on this point. See J. M. Keynes, "Professor Tinbergen's Method", The Economic Journal, September 1939, pp. 558-568 and his further "Comment" with Professor Tinbergen's "Reply", The Economic Journal, March 1940, pp. 141-156.

[3] D. J. Smyth, "The Inventory and Fixed Capital Accelerators", The Economic Record, August 1960, pp. 414-418. Incidentally we may add that this case is not chosen as an example of unusually bad practice. Indeed the high irregularity of the $I(t)$ and $Y(t)-Y(t-1)$ series combined with the relative simplicity of the choice between the alternatives sought, suggests that it is not.

However it is not obvious that the value of R^2 normally constitutes a particularly powerful way to discriminate between stochastic hypotheses concerning the relationships between time series, or to suggest that any one of them is particularly appropriate. In fact is is obvious that in the general case, involving more than one lag coefficient, the presence of serial correlation in the variables will lead to multicolinearity between the various lagged values used. This leads to a badly conditioned variance-covariance matrix of the lagged variables making the estimates of the lag regression coefficients interdependent and subject to large sampling errors, and hence extremely difficult to interpret. This situation seems particularly unhappy in economic dynamics where our *a priori* information concerning the form of the basic relations is in fact usually extremely vague. Fortunately, with the advent of spectral methods, it appears that some interesting advances may be made in the area of model specification. As Jenkins once remarked in the context of a rather similar statistical problem:[4]

> There is no substitute to the construction of meaningful time series models based on a priori considerations from which meaningful physical parameters may be estimated. However in the construction of such models, empirical analysis, especially spectral analysis may be of *considerable importance* in suggesting possible models.

II. The Frequency Domain Approach

In order to indicate how spectral methods can be of use in deciding what types of linear dynamic models are particularly appropriate, it is necessary to say something about linear filters.

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[4] G. M. Jenkins, "General Considerations in the Analysis of Spectra", *Technometrics*, May 1961, p. 142.

A. Linear Filters

A great advantage of frequency representations is that they diagonalize linear operators. Thus a linear time-invariant relationship (filter) between y(t) and x(t) of the form:

$$x(t) = \sum_s a(s) \cdot y(t+s) \tag{4.4}$$

can be decomposed in terms of frequencies so that the component of x(t), which is accounted for by frequency ω, is dependent only on the component of y(t) at the same frequency. So we can write:

$$k_x(\omega) \cdot \exp(i\,\omega t) = L(\omega) \cdot k_y(\omega) \cdot \exp(i\,\omega t) \tag{4.5}$$

where $L(\omega) = \sum_s a(s) \cdot \exp(i\,\omega s)$.

Since stationary functions of time may be given the Cramér representation as sums and integrals of sinusoids, we know the whole dynamic character of the response of a linear system to stationary inputs when we know how it responds to sinusoids at all frequencies. Thus the frequency representation vastly simplifies the mathematical form of input-output relationships.

A common technological way of measuring the frequency response function $L(\omega)$ of a mechanical or electrical filter is to feed in an input y(t) equal to a fixed sinusoid $\exp(i\,\omega t)$ and then measure the relative amplitude and relative phase of the output x(t). This determines the modulus and phase of the transfer function $L(\omega)$ at frequence ω. The whole process is then repreated for a suitable number of different frequencies to obtain a sufficiently well resolved picture of $L(\omega)$.

Unfortunately in a non-experimental situation, such as is normally encountered in economics, we are simply given the input and output functions y(t) and x(t). These in practice are very complicated mixtures of all frequencies at once. Still it might seem that, by performing a suitable mathematical frequency decomposition of both the input and output data such as we discussed in Chapter Two, it may be possible to make broad band estimates of the transfer function at all frequencies resolvable in the finite discrete data. But it should be remembered that economic data and systems are even further complicated by being corrupted by errors. Consequently it will be necessary to use some statistical ideas to think of averaging out the effect of these.

Goodman has examined stochastic systems of the form shown in Equation 4.1:

$$x(t) = \sum_{s} a(s) \cdot y(t+s) + n(t)$$

where the output is assumed to be affected not only by the input but also by system and measurement errors uncorrelated with the input.[5] He showed that from the estimates of the input spectrum and the input-output cross-spectrum, the transfer function can be estimated simply by:

$$\begin{aligned} L(\omega) &\hat{=} \hat{f}_{yx}(\omega)/\hat{f}_{yy}(\omega) \\ &\hat{=} \hat{G}_{yx}(\omega) \cdot \exp(i\hat{\phi}(\omega)) \end{aligned} \tag{4.6}$$

as in Equation 2.7. Hence we have from Equation 4.4 that:

$$\sum_{s} a(s) \cdot \exp(i\,\omega s) \hat{=} \hat{G}_{yx}(\omega) \cdot \exp(i\hat{\phi}(\omega)) \ . \tag{4.7}$$

[5] N. R. Goodman, "On the Joint Estimation of the Spectra, Cospectrum and Quadrature Spectrum of a Two Dimensional Stationary Gaussian Process", Scientific Paper No. 10, Engineering Statistics Laboratory, New York University, 1957.

Further, as is shown in Appendix II by an argument based on the "instrumental variables" formalism, even if the input series as well as the output series is contaminated by uncorrelated noise terms, the phase of the transfer function is still estimated correctly by Equation 4.7; although the corresponding estimate of the gain is biased downwards.[6] Hence we have, quite robustly for economic data (which we know will be subject to errors), that hypothetical linear invariant relations of the form shown in Equation 4.1 can be checked against the estimates of $\phi(\omega)$ using the relation:

$$\tan^{-1}(\mathcal{I}(L(\omega))/\mathcal{R}(L(\omega)) \hat{=} \hat{\phi}(\omega)$$

or, from Equation 4.7:

$$\tan^{-1}\left(\sum_s a(s)\cdot\sin(\omega s)/\sum_s a(s)\cdot\cos(\omega s)\right) \hat{=} \hat{\phi}(\omega) \qquad (4.8)$$

So, by estimating values of the function $\phi(\omega)$ over some range of ω and comparing these values with those predicted by the left hand side of Equation 4.8, we have a workable, and sometimes sensitive, low information test on the general character of the system response function. This can be illustrated by the following example which compares estimates of the phase difference between Australian income and investment over independent frequency bands with the phase differences predicted by the two different forms of the accelerator function given in Equations 4.2 and 4.3 above.

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[6]The method of instrumental variables is well known in econometric literature as a means of handling errors in variables, see J. D. Sargan, "The Estimation of Economic Relationships using Instrumental Variables", Econometrica, July 1958, pp. 393-415, or for a simpler discussion, see J. Johnston, Econometric Methods (McGraw-Hill, New York, 1963).

B. The Australian Accelerator Function

It can readily be seen from Equation 4.4 that the transfer functions corresponding to the linear operators shown in Equations 4.2 and 4.3 are (apart from a scale factor) of the form:

$$L_1(\omega) = 1 - \exp(-i\omega)$$
$$= 1 - \cos\omega + i\cdot\sin\omega$$
$$= |(1 - \cos\omega)^2 + \sin^2\omega|^{1/2}\cdot\exp(i\cdot\tan^{-1}(\sin\omega/(1-\cos\omega)))$$

and

$$L_2(\omega) = \exp(-i\omega) - \exp(-i2\omega)$$
$$= \cos\omega - \cos 2\omega + i(\sin 2\omega - \sin\omega)$$
$$= |(\cos\omega - \cos 2\omega)^2 + (\sin 2\omega - \sin\omega)^2|^{1/2}\cdot\exp(i\cdot\tan^{-1}$$
$$((\sin 2\omega - \sin\omega)/(\cos\omega - \cos 2\omega)))$$

The values of the two phase functions $\phi_1(\omega)$ and $\phi_2(\omega)$ are shown in Figure 4.1 for the range 0 to Π. It seems appropriate to compare these with some actual estimates of $\phi(\omega)$ obtained from the available data.

Annual National Accounting information, prepared on a well defined basis and comparable from year to year, is available for Australia as far back as the financial year 1948/9. From these money accounts and a suitable price index, it was possible to obtain annual estimates of the real Gross National Product (corrected for changes in stock valuation) and real Private Investment in Fixed Capital excluding dwellings[7] (it was decided to subtract out the very substantial investment in dwellings, as this appears to be governed mainly by factors irrelevant to the notion of adjusting productive capital to the level of output). In all there are some eighteen data points available, and in fact these were reduced to seventeen on taking the usual

[7] See Appendix I.

Figure 4.1 Phase of the Accelerator Functions

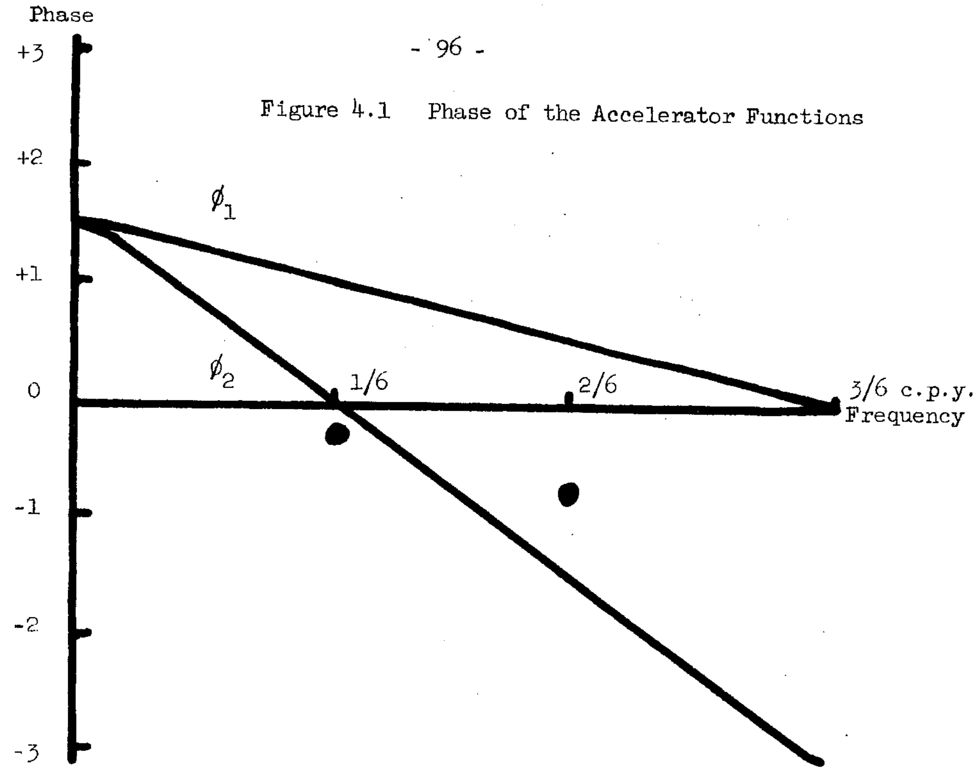
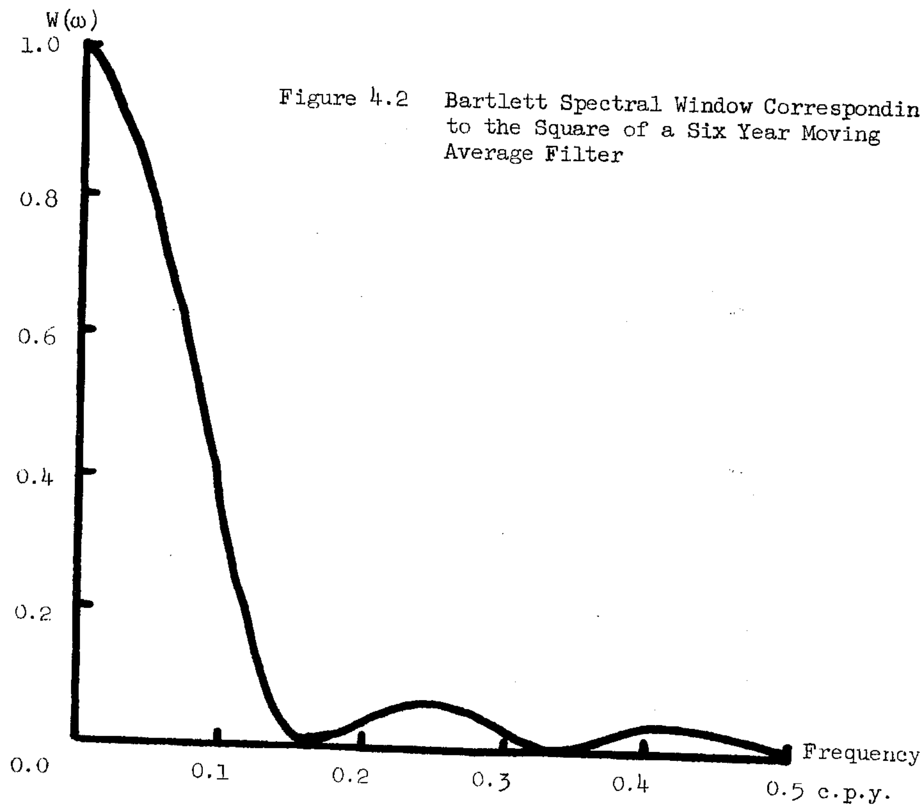


Figure 4.2 Bartlett Spectral Window Corresponding to the Square of a Six Year Moving Average Filter



growth rate transform. As there are not many degrees of freedom to spread over the frequency range 0 to Π, it was decided to divide the fluctuations into just two bands -- "high frequencies" and "low frequencies". More specifically, information over the range 0 to 1/2 cycles per year (c.p.y.), resolvable from the annual data points, was separated into two symmetric parts by complex demodulating at the frequencies 1/6 and 2/6 c.p.y. A simple moving average six years long was used as a low pass filter giving demodulates from which the phase of the cross-spectrum was calculated according to Equation 2.23:

$$\tan \hat{\phi} = \frac{\sum_t |y(t,\omega')| \cdot |x(t,\omega')| \cdot \sin(\theta_y(t,\omega') - \theta_x(t,\omega'))}{\sum_t |y(t,\omega')| \cdot |x(t,\omega')| \cdot \cos(\theta_y(t,\omega') - \theta_x(t,\omega'))}$$

This yielded the estimates shown as large dots in Figure 4.1.

The phase estimates correspond to broad band estimates of the cross-spectrum based on the Bartlett window shown in Figure 4.2, namely:

$$W(\omega) = \left(\frac{\sin 3\omega}{3 \cdot \sin(\omega/2)} \right)^2$$

The two estimates accordingly are "resolved independently" (though only barely so) according to the Raleigh criterion used in the analogous case of physical optics -- "the peak power of the second window is outside the main lobe of the first window". It will be noted that these orthogonally estimated phases correspond more closely with the pattern of $\phi_2(\omega)$ than $\phi_1(\omega)$ and so favor the accelerator shown in Equation 4.3 over that shown in Equation 4.2 for the data in growth rate form (with the mean removed as before).

The above approach seems to count more clearly as an empirical verification of the general form of a hypothesis of economic dynamics than the usual R^2 and t tests on various different formulations of the time domain regression equations, especially so far as medium run fluctuations are concerned. We shall now go on to suggest that it provides a basis for deciding what types of economic dynamic hypotheses can be distinguished from one another by observable data.

C. The Uncertainty Principle for Economic Observables

Perhaps one of the major advantages to be obtained from the use of Fourier methods in econometrics will prove to be conceptual, in that we are shown, as the classical physicists were shown, the confusion and error involved in over-exact specifications of elaborate mathematical models relating observables. To date, dynamic economics and econometrics seem to have aspired to the rigorous mathematical forms of Newtonian mechanics. Consequently it seems opportune to draw attention to the fact that, with the deepening of scientific analysis of real phenomena, such exact specifications have been abandoned in physics in favor of simultaneously limiting uncertainties. Exact specification of any single aspect has come to be regarded not as fundamental but as nominal and inherently unverifiable; and a theory that is inherently unverifiable is to say the least non-operational.

We may recall that the discussion in Section I of Chapter Two indicates that, if a lagged correlation function R(s), or a related set of statistics such as lagged regression coefficients, is to be regarded as operational for the purpose of measuring the dynamic interactions of economic observables recorded within some time interval ΔT, a minimum qualification is that it be possible to consider R(s) identically zero when s is outside certain limits, $\pm$ m for instance. Also m must be considerably less than the span of ΔT so that R(s) can be estimated with a considerable number of degrees of freedom. In the language of Fourier theory, R(s) is required to have a smooth Fourier transform $f(\omega)$ which tends to zero with ω (the smoothness being described by a finite bandwidth $\Delta\omega$ whose magnitude is inversely related to the value of m). However ordinary economic variables typically have a high serial correlation corresponding to predominant low frequency components giving a spectral shape similar to a rectangular hyperbola, and hence do not obviously lend themselves to convergent methods of analysis. But if, as we have argued, the data can normally be considered to represent stationary proportional variations about a steady growth pattern, then a linear analysis of the data in demeaned growth rate form becomes workable and it will be more reasonable to assume that we have satisfied then the statistical requirement of ergodicity. As we have shown, it is then possible to describe the generating process in terms of estimates of frequency domain functions whose successive values are independent of one another provided they are at least $\Delta\omega$ apart. These functions can then be checked for correspondence with the frequency domain mapping of the filter functions corresponding to the usual lagged linear models of the generating process in real time of the form shown in Equation 4.1.

To indicate the uncertainties involved in this procedure for matching theoretical models to observable data, we will assume for simplicity that our data are noise-like and that accordingly our spectra are reasonably flat so that our data can be considered to provide ΔT degrees of freedom distributed fairly evenly over the frequency range 0 to Π. Hence if the estimates are calculated using a window with a band width $\Delta\omega$, then each of them is based on some ν degrees of freedom, where:

$$\nu = \Delta T \cdot \Delta\omega / \Pi$$

hence

$$\nu^{-1} \cdot \Delta T \cdot \Delta\omega / \Pi = 1 \qquad (4.9)$$

This equation deserves to be considered as the fundamental uncertainty principle for econometrics as it indicates the simultaneous uncertainties inherent in operational linear theories describing economic phenomena. Thus ν^{-1} corresponds to the statistical uncertainty arising from the random elements in the model under consideration such as are caused by errors in the data or incompleteness in the specification of the process. Thus it will be recognized as a common term in the sampling variance of linear statistical estimates. ΔT corresponds to the historical uncertainty involved in taking averages over the time span of the data when we know that in practice, economic processes are not perfectly stationary.[8] $\Delta\omega/\Pi$ indicates the limitations on our ability to resolve the fine structure of the filter function corresponding to the specification of the linear dynamic hypothesis describing the process in real time.

It should be observed that however the procedures for estimating the form of linear econometric models may be cast or recast these uncertainties will always be present in some form equivalent to the uncertainty principle of Equation 4.9. So unless we have prior information of the form of our equations not only is it fictional to assume away any of the uncertainties, but also attempts to reduce one of them will only increase one or both of the others till eventually a point will be reached when the whole phenomenon will cease to be resolved. A little reflection on this principle will suffice to show that our operational knowledge of economic dynamics is likely to be rather more uncertain than might seem to be indicated by the high correlations commonly obtained by more traditional

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[8] If we were not in fact already aware of the fact that economic processes are not stationary over long periods of time the complex demodulates shown in Figures 3.5 through 3.8 should provide food for thought. So "more data" is not a universal panacea for uncertainty in econometrics, rather we have to learn to live with uncertainty.

procedures in dynamic econometrics. Such results can probably be explained by the high serial correlation of typical economic data samples. As Orcutt and James have shown, for the type of small sample case normally encountered in econometrics:[9]

> Except in cases where the sample autocorrelations happen to be low, such high correlations between economic time series may be expected by chance that we are unlikely to detect real relations. The distributions given for partial and multiple correlations only accentuate this view.

So presumably the "statistical significance" of standard econometric regression equations will ordinarily be due to the few degrees of freedom corresponding to the important low frequency (deterministic) trends in the data, and so will not really count as powerful statistical evidence for theories concerning short run and medium run interactions.

By the same token however we should remember that with economic data of this (autocorrelated) type we have less degrees of freedom than observations. Hence our uncertainty principle, which was derived to describe operations with noise-like data, gives at best an optimistic view of the sampling variance of the spectral estimates at frequencies corresponding to any signal-like components.

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[9] G. H. Orcutt and S. F. James, "Testing the Significance of Correlation Between Time Series", Biometrica, June 1948, pp. 397-413.

CHAPTER FIVE

CONCLUDING REMARKS

I. Summary

The uncertainty principle of Equation 4.9 shows explicitly the "fuzziness" of models which seek to explain the business cycle in terms of lagged linear relationships whose nature is estimated from the data. Thus it warns us of the high degree of indeterminacy involved in attempts to characterize the laws of economic dynamics as empirically derived stochastic associations between more or less random variables, where, if the values of various lagged values of one variable are known, we have no precise knowledge of the other but merely know that it has a certain probability distribution.

However, data analytic procedures have been explained which take explicit account of this, and which also are particularly appropriate to show if there are any fairly rigid functional relationships between more or less similar fluctuations in different economic time series. Empirical results indicate that this permits a graphic description of the nature and stability of the interactions between the cyclical components of certain variables during the post-war Australian business cycle. They suggest some economically interesting features of this cycle, such as the fact that it appears to be a domestic event -- not imported from abroad via fluctuations in exports or export prices. It has been found that export fluctuations, or more specifically the price fluctuations of the most import export commodity (wool price fluctuations) have an entirely different cyclical pattern, unrelated to Australian conditions, but probably connected with business conditions in the major industrial countries of the Northern Hemisphere.

A data analysis of this type gives direct information concerning the uniformity over time of dynamic interactions between variables. Specifically our methods of complex demodulation break up the sample period into a series of sub-periods in each of which it shows the relative phasing of the variables in question. Hence by examining the time path of relative phases we can see if the dynamic interactions of interest are homogeneous over the different sub-periods. If these results are fairly consistent, then it is suggested that we perhaps have some grounds for making an inductive generalization corresponding, for example, to the mean of our results. We might use this discovery as a basis for projecting the future evolution of the system. In this way it seems possible to specify, or rather put some lower bound on, the uncertainties concerning the stochastic and non-stationary elements of the dynamic interactions in which we are interested. At the same time such results obtained at different frequencies give fairly direct information concerning the nature of any strong lagged linear relationships between our variables.

II. Questions for Further Research

There are a number of theoretical and empirical paths along which this research on fluctuations in the Australian economy could be extended. The economic theory of the linear interactions among variables in growth rate form could be examined, as these seem to be the relationships most clearly tractable from the point of view of empirical investigations using time series data. At the same time the uncertainty principle could be employed to indicate the appropriate degree of precision with which these relationships should be specified, so that they might be operational in terms of data available or likely to be available.

We might then reconsider the empirical phenomena discussed in this study even more carefully, using, for example, disaggregated series and taking greater pains to see that they are as accurate as possible. From this second round of the investigation we might gain information to guide yet another, and hopefully deeper, analysis; and so on in the spirit of an approach advocated by Parzen:[1]

> .... an analysis of time series is not accomplished by adopting a single model; the parameters of which are estimated. Rather it is best carried out by a process of increasing insight from successive analyses.

It may be noted that such a careful and patient analysis of some very simple economic relationships seems a rather modest approach in comparison with the more comprehensive model-building attempts currently in vogue in econometrics. However such methods are not without precedent in more exact sciences, and so one might expect that in dynamic economics also they could at least provide a useful starting point.

Hence this thesis should be considered a beginning, rather than an end, to the analysis of fluctuations in the Australian economy by frequency methods of statistical analysis. However it is suggested that it indicates a worthwhile new approach.

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[1] Emanuel Parzen, "The Role of Spectral Analysis in Time Series Analysis", Technical Report No. 2 (Department of Statistics, Stanford University, Stanford, California, July 1965), pp. 1-2.

APPENDIX I

DATA SOURCES

General Comments

All data are taken in seasonally unadjusted form and, where prices are involved, are expressed in the dollar units of the Australian decimal currency adopted on February 14, 1966. Full references to all the data sources mentioned appear in the Bibliography.

Labor Market Variables

Unfilled Vacancies and Persons Unemployed are as given in the monthly Labour Report of the Department of Labour and National Service. Persons Employed and Net Immigration were taken from the Monthly Review of Business Statistics, while Births, Deaths, and Marriages came from the Quarterly Summary of Australian Statistics.

Wage and Price Indicators

Average Earnings, the Wholesale Price Index, and the Money Supply (measured as cash plus demand deposits held by the public) were obtained from the Statistical Bulletin of the Reserve Bank of Australia. The Share Price Index was taken from the Monthly Review of Business Statistics. The Consumer Price Index was formed by reducing to a common base the old "C Series Index" (for the period 1948 to 1953) and (subsequently) the new "Consumer Price Index"; both series being obtained from the Quarterly Summary of Australian Statistics.

Production Indicators

All Production Indicators were taken directly from the Monthly Review of Business Statistics.

Sales Figures

The Number of New Motor Vehicles Registered is recorded in the Monthly Review of Business Statistics. Figures for the Value of Retail Sales (excluding motor vehicles) from 1951 to date were taken from the Quarterly Summary of Australian Statistics and previous estimates were obtained by blowing up the Sydney Index of Retail Trade (Sales) prepared by the Retail Traders Association of New South Wales and published in the Reserve Bank of Australia Statistical Bulletin. This series deflated by the Consumer Price Index given above was used as a measure of the Volume of Retail Sales.

International Transactions

Imports, Exports, the Export Price Index, and the Wool Export Price Index were taken from the Monthly Review of Business Statistics. Gold and Foreign Exchange Reserves were obtained from the Reserve Bank of Australia Statistical Bulletin.

Wool Market Data

Annual indices of Merino Wool Prices and Commodity Prices since 1865 were taken from the annual report on "Prices of Commodities and Precious Metals" published in the Journal of the Royal Statistical Society. An end of quarter index of Commodity Prices is given in the same source, while Blau has compiled an end of quarter listing of the London Price of Merino 66's for the period 1893 to 1946 in "Wool in the World Economy", Journal of the Royal Statistical Society, September 1946. This series was then extended into the post-war era by data obtained from the Commonwealth Secretariat, Commodities Division, Wool Intelligence and Fibres Supplement and the World Wool Digest of the International Wool Secretariat and the Wool Bureau Incorporated. Corresponding indices for Australian Wool Export Prices and for Boston Prices for Australian Wool were obtained from the Monthly Review of Business Statistics and from the United States Department of Agriculture Wool Statistics and Related Data. Figures for the Wool Production and the Sheep Population of Australia and the States were taken from the Statistical Handbook of the Sheep and Wool Industry of the Commonwealth of Australia, Bureau of Agricultural Economics.

National Income and Investment

Money values of the National Income and Investment were obtained from the annual White Paper on National Income and Expenditure prepared in connection with the Australian Federal Budget. These current value data were reduced to constant 1959-60 values by a deflator made up by averaging the Consumer Price Index with weight 2 and the Wholesale Price Index with weight 1.

APPENDIX II

FREQUENCY ANALYSIS THEOREMS

General Comments

Rather than interrupt the argument of the text with any extensive digressions, we gather together in this Appendix a discussion of some largely formal, but not entirely obvious, results connected with the frequency analysis of data which are invoked in the main text.

A. Low Frequency Bias on Removing a Sample Mean

The effect of sample mean removal on the low frequencies of an infinite stationary process can be understood heuristically in terms of the related linear operator (where 2m+1 equals the sample size n):[1]

$$x(t) = x'(t) - 1/(2m+1) \cdot \sum_{s=-m}^{m} x'(t+s)$$

So given a process with the Cramér representation:

$$x'(t) = \int_{\Pi}^{\Pi} \exp(i\,\omega t) \cdot k_{x'}(\omega) d\omega$$

the above operator would give a new process:

$$x(t) = \int_{\Pi}^{\Pi} \exp(i\,\omega t) \cdot L(\omega) \cdot k_{x'}(\omega) d\omega$$

[1] Although this corresponds exactly to mean removal only for the term t=0 its effect on the very low frequency components of the spectrum will be very similar. For further discussion on this subject see E. J. Hannan, "The Variance of the Mean of a Stationary Process", Journal of the Royal Statistical Society (Series B), June 1957, pp. 282-285.

where,

$$L(\omega) = 1-1/(2m+1) \cdot \sum_{s=-m}^{m} \exp(i\,\omega\, s)$$

Hence from the relationship between the Cramér and spectral representations of a stationary process we see that the spectra of x(t) and x'(t) are related by:[2]

$$f_{xx}(\omega) = |L(\omega)|^2 \cdot f_{x'x'}(\omega)$$

When n = 228, $|L(\omega)|^2$ has the form shown in Figure A.1 for the range 0-1/12. So it will be seen that the spectral estimates of a demeaned series are

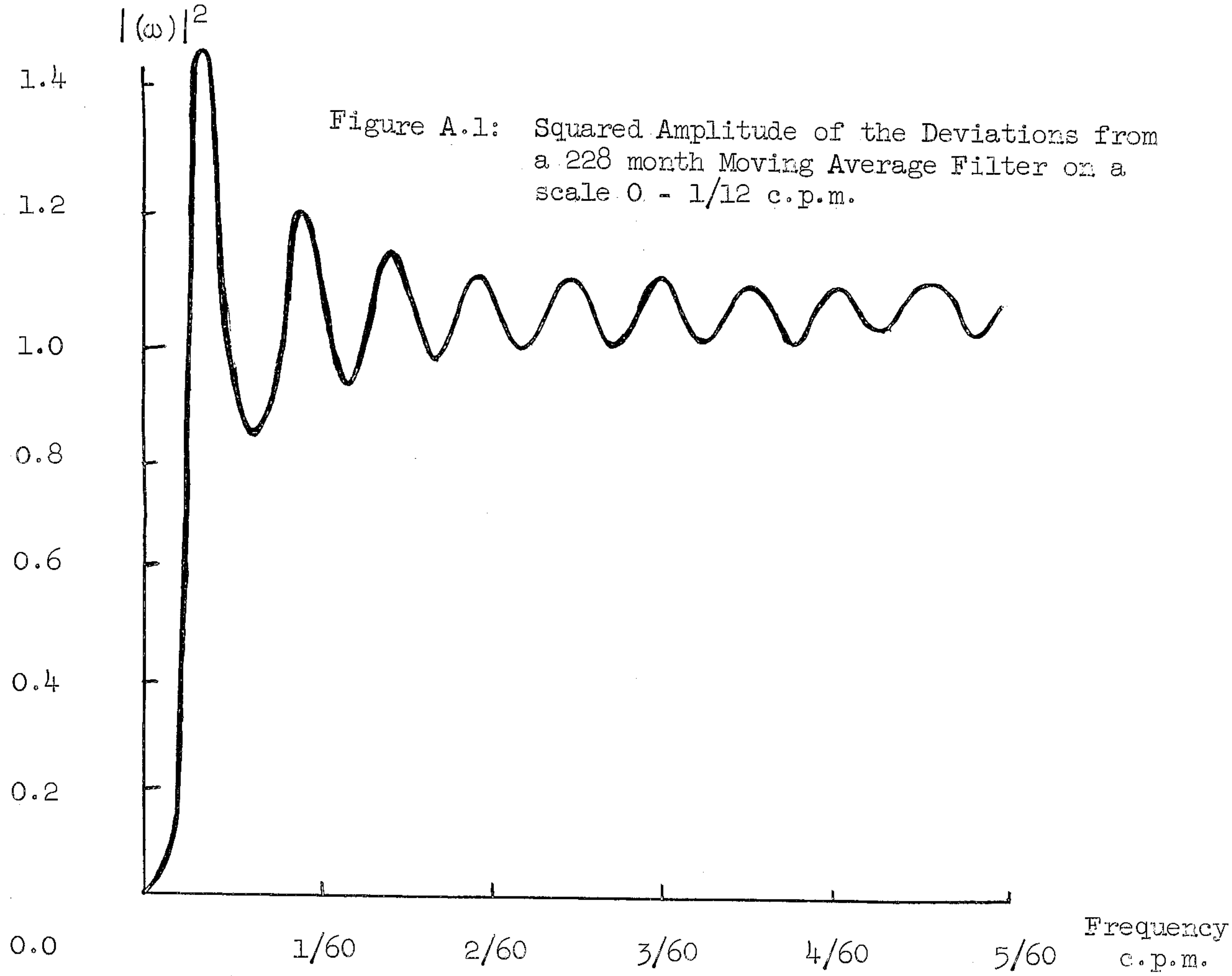

Figure A.1: Squared Amplitude of the Deviations from a 228 month Moving Average Filter on a scale 0 - 1/12 c.p.m.

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[2] H. Cramér, "On Harmonic Analysis in Certain Function Spaces", <u>Arkiv for Mathematik, Astronomi och Fysik</u>, March 1942, pp. 214-230.

strongly biased downwards at frequencies between 0 and 1/n and hence have no obvious meaning at these very low frequencies. One might take note of the ripple in $|L(\omega)|^2$ at higher frequencies. In some cases it may be desirable to attenuate this, perhaps by subtracting out a weighted average, of say the Bartlett or Parzen form, rather than a simple average. Alternatively one might taper the data before mean removal.

B. The Sampling Theorem of Information Theory

In the original form proposed by Shannon the sampling theorem states:[3]

> If a function f(t) contains no frequencies higher than W cycles per second it is completely determined by giving its ordinates at a series of points spaced 1/(2W) seconds apart.

Heuristically this may be seen to be true by considering a function f(t) in its Fourier representation:

$$f(t) = 1/2\Pi \cdot \int_{-\infty}^{\infty} F(\omega)\cdot \exp(i\omega t)d\omega$$

$$= 1/2\,\Pi \cdot \int_{-2\,\Pi W}^{2\,\Pi W} F(\omega)\cdot \exp(i\,\omega t)d\omega$$

(for the case considered by the theorem)

Then for all n = 0,1,2,..............., we have:

$$f(n/2W) = 1/2\Pi \cdot \int_{-2\Pi W}^{2\,\Pi W} F(\omega)\cdot \exp(i\,\omega t)d\omega$$

which corresponds to all the Fourier coefficients of $F(\omega)$ based on the fundamental period -W, W. So $F(\omega)$ is completely specified by all f(n/2W) since a function is completely specified by its Fourier transform, and by the same token if $F(\omega)$ is completely specified so also is f(t).

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[3] C. E. Shannon, "Communication in the Presence of Noise", Proceedings of the Institute of Radio Engineers, January 1949, pp. 10-21.

C. Cross-spectral Description of a Lagged Linear Relation Between Variables that are Observed with Error

Suppose there exists a lagged linear relation between two variables X(t) and Y(t) of the form shown in Equation 4.4:

$$X(t) = \Sigma\, a(s) \cdot Y(t+s) \tag{A.1}$$

but that in practice we can only observe the variables subject to the independent errors u(t) and v(t), i.e. we have:

$$x(t) = X(t) + u(t)$$

and (A.2)

$$y(t) = Y(t) + v(t)$$

Then in terms of the observed variables and their errors Equation A.1 may be written:

$$x(t) = \Sigma\, a(s) \cdot y(t+s) - \Sigma\, a(s) \cdot v(t+s) + u(t) \tag{A.3}$$

Suppose we multiply through by $\exp(i\,\omega t)$, then Equation A.3 becomes:

$$x(t) \cdot \exp(i\,\omega t) = (\Sigma\, a(s) \cdot \exp(i\,\omega s))y(t)\exp(i\,\omega t) - (\Sigma\, a(s) \cdot \exp(i\omega s))v(t) \cdot \exp(i\omega t) + u(t) \cdot \exp(i\,\omega t) \tag{A.4}$$

$$= L(\omega) \cdot y(t) \cdot \exp(i\,\omega t) - L(\omega) \cdot v(t) \cdot \exp(i\,\omega t) + u(t) \cdot \exp(i\,\omega t) \tag{A.5}$$

If $\exp(i\,\omega t)$ represents a strong component of x(t) and y(t) but not of u(t) and v(t) then we may, at least formally, consider $\exp(i\,\omega t)$ as a possible "instrumental variable z(t)"[4] to estimate the complex regression coefficient $L(\omega)$ of Equation A.5. Thus we have the following relation between the theoretical moments:

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[4] Olav Reiersøl, "Identification of a Linear Relation Between Variables which are Subject to Error", Econometrica, October 1950, pp. 375-385.

$$E[x(t)\cdot z(t)] = L(\omega)\cdot E[y(t)\cdot z(t)] \tag{A.6}$$

The method of instrumental variables consists of replacing the theoretical moments in Equation A.6 by estimated moments and hence getting an estimate of $L(\omega)$ as a solution of the equation:

$$L(\omega) = \frac{\sum_{t=1}^{n} x(t)\cdot z(t)}{\sum_{t=1}^{n} y(t)\cdot z(t)} \tag{A.7}$$

If the generating process is ergodic then when n tends to infinity the empirical moments tend to the theoretical moments and so it follows that the estimates obtained by the above procedure tend to the true value of the parameter $L(\omega)$. This seems to be a reasonable procedure when there is a true sinusoid in the true values of the data, but not in their error terms. Generally however this is not true for a stationary process, in which case it is illuminating to consider the use of the related "instrument":

$$z(t) = \exp(i\,\omega t)\cdot \sum_{s=-m}^{m} y(t+s)\cdot\exp(-i\omega\,(t+s))$$

Whence Equation A.6 becomes:

$$E[x(t)\cdot z(t)] = L(\omega)\cdot E[y(t)\cdot z(t)] - L(\omega)\cdot E[v(t)\cdot z(t)] + E[u(t)\cdot z(t)]$$

or,

$$\begin{aligned} & E[x(t)\cdot\exp(i\,\omega t)\cdot \sum_{s=-m}^{m} y(t+s)\cdot\exp(-i\omega\,(t+s))] \\ & = L(\omega)\cdot E[y(t)\cdot\exp(i\,\omega t)\cdot \sum_{s=-m}^{m} y(t+s)\cdot\exp(-i\omega\,(t+s))] \\ & - L(\omega)\cdot E[v(t)\cdot\exp(i\,\omega t)\cdot \sum_{s=-m}^{m} y(t+s)\cdot\exp(-i\omega\,(t+s))] \\ & + E[u(t)\cdot\exp(i\,\omega t)\cdot \sum_{s=-m}^{m} y(t+s)\cdot\exp(-i\omega\,(t+s))] \end{aligned} \tag{A.8}$$

This may be written:

$$\sum_{j=-m}^{m} E[x(t)\cdot y(t+s)]\exp(-i\,\omega s) = L(\omega)\cdot \sum_{j=-m}^{m} E[y(t)\cdot y(t+s)]\exp(-i\,\omega s)$$

$$- L(\omega)\cdot \sum_{j=-m}^{m} E[v(t)\cdot y(t+s)]\exp(-i\,\omega s) \quad \text{(A.9)}$$

$$+ \sum_{j=-m}^{m} E[u(t)\cdot y(t+s)]\exp(-i\,\omega s)$$

Now under the assumptions that u(t) and v(t) are uncorrelated with the true variables X(t) and Y(t) and with one another we have:

$$E[u(t)\cdot y(t+s)] = E[u(t)\cdot(Y(t+s) + v(t+s))] = 0$$

and,

$$E[v(t)\cdot y(t+s)] = E[v(t)\cdot(Y(t+s) + v(t+s))] = E[v(t)\cdot v(t+s)]$$

Hence from Equation A.9 we have:

$$L(\omega) = \frac{\sum_{j=-m}^{m} E[x(t)\cdot y(t+s)]\exp(-i\,\omega s)}{\sum_{j=-m}^{m} E[y(t)\cdot y(t+s)]\exp(-i\,\omega s) - \sum_{j=-m}^{m} E[v(t)\cdot v(t+s)]\exp(-i\,\omega s)}$$

$$= \frac{\underline{f}_{yx}(\omega)}{\underline{f}_{yy}(\omega) - \underline{f}_{vv}(\omega)} \quad \text{(A.10)}$$

(in terms of the spectral and cross-spectral representations of Equations 2.4 and 2.5)

Similarly the estimate provided by Equation A.7 may be written:

$$\hat{L}(\omega) = \frac{\hat{f}_{yx}(\omega)}{\hat{f}_{yy}(\omega)} \quad \text{(A.11)}$$

which corresponds to the cross-spectral estimate of the transfer function given by Equation 2.7. By comparison with Equation A.10 we see that the absolute value of this estimate (the gain) is biased downwards by omitting the term $f_{vv}(\omega)$. However this term is irrelevant to the phase estimate which is given by:

$$\hat{\phi} = \tan^{-1}\frac{\mathcal{I}(\hat{f}_{yx}(\omega))}{\mathcal{R}(\hat{f}_{yx}(\omega))}$$

independently of errors in either variable. Hence this makes it a very useful statistic to use to describe economic relationships, where we know that all variables are observed with error.

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