4.2 $f(x) = \frac{1}{10}$ for $-5 \le x \le 5$ and $= 0$ otherwise

a) $P(x < 0) = \int_{-5}^{0} \frac{1}{10} dx = .5$

b) $P(-2 < x < 2) = \int_{-2}^{2} \frac{1}{10} dx = .4$

c) $P(-2 \le x \le 3) = \int_{-2}^{3} \frac{1}{10} dx = .5$

d) $P(k < x < k+4) = \int_{k}^{k+4} \frac{1}{10} dx = \frac{1}{10}(k+4-k) = .4$

4.4 a) $\int_{-\infty}^{\infty} f(x;\theta)dx = \int_{0}^{\infty} \frac{x}{\theta^2} e^{-x^2/2\theta^2} dx = -e^{-x^2/2\theta^2}|_0^\infty = 0 - (-1) = 1$

b) $P(x \le 200) = \int_{-\infty}^{200} f(x;\theta)dx = \int_{0}^{200} \frac{x}{\theta^2} e^{-x^2/2\theta^2} dx = -e^{-x^2/2\theta^2}|_0^{200} =$
$-.1353 - (-1) = .8647$
$P(x < 200) = P(x \le 200) = .8647$, since x is continuous.
$P(x \ge 200) = 1 - P(x < 200) = 1 - .8647 = .1353$

c) $P(100 \le x \le 200) = \int_{100}^{200} f(x;\theta)dx = .4712$

d) For $x > 0, P(X \le x) = \int_{-\infty}^{x} f(x;\theta)dx = 1 - e^{-x^2/2\theta^2}$.

4.22 a) For $1 \le x \le 2, F(x) = \int_1^x 2\left(1 - \frac{1}{y^2}\right) dy = 2\left(y + \frac{1}{y}\right)|_1^x =$
$2\left(x + \frac{1}{x}\right) - 4$, so

$$F(x) = \begin{cases} 0 & x < 2 \\ 2\left(x + \frac{1}{x}\right) & 1 \le x \le 2 \\ 1 & x > 2 \end{cases}$$

b) $2\left(x_p + \frac{1}{x_p}\right) - 4 = p \Longrightarrow 2x_p^2 - (4+p)x_p + 2 = 0$
$\Longrightarrow x_p = \frac{1}{4}(4 + p + \sqrt{p^2 + 8p})$
To find $\tilde{\mu}$, set $p = .5$ to obtain $\tilde{\mu} = 1.64$.

c) $E(x) = \int_1^2 x \cdot 2\left(1 - \frac{1}{x^2}\right) dx = 2\left(\frac{x^2}{2} - \log x\right)|_1^2 = 1.614$
$E(x^2) = \int_1^2 x^2 \cdot 2\left(1 - \frac{1}{x^2}\right) dx = 2\left(\frac{x^3}{3} - x\right)|_1^2 = \frac{8}{3}$
so, $\mathrm{Var}(X) = .626$

d) Amount left=max$(1.5 - x, 0)$, so
$E(\text{amount left}) = \int_1^2 \max(1.5-x,0)f(x)dx = 2\int_1^{1.5}(1.5-x)\left(1 - \frac{1}{x^2}\right) dx$
$= .061$

4.30 a) $P(x \le 100) = P\left(Z \le \frac{100-80}{10}\right) = P(Z \le 2) = \Phi(2.00) = .9772$

b) $P(x \le 80) = P\left(Z \le \frac{80-80}{10}\right) = P(Z \le 0) = \Phi(0.00) = .5$

c) $P(65 \le x \le 100) = P\left(\frac{65-80}{10} \le Z \le \frac{100-80}{10}\right) = P(-1.50 \le Z \le 2) =$
$\Phi(2.00) - \Phi(-1.50) = .9104$

d) $P(70 \le x) = P(-1.00 \le Z) = 1 - \Phi(-1.00) = .8413$

e) $P(85 \le x \le 95) == P(.50 \le Z \le 1.50) = \Phi(1.50) - \Phi(.50) = .2417$

f) $P(|x - 80| \le 10) = P(-10 \le x - 80 \le 10) = P(70 \le x \le 90) =$
$P(-1.00 \le Z \le 1.00) = .6826$

4.36 $\mu = 43 \quad \sigma = 4.5$

a) $P(x < 40) = P\left(Z < \frac{40-43}{4.5}\right) = P(Z < -.667) = .2514$
$P(x > 60) = P\left(Z > \frac{60-43}{4.5}\right) = P(Z > 3.778) = 0$

b) $43 + (.67)4.5 = 46.015$

4.40 a) $P(\mu - 1.5\sigma \leq x \leq \mu + 1.5\sigma) = P(-1.5 \leq Z \leq 1.5) = \Phi(1.5) - Phi(-1.5) = .8664$

b) $P(x < \mu - 2.5\sigma \text{ or } x > \mu + 2.5\sigma) = 1 - P(\mu - 2.5\sigma \leq x \leq \mu + 2.5\sigma)$
$= 1 - P(-2.5 \leq Z \leq 2.5) = 1 - (\Phi(2.5) - Phi(-2.5)) = 1 - .9876$
$= .0124$

c) $P(\mu - 2\sigma \leq x \leq \mu - \sigma \text{ or } \mu + \sigma \leq x \leq \mu + 2\sigma)$
$= P(\mu - 2\sigma \leq x \leq \mu + 2\sigma) - P(\mu - \sigma \leq x \leq \mu + \sigma)$
$= .9544 - .6826 = .2718$

4.42 a) $P(65 \leq x \leq 75) = P(-1.67 \leq Z \leq 1.67) = .9050$

b) $P(70 - c \leq x \leq 70 + c) = P(-\frac{c}{3} \leq Z \leq \frac{c}{3}) = 2\Phi(\frac{c}{3} - 1 = .95$
$\Longrightarrow \Phi(\frac{c}{3} = .9750) \Longrightarrow \frac{c}{3} = 1.96 \Longrightarrow c = 5.88$

c) $10 \cdot P(\text{a single one is acceptable}) = 9.05.$

d) $p = P(X < 73.84) = P(Z < 1.28) = .9$ so
$P(Y \leq 8) = B(8; 10, .9) = .264$

4.56 a) $\mu = 20, \sigma^2 = 80 \Rightarrow \alpha\beta = 20, \alpha\beta^2 = 80 \Rightarrow \beta = 4, \alpha = 5$

b) $P(x \leq 24) = F\left(\frac{24}{4}; 5\right) = F(6; 5) = .715$

c) $P(20 \leq x \leq 40) = F(10; 5) - F(5; 5) = .411$

4.58 a) $E(x) = \frac{1}{\lambda} = 1$

b) $\sigma = \frac{1}{\lambda} = 1$

c) $P(x \leq 4) = 1 - e^{-\lambda \cdot 4} = 1 - e^{-4} = .982$

d) $P(2 \leq x \leq 5) = 1 - e^{-5\lambda} - \left(1 - e^{-2\lambda}\right) = e^{-2} - e^{-5} = .129$

4.60 mean$= \frac{1}{\lambda} = 25,000$ implies $\lambda = .00004$

a) $P(x > 20000) = 1 - P(x \leq 20000) = 1 - F(20000, .00004) = e^{-(.00004)(20000)} = .449$
$P(x \leq 30000) = F(30000, .00004) = 1 - e^{1.2} = .699$
$P(20000 \leq x \leq 30000) = .699 - .449 = .250$

b) $\sigma = \frac{1}{\lambda} = 25000$, so $P(x > \mu + 2\sigma) = P(x > 75000) = 1 - F(75000, .00004) = .950$
$P(x > \mu + 3\sigma) = P(x > 100000) = .018$

4.62 a) $\{x \geq t\} = A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5$

b) $P(x \geq t) = \prod_{i=1}^{5} P(A_i) = \left(e^{-\lambda t}\right)^5 = e^{-.05t}$, so $F_x(t) = P(x \leq t) = 1 - e{-}.05t, f_x(t) = .05e^{-.05t}$ for $t \geq 0$. Thus X also has an exponential distribution, but with parameter $\lambda = .05$.

c) By the same reasoning, $P(x \leq t) = 1 - e^{-n\lambda t}$, so X has an exponential distribution with parameter $n\lambda$.

5.2 a)

	$p(x,y)$	y = 0	1	2	3	4	
	0	.30	.05	.025	.025	.10	.5
x	1	.18	.03	.015	.015	.06	.3
	2	.12	.02	.01	.01	.04	.2
		.6	.1	.05	.05	.2	

b) $P(x \leq 1 \text{ and } Y \leq 1) = p(0,0) + p(1,0) + p(0,1) + p(1,1) = .56$
$= (.8)(.7) = P(x \leq 1) \cdot P(y \leq 1)$

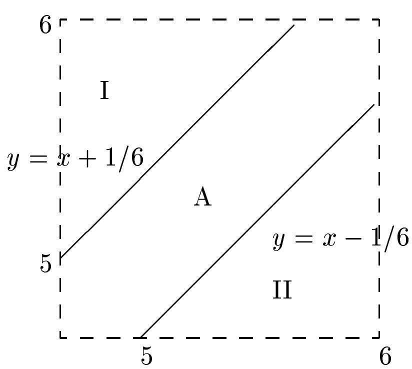
c) $P(x + y = 0) = P(x = 0 \text{ and } y = 0) = p(0,0) = .30$

d) $P(x + y \leq 1) = p(0,0) + p(0,1) + p(1,0) = .53$

5.10 a) $f(x,y) = \begin{cases} 1 & 5 \leq x \leq 6, 5 \leq y \leq 6 \\ 0 & \text{otherwise} \end{cases}$ since $f_X(x) = 1, f_Y(y) = 1$ for $5 \leq x \leq 6, 5 \leq y \leq 6$.

b) $P(5.25 \leq x \leq 5.75, 5.25 \leq y \leq 5.75) = P(5.25 \leq x \leq 5.75) \cdot P(5.25 \leq y \leq 5.75) = (.5)(.5) = .25$

c)



$P((X,Y) \in A) = \iint_A 1 dx dy = \text{area of A}$
$= 1 - (\text{area of I} + \text{area of II})$
$= \frac{11}{36} = .306$

5.12 a) $P(x > 3) = \int_3^\infty \int_0^\infty x e^{-x(1+y)} dy dx = \int_3^\infty e^{-x} dx = .050$

b) The marginal pdf of X is $\int\limits_0^\infty xe^{-x(1+y)}dy = e^{-x}$ for $x \geq 0$; that of Y is $\int\limits_3^\infty xe^{-x(1+y)}dx = \frac{1}{(1+y)^2}$ for $y \geq 0$. It is now clear that $f(x,y)$ is not the product of the marginal pdf's, so the two r.v.'s are not independent.

c) $P(\text{at least one exceeds } 3) = 1 - P(X \leq 3, Y \leq 3)$
$= 1 - \int\limits_0^3 \int\limits_0^3 xe^{-x(1+y)}dydx = 1 - \int_0^3 \left(xe^{-x}\int_0^3 e^{-xy}dy\right)dx = .300$

5.22 a) $E(X+Y) = \sum\limits_x \sum\limits_y (x+y)p(x,y) = (0+0)(.02) + (0+5)(.06) + \cdots + (10+15)(.01) = 14.10$

b) $E(\max(X,Y)) = \sum\limits_x \sum\limits_y \max(x,y)p(x,y) = (0)(.02) + (5)(.06) + \cdots + (15)(.01) = 9.60$

5.30 a) $E(X) = 5.55, E(Y) = 8.55, E(XY) = (0)(.02) + (0)(.06) + \cdots + (150)(.01) = 44.25$,
so $\text{Cov}(X,Y) = 44.25 - (5.55)(8.55) = -3.20$

b) $\sigma_X^2 = 12.45, \sigma_Y^2 = 19.15$, so $\rho_{X,Y} = \frac{-3.20}{\sqrt{(12.45)(19.15)}} = -.207$