

1. a) N_1 is binomial $\sim B(n, p)$
 so, $E(N_1) = np$, $\text{Var}(N_1) = np(1 - p)$
 thus: $E(\hat{p}) = np/n = p$
 $\text{Var}(\hat{p}) = np(1 - p)/n^2 = p(1 - p)/n$
 According to Central Limit Theorem, as n is large enough, the probability distribution of \hat{p} will be normal.
 - b) $E(\hat{p}) = 70/100 = .7$
 The CI is $.7 \pm z_{.05/2} \sigma_{\hat{p}} = (.61, .79)$
 - c) $L = 0.04$
 $n \geq \frac{4Z_{\alpha/2}^2 pq}{L^2}$
 $p = q = .5$ maximizes the right hand side, so
 $n = 2400$

Note: For those considered $L = 0.04p$, $n = 9600$.

2. a) $z = s - 6 = \sum_{i=1}^{12} u_i - 6$ where $u_i \sim U(0, 1)$ and independent.
 so,
 $\mu_z = \mu_s - 6 = 12\mu_U - 6 = 12 * 1/2 - 6 = 6 - 6 = 0$
 $\sigma_z^2 = \sigma_s^2 = 12 * \sigma_U^2 = 12 * 1/12 = 1$
 - b) Step 1: Generate 12 Us, Calc. Z, repeat m times
 Step 2: Calc. Y from the m Zs
 Step 3: If not enough Ys, go to step 1 else stop
 - c) Since Z_i is **symmetrically** distributed around a zero mean, $E(Z_i^3) = 0$.
 $E(Y) = E(\sum Z_i^3) = \sum E(Z_i^3) = \sum 0 = 0$.
 Note: For those who considered $E(Z^3) = E(Z)^3$, you were wrong and couldnot get credits from this part.

The standard deviation of Y will be $\sqrt{m} \sqrt{\text{Var}(Z_1^3)}$. So it is positive relative to \sqrt{m} .

3. a) Since X is normal distributed with unknown variance, we should use t statistic.

	H_0 true	H_0 false
Accept H_0		type II error β
Reject H_0	type I error α	

- b) $H_a : \mu < 100,000$
 Reject H_0 when $|t| \leq |t_{\alpha, n-1}|$
 $t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}} = -2.17$
 $t_{0.01, 16} = -2.583$
 So, cannot reject H_0 .
 - c) There are not enough statistical evidence to accuse the manufacturers of false advertising.

d) $P = t(-2.17) \doteq .023$

P-Value means the value of α at which level we would just reject the hypothesis.

When we increase the sample size, the P-value will be decreased.