5.38 a)

T_0	0	1	2	3	4
$p(T_0)$	.04	.2	.37	.30	.09

b) $\mu_{T_0} = E(T_0) = 2.2 = 2 \cdot \mu$

c) $\sigma^2_{T_0} = E(T_0^2) - E(T_0)^2 = 5.82 - (2.2)^2 = .98 = 2 \cdot \sigma^2$

5.43 The statistic of interest is the fourth spread, or the difference between the medians of the upper and lower halves of the data. The population distribution is uniform with $A = 8$ and $B = 10$. Use a computer to generate samples of sizes $n = 5, 10, 20$, and 30 from a uniform distribution with $A = 8$ and $B = 10$. Keep the number of replictins the same (say for example 500). For each sample compute the upper and lower fourth, then compute the difference. Plot the sampling distribution on seperate histograms for $n = 5, 10, 20$ and 30.

5.44 $\mu = 12\text{cm}$, $\sigma = .04\text{cm}$

a) $n = 16 \qquad E(\bar{X}) = \mu = 12\text{cm}$
$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}} = \frac{.04}{4} = .01\text{cm}$

b) $n = 64 \qquad E(\bar{X}) = \mu = 12\text{cm}$
$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}} = \frac{.04}{8} = .005\text{cm}$

c) $\bar{X}$ is more likely to be within .01 cm of the mean (12 cm) with the second, large, sample. This is due to the decreased variability of $\bar{X}$ with a larger sample size.

5.48 $\mu = 10,000 \qquad \sigma = 500$

a) $n = 40$

$$\begin{aligned} P(9,900 \le \bar{X} \le 10,200) &= P\left(\frac{9,900 - 10,000}{500/\sqrt{40}} \le Z \le \frac{10,200 - 10,000}{500/\sqrt{40}}\right) \\ &= P(-1.26 \le Z \le 2.53) \\ &= \Phi(2.53) - \Phi(-1.26) \\ &= .9943 - .1038 \\ &= .8905 \end{aligned}$$

b) According to the Rule of Thumb given in Section 5.4, n should be greater than 30 in order to apply the C.L.T., thus using the same procedure for $n = 15$ as was used for $n = 40$ would not be appropriate.

5.50 $x - N(8, 1), n = 4$
$\mu_{T_0} = n\mu = (4)(8) = 32$

$\sigma_{T_0} = \sqrt{n}\sigma = (2)(1) = 2$
We desire the 95th percentile: $32 + (1.645)(2) = 35.29$

5.58 Y is normally distributed with $\mu_Y = \frac{1}{2}(\mu_1 + \mu_2) - \frac{1}{3}(\mu_3 + \mu_4 + \mu_5) = -1$, and
$\sigma_Y^2 = \frac{1}{4}\sigma_1^2 + \frac{1}{4}\sigma_2^2 + \frac{1}{9}\sigma_3^2 + \frac{1}{9}\sigma_4^2 + \frac{1}{9}\sigma_5^2 = 3.333, \sigma_Y = 1.826.$
Thus, $P(0 \le Y) = P\left(\frac{0-(-1)}{1.826} \le Z\right) = P(.55 \le Z) = .2912$
and $P(-1 \le Y \le 1) = P(0 \le Z \le 1.10) = .36434.$

5.62 Let $X_1, X_2, \ldots, X_5$ denote morning times and $X_6, \ldots, X_{10}$ denots evening times.

a) $E(X_1 + \cdots + X_{10}) = E(X_1) + \cdots + E(X_{10}) = 5E(X_1) + 5E(X_6) = 37.5$

b) $\text{Var}(X_1 + \cdots + X_{10}) = \text{Var}(X_1) + \cdots + \text{Var}(X_{10}) = 52.083$

c) $E(X_1 - X_6) = E(X_1) - E(X_6) = 2.5 - 5 = -2.5$
$\text{Var}(X_1 - X_6) = \text{Var}(X_1) + \text{Var}(X_6) = \frac{25}{12} + \frac{100}{12} = 10.417$

d) $E(\cdots) = -12.5$
$\text{Var}(\cdots) = 52.083$

5.64 a) With $M = 5X_1 + 10X_2$, $E(M) = (5)(2) + (10)(4) = 50$,
$\sigma_M^2 = (5)^2(0.5)^2 + (10)^2(1)^2 = 106.25$, $\sigma_M = 10.308$

b) $P(75 < M) = P\left(\frac{75-50}{10.308} < Z\right) = P(2.43 < Z) = .0075$

c) $M = A_1X_1 + A_2X_2$ with the A_i's and X_i's all independent, so
$E(M) = E(A_1)E(X_1) + E(A_2)E(X_2) = 50$

d) $E(M^2) = E(A_1^2X_1^2 + 2A_1X_1A_2X_2 + A_2^2X_2^2) = E(A_1^2)E(X_1^2) + 2E(A_1)E(X_1)E(A_2)E(X_2) + E(A_2^2)E(X_2^2) = 2611.5625$
so, $\text{Var}(M) = 2611.5625 - (50)^2 = 111.5625$

e) $E(M) = 50$ still, but now
$\text{VAr}(M) = a_1^2\text{Var}(X_1) + 2a_1a_2\text{Cov}(X_1, X_2) + a_2^2\text{Var}(X_2) = 6.25 + 2(5)(10)(.25) + 100 = 131.25$

5.75 a) $1 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)dxdy = \int_0^{20}\int_{20-x}^{30-x} kxydydx + \int_{20}^{30}\int_0^{30-x} kxydydx = \frac{81{,}250}{3} \cdot k$
$\to k = \frac{3}{81{,}250} = 3.6923\text{E-}5$

b) $$f_X(x) = \begin{cases} \int_{20-x}^{30-x} kxydy & = k(250x - 10x^2) \quad 0 \le x \le 20 \\ \int_0^{30-x} kxydy & = k(450x - 30x^2 + .5x^3) \quad 20 \le x \le 30 \end{cases}$$
and by symmetry $f_Y(y)$ is obtained by substituting y for x in $f_X(x)$.
Since $f_X(25) > 0, f_Y(25) > 0$, but $f(25, 25) = 0$, so X and Y are not independent.

c) $P(x+y\leq 25)=\int_0^{20}\int_{20-x}^{25-x}kxydydx+\int_{20}^{25}\int_0^{25-x}kxydydx=\frac{3}{81{,}250}\cdot\frac{230{,}625}{24}=.355$

d) $E(X+Y)=E(X)+E(Y)=2\left(\int_0^{30}x\cdot f_X(x)dx\right)=25.969$

e) $E(XY)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}xy\cdot f(x,y)dxdy=136.4103$
$\mathrm{Cov}(X,Y)=136.4103-E(X)E(Y)=-32.19$
$E(X^2)=E(Y^2)=\int_{-\infty}^{\infty}x^2\cdot f_X(x)dx=204.6154$
$\sigma_X^2=\sigma_Y^2=204.6154-(12.9845)^2=36.0182$
so, $\rho=\frac{-32.19}{36.0182}=-.894$

f) $\mathrm{Var}(X+Y)=\mathrm{Var}(X)+\mathrm{Var}(Y)+2\mathrm{Cov}(X,Y)=7.66$

5.87 a) with $Y=X_1+X_2$,

$$F_Y(y)=\int_0^y\left(\int_0^{y-x_1}\frac{1}{2^{v_1/2}\Gamma(v_1/2)}\cdot\frac{1}{2^{v_2/2}\Gamma(v_2/2)}\cdot x_1^{\frac{v_1}{2}-1}x_2^{\frac{v_2}{2}-1}e^{-\frac{x_1+x_2}{2}}dx_2\right)dx_1$$

. But the inner integral is equal to

$$\frac{1}{2^{(v_1+v_2)/2}\Gamma((v_1+v_2)/2)}y^{(v_1+v_2)/2-1}e^{y/2},$$

from which the result follows.

b) By (a), $Z_1^2+Z_2^2$ is chi-squared with $v=2$, so $(Z_1^2+Z_2^2)+Z_3^2$ is chi-squared with $v=3$, etc., until $Z_1^2+Z_2^2+\cdots+Z_n^2$ is chi-squared with $v=n$.

c) $\frac{X_j-\mu}{\sigma}$ is standard normal, so the sum of $\left(\frac{X_j-\mu}{\sigma}\right)^2$ is chi-squared with $v=n$.

5.88 a) $\mathrm{Cov}(X,Y+Z)=E(X(Y+Z))-E(X)E(Y+Z)=E(XY)+E(XZ)-E(X)E(Y)-E(X)E(Z)=\mathrm{Cov}(X,Y)+\mathrm{Cov}(X,Z)$

b) $\mathrm{Cov}(X_1+X_2,Y_1+Y_2)=\mathrm{Cov}(X_1,Y_1)+\mathrm{Cov}(X_1,Y_2)+\mathrm{Cov}(X_2,Y_1)+\mathrm{Cov}(X_2,Y_2)=16.$

c) By repeated application of (a),

$$\mathrm{Cov}(\sum_i a_iX_i,\sum_j b_jY_j)=\sum_i\sum_j a_ib_j\mathrm{Cov}(X_i,Y_j).$$