TECHNICAL CHANGE, CAPITAL LONGEVITY

AND

ECONOMIC GROWTH

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ABSTRACT

The specification of a criterion which may be used to determine the optimum longevity of capital has been the subject of some controversy in the literature. The contributions to this problem fall into two distinct categories. One approach to the problem emphasizes micro-economic criteria based on profit maximization or cost minimization. A second approach is concerned with the impact of changes in the longevity of capital on the level and rate of growth of aggregate output and consumption. In this paper these two approaches are compared within the context of a relatively simple macro-economic growth model.

Within the context of this model of economic growth, it is found that these two criteria coincide if and only if the rate of interest, which is determined by a zero-pure-profit condition, is equal to the equilibrium rate of growth of output. Two alternative but equivalent conditions for the two criteria to coincide are also described. The analysis suggests that in designing policies oriented toward increasing the level or rate of growth of output and consumption, it is necessary to consider explicitly the relationship between the investment-income ratio and the longevity of capital. Focussing attention on only one or the other of these magnitudes may be misleading within the context of policy formulation.

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I. INTRODUCTION

The specification of a criterion which may be used to determine the optimum longevity of capital in an economy has been the subject of some controversy in the literature. The contributions to the solution of this problem fall into two distinct categories. One approach to the problem of determining the optimal longevity of capital emphasizes micro-economic criteria based on profit maximization or cost minimization [1,2,12,14]. A second approach emphasizes macro-economic criteria based on the impact of changes in the longevity of capital on the level and rate of growth of aggregate consumption and output [3,4,8,9]. To the extent that economic policy is oriented toward the prevention or elimination or technological obsolescence, it is important to know the relationship between these two criteria.[1] However, these two approaches to the problem of determining an optimal longevity of capital have been developed more or less independently of one another with no consideration having been given to the possibility that these two approaches might lead to identical results.

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[1] A related point which will become more apparent below is that it is important to know the relationship among the level and rate of growth of aggregate consumption and output, the longevity of capital, and the share of investment in output. Policies oriented toward the prevention or elimination of technological obsolescence may well have an impact on the share of investment in output and hence on the level and rate of growth of output. Conversely, policies oriented toward increasing the share of investment in output may well affect the longevity of capital.

It is the purpose of this paper to determine within the context of a neoclassical growth model the conditions which must be satisfied in order for the micro- and macro-economic criteria to lead to similar results. In the next section a macro-economic growth model is specified and the equilibrium properties of this model are described. In Sections III and IV, respectively, the macro-economic and micro-economic capital replacement criteria are explicitly derived. The implications for the life of capital of each of the two criteria are compared in the final section and the conditions which must be satisified for their equivalence are developed.

II. A NEOCLASSICAL MACRO-ECONOMIC GROWTH MODEL

In order to develop the interrelationships among the rate of technical progress, the life of capital, the share of investment in output, and the level and rate of growth of aggregate consumption and output, a well-known macro-economic growth model is introduced in this section [7,8,9,13]. It is assumed that two factors of production, capital and labor, cooperate to produce a single homogeneous output which may be used equally well for either consumption or capital accumulation. In addition the model is characterized by the following properties.

(a) Gross investment at time t is a constant fraction of output at time t and is assumed to embody the latest known production techniques at the time of its construction. In addition, capital is assumed to possess infinite durability and the only factor governing its removal from the production structure is obsolescence.

(b) The supply of labor is given by an autonomous pattern of exponential growth and is assumed to be continuously fully employed.

(c) The capital-labor substitution possibilities at the time capital is constructed are given by a vintage-type Cobb-Douglas production function. After capital has been put into operation, fixed coefficients of production prevail and it is possible to alter the labor-capital ratio only by scrapping old capital and constructing new capital.

Assumption (c) forms the basis of the synthesis of short-run fixed coefficients of production and long-run substitutability of the factors of production [6]. The output obtainable at time t from vintage v capital, $Q_v(t)$, is assumed to be given by

$$(2.1) \qquad Q_v(t) = Ae^{ut + \lambda v} K_v(t)^a L_v(t)^b \qquad a+b = 1$$

where $K_v(t)$ is the stock of capital constructed at time v and still in existence at time t, $L_v(t)$ is the labor operating vintage v capital at time t, μ is the rate of organizational technical progress, λ is the rate of embodied technical progress, a and b are, respectively, the elasticities of output with respect to capital and labor input, and A is a scale factor. At any instant t in time the possibilities for substituting capital for labor are given by (2.1) with $v = t$. However, after capital has been constructed the capital-labor ratio is assumed to be fixed.

The equilibrium growth path of output to which these assumptions give rise may be developed in the following way. Suppose that output is expanding at a constant relative rate equal to β. Since gross investment at time t is a constant fraction of gross output at time t, gross investment must also be expanding at the relative rate β. Since capital is not subject to physical deterioration, capital constructed at time v and still in existence at time t is equal to the gross investment at time v so that

(2.2) $$K_v(t) = I(v) = Ie^{\beta v}$$

where $I(v)$ is gross investment at time v and I is the rate of gross investment at time zero. If the age of the oldest capital in use is θ, then the total stock of physical productive capital in use at time t is given by

(2.3) $$K(t) = \int_{t-\theta}^{t} K_v(t)\,dv = Ig(\beta,\theta)^{-1}e^{\beta t} .$$

In equation (2.3) the expression $g(\beta,\theta)$ is inserted as a shorthand expression for $\beta/(1-e^{-\beta\theta})$.

According to assumption (b), the labor force is growing exponentially and may be written as

(2.4) $$L(t) = Le^{nt}$$

where $L(t)$ is the supply of labor at time t, L is the labor supply at time zero, and n is the relative rate of growth of the labor force. Letting $L_v(t)$ denote the labor which is working with vintage v capital at time t, the total labor force is also equal to

(2.4') $$L(t) = \int_{t-\theta}^{t} L_v(t)dv .$$

Differentiating both (2.4) and (2.4') with respect to t and equating the results gives the equation

(2.5) $$L_t(t) = nLe^{nt} + L_{t-\theta}(t) .$$

This equation states that the labor available at time t to work with new capital is equal to the net growth in the labor force plus the labor released from capital which is retired at time t. This result holds because of the assumption that the labor-capital ratio is fixed after capital is constructed. It is implicitly assumed that if any of the capital of a particular vintage is scrapped, then all of it will be scrapped. The equilibrium solution of (2.5) for the labor which operates vintage v capital is[2]

(2.6) $$L_v(t) = Lg(n,\theta)e^{nv} .$$

Equations (2.1), (2.2), and (2.6) may be combined with the following two equations to obtain an expression for total output at any given point in time in terms of the parameters of the model.

(2.7) $$I(v) = sQ(v)$$

(2.8) $$Q(t) = \int_{t-\theta}^{t} Q_v(t)dv$$

Equation (2.7) states that the constant fraction s of output is devoted to investment at each point in time and (2.8) states that the total output at time t is obtained by integrating over the output of all vintages of capital between t and $t-\theta$. Using (2.1), (2.2), (2.6), (2.7), and (2.8) the equilibrium growth path in terms of the parameters is

(2.9) $$Q(t) = s^{a/b} A^{1/b} Lg(n,\theta)g(\sigma,\theta)^{-1/b} e^{\beta t}$$

where $\sigma = \lambda + a\beta + bn$.

The equilibrium growth path of this model possesses several interesting properties. The equilibrium rate of growth of aggregate output, $\beta = n + (\mu+\lambda)/b$, is independent of both the longevity of capital and the share of output devoted to gross investment. The growth rate is also invariant with respect to changes in

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[2] This equilibrium solution of (2.5) is obtained by noting that $L_{t-\theta}(t) = L_{t-\theta}(t-\theta)$. Making this substitution in (2.5), the resulting difference equation is of the form $\ell(t) - \ell(t-\theta) = nLe^{nt}$ with the equilibrium solution given in (2.6).

the mix of technological progress. Regardless of whether technological change is of the embodied type, entirely of the organizational type, or some mixture of the two, the equilibrium rate of growth of output will be the same provided the level of technical progress (i.e., the sum of the two components) is the same in each case. On the other hand, the level of the equilibrium growth path depends critically on the investment ratio, the longevity of capital, and the mix of technological progress. These properties are such that this model provides a convenient framework within which to compare the macro-economic capital replacement rule derived in the next section with the economic life of capital as determined by profit maximization.[3]

III. CAPITAL ACCUMULATION AND REPLACEMENT

It has been shown above that the equilibrium rate of growth of output is independent of both the investment ratio and the longevity of capital. However, the equilibrium level of output does depend on both of these variables. These properties of the equilibrium growth path of output are utilized in this section to derive a capital replacement rule which must be satisfied in order for the level of the equilibrium consumption path to be a maximum. The longevity of capital which satisfies this rule will be referred to alternatively as the optimal longevity of capital or the optimal capital replacement period. In view of the similarity between the capital replacement rule and the "Golden Rule of Accumulation"

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[3] One of the more important reasons why this model is particularly well suited to this problem is that different sets of values for the longevity of capital and the investment-output ratio do not give rise to intersecting equilibrium growth paths of aggregate output and consumption. For a discussion of this and related points, see [5] and the references there cited.

derived by Phelps [11] for a somewhat simpler growth model, it is of interest to compare the two rules within the context of the model of the preceding section.

Consider first the determination of an optimal investment-output ratio or what has come to be called the "Golden Rule of Accumulation". Since consumption is equal to gross output less gross investment at each point in time, the maximization of the equilibrium growth path of consumption is equivalent to the maximization of

$$C(t) = (1-s)Q(t) \tag{3.1}$$

where $Q(t)$ is given by (2.9). According to (2.9), the equilibrium level of output will be larger the larger is the investment ratio. However, with a larger investment ratio there is a smaller fraction of output available for consumption. Maximizing $C(t)$ with respect to s leads to

$$\bar{s} = a . \tag{3.2}$$

This marginal condition states that the level of the equilibrium time path of consumption in this simple growth model will be maximized provided that the investment ratio is equal to the elasticity of output with respect to the capital stock. It is of interest to note that the golden rule of capital accumulation in this model is independent of the longevity of capital. Regardless of the prevailing longevity of capital, the golden rule states that the percentage of gross output which should be devoted to gross investment in order to maximize the equilibrium level of consumption is exactly the capital elasticity of output.

Suppose now that the investment ratio is given and consider the problem of determining the longevity of capital. With a given investment ratio, it is a matter of indifference as to whether the equilibrium level of consumption or the equilibrium level of output is maximized. In either case the optimal longevity of capital will be the same. For the purposes of exposition it is somewhat simpler to consider the

In order to gain a further understanding of the optimal capital replacement period and the marginal condition by which it is determined, it is instructive to consider the way in which changes in the parameters of the model affect the optimal replacement period. This could be accomplished by subjecting (3.4) to implicit differentiation. However, the same end can be achieved by working out a few numerical examples with plausible values substituted for the parameters. The results of these numerical examples are set out in Table I. Various values of the rate of organizational technical progress (μ), the rate of embodied technical progress (λ), the rate of growth of the labor force (n), and the labor elasticity of output (b) have been assumed. From these values the equilibrium rate of growth of output (β) and the optimal capital replacement period ($\bar{\theta}$) have been derived. In addition, the average age of the capital stock ($m(\bar{\theta})$) and the equilibrium capital-output ratio (K/Q) corresponding to both the optimal capital replacement period and the optimal investment quota have been calculated.

By comparing the entries in lines two and three of Table I, it is easy to see that the optimal capital replacement period and the labor elasticity of output vary inversely. Since the capital elasticity of output varies inversely with the labor elasticity of output, it follows that the optimal capital replacement period varies directly with the elasticity of output with respect to capital input. By comparing lines three and four of Table I, it follows that the optimal longevity of capital and the rate of growth of the labor force vary directly. Similar comparisons of the various entries in Table I show that the optimal capital replacement period varies inversely with both the rate of embodied technical progress and the rate of organizational technical progress.

Table I

The Optimal Capital Replacement Period

μ	λ	n	b	β	$\bar{\theta}$	$m(\bar{\theta})$	K/Q
.00	.02	.005	.70	.034	22	9.7	4.6
.01	.01	.01	.70	.039	33	13.2	5.6
.01	.01	.01	.80	.035	27	11.4	3.5
.01	.01	.005	.80	.03	21	9.4	3.1
.01	.02	.01	.80	.038	22	9.3	3.0
.005	.01	.01	.80	.029	30	12.7	4.0

The basic argument of this section may be summarized very briefly by referring to Figure 1. In this figure time is measured on the horizontal axis and the logarithm of consumption at time t is measured on the vertical axis. The parallel consumption tracks correspond to various pairs of values of the investment ratio and the longevity of capital. These consumption tracks are parallel because they have a common slope equal to the rate of growth of output. The highest attainable consumption track, labeled $\ell nC(\bar{s},\bar{\theta})$, is the consumption track which corresponds to the optimal investment ratio and the optimal longevity of capital. All other possible consumption tracks are lower than this optimal track. Since the optimal investment ratio and the optimal longevity of capital are independently determined, these lower consumption tracks correspond to nonoptimal values for either the investment ratio, the longevity of capital, or both.

IV. THE ECONOMIC LIFE OF CAPITAL

In this section the problem of determining the economic life of capital within the context of the macro-economic growth model set out in Section II is considered. The volume of gross investment at each point in time is considered to be exogenously specified. However, the type of capital which is constructed,

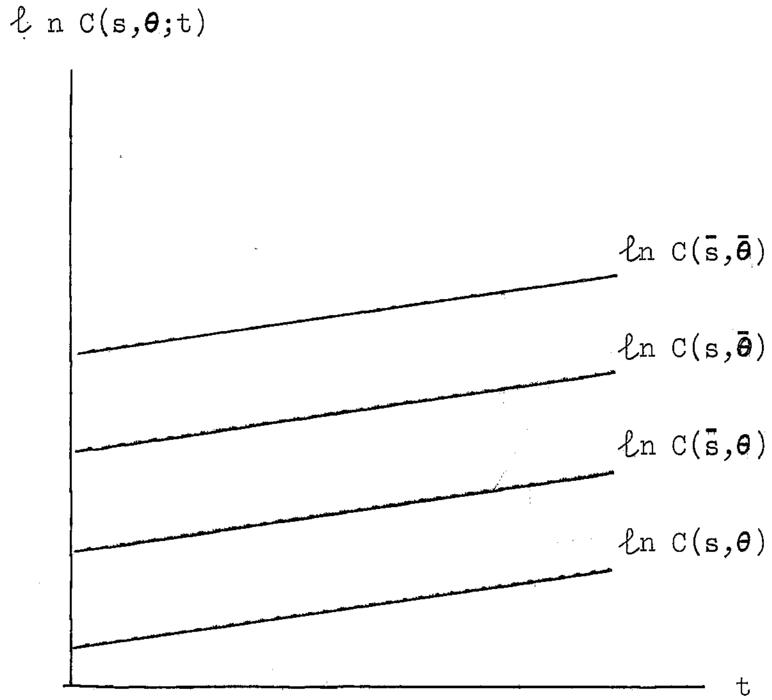

Figure 1. Equilibrium Consumption Tracks for Different Values of the Investment Ratio, s , and the Longevity of Capital, θ .

that is, whether more or less labor is used in conjunction with this capital, and the period of time for which it will be operated are both considered to be endogenous variables. It is assumed that the type of capital which is constructed and the economic life of capital are both determined by maximizing total discounted profit. In order to determine discounted profit, it is necessary to introduce the wage rate and the interest rate explicitly into the analysis. The wage rate is assumed to adjust so that full employment is continuously maintained and the interest rate is assumed to adjust so as to annihilate any pure profit or loss.

Let $R_v(t)$ denote the net return to vintage v capital at time $t \geq v$. Then

(4.1) $$R_v(t) = Q_v(t) - w(t)L_v(t)$$

where $Q_v(t)$ is the output of vintage v capital at time t, $L_v(t)$ is the labor required to operate vintage v capital at time t, and $w(t)$ is the wage rate at time t. The return to vintage v capital at any time is equal to the output less the operating cost at that time. Let $r(t)$ denote the rate of interest at time t. The total discounted profit obtained from the operation of vintage v-capital for θ years is

(4.3) $$P(v) = \int_v^{v+\theta} R_v(t)E(t,v)dt - I(v) \quad .$$

The function $E(t,v) = \exp\left[-\int_v^t r(x)dx\right]$ represents the discount factor which converts the net revenue of time $t \geq v$ to its present value at time v. Total discounted profit is obtained by subtracting the initial cost of vintage v capital, $I(v)$, from the total discounted net revenue obtained from this capital over its life.

The type of capital which will be constructed at time v may be obtained by maximizing (4.2) with respect to $L_v(v)$ $(=L_v(t))$.

$$(4.3) \qquad \int_{v}^{v+\theta} [\partial Q_v(t)/\partial L_v(t)]\, E(t,v)dt = \int_{v}^{v+\theta} w(t)E(t,v)dt$$

This condition states that the amount of labor which is used in conjunction with vintage v capital will be increased up to the point at which the discounted marginal product of labor is equal to the discounted marginal cost of labor. This condition is, of course, similar to the familiar condition that labor will be utilized up to the point at which the marginal product of labor is equal to the wage rate. In fact, (4.3) reduces to this condition if both the marginal product of labor working with vintage v capital and the wage rate are constant over time.

Within the context of the simple growth model of the preceeding section equation (4.3) may be interpreted in another way. Once the interest rate and the longevity of capital are specified, the condition of continuous full employment of the labor force, (2.6), may be used to determine the amount of labor which is available to work with new capital at each point in time. With $L_v(t)$ thus determined by equation (2.6), equation (4.3) may be interpreted as imposing a condition on the time path of the wage rate. Specifically, the wage rate must adjust so that the amount of labor demanded to work with capital of vintage v is equal to the amount of labor actually available to work with vintage v capital. In this way the equilibrium time path of the wage rate is determined.

The economic life of capital may be determined by maximizing total discounted profit with respect to capital longevity. Setting the partial derivative of (4.2) with respect to θ equal to zero yields

$$(4.4) \qquad Q_v(v+\theta) = w(v+\theta)L_v(v+\theta) .$$

This condition states that, in order to maximize profit, capital should be retained in the production structure only so long as the net revenue obtained from that capital is non-negative.

If the time path of the rate of interest is regarded as being exogenously determined, then equations (4.3), (4.4), and (2.6) may be solved simultaneously for the equilibrium values of the time path of the wage rate and the economic life of capital which equilibrate the system. But if the rate of interest is exogenous, there is very likely to be a pure profit or loss resulting from the operation of capital. To preclude this possibility, the interest rate is assumed to adjust so that pure profits or losses are annihilated. This leads to the equilibrium condition

$$P(v) = 0 . \tag{4.5}$$

This condition simply states that the rate of interest adjusts so that the total discounted net revenue obtained from the operation of vintage v capital is equal to the initial cost of this capital. The three equilibrium conditions (4.3), (4.4), and (4.5) together with (2.6) simultaneously determine the equilibrium values of the wage rate, the interest rate, and the longevity of capital.

It is possible to determine the general properties of the equilibrium time paths of the wage and interest rates without actually solving this system of equations explicitly. It may be shown that for the uniform replacement period θ to persist over time, the equilibrium wage rate must be increasing over time at the relative rate Δ/b, where Δ is the sum of the relative rates of embodied and organizational technical change. In addition, the interest rate must be constant over time if the uniform replacement period θ is to obtain. These two properties of the wage and interest rates may be used to simplify conditions (4.3)-(4.5). Substituting $r(t) = r$ and $w(t) = w(v)e^{(\Delta/b)(t-v)}$ into (4.3), performing the indicated integration, and solving for $w(v)$ leads to

$$w(v) = b[Q_v(v)/L_v(v)]\ g(r-\mu,\theta)^{-1}\ g(r-(\Delta/b),\theta) \quad . \tag{4.3'}$$

If there were no technical progress in this model so that $\mu = \Delta = 0$, this equation would state that the wage rate is equal to the marginal product of labor at each point in time. However, since technical progress is present in this model, the wage rate is equal to the marginal product of labor multiplied by the factor $g(r-\mu,\theta)^{-1}g(r-(\Delta/b),\theta)$.

Substituting the value of the wage rate at time $v+\theta$, $w(v+\theta) = w(v)e^{(\Delta/b)\theta}$, into (4.4), and recalling that $Q_v(v+\theta) = Q_v(v)e^{\mu\theta}$ and $L_v(v+\theta) = L_v(v)$, the economic life of capital is obtained by solving

$$e^{\mu\theta} = bg(r-\mu,\theta)^{-1} g(r-(\Delta/b),\theta)e^{(\Delta/b)\theta} \tag{4.4'}$$

for θ . This simplified equilibrium condition which must be satisfied by the economic life of capital states that capital is retained in the production structure only so long as labor's share in output is not greater than total output. Since labor's relative share of vintage v output is increasing more rapidly than is vintage v output, a finite economic life will satisfy this condition.

The third equilibrium condition, the zero profit condition, simplifies to

$$Q_v(v)g(r-\mu,\theta)^{-1} - w(v)L_v(v)g(r-(\Delta/b),\theta)^{-1} = I(v)$$

when $r(t) = r$ and $w(t) = w(v)e^{(\Delta/b)(t-v)}$ are substituted into (4.5). Eliminating the wage rate from this equation by substitution from (4.3') and dividing both sides of the resulting expression by $I(v)$ leads to

$$(a/s)g(\sigma,\theta)g(r-\mu,\theta)^{-1} = 1 \quad . \tag{4.5'}$$

In this equation, $s^{-1}g(\sigma,\theta)$ is the initial output-capital ratio, $Q_v(v)/I(v)$, of vintage v capital, a is vintage v capital's relative share in the output which it produces at time v , and $g(r-\mu,\theta)^{-1}$ is the factor which converts the total output of vintage v capital over its life to its present value. This condition thus states that the total discounted return to each unit of vintage v capital must be equal to the initial cost of each unit of vintage v capital which is, by definition, one.

The formal problem of determining the economic life of capital within the context of the equilibrium growth model of Chapter IV has now been solved. By solving equations (4.3') - (4.5') simultaneously, the equilibrium economic life of capital may be obtained. Fortunately an explicit solution of these equations is not required for the task at hand.[4] By comparing the equations which determine the economic life of capital with that determining the optimal capital replacement period as described in the previous section, the conditions which must be satisfied for their equivalence may easily be determined.

V. THE ECONOMIC LIFE OF CAPITAL AND THE OPTIMAL CAPITAL REPLACEMENT PERIOD

The optimal capital replacement period is obtained by solving

$$(5.1) \qquad e^{-\sigma\bar{\theta}} = bg(n,\bar{\theta})g(\sigma,\bar{\theta})^{-1}e^{-n\bar{\theta}}$$

for $\bar{\theta}$. The economic life of capital is obtained by solving (4.4') for θ. Noting that $\mu = \beta - \sigma$ and $\Delta/b = \beta - n$, (4.4') may be rewritten as

$$(5.2) \qquad e^{-\sigma\theta} = bg(r-\beta+\sigma,\theta)^{-1}g(r-\beta+n,\theta)e^{-n\theta} .$$

By comparing (5.1) and (5.2), the conditions under which the optimal capital replacement period and the economic life of capital coincide may be determined.

By inspection a sufficient condition for the optimal capital replacement period and the economic life of capital to coincide may be determined. Equations

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[4] The existence of a solution to the set of equations (4.3') - (4-5') and comparative static properties of this solution may be shown by an argument analogous to that of Phelps [9].

(5.1) and (5.2) will be identical if the rate of interest is equal to the rate of growth of output. Thus $\theta = \bar{\theta}$ if $r = \beta$. To determine whether this is also a necessary condition, suppose that $\theta = \bar{\theta}$. Then (5.1) and (5.2) together imply that

$$(5.3) \qquad \frac{g(x+\sigma,\theta)g(n,\theta)}{g(x+n,\theta)g(\sigma,\theta)} = 1 ,$$

where x is inserted in place of $r - \beta$. If $x = 0$ is the only solution of this equation, then $r = \beta$ is not only a sufficient condition for θ and $\bar{\theta}$ to coincide; it is also a necessary condition. It may be shown that $x = 0$ is in fact the only solution to (5.3) so that $r = \beta$ is both a necessary and a sufficient condition for $\theta = \bar{\theta}$.

An alternative but equivalent condition for the equality of the optimal capital replacement period and the economic life of capital may be obtained from the zero profit condition. Substituting $\beta-\sigma$ for μ in (4.5'), this equation may be rewritten as

$$(5.4) \qquad (a/s)g(\sigma,\theta)g(r-\beta+\sigma,\theta)^{-1} = 1 .$$

From this zero profit condition it follows that the rate of interest is equal to the rate of growth of output if the investment ratio is equal to a. Moreover, this equation asserts that the rate of interest is equal to the rate of growth only if the investment is equal to a. This means that the economic and optimal lives of capital coincide if and only if the investment ratio is optimal.

A third condition which is implicit in the preceeding two may be obtained for the wage rate. Substituting from (2.1) and (2.6) for $Q_v(v)$ and $L_v(v)$ in (4.3'), the full employment wage rate may be rewritten as

$$(5.5) \qquad w(t) = (s/a)bs^{a/b}A^{1/b}g(\sigma,\theta)^{-1/b}g(r-\beta+n,\theta)e^{(\Delta/b)t} .$$

Here the zero profit condition (4.5') has been used to eliminate $g(r-\mu,\theta)^{-1}$ from (4.3'). The marginal product of labor at time t within the context of the equilibrium growth model may be determined by differentiating (2.9) with respect to $L(t)$. This leads to the expression

$$(5.6) \qquad \partial Q(t)/\partial L(t) = bs^{a/b}A^{1/b}g(n,\theta)g(\sigma,\theta)^{-1/b}e^{(\Delta/b)t} .$$

Comparing (5.5) and (5.6), it is found that the full employment wage rate will be equal to the marginal product of labor if and only if the investment ratio is optimal and hence the interest rate is equal to the equilibrium rate of growth of output. Thus a third condition that $\theta = \bar{\theta}$ is that the full employment wage rate be equal to the marginal product of the total labor force.

VI. SUMMARY AND RELATED PROBLEMS

This paper has been devoted to an examination of two alternative capital replacement criteria which have been advanced in the literature as normative standards for mature economies. These two criteria were compared within the context of a growth model the operation of which may be summarized as follows. At any time there is a certain amount of labor available to work with new capital as determined by the net growth of the labor force and the retirement of old capital. There is also at any time a certain amount of new capital constructed as determined by the investment ratio and the level of output. It was required that the capital be constructed of such a type that full employment is continuously maintained. The equilibrium growth path of gross output was then determined by considering the investment ratio and the age at which capital is retired as parameters, the values of which were specified from outside the model.

A macro-economic replacement rule was derived by maximizing the level of the equilibrium growth path of consumption. For this growth model, the replacement rule was seen to be independent of the investment-income ratio. A second criterion, profit maximization, was used to determine the economic life of capital. These two criteria were seen to yield the same life of capital provided the rate of interest is equal to the rate of growth of output which in turn is true if and only if the investment-income ratio is optimal, i.e., the "Golden Rule of Accumulation" is being followed. The introduction of profit maximization conditions into the macro-economic growth model clearly establishes the inter-dependence of the economic life of capital and the investment-income ratio.

This formulation of the problem discussed in this paper suggests two related questions. The first has to do with the magnitude which is being maximized, in this case, the level of the equilibrium growth path of consumption. It was repeatedly emphasized that this paper was concerned only with alternative equilibrium growth paths of output and consumption. Much more difficult problems arise in the process of comparing the dynamic time paths of these variables while in the process of moving from one equilibrium time path to another. In such cases the problem of evaluating intersecting time paths of the variables must be faced squarely.

A second problem suggested by the discussion of this paper is concerned with the inter-relationship of the investment-income ratio and the longevity of capital. Economic policies oriented toward increasing the rate of growth have been concerned with increasing the investment-income ratio. Very little thought has been given to the possibility that policies which attempt to encourage an increase in the level of investment may be more or less offset by an unforseen change in the longevity of capital. In designing economic policies, both of these parameters must be kept in mind.

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