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# THE ALGEBRA OF REVEALED PREFERENCE

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## Introduction.

Samuelson[1] formulated the principle of revealed preference, which has been investigated further by Houthakker[2] and Uzawa,[3] in application to expenditure systems. An expenditure system is the concept by which a choice of a composition of goods is supposed determined within every expenditure-price restriction. It therefore involves a complete, and thus infinite set of possible choices. But a finite set, or configuration, of choices is all that can be practically observed; so it is important to investigate the revealed preference principle in application to such finite configurations.

While for a complete configuration provided by an expenditure system, analytical methods of the calculus can be applied, for a finite configuration the methods have to be algebraical. It is shown that, in application to a finite configuration, the acyclicity condition defined by Houthakker's Strong Axiom of Revealed Preference has two equivalent algebraical forms, expressed by the consistency of certain systems of linear inequalities. By this means it is shown that, just as the acyclicity condition is, immediately, "the condition for revealed preference" for a complete configuration, so, but with less immediacy, it is for a finite configuration.

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[1] P. A. Samuelson, "Consumption theory in terms of revealed preference," Economica 28 (1948), pp. 243-53.

[2] H. S. Houthakker, "Revealed preference and the utility function," Economica 17 (1950), pp. 159-174.

[3] H. Uzawa, "Preference and rational choice in the theory of consumption," Mathematical Methods in the Social Sciences, Stanford Mathematical Studies in the Social Sciences V (Stanford, 1959).

1. The preference hypothesis

Consider a consumer, in some $k$ occasions $1, \ldots, k$ ; and let it be supposed that for each occasion $r$ , the prices and amounts of some $n$ goods consumed are known, and given by a pair of vectors $(p_r, x_r)$ of order $n$ $(r = 1, \ldots, k)$ .

Then the expenditure in occasion $r$ is determined by the market equation

$$e_r = p_r' x_r \ .$$

Hence, if

$$u_r = \frac{p_r}{e_r} \ ,$$

then

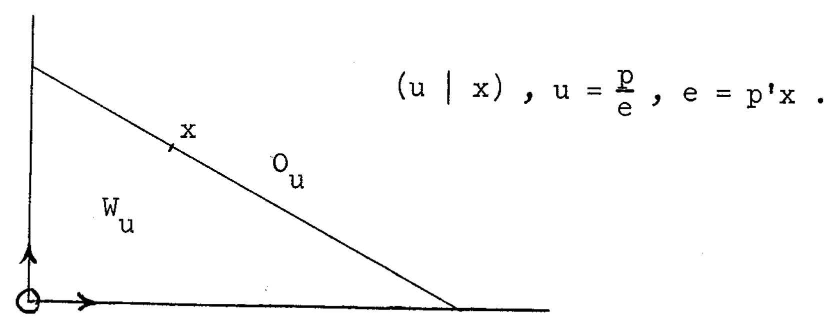
$$u_r' x_r = 1 \ .$$

The vectors $u_r, x_r$ are to define the balance and composition for occasion $r$ ; and they are taken to belong to spaces $B, C$ which are positive orthants of Euclidean spaces of dimension $n$ .

Any composition $x \in C$ is said to be within, on, or over any balance $u \in B$ according as $u'x \leq , = ,$ or $> 1$ . The sets of compositions within and on a balance $u$ are denoted by

$$W_u = \{x \mid u'x \leq 1\} \ , \quad O_u = \{x \mid u'x = 1\} \ .$$

A balance $u$ together with a composition $x$ which is on it forms an expenditure figure, denoted by $(u \mid x)$ , where it is to be automatically understood in this notation that $u'x = 1$ . Any collection of expenditure figures defines an expenditure configuration.



In occasion $r$, there is obtained the expenditure figure $E_r = (u_r \mid x_r)$; and from the $k$ occasions there is obtained the finite expenditure configuration $\mathcal{E} = \{E_r\}_{r = 1,\ldots,k}$.

The compositions $x$ attainable instead of $x_r$ at the same prices $p_r$ for no greater expenditure, are such that $p_r'x \leq p_r'x_r$, and equivalently $u_r'x \leq 1$. They are thus the set $W_{u_r}$ of compositions within the balance $u_r$.

An hypothesis to be applied to the consumer is that there exists an irreflexive, transitive preference order $P$ on the composition space $C$ to the effect that, when the consumer makes an expenditure $e$ at prices $p$, to obtain goods in some composition $x$, necessarily such that $p'x = e$, and thus on the balance $u = \frac{p}{e}$, then $x$ is decided by the condition that it be better, according to $P$, than all other compositions which are within $u$, that is, which could be got instead of $x$ for no greater expenditure at the same prices. Thus

$$x \in O_u \, , \; xP(W_u - x) \, .$$

This describes the general <u>preference hypothesis</u> to be applied to the consumer.

As a special <u>normal</u> form for the order $P$, it could be asked that there exists a numerical function $\varphi(x)$ $(x \in C)$ such that

$$xPy \iff \varphi(x) > \varphi(y) \, .$$

In this case $P$ may be called a <u>scale</u>, for which $\varphi$ is a <u>gauge</u>. There is a unique scale $P$ with any function $\varphi$ as gauge. But any scale $P$ has an infinite class $\Gamma_P$ of gauges, which are all increasing transformations of each other; so each is of the form $\omega(\varphi)$, where $\varphi$ is any representative in $\Gamma_P$ and $\omega(t)$ any increasing function. The <u>normal preference hypothesis</u> for the consumer is obtained, within the framework of the general one, by the further requirement that the hypothetical preference relation $P$ be a scale.

A relation $P$ which in this way decides a composition $x = f_P(u)$ on every balance $u$ will be called a preference system. Any function $f$ determining a composition $x = f(u)$ $(u'x = 1)$, on every balance $u$, defines an expenditure system. If $f = f_P$, that is, $f$ is an expenditure system which can be derived from some preference system $P$, then generally $P$ does not have to be a scale, but it can have a more general structure. However, should $f$ have the regular expansion property

$$\left| f\left(\frac{u}{\rho}\right) - f\left(\frac{u}{\sigma}\right) \right| < M(\rho - \sigma),$$

for some $M > 0$, and all $u \in B$ and $\rho, \sigma > 0$, then $P$ has to be a scale, as appears from the previously cited investigations of Samuelson, Houthakker, and Uzawa. This observation gives a kind of justification for the mathematical convenience of working empirically with the normal, rather than the general form of preference hypothesis, since the regular expansion property is an acceptable supposition. The normal form, moreover, corresponds to the traditional picture, in which there is demarcation of preference by "indifference" surfaces.

Another justification for working with the normal hypothesis is that the admissibility of the general hypothesis on given data, such as could be provided by some finite expenditure configuration $\mathcal{E}$, is no more relaxed a requirement of the data than the admissibility of the normal form of the hypothesis. Thus, just so far as admissibility is concerned, the two forms of hypothesis have equal scope.

This is not the case, however, if $\mathcal{E}$ is a complete, and therefore infinite, configuration, belonging to an expenditure system. For then, if the general preference hypothesis is admitted, it is in a unique fashion, without the indeterminacy which arises with a finite and therefore incomplete configuration. The unique preference order admitted by a complete configuration may or may not be of the normal form, though Houthakker and Uzawa have shown conditions under which it has to be of the normal form.

## 2. Admissibility

Let there now be considered the admissibility of the general, and then of the normal form of the preference hypothesis, on the data $\mathcal{E}$. There has to exist an order P such that

$$x_r P(W_{u_r} - x_r) \quad (r = 1, \ldots, k) .$$

Let

$$P_{\mathcal{E}} = \bigcup_{r=1,\ldots,k} (x_r, W_{u_r} - x_r) ,$$

which is to define the primary preference relations of the expenditure configuration $\mathcal{E}$, where $(x_r, W_{u_r} - x_r)$ stands for the set of ordered couples $(x_r, x)$ with $x \in W_{u_r} - x_r$, that is, with $u_r'x \leq 1$ and $x \neq x_r$. Then the admissibility condition is

$$P_{\mathcal{E}} \subset P .$$

Now, since P is transitive, this is equivalent to

$$\vec{P}_{\mathcal{E}} \subset P ,$$

where $\vec{P}_{\mathcal{E}}$ is the transitive closure of $P_{\mathcal{E}}$. Since P is irreflexive, this implies that $\vec{P}_{\mathcal{E}}$ is irreflexive, and since already transitive, therefore an order.

Conversely, if $\vec{P}_{\mathcal{E}}$ is irreflexive, it is an order containing $P_{\mathcal{E}}$. Hence, if P* is any order which is a refinement of $\vec{P}_{\mathcal{E}}$, sufficiently refined to determine a best composition on any balance, it will be an admissible preference hypothesis for $\mathcal{E}$. It is not immediately apparent that such a P* exists. This is even though, assuming the axiom of choice, every order has a complete order refinement, because that does not mean such a complete order, of maximal refinement though it is, will determine a maximal element on every balance. But it will appear that the irreflexivity of $\vec{P}_{\xi}$ is sufficient for the admissibility of the normal form of hypothesis. Hence, with the general hypothesis implying $\vec{P}_{\xi}$ irreflexive, and this implying the normal hypothesis, which is an instance of the general hypothesis, it appears that the irreflexivity of $\vec{P}_{\xi}$ is necessary and sufficient for the admissibility of both the general and the normal preference hypothesis.

For $\vec{P}_{\xi}$ to be irreflexive, an equivalent condition is that $P_{\xi}$ be acyclic. Any cycle of elements, each of which has the relation $P_{\xi}$ to its successor must be of the form $x_r, x_s, \ldots, x_q, x_r, \ldots$ and such that

$$x_s \in W_{u_r} - x_r \, , \; x_t \in W_{u_s} - x_s, \; \ldots, \; x_r \in W_{u_q} - x_q \, ,$$

or equivalently,

$$x_r \neq x_s, \; x_s \neq x_t \, , \; \ldots, \; x_q \neq x_r$$

and

$$u_r{}'x_s \leq 1, \; u_s{}'x_t \leq 1, \; \ldots, \; u_q{}'x_r \leq 1 \, .$$

Assume, as is likely with observed data, that $x_r \neq x_s \quad (r \neq s)$ ; and take

$$D_{rs} = u_r{}'x_s - 1 \, .$$

Then the absence of any such cycles is equivalent to the absence of any simple cycles, that is without repeated elements, and this is equivalent to the condition

$$D_{rs} \leq 0 \, , \; D_{st} \leq 0 \, , \; \ldots, \; D_{qr} \leq 0 \, ,$$

impossible for distinct $r, s, t, \ldots, q$ .

Were the configuration $\xi$ a complete expenditure configuration derived from an expenditure system, this condition would be the same as Houthakker's Strong Axiom of Revealed Preference. But $\xi$ is a finite configuration, and it is not obvious, though it is plausible, and in fact true, that if the configuration satisfies the acyclicity condition, then it can be embedded in a complete configuration which still satisfies this condition.

It will appear that the acyclicity property is completely extensible, in that any finite configuration which has it can be embedded in a complete configuration which also has it. This embedding can, however, be done in many ways, and to each way there corresponds an admissible preference hypothesis. But a special construction will be made which provides, among the many possibilities, an hypothesis of the normal form.

## 3. The acyclicity condition

The acyclicity condition will now be investigated, first by establishing for it two equivalent forms. The following theorem will be proved.

THEOREM I. The following three conditions on any number $D_{rs}$ $(r \neq s ; r, s = 1, \ldots, k)$ are equivalent:

(I) $D_{rs} \leq 0$, $D_{st} \leq 0$, $\ldots$, $D_{qr} \leq 0$ is impossible for distinct $r, s, t, \ldots, q$.

(II) Numbers $\lambda_r > 0$ exist such that

$$\lambda_r D_{rs} + \lambda_s D_{st} + \ldots + \lambda_q D_{qr} > 0$$

for all distinct $r, s, t, \ldots, q$.

(III) Numbers $\lambda_r > 0$ and $\varphi_r$ exist such that

$$\lambda_r D_{rs} > \varphi_s - \varphi_r \quad (r \neq s) .$$

It is obvious that (III) $\Longrightarrow$ (II) $\Longrightarrow$ (I). For if

$$\lambda_r D_{rs} > \varphi_s - \varphi_r , \lambda_s D_{st} > \varphi_t - \varphi_s, \ldots, \lambda_q D_{qr} > \varphi_r - \varphi_q ,$$

then, by addition,

$$\lambda_r D_{rs} + \lambda_s D_{st} + \ldots + \lambda_q D_{qr} > \varphi_s - \varphi_r + \varphi_t - \varphi_s + \ldots + \varphi_r - \varphi_q = 0 .$$

Hence (III) $\Longrightarrow$ (II). Now assume, contrary to (I), that

$$D_{rs} \leq 0, D_{st} \leq 0, \ldots, D_{qr} \leq 0 ,$$

for some distinct $r, s, t, \ldots, q$ . If $\lambda_r, \lambda_s, \lambda_t, \ldots, \lambda_q > 0$ , then

$$\lambda_r D_{rs} + \lambda_s D_{st} + \ldots + \lambda_q D_{qr} \leq 0 .$$

Hence, the contrary of (I) implies the contrary of (II), and therefore (II) implies (I).

It will now be proved that (I) $\Longrightarrow$ (II). Condition (I) is equivalent to the existence of a complete order $R$ of the number $1, \ldots, k$ such that

$$D_{rs} \leq 0 \Longrightarrow rRs .$$

Thus, if $D$ is the relation defined by $rDs \equiv D_{rs} \leq 0$ , $r \neq s$ , then this is the condition for its transitive closure $\vec{D}$ to be irreflexive, and thus an order, and $R$ can be any complete order which refines $\vec{D}$ . Without loss in generality, it can be assumed, or by a permutation the elements can be so re-numbered, that $R$ is the natural order, so that

$$D_{rs} \leq 0 \Longrightarrow r < s .$$

Then also

$$r < s \Longrightarrow D_{sr} > 0 .$$

Assume, as an inductive hypothesis, that, at an $(m-1)^{th}$ stage, numbers $\lambda_r > 0$ $(r \leq m-1)$ can be found such that

$$\mu_r D_{rs} + \lambda_s D_{st} + \ldots + \lambda_q D_{qr} > 0 \quad (r, \ldots, q \leq m-1) .$$

Now

$$D_{mr} > 0 \quad (r \leq m-1) .$$

Therefore it is possible to define

$$\mu_m = - \min_{r,\ldots,q<m} \frac{\lambda_r D_{rs} + \lambda_s D_{st} + \ldots + \lambda_q D_{qm}}{D_{mr}} .$$

Take any $\lambda_m > \max(0, \mu_m)$ . Then the $m^{th}$ stage is attained from the $(m-1)^{th}$ . The second stage is obviously attainable. For, with any $\lambda_1 > 0$ , there only has to be taken a $\lambda_2 > 0$ such that $\lambda_1 D_{12} + \lambda_2 D_{21} > 0$ , which is possible, since $D_{21} > 0$ . It follows by induction that the $k^{th}$ stage is attainable. That is, the numbers $\lambda_r > 0$ $(r \leq k)$ can be found as required.

To prove (II) $\Longrightarrow$ (III), let $a_{rs} = \lambda_r D_{rs}$ . Then, given

$$a_{rs} + a_{st} + \dots + a_{qr} > 0 ,$$

for all distinct $r, s, t, \dots, q$ , it will be shown that there exist numbers $\varphi_r$ such that

$$\text{(a):} \quad a_{rs} > \varphi_r - \varphi_s \quad (r \neq s) .$$

Let

$$a_{r\ell m \dots ps} = a_{r\ell} + a_{\ell m} + \dots + a_{ps} ,$$

and let

$$A_{rs} = \min_{\ell, m, \dots, p} a_{r\ell m \dots ps} .$$

Then

$$a_{rs} \geq A_{rs} .$$

Also,

$$A_{rs} + A_{sr} > 0 \quad \text{and} \quad A_{rs} + A_{st} \geq A_{rt} .$$

Consider the system

$$\text{(A):} \quad A_{rs} > \varphi_r - \varphi_s \quad (r \neq s) .$$

Any solution $\varphi_r$ of (A) is a solution of (a) , since $a_{rs} \geq A_{rs}$ . Also, any solution $\varphi_r$ of (a) is a solution of (A) . For

$$a_{r\ell} > \varphi_r - \varphi_\ell \ , \ a_{\ell m} > \varphi_\ell - \varphi_m, \ \dots, \ a_{ps} > \varphi_p - \varphi_s ;$$

whence, by addition,

$$\begin{aligned} a_{r\ell m \dots ps} &> \varphi_r - \varphi_\ell + \varphi_\ell - \varphi_m + \dots + \varphi_p - \varphi_s \\ &= \varphi_r - \varphi_s , \end{aligned}$$

and therefore

$$A_{rs} > \varphi_r - \varphi_s .$$

It follows that the consistency of (a), which has to be shown, is equivalent to that of (A), which will be shown now.

The proof depends on an extension property of solution of the subsystems of (A). Thus, assume a solution $\varphi_r$ $(r < m)$ has been found for the subsystem

$$(A, m-1): \quad A_{rs} > \varphi_r - \varphi_s \quad (r \neq s \; ; \; r, s < m) \; .$$

It will be shown that it can be extended by an element $\varphi_m$ to a solution of $(A, m)$ .

Thus, there is to be found a number $\varphi_m$ such that

$$A_{rm} > \varphi_r - \varphi_m \; , \; a_{ms} > \varphi_m - \varphi_s \quad (r, s < m) \; ,$$

that is,

$$A_{ms} + \varphi_s > \varphi_m > \varphi_r - A_{rm} \; .$$

So the condition that such a $\varphi_m$ can be found is

$$A_{mq} + \varphi_q > \varphi_p - A_{pm} \; ,$$

where

$$\varphi_p - A_{pm} = \max_r \{\varphi_r - A_{rm}\} \; , \; A_{mq} + \varphi_q = \min_r \{A_{mq} + \varphi_q\} \; .$$

But if $p = q$ , this is equivalent to

$$A_{mq} + A_{qm} > 0 \; ,$$

which is verified; and if $p \neq q$ , it is equivalent to

$$A_{pm} + A_{mq} > \varphi_p - \varphi_q \; ,$$

which is verified, since, by hypothesis,

$$A_{pm} + A_{mq} \geq A_{pq} \; , \; A_{pq} > \varphi_p - \varphi_q \; .$$

Since the system (A, 2) trivially has a solution, it follows by induction that the system (A) = (A, k) has a solution, and is thus consistent.

<u>THEOREM II.</u> <u>The acyclicity of a finite expenditure configuration is necessary and sufficient for the admissibility of both the normal and the general preference hypothesis.</u>

Let $\mathcal{E} = \{E_r\}_{r=1,\ldots,k}$ be a finite expenditure configuration, with figures $E_r = (u_r \mid x_r)$ ; which is acyclic, so that, with $D_{rs} = u_r' x_s - 1$ ,

$$D_{rs} \leq 0 \ , \ D_{st} \leq 0 \ , \ \ldots, \ D_{qr} \leq 0$$

is impossible for distinct $r, s, t, \ldots, q$ . Then, by Theorem I, there exist numbers $\lambda_r$ and $\varphi_r$ such that

$$(1) \qquad \lambda_r > 0 \ , \ \lambda_r D_{rs} > \varphi_s - \varphi_r \quad (r \neq s \ ; \ r, \ s = 1, \ldots, k) \ .$$

Let $g_r = u_r \lambda_r$ . Then (1) is equivalent to

$$(2) \qquad g_r > 0 \ , \ (x_s - x_r)' \, g_r > \varphi_s - \varphi_r \ .$$

Consider the function

$$\varphi(x) = \min_{r=1,\ldots,k} \{\varphi_r + (x - x_r)' \, g_r\} \ .$$

It is a concave function, strictly increasing, since $g_r > 0$ . Hence it is a gauge for a normal preference scale. Also, since $\varphi_s < \varphi_r + (x_s - x_r)' \, g_r$ ,

$$\varphi(x_s) = \min_{r=1,\ldots,k} \{\varphi_r + (x_s - x_r)' \, g_r\} = \varphi_s \ .$$

If

$$C_r = \{x \mid \varphi(x) = \varphi_r + (x - x_r)' \, g_r\} \ ,$$

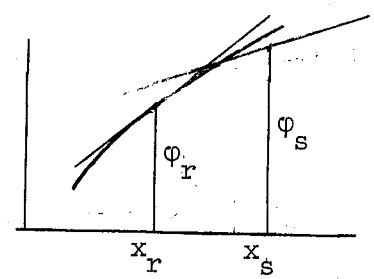
these are cells in a polyhedral dissection of $C$ , with $x_r$ in the interior of $C_r$ , and in the exterior of every $C_s$ $(s \neq r)$ . The gradient $g$ of $\varphi$ exists and is constant in the interior of each cell. Thus,

$$g(x) = g_r \quad (x \in C_r) \ .$$

For $\varphi$ to be the gauge for a preference scale $P$ which is an admissible hypothesis for $\mathcal{E}$ , Gossen's Law, that preference and price directions coincide in equilibrium, has to be satisfied. That is, $g(x_r) = u_r\lambda_r$ ; and this is the case, since $g(x_r) = g_r$ and $g_r = u_r\lambda_r$ .

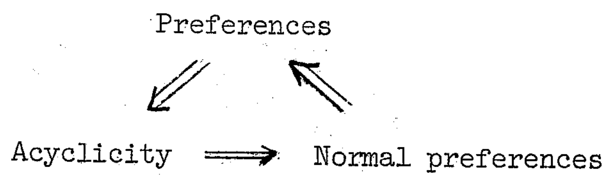
The present function suffices to prove the theorem if a normal hypothesis is understood in a weaker sense, which requires determination of at least one composition, possibly in a larger convex set, which is merely not worse than, instead of strictly better than, every other composition within any given balance $u$ . But to have a function which determines a unique maximum, instead of generally a convex set of maxima, within every balance, a "curving" process has to be applied to $\varphi$ , leaving its gradient undisturbed in direction at the points $x_r$ .

Replace the hyperplane $\varphi = \varphi_r + (x - x_r)' g_r$ in $(x, \varphi)$-space by a sphere touching this hyperplane at $(x_r, \varphi_r)$ , with centre on the side $\varphi < \varphi_r + (x - x_r)' g_r$ of the hyperplane, and with a suitably large radius $R_r$ . Since all the points $(x_s, \varphi_s)$ $(s \neq r)$ , lie on this side of the hyperplane, because $\varphi_s < \varphi_r + (x_s - x_r)' g_r$ , they will lie within this sphere if $R_r$ is sufficiently large. Again, given any compact region in $C_o$ , part of this sphere will be the graph of a strictly concave increasing function defined in $C_o$ , provided $R_r$ is sufficiently large, so as to make the sphere lie sufficiently close to the hyperplane in this region. Therefore, given $C_o$ , let all radii $R_r$ $(r = 1,\ldots,k)$ be chosen sufficiently large to have these properties.

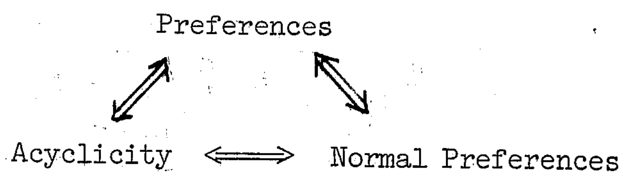



Now the graph of the function $\varphi$ is the polyhedron which is the boundary of the region defined by $\varphi \leq \varphi_r + (x - x_r)' g_r$ $(r = 1,\ldots,k)$ , with $k$ faces lying in the hyperplane $\varphi = \varphi_r + (x - x_r)' g_r$ , with the points $(x_r, \varphi_r)$ in the interior of each. The operation of replacing the hyperplanes by the spheres, of sufficiently large radius, just curves these faces, and leaves a function which, in $C_o$ , is strictly increasing, and now strictly convex, and preserves the condition $\varphi(x_r) = \varphi_r$ , $g(x_r) = g_r$ .

The admissibility of the general preference hypothesis has already been seen to imply acyclicity. Now acyclicity is proved sufficient for the admissibility of the normal hypothesis, which is a special form of the general hypothesis. Hence there is the scheme




from which follows




4. Revealed preference

The acyclicity condition on an expenditure configuration $\mathcal{E}$ is equivalent to the condition that $\vec{P}_{\mathcal{E}}$ be an order. If $\mathcal{E}$ is finite, it is also equivalent to the condition that the class $\mathcal{P}_{\mathcal{E}}$ of admissible preference hypotheses be non-empty. The principle of revealed preference formulated by Samuelson has the interpretation that if a preference system $P$ is admissible, then it is revealed to the extent of its containing $P_{\mathcal{E}}$ ; and this is equivalent to its being a refinement of the order $\vec{P}_{\mathcal{E}}$ ; that is, $P_{\mathcal{E}} \subset P$ , and equivalently $\vec{P}_{\mathcal{E}} \subset P$ . But the existence of such a $P$ , as assumed by Samuelson in formulating his principle of revealed preference, is the condition for revealed preference. If no such $P$ exists, then the primary and derived relations $P_{\mathcal{E}}$ and $\vec{P}_{\mathcal{E}}$ cannot be interpreted as relations of revealed preference, since the preference hypothesis is rejected.

If $\mathcal{E}$ is complete and acyclic, it is trivial that $\mathcal{P}_{\mathcal{E}}$ is non-empty, and has $\vec{P}_{\mathcal{E}}$ as its unique member. The work of Houthakker and Uzawa just shows that if certain further conditions are assumed, then $\vec{P}_{\mathcal{E}}$ has a certain special form. They use methods of the calculus and have results which are inapplicable to a finite configuration, for which different questions have to be asked, and for which the methods have to be algebraical.

Houthakker's Strong Axiom of Revealed Preference, which here is the acyclicity condition, is just as applicable to a finite configuration as a complete one; and in this application it has been shown to have two equivalent forms, expressed by the consistency of certain systems of linear inequalities. By this means it has been shown to imply the existence of an admissible preference structure. Thus, even for a finite configuration, acyclicity is still the condition for revealed preference; that is, the existence of some preference system $P$ which can be taken as revealed by, to the extent of its being a refinement of, the order $\vec{P}_{\mathcal{E}}$.

This investigation for finite configurations can be continued in an analysis of indeterminacies which arise through there being, if any, an infinite number of members in $\mathcal{P}_{\mathcal{E}}$. The analysis can then be applied to questions such as the classical one on the indeterminacy of a cost-of-living index based on finite data.[1]

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[1] P. A. Samuelson. Foundations of Economic Analysis (Harvard, 1947).