5.32 There is a difficulty here. Existence of ρ requires that both X and Y have finite means and variances. Yet since the marginal pdf of Y is

$$\frac{1}{(1+y)^2} \text{ for } y \geq 0, E(Y) = \int_0^\infty \frac{y}{(1+y)^2} dy = \int_0^\infty \frac{1}{1+y} dy - \int_0^\infty \frac{1}{(1+y)^2} dy$$

, and the first integral is not finite. Thus, ρ itself is undefined.

5.35 a) $\text{Cov}(aX+b, cY+d) = E[(aX+b)(cY+d)] - E(aX+b)\cdot E(cY+d)$
$= E(acXY + adX + bcY + bd) - (aE(X)+b)(cE(Y)+d)$
$= acE(XY) - acE(X)E(Y) = ac\text{Cov}(X,Y)$

b) $\text{Corr}(aX+b, cY+d) = \frac{\text{Cov}(aX+b,cY+d)}{\sqrt{\text{Var}(aX+b)}\sqrt{\text{Var}(cY+d)}}$
$= \frac{ac\text{Cov}(X,Y)}{|a||c|\sqrt{\text{Var}(aX+b)\cdot\text{Var}(cY+d)}}$
$= \text{Corr}(X,Y)$ when a and c have the same signs.

c) When a and c differ in sign, $\text{Corr}(aX+bY) = -\text{Corr}(X,Y)$.

5.56 a) $E(27X_1 + 125X_2 + 512X_3) = 27E(X_1) + 125E(X_2) + 512E(X_3)$
$= 27(200) + 125(250) + 512(100) = 87,850$

$V(27X_1+125X_2+512X_3) = (27)^2V(X_1)+(125)^2V(X_2)+(512)^2V(X_3)$
$= (27)^2(10)^2 + (125)^2(12)^2 + (512)^2(8)^2 = 19,100,116$

b) The expected value is still correct, but the variance is not because the covariances now also contribute to the variance.

5.70 Note 1: $T = X_1 + X_2 + X_3 + X_4$, but not $T = X_1 + X_2 + X_3 + 3X_4$.
Note 2: Use $\Phi(.99) = 2.33$, but not $Phi(0.01) = -2.33$.

5.78 a) Let $X_1, \ldots, X_{12}$ denote the weights for the business-class passengers and $Y_1, \ldots, Y_{50}$ denote the tourist-class weights. Let T=Total weight.
$T = X_1 + \ldots + X_{12} + Y_1 + \ldots + Y_{50}$
$E(T) = 12E(X_1) + 50E(Y_1) = 12 * 25 + 50 * 40 = 2300$
$\text{Var}(T) = 12\text{Var}(X_1) + 50\text{Var}(Y_1) = 5432$
$\sigma_T = \sqrt{5432} = 73.7021$
Note: T is not $12X_1 + 50Y_1$, so $\text{Var}(T)$ is not $(12)^2\text{Var}(X_1) + \ldots$.

b) $P(T \leq 2500) = P(Z \leq \frac{2500-2300}{73.7021}) = P(Z \leq 2.71) = .9966$

6.32 a) for $0 \leq y \leq \theta$,
$F_Y(y) = P(Y \leq y) = P(X_1 \leq y, X_2 \leq y, \ldots, X_n \leq y)$
$= P(X_1 \leq y)\cdot P(X_2 \leq y)\cdot \ldots \cdot P(X_n \leq y)$
$= \frac{y}{\theta}\cdot\frac{y}{\theta}\cdot\ldots\cdot\frac{y}{\theta}$
$= \frac{y^n}{\theta^n}$

so,

$$f_Y(y) = \begin{cases} \frac{ny^{n-1}}{\theta^n} & 0 \le y \le \theta \\ 0 & \text{otherwise} \end{cases}$$

b) $E(Y) = \int\limits_0^\theta y \frac{ny^{n-1}}{\theta^n} dy$

$= \frac{n}{n+1} \frac{y^{n+1}}{\theta^n} |_0^\theta = \frac{n}{n+1}\theta$

$\neq \theta$

so mle is biased. And $\frac{n+1}{n} Y = \frac{n+1}{n} \max(X_i)$ is unbiased.

6.36 Medium of sample X is $\tilde{X} = 28.2$. The series $\{|X_i - \tilde{X}\}$ is:

$$\{1.9, 6.9, 5.2, 0.2, 3.4, 2.7, 3.0, 0.2, 0.9, 1.0\}$$

Medium of this series is $\frac{1.9+2.7}{2} = 2.3$. So the estimator for σ is $\frac{2.3}{.6745} = 3.41$.

$s = \sqrt{\frac{\sum X_i^2 - (\sum X_i)^2/n}{n-1}} = 3.45$

Note: If you have question about problems not shown here and want it badly, email me. You can come to my office at office hour or I can add the solution here later. Leihai