

$$12.6 \text{ a) } \hat{\beta}_1 = \frac{9(40.968) - (12.6)(27.68)}{9(18.24) - (12.6)^2} = 3.693$$

$$\hat{\beta}_0 = \frac{27.68 - 3.693(12.6)}{9} = -2.095$$

$$\text{b) } \hat{y} = -2.095 + 3.693(1.5) = 3.44$$

$$\text{c) } \hat{\mu}_{Y \cdot 1.75} = -2.095 + 3.693(1.75) = 4.37$$

$$\text{d) } \text{residual} = y_i - \hat{y}_i = 3.39 - 3.44 = -0.05$$

$$12.10 \text{ a) } \hat{\beta}_0 = .3651, \hat{\beta}_1 = .9668$$

$$\text{b) } .8485$$

$$\text{c) } \hat{\sigma} = .1932$$

$$\text{d) } \text{SST} = 1.4533, 71.7\% \text{ of this variation can be explained by the model.}$$

Note:  $\frac{\text{SSR}}{\text{SST}} = \frac{1.0427}{1.4533} = .717$  which matches R-squared on output.

$$12.12 \text{ a) } \text{SSE} = 93.3448 - (-2.095)(27.68) - (3.683)(40.968) = 0.0290$$

$$\text{b) } \hat{\sigma}^2 = \frac{\text{SSE}}{n-2} = .004142, \quad \hat{\sigma} = s = \sqrt{.004142} = .06436$$

$$\text{c) } \text{SST} = 93.3448 - \frac{(27.68)^2}{9} = 8.213, \text{ so}$$

$$r^2 = 1 - \frac{\text{SSE}}{\text{SST}} = 1 - \frac{.0290}{8.213} = .996. \text{ Thus } 99.6\% \text{ of observed } y \text{ variation}$$

can be attributed to the approximate linear relationship between  $x$  and  $y$ .

$$12.16 \text{ a) } \text{Plot omitted. Based on a scatterplot of the data, a linear relationship does seem reasonable for this data.}$$

$$\text{b) } n = 6, \quad \sum x_i = 272, \sum y_i = 3360, \sum x_i^2 = 18,538, \sum y_i^2 = 2,425,394, \sum x_i y_i = 210,120, \text{ so } \hat{\beta}_1 = 9.312, \hat{\beta}_0 = 137.9. \text{ The estimated equation is}$$

$$y = 137.9 + 9.312x.$$

$$\text{c) } \text{SSE} = 5585.4, \text{ SST} = 543,794, \text{ and } r^2 = 1 - .010 = .990.$$

$$\text{d) } \text{The new } \hat{\beta}_1 = 7.5515 \text{ and } \hat{\beta}_0 = 190.35.$$

x	14	18	40	43	45	112
$y_b$	268	305	510	538	557	1181
$y_d$	296	326	492	515	530	1036

This suggests that the observation (112,1200) is very influential.

- 12.25 a)  $H_0 : \beta_1 = 0$   
 $H_a : \beta_1 \neq 0$   
RR:  $|t| > t_{\alpha/2, n-2} = 3.106$   
 $t = 5.29$   
so reject  $H_0$ , the slope differs significantly from 0, and the model appears to be useful.
- b) At the level  $\alpha = 0.01$  reject  $H_0$  if the P-value is less than 0.01. In this case reported P-value was 0.000, therefore reject  $H_0$ . Conclusion same as part a.
- c)  $H_0 : \beta_1 = 1.5$   
 $H_a : \beta_1 < 1.5$   
RR:  $t < -t_{\alpha, n-2} = -2.718$   
 $t = -2.92$   
so reject  $H_0$ . The data contradicts the prior belief.
- 12.26 a)  $\hat{\beta}_1 \pm t_{\alpha/2, n-2} \cdot s_{\hat{\beta}_1}$   
 $0.5549 \pm (2.306)(0.3101)$   
 $(-0.16, 1.27)$   
The interval contains the value 0, indicating 0 is a possible value for  $\beta_1$ , and showing little usefulness of the model.
- b) The P-value associated with the model is 0.11, which exceeds standard levels of  $\alpha$ . This indicates least squares is not a good way to predict age from transparent dentine content.
- 12.50  $H_0 : \rho = 0$  versus  $H_a : \rho \neq 0$ ,  $t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$ .  
RR:  $|r| \geq t_{.005, 22} = 2.819$   
 $r = .5778$ ,  $t = 3.32$ . So  $H_0$  should be rejected. There appears to be a non-zero correlation in the population.
- 12.52 a)  $H_0 : \rho = 0$  versus  $H_a : \rho \neq 0$ ,  $t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$ .  
RR:  $|r| \geq t_{.025, 12} = 2.179$   
 $t = 1.74$ , failed to reject  $H_0$ . the data does not suggest that the population correlation coefficient differs from 0.
- b)  $(.449)^2 = 0.20$  so 20 percent of the observed variation in gas porosity can be accounted for by variation in hydrogen content.
- 12.56 a) Because P-value = .00032 <  $\alpha = .001$ ,  $H_0$  should be rejected at this significance level.

- b) Not necessarily. For this  $n$ , the test statistic  $t$  has approximately a standard normal distribution when  $H_0 : \rho = 0$  is true, and a P-value of .00032 corresponds to  $z = 3.60$ . Solving  $3.60 = r\sqrt{498}/\sqrt{1 - r^2}$  for  $r$  yields  $r = .159$ . This  $r$  suggests only a weak linear relationship between  $x$  and  $y$ , one that would typically have little practical import.
- 13.16 a)  $\sum x_i = 9.72, \sum y'_i = 313.10, \sum x_i^2 = 8.0976, \sum y'^2_i = 8182.8, \sum x_i y'_i = 255.11$  where  $y'_i = \ln(y_i)$ . So  $\hat{\beta} = \text{beta}_1 = 6.667$  and  $\hat{\alpha} = e^{\hat{\beta}_0} = e^{20.6917} = 968927163$ .
- b) We first predict  $y'$  using the linear model and then exponentiate.  $\hat{y} = 1.438E11$ .
- c) We first compute a prediction interval for the transformed data and then exponentiate.  
 With  $t_{0.025,10} = 2.228, s = .5946$ , and  $\left[1 + \frac{1}{12} + \frac{12(.95 - \bar{x})^2}{12 \sum x^2 - (\sum x)^2}\right]^{1/2} = 1.082$ , the prediction interval for  $y'$  is  $27.0251 \pm (2.228)(.5946)(1.082) = 27.0251 \pm 1.4334 = (25.5917, 28.4585)$ . The P.I. for  $y$  is then  $e^{25.5917}, e^{28.4585}$ .
- 13.18 a)  $Y = \beta_0 + \beta_1 \ln(x) + \varepsilon$  where  $x$  = concentration and  $y$  = critical minimum pH. With  $x' = \ln(x)$  we can find  $\hat{\beta}_1 = .483$  and  $\hat{\beta}_0 = 7.29$ .
- b) the P.I is  $7.29 \pm (4.303)(.6496)(1.360) = (6.91, 7.67)$ .
- 13.22 a) the model is  $\frac{1}{y} = \alpha + \beta x + \varepsilon$ .
- b) the model is  $\ln\left(\frac{1}{y} - 1\right) = \alpha + \beta x + \varepsilon$  or  $Y = \frac{1}{1 + e^{\alpha + \beta x + \varepsilon}}$ .
- c) the model is  $\ln \ln Y = \alpha + \beta x + \varepsilon$  or  $Y = e^{e^{\alpha + \beta x + \varepsilon}}$ .
- d) This function cannot be linearized.