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JUDGING QUALITY BY PRICE, SNOB APPEAL, AND THE NEW CONSUMER THEORY*

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1. Introduction

Standard presentations of the theory of demand begin with the drawing of a negatively sloped curve that is identified as a demand curve. Of course, traditional economic theory recognizes the possibility of positively sloping demand curves. Judgment of quality by price, (price) snob appeal, and Giffen goods are indicated as instances in which demand and price move in the same direction. It is only the last of these possibilities that is treated analytically by the traditional approach to consumer theory.[1]

While some may characterize judgment of quality by price and/or snob appeal as irrational, their presence in the real world cannot be denied. We shall argue that, in fact, the former may not be irrational in a complex econom Thus, it seems desirable on both counts that we be able to demonstrate

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1 See any standard textbook for statements disclaiming any attempt to cope with judgment of quality by price or snob appeal. For example, William J. Baumol, Economic Theory and Operations Analysis, 2nd edition, Prentice-Hall Inc., Englewood Cliffs, N. J., 1965, pp. 169-196, especially n. 12, p. 196.

analytically how judgment of quality by price and/or snob appeal can lead to upward-sloping demand curves.

A recent important contribution to consumer theory provides the foundation for such a demonstration.[2] The essence of Lancaster's novel approach to consumer theory is that the individual's utility level is a function of characteristics rather than commodities. Consumption is conceived of as an activity with goods being the inputs and a bundle of characteristics the output. Most consumption activities will be characterized by joint outputs since even single goods, which constitute the simplest consumption activities, typically possess more than one attribute. Similarly, many characteristics will be shared by more than one good.

It is the purpose of this paper to provide an analytical treatment of judgment of quality by price and snob appeal. The next section summarizes the relevant aspects of Lancaster's approach. Section 3 shows how snob appeal and judgment of quality by price can be incorporated into the "new consumer theory" Then, in Section 4, making use of this integration, we derive analytically the conditions under which positively sloping demand curves will result. In the course of this demonstration, the new consumer theory provides a vehicle for including prices in the utility functions of traditional theory.

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[2] See Kelvin J. Lancaster, "A New Approach to Consumer Theory", Journal of Political Economy, Vol. LXXIV, No. 2, April 1966, pp. 132-157; Kelvin J. Lancaster, "Change and Innovation in the Technology of Consumption", American Economic Review, Papers and Proceedings, Vol. LVI, No. 2, May 1966, pp. 14-23; Richard E. Quandt, "Some Remarks on the New Consumer Theory", mimeographed, Spring 1966.

2. The New Approach to Consumer Theory

In the Lancaster formulation, the relationships among commodities and characteristics can be summarized by the following system of linear equations

(1) $$z = Bx .$$

Notationally,

x is an n x 1 vector of quantities of goods consumed,

z is an r x 1 vector of quantities of the various characteristics obtained, and

B is an r x n matrix of coefficients with b_{ij} representing the amount of the i^{th} characteristic yielded by one unit of good j.[3]

Note how this relationship differs from the relationship assumed in traditional consumer theory. In the latter, there is a one-to-one relationship between goods and attributes. That is, only if the B matrix is a permutation of a diagonal matrix is Lancaster's model identical with the traditional one.

Consider a situation in which there are three goods and two attributes ($n = 3$, $r = 2$). This is presented in Figure 1 where points D, E, and F represent the combinations of characteristics (z_1, z_2) obtainable if the consumer' entire given budget, M, is spent on good x_1, x_2, or x_3, respectively. Note that the slopes of the rays x_1, x_2, x_3 are determined by the coefficients in (1). In this example, (1) becomes

[3]Lancaster's most general formulation consists of two systems of linear equations. One relates goods to activities while the second relates characteristics to activities. But he conducts most of his analysis under the assumption that the matrix relating goods to activities is a permutation of a diagonal matri That is, he posits a one-to-one correspondence between commodities and activities. We also make this assumption, primarily for ease of manipulation, and hence consider only the relationships among goods and characteristics.

$$
\text{(2)} \quad \begin{aligned} z_1 &= b_{11}\,x_1 + b_{12}\,x_2 + b_{13}\,x_3 \\ z_2 &= b_{21}\,x_1 + b_{22}\,x_2 + b_{23}\,x_3 \,. \end{aligned}
$$

The slopes of x_1, x_2, x_3 are thus b_{21}/b_{11}, b_{22}/b_{12}, b_{23}/b_{13}, respectively.

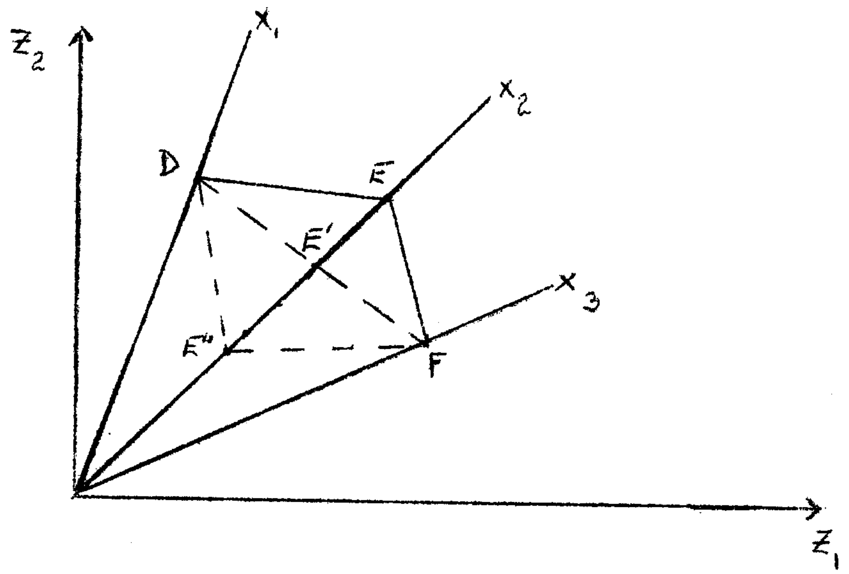

Figure 1.

The broken line segment DEF is the efficiency frontier for characteristics. An efficient consumer will only choose combinations of goods yielding bundles of characteristics represented by points on this frontier. In the present example he will not consume both x_1 and x_3 since any set of characteristics attainable from joint consumption of these goods is dominated by a set of characteristics attainable by consuming x_1 and x_2 or x_2 and x_3. The consumer's actual position on the frontier will be determined, of course, by his utility function in characteristics. Thus, the consumer makes two choices: an efficiency choice and a private utility choice.[4]

[4] See Lancaster, <u>op. cit.</u>, <u>JPE</u>, pp. 133-140 or Lancaster, <u>op. cit.</u>, <u>AER</u>, pp. 13-17.

Similarly, in the new consumer theory, the substitution effect consists of both an efficiency substitution and a utility substitution. If there is an increase in the relative price of, say, x_2 then the point, that represents spending the whole budget on x_2, E, will shift toward the origin. If the increase is great enough to shift E to E'' then the efficiency frontier will be DF and only combinations of x_1 and x_3 will be efficient.[5] Substitutions of this type are efficiency substitutions and will always be non-positive.[6]

The second part of the substitution effect, the utility substitution due to a relative-price change, is the change in the consumption of goods determined by the individual's utility function. It results from using the same utility function to select a new equilibrium point on the new efficiency frontier. This utility substitution is also non-positive for the good whose price has changed. The efficiency and utility substitution effects are independent but work in the same direction. Hence, when they both occur -- as we would generally expect - they tend to reinforce one another. "Thus, the consumer model presented here in the context of an advanced economy, has, in a sense, more substitution than the traditional model."[7] Our confidence in the negative slope of individuals' demand curves is thus strengthened.

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[5] If the price change moves E to E' then any pair of goods (x_1 and x_2, x_2 and x_3, <u>and</u> x_1 and x_3) can constitute an efficient combination.

[6] In the case we are considering, that of a one-to-one relationship among good and activities, the efficiency substitution effect will result either in no change or in a complete switch from consumption of one good to the consumption of another good. See Lancaster, <u>op. cit. JPE</u>, p. 142. The preceding discussion is based upon pp. 140-142 in that article.

[7] Lancaster, <u>op. cit.</u>, <u>JPE</u>, p. 143.

3. The Price Attribute and the New Consumer Theory

When a consumer judges quality by price or practices price-snob appeal the price of a commodity has a greater effect on his consumption decision than its place in his budget constraint would suggest. In each case price not only constrains his consumption but also constitutes an attribute which is desired in and of itself. While the introduction of the price attribute into the analysis is the same in both instances the two phenomena require different interpretations of the new characteristic. In the case of snob appeal it is assumed that all relevant measurable characteristics have been enumerated and are included in the utility function. The price attribute is one of these relevant characteristics. In other words, in the case of snob appeal the price attribute is a real attribute.

In contrast, judgment of quality by price means that the price attribute is a proxy (perhaps an imperfect one) for other characteristics. The price attribute is included in the utility function as a surrogate for some relevant characteristics which have not been enumerated. When the attributes are large in number or extremely difficult to measure some such substitution is not unreasonable. That is to say, when difficulties in collecting or interpreting reliable information are substantial it may be quite reasonable to introduce some function of price, since prices are common knowledge and hence easily obtainable, as a proxy for "quality".

Keeping in mind the fact that the interpretation of the price attribute depends on which of the two phenomena we are discussing, let us now introduce such an attribute. Specifically, suppose one of the characteristics, call it z_p, is a price attribute. That is, z_p is some function of the prices of goods x_j, $j = 1, \ldots, n$ ($n = 3$ in the model presented here). It follows from the definition in (1) that

(3) $$z_p = \sum_{j=1}^{n} b_{pj} x_j \quad ,$$

where $b_{pj} = k_j p_j$, $j = 1, \ldots, n$.

Since we want to deal with the case where an increase in price is considered desirable (where quality is judged by price or price-snob appeal exists), we assume that each b_{pj} is non-negative: a rise in the price of any good either increases the value of the price characteristic or does not affect it at all. Each good generates the attribute "quality" (as measured by price) in quantities described by (3).

The influence of "quality" on the consumer's decisions will, however, depend on the weight assigned this price characteristic in the individual's utility function. This is exactly analogous to the role played by any other characteristic. The signs and magnitudes of the partials of the individual's utility function determine the effects of the attributes on the consumer's choice. The general price attribute, z_p, will have positive marginal utility $(\frac{\partial u}{\partial z_p} > 0)$ for someone judging quality by price and zero marginal utility otherwise. In an extreme case, an individual might prefer a combination of goods offering more price benefit and less z_i, say nutrition, to one with less z_p and greater nutrition

Restricting ourselves, as we have been doing, to three commodities and two characteristics, the consumption technology is now defined by

$$(4)\qquad \begin{aligned} z_1 &= b_{11}\, x_1 + b_{12}\, x_2 + b_{13}\, x_3 \\ z_p &= b_{p1}\, x_1 + b_{p2}\, x_2 + b_{p3}\, x_3 \end{aligned}$$

Figure 2 depicts this situation where b_{p1}/b_{11}, b_{p2}/b_{12}, and b_{p3}/b_{13} are the slopes of rays x_1, x_2, and x_3, respectively, and DEF is the efficiency frontier. Now an increase in the price of x_2 relative to the prices of x_1 and x_3[8] will do more than shift E toward the origin. It will also change the slope of ray x_2. This is true because the slope of ray x_2 equalt b_{p2}/b_{12} which is equal to $k_2\, p_2/b_{12}$. (Recall b_{p2} is, by definition, $k_2\, p_2$.) The slope of ray x_2 thus varies directly with p_2. Similarly, the slope of ray x_1 varies directly with p_1

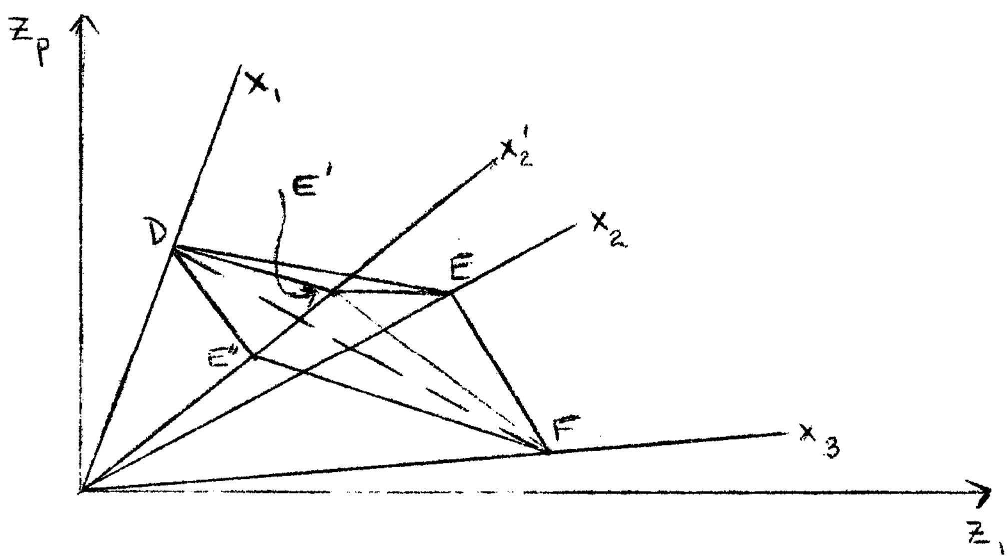

Figure 2.

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[8] A change in the price of good i relative to that of good j can always be expressed as $(g_{ij}\, p_i^{o})/(p_j^{o})$ where g_{ij} is some positive constant and p_i^{o} and p_j^{o} are the initial prices. We will restrict ourselves to relative-price changes in which all the g_{ij} coefficients are equal. That is, we consider the price of good 2 to increase to g times its original value while the prices of all other goods remain constant.

and that of x_3 with p_3. An increase in the relative price of good 2 (as we are considering it here) will thus pivot ray x_2 through the origin toward the z_p axis.

If the new ray is x_2', then the new frontier would seem to be something like DE''F. (As drawn in Figure 2 it would thus actually become DF.) It can be shown, however, that the ordinate of the new frontier vertex on ray x_2' must in fact, be the same as that of E, while the abscissa of the new point must be less than that of E. Therefore, the new frontier must be DE'F.

To see this, note first that the equation of ray x_2 is

(5) $$x_2: \qquad z_p = \frac{b_{p2}}{b_{12}} z_1 = \frac{k_2 \, p_2}{b_{12}} z_1$$

Assuming the individual has M dollars to spend, his budget constraint is

(6) $$p_1 x_1 + p_2 x_2 + p_3 x_3 \leqq M \, .$$

From (4), (5), and (6) it follows that the coordinates of the original frontier vertex on ray x_2, namely E, are

(7) $$E: \qquad \left(\frac{b_{12} \, M}{p_2}, \; k_2 \, M\right) .$$

Hence a change in the price of good 2 affects only the abscissa of the frontier point. The condition may be stated even more strongly: a change in the price of good 2 from p_2 to gp_2 will change the abscissa of the frontier vertex of ray x_2 to $1/g$ times its original value, leaving its ordinate unchanged.

The explanation of the unchanged ordinate (the unchanged z_p) is really quite simple. Recall that the contribution of good j to price benefit (measure of "quality") is defined to be $k_j \, p_j$. Hence, while a g-fold increase in the price of good j increases the magnitude of its contribution g times, it simultaneously reduces by a factor of g the number of units of good j the consumer can buy if he spends all his income on that good. The amount of price benefit derived from consuming good j alone is thus unchanged.

A general price attribute can thus be introduced rather smoothly into the new consumer theory. This is not very surprising since there is no inheren difference between such a price attribute and any of the conventional attributes, for example, nutritional value, taste, size, color. But the consumption technology is no longer constant with respect to changes in the prices of commodities. The functional dependence of the general price attribute on the individual commodity prices is, however, known and constant. Hence, since it is only because of changes in commodity prices that the technology is altered, all changes in the technology are predictable and analytically tractable.

Furthermore, the substitution effect still consists of two parts, an efficiency substitution and a utility substitution. Although the efficiency substitution is now the result of two simultaneous changes in the frontier, a shift of the vertex and a pivoting of the commodity ray, it remains non-positive as in the case where no price attribute exists. It will be negative if the ray corresponding to commodity 2 pivots so that x_2^{*} lies to the left of x_1 <u>or</u> if it pivots so that DE' lies below DF. In either of these two cases, the consumer would no longer purchase good 2. He would switch from consuming x_1 and x_2 or x_2 and x_3 to consuming x_1 and x_3.

On the other hand, one would expect that the other part of the substitution resulting from the price change, the utility substitution, might deviate from its conventional form with, perhaps, interesting implications for the total substitution effect. It is to this phase of the analysis that the discussion now turns.

4. The Traditional Substitution Effect and Positively Sloped Demand Curves

In turning to the analysis of the traditional substitution effect we restric ourselves to a two-commodity world. Why? Because the existence of an efficiency substitution clearly requires that there be fewer characteristics than activities.[9] Since we have assumed that the number of activities equals the number of goods, this means for an efficiency substitution to occur, the numbe of goods must exceed the number of characteristics. By equating the number of commodities (2) to the number of characteristics (2) we thus eliminate the possibility of any efficiency substitution and we can concentrate on the traditional substitution effect. To further simplify the analysis we assume the existence of an interior maximum.

The classical problem confronting the consumer can be restated in terms of the new consumer theory as follows.

$$\text{(8)} \quad \begin{aligned} &\text{Maximize} && U = U(z_1, z_2) \\ &\text{Subject to:} && p_1 x_1 + p_2 x_2 \leqq M \\ &&& b_{11} x_1 + b_{12} x_2 = z_1 \\ &&& b_{21} x_1 + b_{22} x_2 = z_2 \, . \end{aligned}$$

Substituting for attributes in the utility function their equivalents in terms of commodities yields

$$\text{(9)} \quad \text{Maximize} \quad U = U(b_{11} x_1 + b_{12} x_2, b_{21} x_1 + b_{22} x_2)$$

[9] Lancaster, op. cit., JPE, p. 142.

Subject to: $$p_1 x_1 + p_2 x_2 \leq M .$$

It is assumed that each good is perfectly divisible and beneficial to the consumer. Together with the assumption that an interior maximum exists, this implies that the budget constraint will hold as an equation. This problem can therefore be solved by the technique of Lagrange multipliers. We form the function

$$(10) \qquad \Phi = U(b_{11}x_1 + b_{12}x_2 \, , \, b_{21}x_1 + b_{22}x_2) + \lambda(M - p_1x_1 - p_2x_2) \, ,$$

and set the first partial derivatives of Φ equal to zero.[10] Equilibrium requires that the ratio of the marginal utilities of goods expressed in terms of the marginal utilities of attributes be equal to the price ratio. That is, we find that in equilibrium

$$(11) \qquad \frac{p_1}{p_2} = \frac{b_{11} U_1 + b_{21} U_2}{b_{12} U_1 + b_{22} U_2} \, .$$

Moreover, as in traditional theory, it can be shown that the own-substitution effects $\left[\left(\frac{\partial x_j}{\partial p_j}\right)_{\text{utility constant}}\right]$ are negative and that the signs of the own-income effects $\left[- x_j\left(\frac{\partial x_j}{\partial M}\right)_{\text{prices constant}}\right]$ are indeterminate.[11]

Now suppose that one of the characteristics in this two-attribute world is the price attribute defined by equation (3) with $n = 2$. If the price attribute

[10] For the traditional approach see James M. Henderson and Richard E. Quan Microeconomic Theory; A Mathematical Approach, McGraw-Hill Book Company, Inc., New York, 1958, pp. 16, 24-27.

[11] The apparent discrepancy between this result and the result of section 2 (that substitution effects, utility and efficiency, are non-positive and hence that the total substitution effect is non-positive) arises because this analysis assumes continuous functions and partial derivatives. On the other hand, the graphical analysis allowed functions with discontinuities and/or kinks. It is in these latter instances that the substitution effect may equal zero.

arises from snob appeal there exist only two real attributes in this world: z_1 and z_p. If, on the other hand, we are discussing judgment of quality by price there are at least two (but probably more) real attributes in the world: z_1 and all the non-enumerated characteristics for which z_p is a proxy.

The consumer now desires to

$$\text{Maximize} \quad U = U(z_1, z_p)$$

$$\text{(12)} \qquad \text{Subject to:} \quad \begin{aligned} p_1 x_1 + p_2 x_2 &= M \\ b_{11} x_1 + b_{12} x_2 &= z_1 \\ k_1 p_1 x_1 + k_2 p_2 x_2 &= z_p \ . \end{aligned}$$

Proceeding by substituting the expressions in terms of goods for z_1 and z_p into the utility function, we form the appropriate Lagrangian expression, and set its first partials equal to zero.

$$\text{Maximize} \quad \Phi = U(b_{11}x_1 + b_{12}x_2, k_1p_1x_1 + k_2p_2x_2) + \lambda(M - p_1x_1 - p_2x_2)$$

$$\text{(13)} \qquad \begin{aligned} \frac{\partial \Phi}{\partial x_1} &= U_1 b_{11} + U_p k_1 p_1 - \lambda p_1 = 0 \\ \frac{\partial \Phi}{\partial x_2} &= U_1 b_{12} + U_p k_2 p_2 - \lambda p_2 = 0 \\ \frac{\partial \Phi}{\partial \lambda} &= M - p_1 x_1 - p_2 x_2 = 0 \ , \end{aligned}$$

where $U_1 = \frac{\partial U}{\partial z_1}$ and $U_p = \frac{\partial U}{\partial z_p}$.

The way in which the new approach to consumer theory facilitates the discussion of judgment of quality by price and snob appeal should now be clear. It enables us, by not-so-devious means, to introduce prices into the traditional utility function. Equilibrium again requires that the ratio of the marginal utilities of goods, expressed in terms of the marginal utilities of attributes, equal the ratio of the prices of the goods,

$$\text{(14)} \qquad \frac{(\partial U/\partial x_1)}{(\partial U/\partial x_2)} = \frac{U_1\, b_{11} + U_p\, k_1\, p_1}{U_1\, b_{12} + U_p\, k_2\, p_2} = \frac{p_1}{p_2} \; .$$

Differentiating system (13) totally yields

$$\text{(15)} \qquad \begin{aligned}
&\left(b_{11}\frac{\partial U_1}{\partial x_1} + k_1 p_1 \frac{\partial U_p}{\partial x_1}\right)dx_1 + \left(b_{11}\frac{\partial U_1}{\partial x_2} + k_1 p_1 \frac{\partial U_p}{\partial x_2}\right)dx_2 - p_1\, d\lambda = \\
&\qquad \left(\lambda - b_{11}\frac{\partial U_1}{\partial p_1} - k_1 U_p - k_1 p_1 \frac{\partial U_p}{\partial p_1}\right)dp_1 - \left(b_{11}\frac{\partial U_1}{\partial p_2} + k_1 p_1 \frac{\partial U_p}{\partial p_2}\right)dp_2 \\
&\left(b_{12}\frac{\partial U_1}{\partial x_1} + k_2 p_2 \frac{\partial U_p}{\partial x_1}\right)dx_1 + \left(b_{12}\frac{\partial U_1}{\partial x_2} + k_2 p_2 \frac{\partial U_p}{\partial x_2}\right)dx_2 - p_2\, d\lambda = \\
&\qquad -\left(b_{12}\frac{\partial U_1}{\partial p_1} + k_2 p_2 \frac{\partial U_p}{\partial p_1}\right)dp_1 + \left(\lambda - b_{12}\frac{\partial U_1}{\partial p_2} - k_2 U_p - k_2 p_2 \frac{\partial U_p}{\partial p_2}\right)dp_2 \\
&- p_1\, dx_1 \qquad\qquad - p_2\, dx_2 \qquad = - dM + x_1\, dp_1 + x_2\, dp_2 \; .
\end{aligned}$$

If we regard the right-hand side of (15) as a vector of constants we can solve for dx_1, dx_2, and $d\lambda$. We will proceed only with dx_1 since the results for dx_2 follow analogously. Denote the determinant of the system in (15) by A and denote the cofactor of the i, j^{th} element of A by A_{ij}. Moreover, to simplify matters let us note that

$$\text{(16)} \qquad \begin{aligned}
&\frac{\partial^2 U}{\partial p_1 \partial x_1} = b_{11}\frac{\partial U_1}{\partial p_1} + k_1 U_p + k_1 p_1 \frac{\partial U_p}{\partial p_1}, \quad \frac{\partial^2 U}{\partial p_1 \partial x_2} = b_{12}\frac{\partial U_1}{\partial p_1} + k_2 p_2 \frac{\partial U_p}{\partial p_1} \\
&\frac{\partial^2 U}{\partial p_2 \partial x_1} = b_{11}\frac{\partial U_1}{\partial p_2} + k_1 p_1 \frac{\partial U_p}{\partial p_2}, \quad \frac{\partial^2 U}{\partial p_2 \partial x_2} = b_{12}\frac{\partial U_1}{\partial p_2} + k_2 U_p + k_2 p_2 \frac{\partial U_p}{\partial p_2} \; .
\end{aligned}$$

Using the relationships in (16) we find that

$$\text{(17)} \qquad dx_1 = \frac{1}{A}\left\{\left[\left(\lambda - \frac{\partial^2 U}{\partial p_1 \partial x_1}\right) dp_1 - \frac{\partial^2 U}{\partial p_2 \partial x_1} dp_2\right] A_{11} + \left[- \frac{\partial^2 U}{\partial p_1 \partial x_2} dp_1 + \left(\lambda - \frac{\partial^2 U}{\partial p_2 \partial x_2}\right) dp_2\right] A_{21} + [- dM + x_1 dp_1 + x_2 dp_2] A_{31}\right\}$$

We now want to investigate, in the traditional manner, the way in which the demand for a good changes when its price, and only its price, changes. That is, we set $dp_2 = dM = 0$, leave all other elements of the technology matrix except p_1 - unchanged and calculate $\frac{\partial x_1}{\partial p_1}$. It is important to understand what our assumptions mean in terms of the attributes model. A change in the price of a good may be accompanied by a change in its per-unit contribution to some enumerated non-price attribute. For example, the price of an automobile will generally be increased when the horsepower of the engine increases. This is not the case with which we are concerned. Instead, under our assumptions, the good's per-unit contribution to all other enumerated characteristics remains constant. In the case of snob appeal this means that its per-unit contribution to all relevant non-price characteristics remains unchanged. On the other hand, if the individual is not a price-snob but simply judges quality by price the good's per-unit contribution to some non-enumerated but relevant characteristics may actually change. But his perception of this change is reflected only in its effect on the good's per-unit contribution to the price attribute.[12]

[12] In the case of judgment of quality by price or price-snob appeal, this market experiment is equivalent to the substitution of a new good for an old good in the system. The new good is identical with the old good in all enumerated characteristics except the price one.

Dividing both sides of (17) by dp_1 (and setting $dp_2 = dM = 0$) we obtain

(18) $$\frac{\partial x_1}{\partial p_1} = \left(\lambda - \frac{\partial^2 U}{\partial p_1 \partial x_1}\right)\frac{A_{11}}{A} - \frac{\partial^2 U}{\partial p_1 \partial x_2}\frac{A_{21}}{A} + x_1 \frac{A_{31}}{A}$$

The traditional income effect is $x_1 \frac{A_{31}}{A}$. But the traditional substitution effect now consists of two terms, the first two terms on the right-hand side of (18). Substituting for A_{11} and A_{21} in (18) we get

(19) $$\frac{\partial x_1}{\partial p_1} = \left(\lambda - \frac{\partial^2 U}{\partial p_1 \partial x_1}\right)\left(-\frac{p_2^2}{A}\right) - \left(\frac{\partial^2 U}{\partial p_1 \partial x_2}\right)\left(\frac{p_1 p_2}{A}\right) + x_1 \frac{A_{31}}{A} .$$

The determinant A is greater than zero by the second-order conditions for a constrained maximum. And the income effect is again indeterminate without knowledge of the magnitudes of the components of A_{31}. But the conclusion about the substitution effect differs from the usual one. The traditional substitution effect is presented in equation (20).

(20) $$S = -\frac{p_2}{A}\left[p_2\left(\lambda - \frac{\partial^2 U}{\partial p_1 \partial x_1}\right) + p_1 \frac{\partial^2 U}{\partial p_1 \partial x_2}\right] .$$

While $-\frac{p_2}{A}$ is negative the sign of the term in brackets is ambiguous. The traditional substitution effect will be negative if and only if

(21) $$p_2\left(\lambda - \frac{\partial^2 U}{\partial p_1 \partial x_1}\right) + p_1 \frac{\partial^2 U}{\partial p_1 \partial x_2} > 0 .$$

Dividing by the positive quantity $p_1 p_2$, (21) can be rewritten as

(22) $$\frac{1}{p_1}\left(\lambda - \frac{\partial^2 U}{\partial p_1 \partial x_1}\right) + \frac{1}{p_2}\frac{\partial^2 U}{\partial p_1 \partial x_2} > 0 .$$

One more simple manipulation will reduce the necessary and sufficient condition for a negative substitution effect to an easily interpreted form. Moving everything to the right-hand side but $\frac{\lambda}{p_1}$, we get

$$\frac{\lambda}{p_1} > \frac{1}{p_1}\frac{\partial^2 U}{\partial p_1 \partial x_1} - \frac{1}{p_2}\frac{\partial^2 U}{\partial p_1 \partial x_2} \tag{23}$$

The substitution effect will be negative if and only if the optimal marginal utility of income per dollar spent on x_1 is greater than the difference between the per-dollar change in the marginal utility of x_1 as p_1 changes and the per-dollar change in the marginal utility of x_2 as p_1 changes.

Thus the sign of the substitution effect is not necessarily negative. Within the framework of the traditional study of consumer behavior, demand curves are positively sloped if and only if the income effect is positive and greater in absolute value than the substitution effect. In contrast, when judgment of quality by price and snob appeal are introduced we are able analytically to indicate additional reasons for rising demand curves. Both the traditional substitution and income effects may be positive, or one may be positive and the other negative with the positive effect greater in absolute value.

5. Conclusion

The analysis contained in this paper has served two purposes. First, it has demonstrated the richness of "the new consumer theory" by showing how this new approach can incorporate the questions of judging quality by price and price-snob appeal. And second, the paper has derived analytically the conditions under which these phenomena will yield a positively sloping demand curve in a two-commodity, two-attribute world.

In traditional theory the consumer's response to a change in the price of a good is conceptually divided into an income effect and a substitution effect. The new approach to consumer theory partitions the consumer's response differently, namely, into an efficiency substitution and a utility substitution. When a general price attribute is introduced each of these divisions retains its individual validity. The conclusion that the efficiency substitution is always non-positive also remains valid. But, the traditional substitution effect may now be positive and of sufficient magnitude to make the <u>entire</u> response to an own-price change <u>positive</u>, $\frac{\partial x_j}{\partial p_j} > 0$. We have shown analytically, then, as the verbal discussions of traditional consumer theory lead us to believe, that judgment of quality by price and price-snob appeal can result in positively sloped demand curves.