Quantum Gravity in a Model Universe

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Abstract
This report describes a hypothetical universe termed Simple Absorption (SA). SA is based on one underlying principle: a specific definition of absorption. Two distributed entities or “fogs” can absorb each other to produce a third one. Starting from the equations governing absorption in SA, it is possible to derive basic features of general relativity and of quantum mechanics. The two sets of properties are manifestations of the same underlying rules of SA. It is suggested here that the hypothetical, simplified universe SA might provide a new perspective on the relationship between gravity and quantum mechanics.

1. INTRODUCTION
I would like to start with a disclaimer. I am not a physicist. I am a neuroscientist with a background in physics. A few years ago, as a hobby, I defined a simple, hypothetical universe on the basis of one underlying principle: a specific definition of absorption. I called that universe Simple Absorption (SA) and explored some of its physical properties. The universe turned out to have an unexpectedly rich physics. It was possible to show that elements in SA behaved in a way consistent with general relativity. Moreover, elements in SA behaved in a way consistent with quantum mechanics. The two sets of properties were related to each other in a simple, internally consistent manner. SA is defined in a manner that appears superficially to have nothing to do with the standard treatments of general relativity or quantum mechanics [1-5]. It seems more related to the interaction of fluids or gases [6] or to the reaction rate in a mixture of two chemicals [7]. Yet when the properties of SA are explored, some of the most basic aspects of general relativity and quantum mechanics emerge naturally. SA therefore provides a novel way to think about familiar topics in physics. The purpose of this report is to describe SA and some of its physical properties.
SA grew out of a consideration of absorption. Consider two entities or “fogs” that are distributed in space and time, fog1 and fog2. (Throughout this report, the notation “X1” refers to “the property X of fog1,” “X2” refers to “the property X of fog2,” and so on.) The two fogs absorb each other and thereby create a third entity, fog3. How can this process be described?

As a first intuitively reasonable guess, in a non-relativistic universe, one might describe a fog as having a density at each location in space-time. It is thicker here and thinner there. In any infinitesimal volume of space-time, the extent of overlap between fog1 and fog2 can be quantified as the product of their densities. Suppose that where fog1 and fog2 overlap in space-time, they absorb each other and produce some quantity of fog3. The greater the extent of overlap between fog1 and fog2 at a point in space-time, the more absorption occurs there. As they absorb each other, some amount of fog1 and fog2 disappears (fog1 and fog2 experience sinks) and a corresponding amount of fog3 appears (fog3 experiences a source). This description resembles the rate equation for the reaction of two chemical substances [7]. The combining of two chemical substances is, microscopically, a discontinuous process. In the hypothetical universe being built up here, however, a fog is not a collection of particles. It is a hypothetical, single, continuous entity defined by means of a density that varies in space-time.

In a relativistic universe, density cannot by itself be an adequate description of a fog. Density is not a scalar quantity, but is instead one component of a vector, the flux-density four-vector here termed $\vec{F}$ [2,8]. At each point in space-time, the extent of overlap between fog1 and fog2 is not the product of their densities, but instead the dot product of their flux-density vectors. Therefore if one is to build the intuitively simple description of absorption that is outlined above, but ensure that it is correct in relativistic space-time, one needs the following prescription: in any infinitesimal volume of space-time, the amount that fog1 and fog2 absorb each other, and therefore the sink in fog1 and fog2, should be proportional to the dot product of their flux-density vectors in four-dimensional Lorentzian space-time. The source in fog3 should be the sum of the sinks in fog1 and fog2. The equations for the three fogs should therefore be:

$$\text{(for fog1)} \quad \text{div} \vec{F}_1 = -k \vec{F}_1 \cdot \vec{F}_2$$  
$$\text{(for fog2)} \quad \text{div} \vec{F}_2 = -k \vec{F}_1 \cdot \vec{F}_2$$  
$$\text{(for fog3)} \quad \text{div} \vec{F}_3 = 2k \vec{F}_1 \cdot \vec{F}_2$$

where $\text{div}$ refers to a four-dimensional divergence that represents the sources and sinks of a fog, $\vec{F}$ refers to the flux-density vector of a fog, and $k$ is an absorption constant.
The Simple Absorption universe, SA, is based on the above description of two gas-like entities absorbing each other in a relativistic space-time. All the physics in SA is contained in equations 1-3. Through the remainder of this paper, nothing will be added to the physics. Instead the equations will be re-written in different ways using different mathematical formalisms to explore otherwise non-obvious properties of SA. In this way it will be seen that general relativity and some basic elements of quantum mechanics are implied by equations 1-3.

2. DEFINITION OF SA

SA is defined by the following two statements:

1. SA contains elements termed “fogs.” Each fog is described by a vector field, the flux-density four-vector $\vec{F}$ that varies in space-time.

2. Fogs interact by absorption. Absorption in SA is when two fogs combine to produce a new fog. Fog1 and fog2 absorb each other to produce fog3. In this process, where fog1 and fog2 overlap in space-time, they experience a sink and fog3 experiences a corresponding source. The sinks and sources obey equations 1-3. For simplicity, in SA, the absorption constant $k=1$.

Two key points about SA should be noted at the outset.

The first point concerns emissions in SA. How can emissions be incorporated into SA, or is it a universe solely of absorptions? In SA, absorption and emission are simply different names for the same thing. Consider again the case of fog1 and fog2 absorbing each other to produce fog3. All three fogs overlap in space and time. There is no definite time and place at which the absorption occurs; there is no distinct “before the absorption” or “after the absorption.” Instead the absorption is a process that takes place incrementally over all of space and time. Because of the spatial-temporal overlap, one could just as well say that fog3 emitted fog1 and fog2. The equations do not distinguish. Whether an interaction “looks” more like an emission or an absorption depends on the particular spatial-temporal properties of the fogs involved and whether the dot product between their flux-density vectors is positive or negative. SA in this sense contains a complete set of interactions. In the following sections the term absorption is used, but in SA there is no conceptual difference between emission and absorption.

The second key point is that, although all three fogs can overlap in space-time, there is an interaction constraint. Fog1 and fog2 do not absorb fog3 to produce yet other fogs. This constraint is implicit in equations 1-3 in that they lack sink terms corresponding to absorptions.
between fog1 and fog3 or between fog2 and fog3. If SA is “seeded” with two fogs, they will absorb each other, produce fog3, and no other interactions will be possible. SA forms a closed set. Two worlds are effectively present. Fog1 and fog2 belong together in world A. Fog3 belongs to world B. At any point in space-time, it is simultaneously true that the absorption has not occurred (world A) and that the absorption has occurred (world B). Most of the analysis below is limited to this extremely simple case of a three-fog universe. However, the same rules can in principle be extended to a more complicated case in which more than three fogs are present.

3. CURVED SPACE IN SA

The present section describes how the equations of SA (equations 1-3) can be re-written as wave equations within a curved space-time. It is important to understand that as equations 1-3 are re-written, they remain mathematically the same. The behavior of fogs in SA remains unchanged. Only the mathematical formalism used to express equations 1-3 is explored here.

Let Ψ be a scalar field whose partial derivatives, when expressed in contravariant form, provide the components of $\tilde{F}$. Using the Einstein summation convention and the convention that a comma refers to differentiation, $F^\alpha = \eta^{\alpha\beta} \Psi_{,\beta}$. This equation incorporates the metric tensor for the Lorentzian space-time manifold:

$$\eta_{\alpha\beta} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

With this definition of Ψ, equations 1-3 become:

$$(\eta^{\mu\alpha} \Psi_{1,\mu})_{,\alpha} = -(\eta^{\beta\alpha} \Psi_{1,\beta}) \Psi_{2,\beta} \quad (4)$$

$$(\eta^{\mu\alpha} \Psi_{2,\mu})_{,\alpha} = -(\eta^{\beta\alpha} \Psi_{1,\beta}) \Psi_{2,\beta} \quad (5)$$

$$(\eta^{\mu\alpha} \Psi_{3,\mu})_{,\alpha} = 2(\eta^{\beta\alpha} \Psi_{1,\beta}) \Psi_{2,\beta}. \quad (6)$$

Consider equation 4, the equation for fog1. It is a type of wave equation. The left side is a four-dimensional divergence. The right side is a more complicated expression that looks superficially like the extra terms that might be introduced by a covariant derivative. The possibility arises, therefore, that the equation might be interpretable as a tensor equation in a
space-time that has some intrinsic curvature. Fog1 might follow a wave equation in which wave crests move along geodesics. If so, then the equation for fog1 would have the following form:

\[(g^{\alpha\beta} \Psi_1^\beta)_{;\alpha} = 0\]  

(7)

where the semicolon indicates a covariant derivative. In order for equation 7 to be equivalent to equation 4, there would need to exist a metric tensor g for a curved space-time such that, when the covariant derivatives are computed, equation 7 algebraically matches equation 4. A metric tensor with this property is:

\[g_{\alpha\beta} = e^{-\Psi} \eta_{\alpha\beta}.\]  

(8)

That equation 7 is algebraically equivalent to equation 4 can be verified in the following manner. First, to manipulate derivatives correctly in curved space, it is necessary to compute the Christoffel components, or \(\Gamma^\alpha_{\beta\gamma}\), associated with that curvature [3]. The metric tensor in equation 8 can be used to compute the \(\Gamma^\alpha_{\beta\gamma}\) by means of the standard equation:

\[\Gamma^\gamma_{\beta\mu} = \frac{1}{2} g^{\alpha\gamma} (g_{\alpha\mu,\beta} + g_{\alpha\beta,\mu} - g_{\beta\mu,\alpha}).\]  

(9)

The 64 Christoffel components, computed according to equation 9, are given in Appendix A. Rewriting equation 7 with the \(\Gamma^\alpha_{\beta\gamma}\) explicit yields:

\[0 = (g^{\alpha\mu} \Psi_1^\mu)_{;\alpha} = (g^{\alpha\mu} \Psi_1^\mu)_{;\alpha} + (g^{\beta\alpha} \Psi_1^\beta)_{;\alpha} \Gamma^\alpha_{\beta\gamma}.\]  

(10)

Inserting the expression for g from equation 8 yields:

\[0 = (e^{-\Psi} \eta^{\alpha\mu} \Psi_1^\mu)_{;\alpha} + (e^{-\Psi} \eta^{\beta\alpha} \Psi_1^\beta)_{;\alpha} \Gamma^\alpha_{\beta\gamma}.\]  

(11)

Inserting the expressions for \(\Gamma^\alpha_{\beta\gamma}\) from Appendix A, and cancelling and re-arranging terms, yields equation 4. In this way it is confirmed that equation 7 is algebraically equivalent to equation 4, given the metric tensor in equation 8.

It is important to note that equation 4 was not changed into equation 7 by means of a coordinate transformation from flat space to curved space. Instead, equation 4 is already a tensor equation in a space-time that has an intrinsic curvature. The intrinsic curvature is merely made explicit in equation 7.

There are two mathematically equivalent ways to describe the behavior of fog1. One way, explicit in equation 1, is that its behavior is warped by fog2 because it is being absorbed by fog2 and therefore experiences a sink that varies in space and time. Another way, explicit in
equation 7, is that it is moving according to a simple wave equation, and that any warping of its behavior is attributable to an intrinsic curvature of the space-time in which it moves.

This curvature of space-time in which fog1 moves is created by fog2. The metric tensor from equation 8 is $e^{\Psi^2} \eta$. As the magnitude of $\Psi^2$ approaches zero in any region of space-time, the metric tensor approaches $\eta$, the metric for flat space-time, and fog1 follows the homogeneous wave equation $(\eta^{\alpha\beta} \Psi_{\alpha\beta})_{,\alpha\beta} = 0$. As $\Psi^2$ becomes non-negligible, fog1 behaves as if it were following the same wave equation but in a curved space-time with metric tensor $g$. The larger $\Psi^2$, the greater the curvature. Fog1 is essentially a collection of ripples living in a curved space-time created by fog2. A symmetric argument can be made: Fog2 is ripples in a curved space-time created by fog1.

4. SOME FEATURES OF GENERAL RELATIVITY IN SA

As shown in the previous section, fog2 effectively produces a curved space-time in which fog1 moves. This curvature can be described in a number of ways in the formalism of differential geometry [2,3]. One way is by the metric tensor, as detailed in the previous section. A second way is by the Riemann curvature tensor. A third way is by a contraction of the Riemann curvature tensor called the Einstein Tensor, $G$. Given the metric tensor (from equation 8), it is possible to compute the Riemann tensor, and then to perform a contraction on the Riemann tensor thereby computing the Einstein tensor [2,3]. The 16 components of $G$, computed in this way from the metric tensor using standard formulas, are given in Appendix B. This tensor, $G$, is one way to summarize the curvature of space-time that is effectively produced by fog2.

To simplify the math, consider the case in which fog1 and fog2 have vector fields of very unequal magnitude: $|\vec{F}_1| \ll |\vec{F}_2|$ in the region of space-time that is of interest. As a result, fog2 is essentially unperturbed by fog1. We are effectively dealing with a single fog, fog2, and trying to characterize the curvature of space-time produced by that fog. Given this approximation, the equation of motion for fog2 (equation 5) can be simplified:

$$(\eta^{\alpha\beta} \Psi^2_{,\alpha\beta})_{,\alpha\beta} = 0.$$  \hfill (12)

Given this relationship, it is possible to further simplify the components of $G$. As can be seen in Appendix B, some of the terms reduce with the application of equation 12. The result is that $G$ can be written as follows:
\[ G_{\alpha\beta} = \frac{1}{2} \Psi_{2, \alpha} \Psi_{2, \beta} - \Psi_{2, \alpha, \beta} + \frac{1}{4} \Psi_{2, \mu} \Psi_{2, \mu} \eta^{\mu\nu} \eta_{\alpha\beta}. \]  

Equation 13 can be interpreted by following several different approaches. Here one approximation is discussed. If \( \Psi_2 \) is assumed to be very small everywhere, then second order terms in \( \Psi_2 \) can be ignored. The equation reduces to:

\[ G_{\alpha\beta} = -\Psi_{2, \alpha, \beta}. \]  

Let \( P_\alpha = \frac{\hbar}{i} \frac{\partial}{\partial x_\alpha} \). The operator \( P_\alpha \) is the quantum-mechanical operator for momentum. Furthermore, assume natural units in which \( \hbar = 1 \). Equation 14 can then be re-written as \( G_{\alpha\beta} = P_\alpha P_{\beta}(\Psi_2) \). This equation indicates that the curvature of space-time produced by fog2 is determined ultimately by a property of fog2, \( P_\alpha(\Psi_2) \), that exactly corresponds to the quantum-mechanical momentum. Specifically, fog2 has a stress-energy tensor \( P_\alpha P_{\beta}(\Psi_2) \). Following convention, the stress-energy tensor is notated here as \( T_{\alpha\beta} \) and equation 14 becomes:

\[ G_{\alpha\beta} = T_{\alpha\beta}. \]  

This equation matches the central equation of general relativity, \( G = KT \), in which the constant \( K = 1 \). Note that in universe SA, the relevant constants turn out to be extremely simple: \( c = 1, K = 1, \text{ and } \hbar = 1 \). The importance of equation 15 in SA is that it relates quantum-mechanical momentum to the gravitational curvature of space-time.

5. SOME FEATURES OF QUANTUM MECHANICS IN SA

This paper focuses on how fogs, elements in universe SA, combine by absorption. Fog1 and fog2 absorb each other in a graded manner over space and time to produce fog3. The previous sections focused on how fog1 affects fog2 and vice versa. This mutual influence can be compared to gravity. Fog1 and fog2 exert an effect on each other that is equivalent to a curvature of space-time and that follows the equations of general relativity. In analyzing that process, in the previous section, it was found that a fog in SA acts as though it has a four-momentum. The four-momentum, described by means of the quantum-mechanical momentum operator, ultimately leads to a stress-energy tensor and hence to a gravitational field.
The present section ignores the gravitational effect of fog1 and fog2 on each other, and focuses instead on the behavior of fog3. How do the properties of fog3 depend on the properties of fog1 and fog2? In particular, is the four-momentum of fog3 the sum of the four-momenta of fog1 and fog2? If so, then momentum is conserved during an absorption, and momentum has the same properties in SA as in the real universe.

In the real universe, if one quantum particle such as an electron absorbs another such as a photon, and we consider how the momenta of the two original particles are combined in the resultant particle, we would normally consider the gravitational interaction between the two particles to be negligible. Here we will do the same. Consider an approximation in which gravitation-like effects in SA are ignored. The equations for fog1 and fog2 (from equations 4 and 5) become:

\[
(\eta^{\alpha\mu}\Psi^1_{\alpha\mu}) = (\eta^{\alpha\mu}\Psi^2_{\alpha\mu}) = 0. \tag{16}
\]

A solution for \( \Psi^1 \) that satisfies equation 16 is a Fourier sum of terms for which each term has the form:

\[
\Phi_1 = A_1 e^{i(p_1 \cdot x^n)}. \tag{17}
\]

In this expression, three new symbols are introduced. First, \( \Phi_1 \) represents a single term in a sum. Second, \( A_1 \) is the amplitude of that term relative to other terms in the sum. Third, \( p_1 \) is the eigenvalue of the momentum operator, with respect to that term in the sum. A similar treatment applies to fog2.

Now consider the behavior of fog3, the result of the absorption between fog1 and fog2. The equation for fog3 (from equation 6) is:

\[
(\eta^{\alpha\mu}\Psi^3_{\alpha\mu}) = 2(\eta^{\beta\mu}\Psi^1_{\alpha\mu})\Psi^2_{\beta\cdot}. \tag{18}
\]

A solution that satisfies equation 18 is:

\[
\Psi^3 = \Psi^1\Psi^2. \tag{19}
\]

Let one particular term in \( \Psi^1 \) be represented by \( \Phi_1 = A_1 e^{i(p_1 \cdot x^n)} \) (from equation 17). Likewise let one particular term in \( \Psi^2 \) be represented by \( \Phi_2 = A_2 e^{i(p_2 \cdot x^n)} \). By equation 19, \( \Psi^3 \) contains a sum of terms one of which is:

\[
\Phi_3 = A_3 e^{i(p_3 \cdot x^n)} = A_1 A_2 e^{i(p_1 + p_2 \cdot x^n)}. \tag{20}
\]

In equation 20, \( p_3 = p_1 + p_2 \) and \( A_3 = A_1 A_2 \).
In quantum mechanics one would say that $\Phi_1$ is one possible momentum state of particle 1, has a probability amplitude of $A_1A_1^*$, and has a four-momentum of $p_1^\alpha$; $\Phi_2$ is one possible state of particle 2, has a probability amplitude of $A_2A_2^*$, and has a four-momentum of $p_2^\alpha$; and $\Phi_3$ is one possible state of particle 3, has a probability amplitude of $A_3A_3^* = A_1A_1^*A_2A_2^*$, and has a four-momentum of $p_3^\alpha = p_1^\alpha + p_2^\alpha$. The probabilities multiply and the momenta add. Therefore at least some aspects of quantum mechanics emerge naturally in universe SA. Objects in SA carry a quantum-mechanical momentum; when one object is absorbed by another, their momenta are added together in the resultant object, and hence momentum is conserved in an absorption; and momentum states can be thought of as probabilistic, each object with many momentum states, each momentum state with an amplitude, and the amplitudes combining in a quantum-mechanical manner.

6. MORE THAN THREE FOGS

Thus far in this account a specific example was analyzed: fog1 and fog2 absorb each other to produce fog3. Consider a more general case. Suppose there are N fogs such that fog1 can be absorbed by fogs 2-N. Each absorption results in a sink in fog1. Let $\bar{\Psi}_1 = \sum_{n=2-N} \Psi_n$. One can read $\bar{\Psi}_1$ as “the sum of the scalar fields of all fogs that can absorb fog1.” The same logic used in sections 1-3 implies:

$$ (\eta^{\alpha\mu}\Psi_{1,\mu})_{,\alpha} = -\eta^{\beta\mu}(\Psi_{1,\mu})_{,\beta} \bar{\Psi}_1. $$

Equation 21 indicates that the total sink in fog1 is a sum of sinks caused by the absorption of fog1 by a set of other fogs. For the same reasons outlined in section 3, equation 21 is equivalent to the equation $(g^{\alpha\beta}\Psi_{1,\alpha})_{,\beta} = 0$ in a curved space-time in which the metric tensor is:

$$ g_{\alpha\beta} = e^{\Phi_1} \eta_{\alpha\beta}. $$

Here the metric tensor depends on $\bar{\Psi}_1$, the sum of the scalar fields of all fogs that can absorb fog1. Equation 22 implies that fogs 2-N collectively create a curved space-time for fog1.

Consider now the most general case in SA, a fog that has both a source and a sink. Suppose fog1 and fog2 absorb each other to produce fog3; but the triplet is embedded in a larger universe that includes other fogs that can absorb fog3. Let $\bar{\Psi}_3$ be the sum of the scalar fields of
all fogs that can absorb fog3. In this case, fog3 has both a source (caused by $\Psi_1$ and $\Psi_2$) and a sink (caused by $\Psi_3$). Its equation is therefore:

$$\eta^{\alpha\mu} \Psi_3^{,\alpha} = 2(\eta^{\beta\nu} \Psi_1^{,\mu}) \Psi_2^{,\beta} - (\eta^{\beta\nu} \Psi_3^{,\mu}) \Psi_3^{,\beta}.$$  

(23)

By a similar derivation as in section 3, equation 23 is equivalent to the equation:

$$g^{\alpha\mu} \Psi_3^{,\alpha} = 2(g^{\beta\nu} \Psi_1^{,\mu}) \Psi_2^{,\beta}$$

(24)

in a curved space in which $g_{\alpha\beta} = \epsilon^{\Psi_3}_{,\alpha\beta}$. In equation 24, the sink term is expressed as a space-time curvature, and the source term remains valid in that curved space-time.

Equation 24 is the most general equation for a fog in SA. In essence, in SA, fogs can experience sinks and sources as a result of interacting. The sink terms are equivalent to a curved space-time in which the fogs move, and are ultimately responsible for gravity in SA. The source terms supply the conservation of momentum and some basic properties of quantum mechanics.

7. SUMMARY AND FURTHER DIRECTIONS

Quantum gravity has been studied in an enormous variety of ways [9]. The hypothetical universe SA has its own version of quantum gravity, in which quantum mechanics and general relativity emerge from a simple underlying definition of absorption. SA is nothing more than a precise definition of how two distributed entities absorb each other in space-time. Fog1 and fog2 absorb each other to produce fog3.

The absorption includes two related processes: sinks and sources.

In the first process, fog1 and fog2 experience sinks as they absorb each other. This process results in general relativity, in which each fog effectively produces a curvature of space-time that can affect other fogs. One of the more peculiar properties of SA is that its version of quantum gravity has no gravitons. That statement may seem contradictory, but it is not once SA is understood. In SA, gravity is not the result of momentum being carried from place to place via quantized particles. It is the sink in fog1 and fog2 as they absorb each other. Fog1 and Fog2 do not exchange force-carrying particles, and yet they exert a gravitational effect on each other.

In the second process in SA, fog1 and fog2 provide a source for fog3. In particular, the momentum of fog3 is the sum of the momenta of fog1 and fog2. In this sense fogs in SA act like force-carrying particles, absorbing each other and thereby transferring momentum to each other. SA could be said to contain two forces that are necessary flip-sides of each other. One is gravity
that acts as a graded curvature of space-time, and the other is a more standard quantized force carried by particles. Both emerge from the same underlying process of absorption, and SA cannot have one without the other.

The properties of SA described in the present report are not exhaustive. Many more physical properties that may be of interest are not pursued here. In the examples provided above, each fog in SA is described by a wave equation with a wave speed of 1. In this sense a fog is similar to a zero-rest-mass particle. However, because of the special nature of gravity in SA, it is possible to construct an object that effectively has a rest mass. Consider again the case of two fogs, fog1 and fog2, that absorb each other to produce fog3. The interaction between fog1 and fog2 can be described as a gravitational interaction. Suppose that the scalar field of fog1 is spatially peaked in its magnitude. In principle, it could be so sharply peaked as to create a local, gravitational event horizon from which other fogs cannot escape. Suppose that fog2 is also so sharply peaked spatially as to produce a gravitational event horizon around its peak. Now consider the interaction of the two fogs. These two peaks could in principle capture each other gravitationally and create a single event horizon. This joining of two gravitational event horizons is reminiscent of the absorption of one string loop by another in string theory [10]. The entity contains the combined four-momenta of fog1 and fog2, incorporates complicated vibration and spin properties, and can be at rest in some coordinate frame. It is an exotic particle constructed out of two fogs that are gravitationally interacting with each other. Other particles could be constructed in similar ways. The equations to describe such objects are not pursued here. The point of this example is that a complicated chemistry can exist in SA. From its simple initial premis of absorption, enormous complexity can emerge.

Appendix A. Table for $\Gamma$ given $g_{\alpha\beta} = e^{\Psi_2} \eta_{\alpha\beta}$. $\Gamma$ is computed from the metric tensor based on standard formulas from general relativity [2,3].

\[
\begin{align*}
\Gamma^0_{00} &= \frac{1}{2} \Psi_2, \\
\Gamma^0_{01} &= \frac{1}{2} \Psi_2, \\
\Gamma^0_{02} &= \frac{1}{2} \Psi_2, \\
\Gamma^0_{03} &= \frac{1}{2} \Psi_2, \\
\Gamma^0_{10} &= \frac{1}{2} \Psi_2, \\
\Gamma^0_{11} &= \frac{1}{2} \Psi_2, \\
\Gamma^0_{12} &= 0, \\
\Gamma^0_{13} &= 0.
\end{align*}
\]
\[ \gamma^0_{20} = \frac{1}{2} \psi_{2\nu} \]
\[ \gamma^0_{21} = 0 \]
\[ \gamma^0_{22} = \frac{1}{2} \psi_{2\nu} \]
\[ \gamma^0_{23} = 0 \]

\[ \gamma^0_{30} = \frac{1}{2} \psi_{2\eta} \]
\[ \gamma^0_{31} = 0 \]
\[ \gamma^0_{32} = 0 \]
\[ \gamma^0_{33} = \frac{1}{2} \psi_{2\nu} \]

\[ \gamma^1_{00} = \frac{1}{2} \psi_{2\nu} \]
\[ \gamma^1_{01} = \frac{1}{2} \psi_{2\eta} \]
\[ \gamma^1_{02} = 0 \]
\[ \gamma^1_{03} = 0 \]

\[ \gamma^1_{10} = \frac{1}{2} \psi_{2\nu} \]
\[ \gamma^1_{11} = \frac{1}{2} \psi_{2\eta} \]
\[ \gamma^1_{12} = \frac{1}{2} \psi_{2\nu} \]
\[ \gamma^1_{13} = \frac{1}{2} \psi_{2\eta} \]

\[ \gamma^1_{20} = 0 \]
\[ \gamma^1_{21} = \frac{1}{2} \psi_{2\nu} \]
\[ \gamma^1_{22} = -\frac{1}{2} \psi_{2\eta} \]
\[ \gamma^1_{23} = 0 \]

\[ \gamma^1_{30} = 0 \]
\[ \gamma^1_{31} = \frac{1}{2} \psi_{2\nu} \]
\[ \gamma^1_{32} = 0 \]
\[ \gamma^1_{33} = -\frac{1}{2} \psi_{2\eta} \]

\[ \gamma^2_{00} = \frac{1}{2} \psi_{2\nu} \]
\[ \gamma^2_{01} = 0 \]
\[ \gamma^2_{02} = \frac{1}{2} \psi_{2\nu} \]
\[ \gamma^2_{03} = 0 \]

\[ \gamma^2_{10} = 0 \]
\[ \gamma^2_{11} = -\frac{1}{2} \psi_{2\nu} \]
\[ \gamma^2_{12} = \frac{1}{2} \psi_{2\eta} \]
\[ \gamma^2_{13} = 0 \]

\[ \gamma^2_{20} = \frac{1}{2} \psi_{2\nu} \]
\[ \gamma^2_{21} = \frac{1}{2} \psi_{2\eta} \]
\[ \gamma^2_{22} = \frac{1}{2} \psi_{2\nu} \]
\[ \gamma^2_{23} = \frac{1}{2} \psi_{2\eta} \]

\[ \gamma^2_{30} = 0 \]
\[ \gamma^2_{31} = 0 \]
\[ \gamma^2_{32} = \frac{1}{2} \psi_{2\nu} \]
\[ \gamma^2_{33} = -\frac{1}{2} \psi_{2\eta} \]

\[ \gamma^3_{00} = \frac{1}{2} \psi_{2\nu} \]
\[ \gamma^3_{01} = 0 \]
\[ \gamma^3_{02} = 0 \]
\[ \gamma^3_{03} = \frac{1}{2} \psi_{2\nu} \]
\( \Gamma^3_{10} = 0 \quad \Gamma^3_{11} = -\frac{1}{2} \Psi_{2,3} \quad \Gamma^3_{12} = 0 \quad \Gamma^3_{13} = \frac{1}{2} \Psi_{2,1} \)

\( \Gamma^3_{20} = 0 \quad \Gamma^3_{21} = 0 \quad \Gamma^3_{22} = -\frac{1}{2} \Psi_{2,3} \quad \Gamma^3_{23} = \frac{1}{2} \Psi_{2,2} \)

\( \Gamma^3_{30} = \frac{1}{2} \Psi_{2,0} \quad \Gamma^3_{31} = \frac{1}{2} \Psi_{2,1} \quad \Gamma^3_{32} = \frac{1}{2} \Psi_{2,2} \quad \Gamma^3_{33} = \frac{1}{2} \Psi_{2,3} \)

**Appendix B.** Table for Einstein tensor \( G_{\alpha\beta} \) given the metric tensor \( g_{\alpha\beta} = e^{\psi} \eta_{\alpha\beta} \). \( G \) is computed from the metric tensor using standard formulas from general relativity [2,3].

\[ G_{00} = \frac{1}{2} \Psi_{2,0} \Psi_{2,0} - \Psi_{2,1,1} - \Psi_{2,2,2} - \Psi_{2,3,3} + \frac{1}{4} \Psi_{2,0} \Psi_{2,0} - \frac{1}{4} \Psi_{2,1} \Psi_{2,1} - \frac{1}{4} \Psi_{2,2} \Psi_{2,2} - \frac{1}{4} \Psi_{2,3} \Psi_{2,3} \]

\[ G_{01} = \frac{1}{2} \Psi_{2,0} \Psi_{2,1} - \Psi_{2,0,1} \]

\[ G_{02} = \frac{1}{2} \Psi_{2,0} \Psi_{2,2} - \Psi_{2,0,2} \]

\[ G_{03} = \frac{1}{2} \Psi_{2,0} \Psi_{2,3} - \Psi_{2,0,3} \]

\[ G_{10} = \frac{1}{2} \Psi_{2,1} \Psi_{2,0} - \Psi_{2,1,0} \]

\[ G_{11} = \frac{1}{2} \Psi_{2,1} \Psi_{2,1} + \Psi_{2,2,2} + \Psi_{2,3,3} - \frac{1}{4} \Psi_{2,0} \Psi_{2,0} + \frac{1}{4} \Psi_{2,1} \Psi_{2,1} + \frac{1}{4} \Psi_{2,2} \Psi_{2,2} + \frac{1}{4} \Psi_{2,3} \Psi_{2,3} \]
\[ G_{12} = \frac{1}{2} \psi_{2,1} \psi_{2,2} - \psi_{1,2} \]
\[ G_{13} = \frac{1}{2} \psi_{2,2} \psi_{2,3} - \psi_{1,3} \]
\[ G_{20} = \frac{1}{2} \psi_{2,2} \psi_{2,0} - \psi_{2,2,0} \]
\[ G_{21} = \frac{1}{2} \psi_{2,2} \psi_{2,1} - \psi_{2,2,1} \]
\[ G_{22} = \frac{1}{2} \psi_{2,2} \psi_{2,2} - \psi_{2,0,0} + \psi_{2,1,1} + \psi_{2,3,3} - \frac{1}{4} \psi_{2,2} \psi_{2,2} + \frac{1}{4} \psi_{2,2} \psi_{2,2} + \frac{1}{4} \psi_{2,2} \psi_{2,2} + \frac{1}{4} \psi_{2,2} \psi_{2,2} \]
\[ G_{23} = \frac{1}{2} \psi_{2,2} \psi_{2,3} - \psi_{2,2,3} \]
\[ G_{30} = \frac{1}{2} \psi_{2,3} \psi_{2,0} - \psi_{2,3,0} \]
\[ G_{31} = \frac{1}{2} \psi_{2,3} \psi_{2,1} - \psi_{2,3,1} \]
\[ G_{32} = \frac{1}{2} \psi_{2,3} \psi_{2,2} - \psi_{2,3,2} \]
\[ G_{33} = \frac{1}{2} \psi_{2,3} \psi_{2,3} - \psi_{2,0,0} + \psi_{2,1,1} + \psi_{2,2,2} - \frac{1}{4} \psi_{2,2} \psi_{2,2} + \frac{1}{4} \psi_{2,2} \psi_{2,2} + \frac{1}{4} \psi_{2,2} \psi_{2,2} + \frac{1}{4} \psi_{2,2} \psi_{2,2} \]
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