

DISTRIBUTED LAGS AND THE EFFECTIVENESS OF MONETARY POLICY

E. Philip Howrey

Econometric Research Program
Research Paper No. 21
May 1968

The research described in this paper was supported by the National Science Foundation (GS 1840) and the Office of Naval Research (N00014-67 A-0151-0007, Task No. 047-086).

Reproduction in whole or in part is permitted for any purpose of the United States Government.

Princeton University
Econometric Research Program
207 Dickinson Hall
Princeton, New Jersey

DISTRIBUTED LAGS AND THE EFFECTIVENESS OF MONETARY POLICY

E. Philip Howrey

ABSTRACT

As a result of an apparent revival of interest in monetary theory and policy, a considerable amount of effort has been devoted recently to an analysis of the operational lags to which monetary policy is subject. Underlying this interest in the time-form of the response of income to changes in monetary policy is the notion that the (potential) effectiveness of monetary policy is a function of the speed with which income responds to changes in the money supply. It seems to be accepted, more in general, perhaps, than in any particular case, that if aggregate demand responds with a long distributed lag to changes in the money supply, the scope for contracyclical monetary management may be quite limited. This paper is devoted to a formal analysis of the relationship between the speed of adjustment of income to changes in the money supply and the effectiveness of contracyclical monetary policy.

Within the context of a stochastic model of income determination in which aggregate product demand and the demand for money exhibit distributed lag responses to changes in income and the interest rate, it is shown that it is not true in general that monetary policy is likely to be more effective the more rapidly income responds to changes in the money supply. Indeed, it is entirely possible for a well-conceived stabilization scheme

to effect a greater reduction in the variance of income about some target level if income responds slowly to money-supply changes than if it responds quickly. It is also shown that the sensitivity of the system to miscalculations on the part of the stabilization authority is increased the more quickly income responds to changes in the money supply. Thus a rapid response of income to changes in monetary policy is not a necessary condition for stabilization policy to be effective.

DISTRIBUTED LAGS AND THE EFFECTIVENESS OF MONETARY POLICY

E. Philip Howrey*

In a fairly recent paper [5] Professor D. P. Tucker has ably demonstrated that the responsiveness of aggregate demand to monetary policy depends not only on the time-form of the response of investment expenditure to changes in the interest rate but also on the response of the demand for money to interest-rate changes. In particular, it was shown that a long distributed lag in investment response does not preclude the possibility that aggregate demand responds quickly to changes in the money supply. This argument was put forward to counter the prevalent notion that if investment expenditure responds only slowly to changes in the interest rate, the scope for contracyclical monetary management may be quite limited. While Tucker did show that a long distributed product-demand lag is consistent with a rapid response of income to changes in monetary policy, the implication of this finding for the effectiveness of monetary policy was not explored.

The purpose of this paper is to consider rather formally the relationship between the speed of adjustment of income to changes in the money supply and the effectiveness or potential effectiveness of contracyclical management of the money supply. This may seem like a minor point

* This research was performed in the Econometric Research Program at Princeton University with support from the National Science Foundation (GS 1840) and the Office of Naval Research (N00014-67 A-0151-0007, Task No. 047-086). While retaining full responsibility for any errors, the author would like to thank W. J. Baumol, R. E. Quandt, L. G. Telser, and D. P. Tucker for their helpful comments on an earlier draft of this paper. Reproduction in whole or in part is permitted for any purpose of the United States Government.

for it might appear to be intuitively obvious that the more quickly income responds to variations in monetary policy, the more effective is contracyclical monetary management likely to be, provided, of course, the stabilization policy is designed properly. But it has become increasingly apparent to students of economic dynamics that intuition can be very unreliable when it comes to the analysis of dynamic models. For this reason alone it seems desirable to subject this proposition to formal analysis. Moreover, as will be demonstrated by the analysis that follows, the proposition that monetary policy is more effective the more quickly income responds to changes in monetary policy is not true in general.

The relationship between the speed and effectiveness of monetary policy is developed here within the context of a simple stochastic model of income determination. The basic properties of this model are described briefly in Section I. The stabilization problem is explicitly formulated in Section II. It is assumed that the money supply is varied in such a way that optimal feedback control is applied to the system in order to mitigate as quickly as possible the effects of random disturbances. The residual variance, that is, the variance of income about some target level that remains even when the money supply is varied to minimize instability, is derived and expressed as a function of the lag parameters of the system. An examination of this expression reveals that it is not true in general that the residual variance is smaller the more quickly income responds to changes in the money supply. Indeed, within certain limits, just the reverse is true: the residual variance is larger the more quickly income responds to changes in the money supply.

In Section III, the way in which the speed of income response affects the relative reduction in variance that can be achieved by a well-conceived stabilization scheme is determined. In addition, in order to explore the possibility that monetary policy might actually be destabilizing, the sensitivity of the system to miscalculations in contracyclical management of the money supply is examined. The major conclusions about the relationship between the effectiveness and the speed of monetary policy are summarized in the final section of the paper.

I. A Stochastic Model of Income Determination

In order to explore the relationship between the speed with which income responds to money-supply changes and the (potential) effectiveness of monetary policy, it is necessary to introduce a dynamic model of income determination. The model that will be analyzed here is Tucker's expositional model in which distributed lags are included in the behavioral relationships. However, rather than retain the deterministic system as it was originally advanced, random disturbances are explicitly introduced into the behavioral equations. The resulting stochastic system is somewhat more realistic [2] and provides a more suitable framework within which to discuss stabilization policy [4] than its deterministic counterpart.

The equations of the model are as follows [5, p. 435]:

$$(1) \quad I_t = (1-j) (a_1 + b_1 Y_t + dr_t) + j I_{t-1} + u_{1t}$$

$$(2) \quad C_t = (1-j) (a_2 + b_2 Y_t) + j C_{t-1} + u_{2t}$$

$$(3) \quad Y_t = C_t + I_t$$

$$(4) \quad M_t^* = (1-m) (e + f Y_t + g r_t) + m M_{t-1} + v_t$$

$$(5) \quad M_t = M_t^*$$

where

C = consumption expenditure;

I = investment expenditure;

Y = income;

M^* = desired money balances;

M = money supply;

r = interest rate;

t = time subscript;

j and m are lag parameters with $0 < j, m < 1$;

$u_1, u_2,$ and v are random disturbances.

Since price and wage changes are ignored, this model is concerned with the impact of monetary policy on income and employment and thus applies to a situation in which income can expand without encountering a labor-force constraint. Both product demand ($C + I$) and the demand for money (M^*) are assumed to be stochastic rather than purely deterministic functions and each of these demands responds with a distributed lag to changes in the interest rate. The equilibrium conditions (3) and (5) require income and the interest rate to adjust each period so that the stochastically given product and money demands are satisfied.

The dynamic characteristics of this system are determined by the simple distributed lags that are included in the demand equations (1), (2), and (4) of the model. The lag parameters j and m have been introduced in such a way that the static and comparative static properties of the system are not affected by changes in these parameters. For example, if the random variable u_{1t} is suppressed for the moment and investment

expenditure is set equal to its lagged value, it is clear from (1) that the steady-state value of I does not depend on the lag parameter j . Although the equilibrium response of investment to a change in the interest rate does not depend on the lag parameter j , the speed with which investment approaches its equilibrium value does depend on this parameter. The mean lag implied by the relationship between investment and the interest rate is $j/(1-j)$.¹ This expression for the mean lag indicates that the response of investment to a change in the interest rate is quite rapid if j is close to zero and is much slower if j is close to one.

¹The partial relationship between investment and the interest rate implicit in equation (1) can be written as

$$(A) \quad (1-jL) I_t = (1-j) d r_t$$

or

$$(B) \quad I_t = (1-j) d (1+jL + j^2L^2 + \dots) r_t$$

where L , the lag operator, is defined by $L^k y_t = y_{t-k}$. It is clear from (A) that the equilibrium change in investment corresponding to a unit change in the interest rate is d . From (B) it follows that $(1-j)$ of this change takes place in the same period that the interest rate is changed, $(1-j)j$ takes place in one period after the interest-rate change and, in general, $(1-j)j^n$ of the change takes place in the n^{th} period after the change in the interest rate. The mean lag of the distribution is therefore

$$(C) \quad \bar{\theta} = (1-j) \sum_{n=0}^{\infty} n j^n = j/(1-j).$$

In terms of the generating function of the lag distribution, $W(\lambda) = (1-j)d/(1-j\lambda)$, the mean lag is given by

$$(D) \quad \bar{\theta} = W'(1)/W(1)$$

where $W'(1)$ is the derivative of the generating function evaluated at $\lambda = 1$.

The final form of the equation for income can be obtained from the system of equations (1) - (5) by solving for Y in terms of M and the disturbance terms:

$$(6) \quad Y_t - AY_{t-1} = B + D(M_t - m M_{t-1}) + \epsilon_t$$

where

$$(7) \quad A = \frac{j}{1 - (1-j)(b_1 + b_2 - df/g)}$$

$$(8) \quad B = (1-j)(a_1 + a_2 - d e/g) A/j$$

$$(9) \quad D = \frac{(1-j) d A}{(1-m) g j}$$

$$(10) \quad \epsilon_t = \frac{A}{j} u_t - D v_t$$

$$(11) \quad u_t = u_{1t} + u_{2t}$$

It is clear from (6) that the time path of Y will depend on the path of M as well as the values assumed by the random variables u in the product market and v in the money market. In what follows it will be assumed that these random variables are mutually and serially independent with zero means and constant variances denoted by σ_u^2 and σ_v^2 .

Within the context of this model it is now possible to indicate quite precisely how the speed with which income responds to changes in the money supply depends on the lag parameters j and m. Suppose that M_t exhibits a unit increase at $t = 1$. The expected income response to this change in the money supply is

$$(12) \quad Y_t^i = D(1-m)/(1-A) + A^{t-1}(m-A)/(1-A) \quad t=1,2,\dots$$

This is the path that income would follow if there were no disturbances in the system, that is, if $u_t = v_t = 0$ for all t . Now as Tucker has pointed out, the second term on the right-hand side of (12) depends on the difference $m-A$. If m is equal to A , the response of income to a change in the money supply is complete in the same period that the change in the money supply takes place. And this is true despite the fact that the product-demand lag may be quite long (i.e., j and therefore A may be close to unity²) provided the money-demand lag is correspondingly long. This constitutes Tucker's disproof of the conjecture that a long product-demand lag implies that income responds slowly to changes in the money supply.³

II. Income Response and the Effectiveness of Monetary Policy

We now come to the crucial question of this paper: Is there any reason to believe that stabilization policy is likely to be more effective

²From the definition of A in (7) it is easy to verify that $A = 0$ when $j = 0$ and $A = 1$ when $j = 1$. Moreover, $\text{sign} [\partial A / \partial j] = \text{sign} [1 - b_1 - b_2 + d f / g]$ for all j in the closed interval $[0, 1]$. Therefore $\partial A / \partial j > 0$ and A can be taken as a proxy for j . This justifies the designation "generalized product-demand lag parameter" for the parameter A .

³This result can also be obtained by calculating the mean lag of the relationship between income and the money supply given in (6). It is not difficult to verify that

$$\bar{\theta} = W'(1)/W(1) = A/(1-A) - m/(1-m)$$

where $W(\lambda) = D(1-m\lambda)/(1-\lambda)$ is the generating function of the lag relationship between Y and M . This expression indicates that in this model the mean lag between income and the money supply is the difference between the mean product-demand lag $A/(1-A)$ and the mean money-demand lag $m/(1-m)$.

the more quickly income responds to changes in the money supply? In terms of the model expressed in equations (1) - (5), it would be desirable to know if a small difference between the generalized product-demand lag (A) and the money-demand lag (m) enhances the (potential) effectiveness of contracyclical monetary policy. One way to answer this question is to modify the basic system to include a plausible contracyclical money supply equation and then to compare the resulting time paths of income for different pairs of values of the lag parameters j and m . It is clear that the time path of income and hence the effectiveness of stabilization policy in general depends upon how well the specific policy that is followed is designed as well as the inherent limitations imposed by the economic system. In order to separate these two aspects of the problem as completely as possible, it is essential to include a control equation that is not determined on an ad hoc basis, but rather is designed to effect optimal control.

Within the context of linear stochastic systems, it seems plausible to consider linear stabilization policy that is designed to minimize the variance of income. Returning to equation (6), the final form of the income equation, it is necessary to determine a control loop on M_t which minimizes the variance of income.⁴ In order to derive a minimum variance control equation, we note that the expected value of income in period t ,

⁴Here we abstract from the fact that it is not possible to control the supply of money directly. Since the monetary authority has only indirect control over the amount of money in existence, it might be reasonable to add a disturbance term to this equation to reflect the fact that the desired change in the money supply is imperfectly achieved. This will be considered in some detail in Section III.

\bar{Y}_t , is determined by

$$(13) \quad \bar{Y}_t = A\bar{Y}_{t-1} + B + D(\bar{M}_t - m\bar{M}_{t-1})$$

where \bar{M}_t is the expected value of M_t . This equation can be used to determine the "mean control" to be applied to the system. If, for example, the mean-square of the deviations of income from its full-employment level Y_t^f is to be minimized, unbiased control requires that \bar{M}_t be set so that $\bar{Y}_t = Y_t^f$. Inserting Y_t^f for the mean value of income in (13), the mean value of the money supply is found to be

$$(14) \quad \bar{M}_t = m\bar{M}_{t-1} + (Y_t^f - A Y_{t-1}^f - B)/D.$$

If the monetary authority varies the money supply according to this equation, the full-employment gap will be zero on average. It should be noted that this mean-correction rule assumes that the full-employment level of income is known in advance so that the money supply of the current period can be adjusted to a level appropriate for the current full-employment level of income.

Now the deviations of income from its mean value, $y_t = Y_t - \bar{Y}_t$, are given by

$$(15) \quad y_t - A y_{t-1} = D(N_t - m N_{t-1}) + \epsilon_t$$

where $N_t = M_t - \bar{M}_t$ is the deviation at time t of the money supply from its mean value. In order for the variance of y_t to assume a

minimum, it is necessary for the money supply to deviate from its mean value in period t by an amount⁵

$$(16) \quad N_t = m N_{t-1} - A y_{t-1}/D .$$

This control equation assumes that the most recent observations that are available on the variables in the system are those of last period and

⁵This result is obtained in the following way. First, write y_t in terms of a moving average of current and past values of N and ϵ :

$$y_t = \sum_{k=0}^{\infty} A^k [H_{t-k} + \epsilon_{t-k}]$$

where $H_t = D(N_t - m N_{t-1})$. The variance of y_t can now be expressed as

$$\begin{aligned} E[(y_t)^2] &= E[(\epsilon_t + \{ \sum_{k=0}^{\infty} A^k H_{t-k} + \sum_{k=1}^{\infty} A^k \epsilon_{t-k} \})^2] \\ &= E[(\epsilon_t)^2] + 2 E[\epsilon_t \{ \sum_{k=0}^{\infty} A^k H_{t-k} + \sum_{k=1}^{\infty} A^k \epsilon_{t-k} \}] \\ &\quad + E[\{ \sum_{k=0}^{\infty} A^k H_{t-k} + \sum_{k=1}^{\infty} A^k \epsilon_{t-k} \}^2] . \end{aligned}$$

Since M_t and hence H_t is to depend linearly on y_{t-1}, y_{t-2}, \dots , H_t will depend implicitly on $\epsilon_{t-1}, \epsilon_{t-2}, \dots$. This means that the expression in braces {...} is a linear function of $\epsilon_{t-1}, \epsilon_{t-2}, \dots$ and since the ϵ_t are assumed to be serially uncorrelated, the middle term in this expanded expression for the variance of y_t is zero. This leaves the first term which cannot be influenced by stabilization policy and the last term which is clearly minimized by setting it equal to zero. This yields

$$H_t = \sum_{k=1}^{\infty} A^k (H_{t-k} + \epsilon_{t-k}) = -A y_{t-1}$$

which in turn implies that N_t should be varied according to equation (16).

hence incorporates a one-period "inside" lag. Combining this equation with (14), the control equation can be written as

$$(17) \quad M_t = m M_{t-1} + (Y_t^f - A Y_{t-1} - B)/D .$$

With this control equation, the equation for income reduces to

$$(18) \quad Y_t = Y_t^f + \epsilon_t$$

so that actual income differs from full-employment income by an amount that depends on the unpredictable disturbances that occur in the current period.

From this last expression, it is clear that the variance of income with optimum control is simply the variance of the composite disturbance term ϵ_t defined in (10). The variance of ϵ_t provides one measure of the potential effectiveness of stabilization policy; the smaller the variance of ϵ_t , the more effective is the stabilization policy. In terms of the parameters of the model specified in (1)-(5), the variance of ϵ_t is given by

$$(19) \quad \sigma_{\epsilon}^2 = (A/j)^2 \sigma_u^2 + D^2 \sigma_v^2$$

where σ_u^2 (σ_v^2) is the variance of the disturbance term in the product-demand (money-demand) equation. By differentiating this variance expression with respect to the lag parameters j and m , it is possible to determine the relationship between the effectiveness of monetary policy and the speed with which income adjusts to changes in the money supply. It is easy to verify that

$$(20) \quad \partial \sigma_{\epsilon}^2 / \partial m = 2 D^2 \sigma_v^2 / (1-m)$$

which is clearly positive. This shows that the residual variance, that is, the variance of income which remains with the most timely stabilization policy, increases as the money-demand lag increases.

This is sufficient to demonstrate that it is not true in general that monetary policy is more effective the more rapidly income responds to changes in the money supply. In order to see this, consider two economies which are characterized by the system of equations (1)-(5). Suppose that the only difference between the two economies is that money demand responds more quickly to changes in the interest rate in one economy than in the other, i.e., $m_1 < m_2$. If A , the generalized product-demand lag parameter, is greater than m_2 , it follows from equation (12) that in economy two income responds more quickly to changes in the money supply than it does in economy one.⁶ However, we have just seen that the residual variance of income is larger the larger is the lag parameter m so that with optimal monetary control the variance of income is larger in economy two than in economy one. Therefore, monetary policy is more effective in economy one than in economy two even though income responds more slowly to changes in the money supply in economy one. This shows that a slow income response can actually increase the effectiveness of monetary policy.

⁶In terms of the mean lag defined in footnote 3, the average lags in the effect of monetary policy for the two economies are

$$\begin{aligned} \theta_1 &= A/(1-A) - m_1/(1-m_1) \\ \text{and} \\ \theta_2 &= A/(1-A) - m_2/(1-m_2) . \end{aligned}$$

With $m_1 < m_2 < A$, it follows that $\theta_1 > \theta_2 > 0$. For example, if $A = .9$, $m_1 = .5$, and $m_2 = .8$, the corresponding average lags are $\theta_1 = 8$ and $\theta_2 = 5$ so that monetary policy works more quickly in the mean-lag sense in economy two than in economy one.

III. The Potential Effectiveness of Monetary Policy

There is another way to measure the potential effectiveness of stabilization policy which also has certain interesting implications. We might ask by how much the variance of income can be reduced by activating a well-conceived stabilization scheme. If a large reduction in variance can be effected by the stabilization policy, then the potential effectiveness of monetary management is greater than if only a small reduction in variance can be achieved. The reduction in variance as a function of the lag parameters in the model can then be analyzed to determine whether the potential effectiveness of stabilization policy is increased the more quickly aggregate demand responds to changes in the money supply.

If we suppose that monetary policy is used to apply only a mean correction to the system, the final form of the income equation is obtained by substituting the expression for \bar{M}_t given in (14) for M_t in equation (6). When this is done the deviations of income from the full-employment level are generated by

$$(21) \quad y_t = A y_{t-1} + \epsilon_t$$

where ϵ_t is given by equation (10). The variance of y_t with mean correction only is

$$(22) \quad \sigma_y^2 = \sigma_\epsilon^2 / (1 - A^2)$$

With optimal control the variance is σ_ϵ^2 so the relative reduction in variance is

$$(23) \quad R = \frac{\sigma_y^2 - \sigma_\epsilon^2}{\sigma_\epsilon^2} = A^2$$

Using this measure we find that the potential effectiveness of stabilization policy is independent of the money-demand lag. As already noted (footnote 2) A lies between zero and one and varies directly with the product-demand lag j so that the potential effectiveness of contracyclical monetary policy in this model is greater the longer is the product-demand lag. From this we can conclude that for this model the potential effectiveness of contracyclical monetary policy is independent of the speed with which income responds to changes in the money supply.

Up to this point it has been assumed that the monetary authority exercises its discretionary power in such a way as to minimize the variance of deviations of income from the full-employment level of output. There has been no question of what level of income corresponds to full employment nor is there any question about the structure and parameter values of the system in which the authority operates. Since these are all rather heroic assumptions it might be interesting to see how much scope there is for error in this system. Suppose that the control equation is now modified to include a random disturbance w_t so that the monetary authority makes the correct move only on average. In this case equation (17) is modified to read

$$(24) \quad M_t = mM_{t-1} + (Y_y^f - A Y_{t-1} - B)/D + w_t .$$

Combining this with the income equation (6), income is now determined by

$$(25) \quad Y_t = Y_t^f + \epsilon_t + D w_t .$$

The residual variance of income with disturbed or "almost-optimal" control is now $\sigma_\epsilon^2 + 2D\sigma_{\epsilon w} + D^2\sigma_w^2$. Unless w and ϵ exhibit a large negative covariance, the variance of income will be larger than it would have been

with perfect (undisturbed) control. One might suppose that a reasonable assumption is that w and e are uncorrelated in which case the residual variance is unambiguously increased as expected.⁷

With disturbed control it is possible for the variance of income to be even larger than it would have been with mean control alone. On the assumption that w and e are independent, the stabilization policy in (24) will actually be destabilizing if

$$(26) \quad \sigma_e^2 + D^2 \sigma_w^2 > \sigma_e^2 / (1-A^2)$$

or, solving for σ_w^2 and substituting for σ_e^2 from (19),

$$(26') \quad \sigma_w^2 > \frac{A^2}{1-A^2} \left[\frac{(1-m)^2 g^2}{(1-j)^2 d^2} \sigma_u^2 + \sigma_v^2 \right] .$$

Conversely, the variance of the random disturbances in the control equation must be smaller than the critical value on the right-hand side of the inequality in (26') in order for the stabilization policy to reduce the variance of income below what it would be with mean control only. It is clear that this critical value is a decreasing function of the lag parameter m . Therefore in order to avoid increasing the variance of income, σ_w^2 must be smaller the larger is the money-demand lag parameter.

⁷It should be emphasized that we are here comparing the variance of income with optimal control with the variance of income with "almost-optimal" control. Friedman [3, p. 123] gives a formula similar to the one shown above but his formula compares the variance of income with control with the variance of income without control. In the model used here, the variance of income without contracyclical stabilization (i.e., mean control only) is $\sigma_y^2 = \sigma_e^2 / (1-A^2)$. The variance with almost-optimal control is $\sigma_e^2 + 2D\sigma_{ew} + D^2\sigma_w^2$ and the variance of income with optimal control is σ_e^2 . Clearly $\sigma_e^2 < \sigma_y^2$ so it is impossible for optimal control to be destabilizing. However, as y shown below, almost-optimal control can be destabilizing.

Returning to the two-economy example considered above, suppose that the two economies are identical except that again $m_1 < m_2 < A$ so that income responds more quickly to changes in the money supply in economy two than in economy one. According to (26') there will be less scope for error in contracyclical stabilization policy in economy two than in economy one. Thus in spite of the fact that income responds quickly, the monetary authority in economy two must use a more finely tuned contracyclical policy in the high-speed economy than in the low-speed economy if it is to avoid increasing the variance of income.

IV. Concluding Remarks

From this analysis of a linear stochastic model in which investment demand and the demand for money exhibit distributed lag responses to changes in the interest rate, we conclude that it is not true in general that monetary policy is likely to be more effective the more rapidly income responds to changes in the money supply. It is entirely possible for a well-conceived stabilization scheme to effect a greater reduction in the variance of income about some target level if income responds slowly to money-supply changes than if it responds quickly. Thus a rapid response of income to changes in monetary policy is not a necessary condition for stabilization policy to be effective.

Two additional conclusions emerge from the analysis of this model. First, the relative (percentage) reduction in the variance of income that can be achieved by a well-designed stabilization policy does not depend on

the money demand lag at all. Regardless of the length of the money-demand lag, a greater percentage reduction in the variance is possible the longer the mean product-demand lag. Second, the sensitivity of the system to miscalculations in the stabilization policy is increased the more quickly income responds to changes in the money supply. Thus if the monetary authority pursues a contracyclical stabilization scheme that is correct only on average, the possibility that this policy will actually be destabilizing is greater the more quickly income responds to changes in the money supply. It should be emphasized that these last two conclusions are specific to the model analyzed here. They are included to emphasize the basic point that there need not be a simple, straightforward relationship between the effectiveness or potential effectiveness of monetary policy and the length of the distributed operational lag in the effect of monetary policy.

REFERENCES

1. W. J. Baumol, "Pitfalls in Contracyclical Policies: Some Tools and Results," Rev. Econ. Stat., Feb. 1961, 43, 21-26.
2. G. H. Fisher, "Some Comments on Stochastic Macro-Economic Models," American Econ. Rev., Sept. 1952, 42, 528-39.
3. M. Friedman, "The Effects of a Full-Employment Policy on Economic Stability: A Formal Analysis," in Essays in Positive Economics. Chicago 1953.
4. E. P. Howrey, "Stabilization Policy in Linear Stochastic Systems," Rev. Econ. Stat., Aug. 1967, 49, 404-11.
5. D. P. Tucker, "Dynamic Income Adjustment to Money-Supply Changes," American Econ. Rev., June 1966, 56, 433-49.