ON SOME CRITICISMS OF GAME THEORY

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The following speech was made at the end of the "NATO Symposium on
Operations Research" on "Game Theory and its Military Applications",
Toulon (France), June 29--July 3, 1964.

ABSTRACT

Game theory is a mathematical model of certain situations of human
conflict and cooperation. As in all models there are severe simplifi-
cations; if these are found to be undesirable they have to be replaced
by others or additional elements have to be introduced, thereby compli-
cating the model, possibly beyond the degree of complication which we
are able to manage at present. The demand to consider such other ele-
ments is typical of certain critics such as Zuckerman, Blackett etc. but
they have not developed any methods with which to study additional com-
plications. Instead, recourse is had to exhortations, to appeal to
"intuition", "experience", "judgement" etc. which remain unanalyzed and
therefore have little operational meaning. If a rigorous theory is re-
jected as inadequate for analyzing a problem a less rigorous procedure
can lead to better or equivalent results only by accident. There are
many genuine, open problems in game theory, especially there exists the
need to give an existence proof of solutions for arbitrary large n-person
games. The author outlines many other unsolved problems but expresses
confidence that they too will find answers as have those which charac-
terize the development of the theory since 1944.

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Mr. President:

I am very glad to be able to say a word here on this occasion, where so many interesting and important communications have been made on game theory and its applications.

My long association with this theory puts me naturally somewhat on the spot. I am impressed by the enormous amount of work which has been going on over the years. This surely is a sign that game theory is a living thing and I believe it still has a lot of life in it for many years to come. This is partly due to the fact that there are many problems which are not solved. Therefore, it is very important to realize what kind of problems there are and perhaps also to distinguish between those which are real problems connected with the theory and other problems which are not true problems having much to do with the theory but rather are superimposed on the thought which has been developed.

Let us first of all remember very briefly what game theory really is and wants to do as originally conceived. Clearly, it is a mathematical descipline and tries to analyse mathematically--and to develop proper computational and other techniques--conflict situations and situations of cooperation, thinking that such situations are really fundamental in attempting to describe social behavior. I wish it to be understood that game theory offers a model for social situations, for social behavior and therefore some of these problems which we encounter in actual life actually

might fall under this particular theory. The theory can be conceived as both normative and as descriptive. It is normative in the sense that it wishes to indicate what the optimal course of action is in different situations: for example, in the zero-sum situation for two persons how to behave in a manner which will guarantee a certain outcome. These situations may be such that an ordinary approach, any common sense approach, fails. Although naturally, there will be relations to common sense solutions of the very simplest nature. It wishes to be descriptive in so far as it can be conceived to function as a model; for example, to explain what actually happens in a market as far as economic applications are concerned, or perhaps regarding conflict or cooperative situations as they might occur, let us say, in military operations of certain types.

If game theory is to be thought of in the latter sense, then it is really a model, and like all models must be looked upon as such. That is to say a model must fulfill certain requirements: it must be similar to reality, it must be simple enough to be mathematically manageable, it must not be too simple in order to be at least interesting, it must not be too complicated so that one can, with our present means, actually do something with it. If it is not similar then we will reject it at least as a model. Now I think it is important for everybody to realize that there are many situations for which game theory is a good model; but the question arises whether it is a model for every conflict situation or every decision situation which might occur. Of course, it has never been asserted by anybody who seriously writes on game theory that every single decision is a game situation. Certainly a judge does not engage himself in playing a game. He makes a legal decision and that is a very different thing. He has to classify; he has to judge. That is totally different.

Another point which I would like to make and which expresses an experience of human development, is that where mathematics has once entered in a really truly significant way, it has never been driven out again. I think this has happened with game theory and is due to the great guiding spirit of John von Neumann. I think it is a safe prediction--unless we run altogether into a new dark age--that mathematics of the type of game theory will not be driven out from the analysis of social and economic situations, or other situations which might still come under the theory.

Now there are always new questions, but as I said we must see whether the new questions are really the right questions and let us not forget that scientific progress very often begins precisely by asking the right questions and that very often scientific progress is stifled by asking the wrong questions, totally irrelevant questions.

Now in what I shall say, I shall talk about the constructive criticisms which have led to further progress, and about some which not yet having been fully answered and taken care of, yet will lead, I am sure, to further progress. Then I will say something about the--in a sense more immediate--non-constructive objections and criticisms of the theory, and then I shall say something about what I think might be considered as the nature of future work. As for constructive criticisms: in the original work, only finite numbers of strategies were considered. It is natural to ask what happens if you have infinitely many strategies.

It is very good to see that the main theorem goes through to them too, and this can be done in many ways and there is a good literature on this particular subject. We had at least one fine paper here dealing with the latter case. It is however astonishing to see that, as I was

told this morning, the contrary assertion was made-namely that one does

not deal with infinitely many strategies or cannot deal with them. That

just is not so. Another point which was raised was, for example, that

if you have very many players in an n-persons game, you have problems of

communication and possibly there might be a cost of communications and

this might have an influence on the behavior of people. This is an im-

portant, interesting, significant problem, and as such absorbable in

the theory. There is further the problem of going over to a large num-

ber of players; this has been studied in a highly significant way. The

question arises of finding solutions when one goes to the limits. For

example will one then approach a certain stability and simpler sets of

imputations? This would be interesting for economic applications. Al-

ready a long time ago there was the problem of whether the originally

offered solution concept was too narrow or otherwise restrictive, so new

concepts of solutions have been offered -- some of them perhaps of purely

mathematical interest; others perhaps relevant also in their bearing upon

the reality - the subject matter with which one deals.

There is the further question that the theory is unable to deal with

mistakes made by opponents, an issue that was mentioned originally in the

book by von Neumann and myself. The indication was, at least at that time,

that one really would have to have fundamentally new ideas in order to deal

with this situation. But it is a true problem, and possibly somebody has

really a new idea, which so far I have not yet seen; but I may be imper-

fectly informed. Here one could take an important step forward.

There is another point, which we also mentioned in the original publi-

cation, namely that the theory is in a certain sense static -- although it

is not so clear what is meant by the distinction of "static" and "dynamic"

as long as one talks about game theory. To establish an analogy with physical ideas where these concepts were evolved is not necessarily help-ful. In fact any game as we know it, especially if we consider it in its extensive form, is what one would normally consider a rather dynamic affair with much give and take and therefore movement, yet it is adequately treated by a "static" theory. There is the problem raised by the assumption made in the theory that utility is strictly transferable. That is a very severe restriction and it would be nice if one could drop it without upsetting the positive achievements which have been gained. There is the possibility of perhaps going over and nearly clearing up the very troublesome question as to what to do when you do not have a numerical probability instead only have subjective probabilities. As you know, a great deal has been written on this, and so it goes on and I shall not take your time by a further enumera-tion.

Many of these questions have been referred to already in the original publication by von Neumann and myself. We were well aware at that time that not everything could be said at once and that the theory is imperfect; therefore lots of further problems are stated there as an inducement to go on with the work. Our book contains many places where open problems are specifically identified. Not all of them have, as yet, been considered by more recent writers.

Now, another point where I think a very important development has occurred is that experiments have been made. Many concern themselves par-ticularly with a side issue, namely utility theory which is brought in be-cause as we know we can't have game theory unless we have numbers. And how do we get these numbers? And so a proposal was made by us to obtain a numerical utility. Now many experiments have been made, and I think their

value lies in the fact that they are, to my knowledge, the first really strictly controlled experiments in economics. Of course, there are many experiments in sociology and elsewhere, but not from the basis of firm theory. Certainly in this narrow and sharply defined field of utility a precisely, axiomatically formulated system was available and as one would expect, such experiments have immediately uncovered many other problems. Among those is for example, the question of how people react to given certain numbers, whether they like them, how they react subjectively to the same uncertain situation if differently described etc. One has broached this particular topic and progress has been made.

In general, however, I would say that experiments with games proper are still in infancy although there are highly significant beginings. For example, Dr. Maschler has made experiments about bargaining and they look very interesting; we have heard of others here at this meeting. I think, in general, we are not at present in the very simple situation as in logic where if you make a statement and say "all swans are white" and you are shown one black swan, your statement is false--and it is that simple to overthrow a theory that leads to this kind of "all" statement. Nor are we in the situation of relativity theory where you may make one single new measurement and as a consequence the whole relativity theory could collapse. We are in neither of the two situations. We are somewhere in between and I think the setting up of experiments to decide whether a theory is true or not is a very much more complicated affair, especially for social situations. The history of the natural sciences shows that it is very, very difficult to make decent experiments, to interpret them correctly, to synthetize the findings, to show what they mean. Therefore we must be very careful in either taking some experiments as a

confirmation or taking some as a rejection of the theory of games as it exists at present. History will tell us that the situation in the social sciences is possibly more complicated as far as experiments are concerned than the experiments in the physical sciences. The experimental effort made in the physical sciences is very expensive and of course enormously large. So great care is taken in making experiments and there exists a tradition of hundreds of years. We on the other hand have a tremendously long way to go.

Now turning to the non-constructive, or what I consider non-constructive, criticisms I think there is a great deal of that sort of thing afloat and understandably we have heard some such criticisms even in this meeting. They should not be taken too much to heart. Dr. Rapoport who, unfortunately, is not here but whose paper (based on views developed in his recent book "Strategy and Conscience") was presented yesterday, surprisingly illustrates some of the misconceptions and perhaps it is necessary to say a few words about this. I think what he says about utility theory is based on profound misunderstanding, but I shall not dwell on this issue.

Before going on I would like to quote the German philosopher Kant in two respects: (1) he has said that one must be very careful in regard to the "exaggerated use" of theory, and he has warned very severely and rightly against this on his own behalf. (2) He has also said that a single man can ask more questions than 1000 wise men can answer. The mere asking of questions is not necessarily a contribution to the development of sciences. As I said before, it has to be the right question which is to be answered and the right question is often difficult to find. Merely to make large enumerations of all sorts of questions is nothing; anybody can do that. It is a question of what is the question, what is the next question which must be asked.

Now, among those misunderstandings, there is the following: you encounter in the literature time and again, and I referred to this here also, that game theory makes the assumption of rational behaviour on the part of the participants. But what does that mean? There is a total misunderstanding. The theory is not making any such assumption; the theory is and was designed to give meaning to what common sense vaguely calls "rational behaviour." What does it mean to be rational in a situation where the outcome does not depend on you alone, nor on chance alone, but on others also who have opposite aims but likewise lack the ability to achieve them because they depend on your actions too? What does it mean to assume "rationality" then? But once the theory has given meaning to this notion then one can begin to ask what an extension of such a particular question might be. To say that rational behavior is imposed, introduced at the beginning into the theory, to my mind misses the point completely.

I was also astonished to hear yesterday that there is no non-zero sum game theory; this is most extraordinary and I just cannot understand how anybody can say such a thing. In the original book by von Neumann and myself, there are 100s of pages devoted to the n-person games, many of which certainly are of non-zero-sum and variable sums. In those chapters notions are introduced of threats and of the cost of threats to various parties; there it is examined in detail what kind of cooperation between people, engaged in non-zero-sum situations, is possible, etc. I think people who make such remarks not only do not know the original work, but they also do not know the great development which has taken place in this respect since 1944.

Dozens and dozens of papers have been published and valuable work has been done in the true mathematical spirit of the theory itself which demonstrates the viability of the original attempts. Take such things, in order to quote some remoter fields, (although why they should be remote I do not know) such as the problem of composition and decomposition of games. Or consider the problem of the intensely interesting phenomenon in some simple games where you have substitutability of players but can assign no value to these players, because the relationship of their interchange depends on the structure of the situation and not on values. Or consider the study of the problem of formation of coalitions of different strength which is again treated amply, or the problem of removable players, etc. to name matters which are not so obvious. That the late Mr. Strachey has learned about the existence of non-zero-sum games only after five years reading Prof. Schelling's book as Prof. Schelling told us here yesterday, is a matter which rests with Mr. Strachey but not with the theory of games, and I shall not discuss this any further. He was a valuable man and died too soon, and his last book is very interesting, but that particular point is surely completely missing the issue.

To call the present thinking about non-zero-sum games extremely primitive, as was done yesterday by Professor Schelling means applying a standard with which I am not familiar; it is a standard so high that, I would say, it almost jumps a whole development of mathematics which I would be happy to participate in, but I have not had any occasion to see happen. It is, of course, arbitrary what one considers "primitive" but I wonder whether many here would apply this predicate say to the analysis of the simple game with the "chief player" (for arbitrarily large n) who wins when he finds at least one ally and is defeated by a coalition of all others against him.

It is rather surprising that another criticism has not been made of the theory and that is that we have as yet no general existence theorem, no proof of the existence of solutions for all arbitrarily large n-person zero-sum cases. If that had been said, I would heartily say yes that is a very serious drawback; but here in the audience are so many brilliant young people that we may hope that one of them perhaps will provide this proof. As you know the fact is that wherever one has analyzed any arbitrarily large n-person game, one has found a solution when one has gone deeply into the matter. And there are some strong signs that this existence proof will be given in the near future. Of course when it is given, it is likely to be only an existence proof. It would probably not be constructive at first and this would still be a drawback, but the sequence of existence proof to constructive proof again is in the nature of things.

Now another point which is raised time and again, for example, in the recent writings of Rapoport and others, is that the theory is "immoral." But what does that mean? It is very hard to understand and give any meaning to such a statement. No theory is moral and no theory is immoral. The question of morality does not apply - it just does not arise.

If you wish to read a paper which takes care of this whole issue, and use your valuable time on it, read the paper by Mr. Wohlstetter in the readings which Dr. Shubik has recently published, called "Readings in Game Theory and Related Approaches" (1964). This excellent paper is called "Sin and Games in America" and as sin may also occur elsewhere, it is possible that what he says is applicable also to other countries. In fact, it is, because, he quotes Mr. Blackett who is now of the opinion that game theory is immoral, though a few years earlier - in the meantime having forgotten what he said - he stated that game theory was one of the most

significant and valuable developments of our times. So you can have it one way or you can't, just as you wish to choose.

This question of immorality to my mind is a sign of an obscurantism which is spreading under the guise of some sort of intellectual occupation. I shall give you an example of that. The example which I shall pick is a publication in Foreign Affairs, an American periodical which is written, frequently ghost-written however, on international affairs; people like Khrushchev, Stevenson, MacMillan and others of this stature write in it, or sign papers in it, so it has a broad appeal. There is a paper there by Sir Solly Zuckerman entitled "Judgment and Control in Modern Warfare," (1962). And that brings me to the last point I want to make to-day: What is the value of game theory in regards to the analysis of world political situations. Sir Solly Zuckerman is, I am told, a very worthy zoologist and in particular a great authority on monkeys. He is also, at the present time, the chief defense advisor to the British government.

He writes in this paper the usual things; that one cannot rely on theory too much; that one must have judgment; that one cannot eliminate the human element from decisions--as if anybody wanted to do that. He writes on game theory, for example the following "that game theory burst upon the world a few years ago" and goes on to say: "I do not pretend to understand the intricacies of either the mathematics or the logical symbolism one finds in the writings. As I understand it, game theory is based upon the interaction of sequences of probabilities, and it is assumed that you can make the best choice even when one randomizes one's opponent moves," and in this style it goes on. As you see immediately-- this nonsense has absolutely nothing whatsoever to do with game theory. He says furthermore: "the theory includes such matter as the enemy's

intentions," while in the two-person zero-sum theory the critical point is exactly that one eliminates consideration of the intentions of the other side, that instead one wishes to make decisions under conditions which make it irrelevant what the other fellow thinks. One has finally gotten away from this 'I think, he thinks, I think he thinks' type of argument although the argument again appears in some of the critical literature.

In this manner, it goes on and I will not bother you with further details from this paper to which the periodical refused to take a rejoinder. Now this author asks for intuition instead of mathematics and I'll not say anything against intuition. I think intuition is a terribly important thing -- for example, precisely in mathemtatics. It is very nice to have mathematical intuition. I wish I had some. And intuition is indispensable in music, in art, and especially in all sorts of things where a great deal of experience has been accumulated in the man's mind and which at the right time is pulled out, in the right manner, in the right situation. But, here, the idea is to get beyond intuition and to look at those explicit facts which are analyzable and fall under the scope of game theory, and not to try to identify game theory with things which have nothing to do with it.

Now a few words about future work. This is the sort of characterization I feel I have to add to that kind of criticism which I considered as really non-constructive. There is an excellent paper which I would like to recommend to you written by a former student of mine, Anthony Koo, a young Chinese, now professor in Columbia University. His paper, "Recurrent Objections to the Minimax Strategy," The Review of Economics and Statistics, Vol. XLI, No. I, February 1959, concerns some recurrent misinterpretations of game theory. He analyzes precisely some of the things I have indicated

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here, where the matrices have been wrongly formed, imperfectly interpreted, etc. This paper appeared several years ago at Harvard, where there is a modicum of interest in game theory also. It can be read with considerable profit but time forbids that I dwell on it further.

Now the idea that the theory is unable to take care of many problems which are genuine problems leads to the dangerous statement that it is basically unable to do so merely because these problems are not being taken care of at the present time. I would like to remind you that our experience tells us that it is very dangerous to make negative statements, because to-day you may say that something is impossible, and to-morrow somebody comes and does it, unless you can prove now that it is impossible in principle, such as squaring the circle, using ruler and compass only; or you might perhaps even go further to say that it is in principle impossible to decide in certain situations whether one thing or the other can be decided, as in the case of Gödel's proof.

We are a long way away from that, but who knows, we may be moving in that direction. I do think that we may have insoluble problems in the social field. That is quite possible. To determine whether that is so and whether that can be proved would be a very important thing to do. "Insoluble" might mean that we cannot solve them at present, but it might also mean that they can never be solved in principle; I have the latter possibility in mind.

Let me give you an idea, one indication of where I think possibly work is going on in the wrong direction. That is when people are trying to find unique solutions. As you know, as far as the solution concept is concerned, especially for cooperative situations, and in particular applicable to variable sums, one finds that one gets solutions which consist of alternative

sets of numbers, i.e. imputations. There are sometimes solutions with infinitely many and there may be many solutions, even infinitely many solutions, each with infinitely many imputations. This is a most uncomfortable world to live with. Granted, but it may be the world we live in. Therefore the idea that we must under all circumstances find unique solutions in game situations may be completely wrong and efforts be correspondingly misdirected.

It is therefore not a characterization of an imperfection of the theory if it does not yield uniqueness. Rather it is a characterization of a concrete realistic phenomenon with which we are forced to deal. It is reality which produces such peculiar products. In a sense, although I am far from wishing to make the comparison by value or importance, but merely as a characterization of an intellectual situation, I have a certain feeling that regarding this craving for uniqueness one is reminded of the models which existed in physics at the time before quantum mechanics was created. It took physicists thirty or forty years to get used to indeterminateness and they are still unhappy about quantum mechanics and what it means, and about the absence of the nice classical solutions. Even statistics and statistical mechanics do not conform to quantum mechanics and that is most unpleasant. Physicists are in just this sort of trouble. The social scientist is in a much greater trouble; he will have to adjust to the present malaise which this situation causes. It is most likely caused by the nature of the empirical situation we have to face and is not an imperfection of the theory as it is sometimes interpreted.

Now a final word on application to war. First of all, we know that there is even in war a mutual interest of the two parties: that is a matter that goes without saying. We don't have to be told about it as if

this were some new discovery. It is also clear that game theory poses issues and that they are not exhaustive. That is particularly due to the fact that the model is incomplete, and the model may have to be enlarged. To make it more complicated is possibly beyond our present capability. But I would say that game theory, whatever little contribution it may have made to the analysis of the present disastrous opposition of interest, by showing anew the common interest of not destroying each other, has done precisely something which was lacking in previous situations.

Take the time before 1914; at that time you could open any book on war and you would find it said that war is not possible and would never happen again. Now we think that this is not so in our time. When war broke out in 1914, the people volunteered to be slaughtered; with flowers they were sent by their wives and sweethearts to their death. Now we know that nobody would do that at present because we have investigated, though probably in a very inadequate way, the absolute horror which would prevail in the world if a conflagration of a great kind would occur. I think a part of this realization is to be attributed to the fact that the people are taking a strong look, and another look, and another look at the possibility and the consequences of such a conflagration. Now, maybe, there is a certain new fact here, a consciousness and an awareness which perhaps would not have been produced if various applications of game theory had not been made to war, no matter how poor and how weak they may have been compared to the needs of the total problem. And so, in that sense, I think there is possibly a positive contribution, although the war problem as such is so big and so immense, and although the simple theory we have today is far from adequate of making a decisive contribution of giving us

the right answer. Perhaps we have not even asked the right questions? But do we have anything better? Is the appeal to 'judgement' and 'intuition' appealing? Does that give us greater security?

I have taken the liberty of presenting some of the thoughts that occurred to me listening to the very many interesting papers at this conference and looking upon the literature, although I must say I am no longer familiar with the whole literature, because it is so rich. Thus much of what I have said may be superseded; I hope it is by what is already published or is already in your minds.