ON THE LONG-SWING HYPOTHESIS

Michio Hatanaka and E. Philip Howrey

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Princeton University
Econometric Research Program
92-A Nassau Street
Princeton, New Jersey 08540

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I. Introduction

In a recent study by Professor Adelman [3] spectral analysis has been applied to historical economic time series to investigate the existence of long swings. The purpose of that investigation was to determine whether the smoothing devices used in traditional techniques of analysis constitute a sufficient explanation of long cycles which have been isolated by earlier investigations.[2] It was concluded that the estimates of the spectrum "offer no evidence for the existence of a long-cycle component in the business fluctuations of the U.S. economy since 1890", and it was suggested that the long swings which have been observed "...are due in part to the introduction of spurious long cycles by the smoothing process, and in part to the necessity for averaging over a statistically small number of random shocks." [3, p. 459]

A careful study of Adelman's work suggests that the interpretation of the long-swing hypothesis in terms of the spectrum and the statistical inferences

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[1]This is a revised version of Research Memorandum No. 77 of the Econometric Research Program, Princeton University [11]. An earlier version of this paper was presented at the 1965 meetings of the Econometric Society in New York. We have benefitted from the comments made on earlier drafts by Professors Richard A. Easterlin, George Fishman, and Michael D. Godfrey. We are grateful to Mr. Anthony Hughes who provided the results of small-sample experiments on the properties of spectral estimates made at Nottingham, England.

[2]It may be that the crucial test of the long-swing hypothesis involves an investigation of the relationship among different economic variables. So far as we know, this has not yet been done.

from the spectral calculations may be deficient. We have concluded that Adelman's conclusion cited above does not follow from the estimate presented in her paper. Our application of spectral analysis and our conclusions on the existence of long swings will be presented in the main text of the present paper. This paper is reasonably self-contained with references to Adelman's study relegated to footnotes. A more detailed discussion of Adelman's paper is contained in [11].

II. Formulation of the Long-Swing Hypothesis

The long-swing hypothesis has been formulated alternatively in terms of levels, deviations from trend, and rates of growth of an economic variable. In each of these forms the hypothesis is concerned with the existence of a class of fluctuations which are longer in duration than the business cycle but of shorter-term nature than the secular trend [17, chapter 2]. Following [3], we shall confine our discussion to the deviation-from-trend variant of the hypothesis.[3] In formulating the long-swing hypothesis in such a way that it is amenable to spectrum analysis, there are at least two important specifications to be made: (1) the frequency band which contains the long swing, and (2) the criteria which may be used to judge the relative importance of the long-swing frequency band.

For the purpose of making a tentative specification of the long-swing frequency band it is instructive to consider the lower and upper limits of the

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[3] A study of the annual rate of growth has been made by one of the authors in [13].

duration of the long swing which have been suggested by previous investigators.[4] Abramovitz [1, p. 142] suggests ten years and twenty years as the lower and upper limits. Kuznets [17, chapter 7 and p. 423] seems to include movements of longer durations in the long swing, taking twenty years as the average duration rather than the maximum duration.

The lower limit to the long-swing duration should be at least ten years in order to distinguish it from other shorter cycles.[5] Therefore, we shall adopt 1/10 cycles per year (abbreviated as c/y) as the upper limit of the long-swing frequency band. A determination of the lower limit of the frequency band involves a number of subtle points, and any choice would be somewhat arbitrary. The basic motivation underlying an attempt to divide long-term movements into trend and long-swings suggests that the upper limit of the frequency band which contains the trend should be taken as the lower limit of the long-swing frequency band. Although the spectral density function is strictly defined only for a stationary stochastic process, the frequency representation of a trend can be analyzed by using the pseudo-spectrum [10, especially section 6]. The pseudo-spectrum of a linear trend is concentrated in the very low frequencies [7, Figure 8.1, p. 131], but a precise specification of the frequency band which contains the trend is to a certain extent arbitrary. One might take 1/n c/y as

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[4] A danger in using the limits in the previous studies has been indicated in [13]. The danger is essentially due to the effects of filters upon the duration.

[5] It might be argued that the duration of the long-swing should not be as short as ten years on the ground that ten years is nearly the average duration of the so-called major cycle.

the upper limit of the frequency band for trend, where n is the number of years covered by the data, since the band of frequencies [0, 1/n] c/y contains most of the power contributed by the trend.[6]

It might be argued, however, that 1/n c/y is too low for the lower limit of the long-swing frequency band. When time series data run over 60-80 years, as most of the data available for a test of the long-swing hypothesis do, 1/60 or 1/80 c/y as the lower limit of the long-swing frequency band means 60 or 80 years as the upper limit of the duration of the long-swing, and this upper limit might be too long.[7] These considerations suggest a tentative specification of the long-swing band as the interval between 1/10 c/y on the one hand and 1/30 c/y to 1/40 c/y on the other.[8]

It is now necessary to consider in terms of the spectrum what is meant by the "existence" of a cycle in a given frequency band. Several criteria which may be used to determine whether a band of frequencies $[f_1, f_2]$ is important immediately suggest themselves. First, cycles of frequencies $[f_1, f_2]$ can be said to exist if the spectrum exhibits a relative peak in this frequency band. If no relative peak exists in the band, then we reject the hypothesis that the

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[6] The notation $[f_1, f_2]$ is used to denote the band of frequencies the lower and upper limits of which are f_1 c/y and f_2 c/y, respectively.

[7] However, Kuznets [18, p. 25] does not think so.

[8] Adelman takes 1/20 c/y as the lower limit of the long-swing frequency band. Thus a large part of Kuznets' cycles is eliminated from her study. Actually as was shown in [11] by using Hext's interpretation of prewhitening and recoloring [12], Adelman's filtering method is inadequate for the estimation of spectrum at the frequency as high as 1/15 c/y. Therefore, a large part of Abramovitz's cycles and nearly all of Kuznets' cycles are eliminated from Adelman's study.

series contains an important cyclical component in this frequency band. A test based on this existence criterion might be called a <u>two-sided test</u>.

Second, in the particular case of long swings, Kuznets seems to place great importance on the condition that the long-term movements, if meaningful, should not be swamped by the short-term movements [17, chapter 2]. This condition for the "existence" of long swings may be interpreted to mean either (1) that the variance contribution of frequencies centered on the long-swing frequency f_ℓ, $[f_\ell - \delta f, f_\ell + \delta f]$, should exceed the contribution of any higher frequency band of equal length, or (2) that the contribution of $[f_\ell - \delta f, f_\ell + \delta f]$ should exceed the contribution of $[f_\ell + \delta f, 1/2]$ to the variance of the series. Both of the variants of this criterion involve a comparison of the average contribution of the long-swing band with the contribution of higher frequency bands and will therefore be referred to as <u>one-sided tests</u>. With respect to the first variant of the one-sided test, it seems reasonable to emphasize a comparison of the contribution of the long-swing frequency band with that band of frequencies which contains the three-to-four year business cycle. In the rest of the present paper the first version of the one-sided test will be referred to simply as the one-sided test.

III. Spectrum-Analytic Tests of the Hypothesis

Let us consider <u>a priori</u> plausible shapes of true spectrum in order to illustrate what kind of discrimination is required for testing the long-swing hypothesis. In Figure 1(a) is shown a spectrum which satisfies the condition for the existence of long swing in terms of both the two-sided and one-sided tests.[9] In Figure 2(a) is shown a spectrum which shows the existence of long swing on the basis of the one-sided test but not the two-sided test.[10] Figure 3(a) shows a shape of spectrum which rejects the long-swing hypothesis both in the one-sided and two-sided tests.

When the spectral estimation is required to discriminate these different shapes of spectrum, the concept of the spectral window is exceedingly important. With a finite number of observations it is possible to estimate only the average power contained in a set of frequency bands of finite width [4]. The estimates obtained from a finite realization of a process can be thought of as estimates of a weighted average of the true spectrum taken over frequencies. The weighting function which is used in averaging the true spectrum is called the spectral window. The window emphasizes the power contributed by the frequency on which the window is centered and supresses the contribution of frequencies distant from the center frequency. The width of the window is controlled by the parameter called the maximum lag. The Parzen window using the maximum lag 20 is illustrated in Figure 4.

It is now clear that the discrimination among the three figures 1(a) through 3(a) requires a narrow window. The maximum lags 15, 20, 25, and 30 have

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[9] The scale of the abscissa in Figures 1 - 4 is 1/300 c/y so that, for example, the frequency point 0.20 denotes $0.2 \times 10^2/300 = 20/300$ c/y.

[10] A number of investigators using the technique of spectral analysis have suggested that for a large number of economic variables the general contour of spectrum is like Figure 2(a) if the fine details are ignored. Granger [8] has called this the "typical spectral shape".

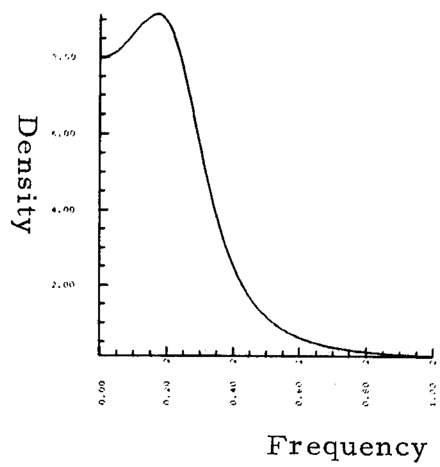

Figure 1a

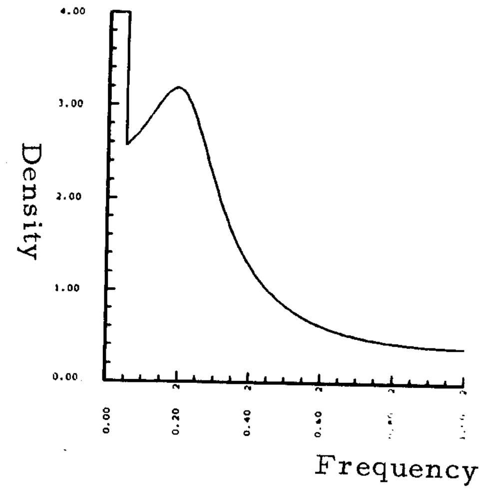

Figure 1b

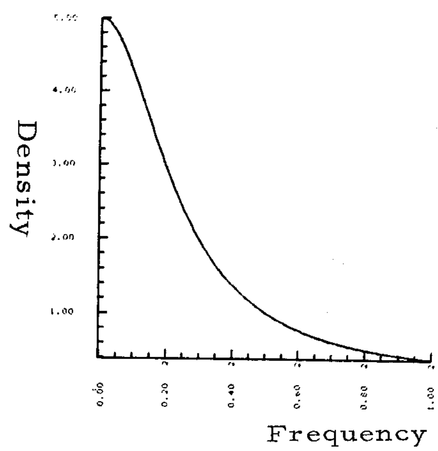

Figure 2a

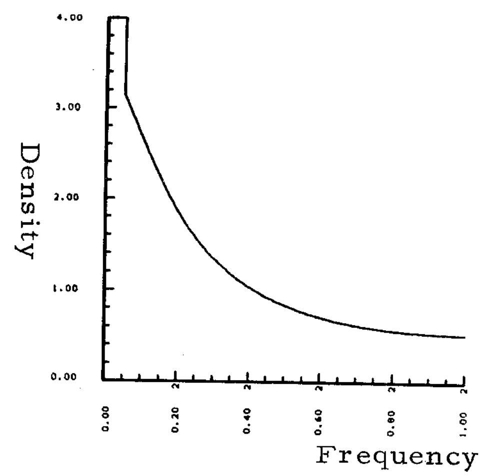

Figure 2b

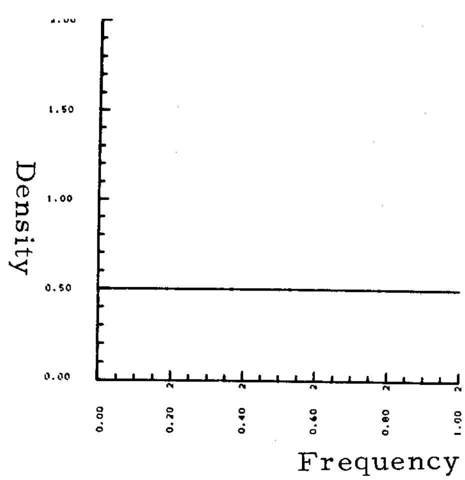

Figure 3a

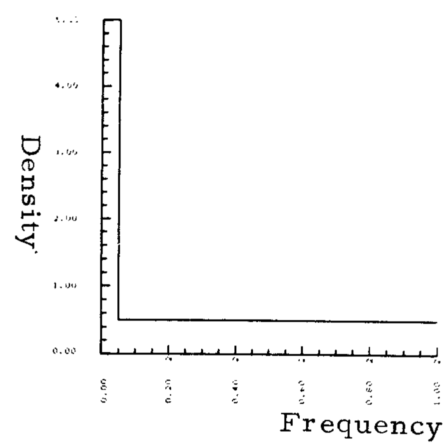

Figure 3b

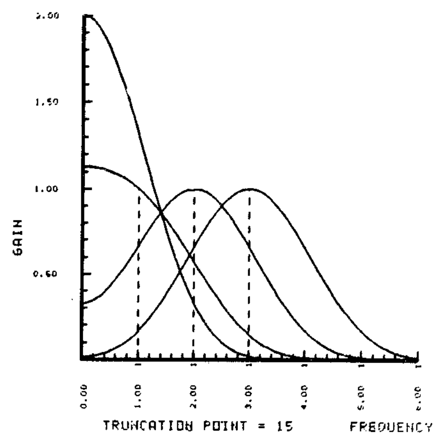

Figure 4. Normalized Parzen Window Centered on 0, 1/30, 2/30, and 3/30 Cycle Per Year

been tried, and it has been discovered that the use of a 20-lag window appears to be sufficient to discriminate between Figures 1(a) and 2(a) in most cases.

It is important to note two factors that might prejudice our judgment of the long-swing hypothesis. First, although we have eliminated the log-linear trend, there is no reason why the trend should be log-linear. Even if we assume that the trend (in the logarithms of economic variables) is represented by a polynomial function of time, we do not know its order. Suppose that after the elimination of log-linear trend the true spectrum is like Figure 1(b), or 2(b), or 3(b), the sharp spike at zero frequency representing the remainder of the trend. Due to the use of spectral window it seems impossible to discriminate between Figures 1(b) and 2(b). One might try a higher order polynomial for trend; however, even if Figure 1(a) emerges, how can we be sure that the result is not due to overadjustment? We do not think there is an easy way to resolve the problem. We have decided to make the conclusion of the present study contingent upon the assumption that the trend is indeed log-linear. This procedure may stack the cards against the long-swing hypothesis.

The second factor which we should consider is the bias in the estimate of spectrum due to the elimination of a polynomial trend of known order. Suppose that the stochastic process which we analyze is represented by

$$\epsilon_t + \sum_{j=0}^{p} a_j t^j$$

where $\{\epsilon_t\}$ is a stationary stochastic process. Suppose that an estimate of trend is obtained by fitting a polynomial of order p to the observed time series by the method of least squares, and that the residual series $\{\hat{\epsilon}_t\}$ has been obtained. One is tempted to use the spectral estimate of $\{\hat{\epsilon}_t\}$ as an estimate of the spectrum of $\{\epsilon_t\}$, but this estimate has a downward bias at and near zero frequency [9]. This stacks the cards in favor of the long-swing hypothesis.

However, the effect of this bias can be investigated by considering the correction factors suggested by Hannan [9] and Hughes [14]. The correction factor suggested by Hannan refers specifically to the (asymptotic) bias introduced by least-squares trend elimination, whereas the correction factor suggested by Hughes' small sample experiments refers to the total bias, i.e., the bias due to trend elimination and the bias due to the windowing operation.[11] Both of these methods have been used to investigate the low-frequency bias in the spectrum estimates presented in the following section.

IV. Results of the Tests

Twenty-two important economic time series[12] have been selected, and the spectra of the residuals from the log-linear trend have been estimated by

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[11] The use of a spectral window introduces a bias at all frequencies at which the spectrum is estimated and can be expressed as $f_{\epsilon,w} - f_{\epsilon}$ where $f_{\epsilon,w}$ refers to the smoothed (windowed) spectrum and f_{ϵ} the true spectrum. The additional bias due to trend elimination, $E[\hat{f}_{\hat{\epsilon},w}] - f_{\epsilon,w}$, can simply be added to the bias introduced by the window to obtain the total bias.

[12] The data which Adelman [3] analyzed and which she found to be representative are also included in our study. However, they form only a portion of the series in our study. Professor R. A. Easterlin pointed out to us that Adelman's study omits a number of important historical time series which have been used in previous studies of the long swing. We have selected from the list of series which he suggested those time series which can be analyzed adequately by our method.

using a Parzen window with maximum lag 20.[13] Table 1[14] shows the location of peak before correcting the bias due to the trend elimination. For those series for which the peak is located off zero frequency but within the long-swing frequency band the correction of this bias has been made. Table 2 shows the ratio between the corrected spectral estimates at zero frequency and at the peak, column (3) using the estimates of bias provided by Hughes and column (4) using Hannan's formula. If the uncorrected estimate shows a peak at zero frequency, the peak remains there after the correction, and hence they are not shown in Table 2.

In terms of the one-sided test the data are overwhelmingly favorable to the long-swing hypothesis. For all the time series which we analyzed the spectral estimates in the long-swing frequency band exceed the estimates about the business cycle frequency band by a factor of between 10 to 1 and 50 to 1.

In terms of the two-sided test all we can say is that the results are mixed. On the one hand, a number of series, especially those in the Building Construction, show spectral estimates like Figure 2(a), which is favorable to the long-swing hypothesis in terms of the two-sided test.[15] Further, at least some of

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[13] Estimations were made at the frequency points j/200 c/y, j = 0, 1, ..., 100.

[14] The more detailed description of the time series as well as their sources are found in the Appendix.

[15] Table 1 is based on what Adelman called unfiltered spectral estimates. Despite the fact that she made no corrections for the bias due to the trend elimination, she did not find any peaks off zero frequency in her unfiltered spectra. She illustrates this by showing a diagram for GNP (Figure 2 in [3]). Our Table 1 shows that the peak for this series is indeed located off zero frequency. As we see it, the difference is due to the maximum lag used in the spectral estimation. She used maximum lag 15 rather than 20 which we used. Her spectral window is not narrow enough to discriminate different shapes of the

Table 1

Time Series (1)	Period (2)	Location of Peak in the unadjusted estimate of spectrum (c/y) (3)
General Economic Activity		
1. GNP in 1929 price	1889 - 1963	7/200
2. Bituminous Coal Output	1841 - 1913	zero
3. Manhours	1889 - 1957	zero
Investment		
4. Gross Investment in 1929 prices	1889 - 1963	6/200
5. Miles of Railroad Built	1830 - 1925	zero
6. Median Index of Incorporations	1860 - 1943	zero
7. Gross New Construction in 1929 prices	1889 - 1959	9/200
8. Residential Building Permit	1868 - 1936	11/200
9. Urban Building Permit in current prices	1853 - 1933	11/200
10. Urban Building Permits in constant prices	1853 - 1933	12/200
11. Gross Capital Expenditures in Regulated Industries, 1929 prices	1890 - 1950	6/200
Capital		
12. Real Capital Stock	1889 - 1953	zero
Prices		
13. Wholesale Price, all commodities	1890 - 1951	5/200
14. Wholesale Price, all commodities	1785 - 1890	zero
15. Wholesale Price, farm products	1786 - 1890	zero
Foreign Trade		
16. Values of Export	1821 - 1957	6/200
17. Values of Import	1821 - 1957	6/200
18. Deflated Import	1820 - 1934	8/200
Demographic Factors		
19. Population	1869 - 1955	zero
20. Immigration	1820 - 1957	zero
Structure		
21. Ratio of Labor Input to Output	1889 - 1957	zero
22. Ratio of Capital Input to Output	1889 - 1957	zero

Table 2

Time Series (1)	$S(f_\ell)/S(0)$ unadjusted* (2)	$S(f_\ell)/S(0)$ adjusted by Hughes' results** (3)	$S(f_\ell)/S(0)$ adjusted by Hannan's formula*** (4)
1. GNP in 1929 Prices	1.14	1.07	0.88
4. Gross Investment	1.04	0.98	0.84
7. Gross New Construction	1.14	1.04	0.81
8. Residential Building	2.49	2.22	1.67
9. Urban Building Permits	2.33	2.07	1.65
10. Urban Building Permits	1.56	1.37	1.09
11. Gross Capital Expenditures in Regulated Industries	1.02	0.96	0.78
13. Wholesale Price	1.04	0.98	0.85
16. Value of Export	1.06	1.02	0.95
17. Value of Import	1.04	1.00	0.94
18. Deflated Import	1.13	1.08	0.93

* f_ℓ: the frequency in the long-swing frequency band at which the peak appears in the spectral estimates.

** Hughes' results for $2n/m = 6.7$ have been used for the time series 1, 4, 7, 8, 9, 10, 11, those for $2n/m = 6.0$ for the time series 13, and those for $2n/m = 13.3$ for the time series 16, 17, 18.

*** Hannan's correction formula $h(\theta)^{-1}$ in [9] has been calculated for the removal of linear trend, using the parameters: n (number of data) = 61, 71, 81, 111, 131 and m (maximum lags) = 20 for Parzen window. Among the above five values of n the one closest to the actual number of data has been used for each series in Table 2.

the series that have peaks at zero frequency according to Table 1 might actually have peaks off zero frequency if a more adequate trend elimination is made. On the other hand, in the case of some important series such as Population the dominance of zero frequency is so strong that one can hardly imagine that it is due to the inadequacy of trend elimination.[16]

Even though peaks emerge in the long-swing frequency band in some time series, the peaks do not appear to be statistically significant in any series. Actually even the business cycles do not appear as a significant peak in the spectral estimates obtained through the presently available length of time series data [7, chapter 12]. It seems that mild peaks are as much as one should hope for, in case one wants to support the long-swing hypothesis.[17,18]

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true spectrum sketched in Figure 1(a) through Figure 3(a). More specifically, her estimate could have been obtained through her window from either Figure 1(a) or Figure 2(a). For a more detailed discussion of this and related points the reader is referred to [11].

[16] This is based on the results obtained through the fitting of log-cubic trends.

[17] The significance test of the peak in the long-swing frequency band is hampered by two difficulties. 1) The spectral estimates about zero frequency and the long-swing frequency are correlated unless an extremely narrow window is used. Use of such a window is undesirable because it yields an extremely large variance of the estimates. The filters used in [11] to make the two estimates uncorrelated have been abandoned because of their adverse effect upon the bias of the estimates. 2) The asymptotic variance of the estimate about zero frequency (relative to the true spectrum) is twice as large as the asymptotic variance of the estimate at a frequency f higher than f_o, f_o being defined as the frequency at which the window centered at zero frequency becomes practically zero. For the frequencies between zero and f_o the factor gradually declines from 2 to 1. With the use of maximum lag 20 in Parzen window f_o may be taken as 3/40 c/y, which is nearly the upper limit of the long-swing frequency band. This makes the significance test more difficult. We owe part of this point to Mr. Anthony Hughes.

[18] Some attempts have been made to eliminate the effects of outliers from the estimates of the auto covariance function and the spectrum. In some time series including GNP the peak tends to shift toward zero frequency when the effect of outliers is eliminated, but there are many series for which the location of the peak is unaffected by the outliers.

We conclude that the evidence provided by the spectral estimates can be interpreted either favorably or unfavorably with respect to the long-swing hypothesis. It depends upon different criteria which one uses and the validity of some assumptions which one cannot test.

APPENDIX

Sources of Data

1. GNP in 1929 prices, 1889-1963, Dept. of Commerce concept; Kendrick's estimates for 1889-1953, in column 11 of Table A-IIa of [16] form the major part of the data.

2. Bituminous Coal Output, 1841-1913, Table B-12 of [20].

3. Manhours in private domestic economy, 1889-1957, the fourth column of Table A-XXII of [16].

4. Gross Private Domestic Investment in 1929 prices, 1889-1963, Dept. of Commerce concept; Kendrick's estimates for 1889-1953, in column 7 of Table A-IIa of [16] form the major part of the data.

5. Miles of Railroad Built, 1830-1925, Table Q-43 of [19].

6. Median Index of Incorporations, 1860-1943, Table 12 of [15].

7. Gross New Constructions in 1929 prices, 1889-1959, series 2 in Table A-1 of [2].

8. Values of Nonfarm Residential Building Permits (Long), 1868-1936, series 17 in Table A-1 of [2].

9. Urban Building Permits per capita (Riggleman-Isard), in current prices, 1853-1933, series 11 in Table A-1 of [2].

10. Urban Building Permits, Index of Value in 1913 prices (Colean-Newcomb), 1853-1933, series 16 in Table A-1 of [2].

11. Gross Capital Expenditures in Regulated Industries in 1929 prices (Ulmer), 1890-1953, series 30 in Table A-1 of [2].

12. Real Capital Stock, equipment in private domestic economy in 1929 prices, 1889-1953, the 9th column of Table A-XXI of [16].

13. Wholesale Price Index (BLS), all commodities, 1890-1951, Table E-13 of [19].

14. Wholesale Price Index (Warren and Pearson), all commodities, 1785-1890, Table E-1 of [19].

15. Wholesale Price Index (Warren and Pearson), farm products, 1786-1890, Table E-2 of [19].

16. Values of U.S. Export, 1821-1957, Table U-1 of [19].

17. Values of U.S. Import, 1821-1957, Table U-2 of [19].

18. Deflated Imports, 1820-1934, estimated by Jeffrey G. Williamson in Table B-5 of [20].

19. Population, 1869-1955, estimated by Simon Kuznets and supplied privately through Richard A. Easterlin.

20. Immigration, 1820-1957, Table C-88 of [19].

21. Labor Input-Output Ratio, private domestic nonfarm economy, 1889-1957, the seventh column of Table A-XXIII of [16].

22. Capital Input-Output Ratio, private domestic economy, 1889-1957, the ninth column of Table A-XXII of [16].

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