6.8 a) A point estimate of the proportion of all such components that are not defective can be $\hat{p} = \frac{80-12}{80} = .85$

b) $P(\text{system work}) = \hat{p}^2 = .72$

6.15 a) Show that $E(X^2) = 2\theta$:

$$\begin{aligned} E(X^2) &= \int_0^\infty x^2 \cdot \frac{x}{\theta} e^{-x^2/2\theta} dx \\ &= 2\theta \cdot \int_0^\infty \frac{x^2}{2\theta} e^{-x^2/2\theta} d\frac{x^2}{2\theta} \\ &= 2\theta \cdot \int_0^\infty y e^{-y} dy \qquad (\text{replace } \frac{x^2}{2\theta} \text{ by } y) \\ &= 2\theta \end{aligned}$$

So a estimator of θ can be:

$$\hat{\theta} = \frac{\sum X_i^2}{2n}$$

To show it is unbiased:

$$\begin{aligned} E(\hat{\theta}) &= E(\frac{sumX_i^2}{2n}) \\ &= \frac{1}{2n} \sum E(X_i^2) \\ &= \frac{1}{2n}(n)E(X^2) \\ &= \frac{1}{2}2\theta \\ &= \theta \end{aligned}$$

b) $\hat{\theta} = 74.505$

6.20 a) $f(X, n; p) = \binom{X}{n} p^X (1-p)^{(n-X)}$
so
$ln[f(X; n, p)] = lnC + Xln(p) + (n-X)ln(1-p)$
so mle of p (given n) satisfies: $\frac{dln(f)}{dp} = 0, \text{i.e.},$
$\frac{X}{p} - \frac{n-X}{1-p} = 0$
$\hat{p} = p = \frac{X}{n}$

b) $E(\hat{p}) = E(\frac{X}{n}) = \frac{E(X)}{n} = \frac{np}{n} = p,$
so it is unbiased.

c) The mle of $(1-p)^5$ is $(1-\hat{p})^5 = (1-\frac{X}{n})^5 = .44.$

6.24 The condition shown here is the same as: there are r-1 flaws in X-1 samples and the Xth sample has flaw. So: $f(X;r,p) = \binom{X-1}{r-1}p^{r-1}(1-p)^{X-1-(r-1)}$. p and
$ln[f(X;r,p)] = lnC + rln(p) + (X-r)ln(1-p)$
We can see it is the same as what we found in 6.20 except the constant term. So the estimate will be the same.

6.28 a) $f(x_1, x_2, \ldots, x_n; \theta) = \prod \frac{x}{\theta} e^{-x_i^2/2\theta}$
$= \frac{x^n}{\theta^n} e^{-\sum x_i^2/2\theta}$
The mle of θ satisfies: $\frac{dln(f)}{d\theta} = 0$, i.e.,
$-\frac{n}{\theta} - (-\frac{\sum x_i^2}{2\theta^2} = 0$
$\theta = \frac{sumx_i^2}{2n}$

b) Note: the median $\tilde{x}$ satisfies:

$$\int_{-\infty}^{\tilde{x}} f(x)dx = 0.5$$

so, $\tilde{x} = \sqrt{2\theta ln2}$

7.8 a) Use the formula: $P\left(-z_{\alpha/2} < \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} < z_{\alpha/2}\right) = 1-\alpha$

b) The inteval will be expanded.

c) Same as (a) since for an exponential random variable, $\mu = \sigma$.

7.10 Same as 7.8 a

7.14 Same as 7.8 a

7.18 Same as 7.8 a

Note: For problem 7.8-7.18, do not forget dividing the $\sqrt{n}$ term.

7.20 Search the table on pg.707 of Devore's book.

7.22 Use the formula: $CI = \left(\bar{x} - t_{\alpha/2,n-1} \cdot \frac{s}{\sqrt{n}}, \bar{x} + t_{\alpha/2,n-1} \cdot \frac{s}{\sqrt{n}}\right)$.

Note: How to decide when to use the method of 7.22 or of 7.10? The criteria are the same as of the validity of CLT, i.e, if n is 'large enough'.