Numerical Simulations for "Matching, Sorting and the Distributional Effects of International Trade" by G. Grossman, E. Helpman and P. Kircher

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This appendix serves three purposes. First, in section 1, it describes the computational algorithm employed to solve the Grossman, Helpman and Kircher (henceforth, GHK) model numerically. Second, in section 2, it documents the Matlab code to reproduce figures in the main text generated by numerical computation. Finally, in section 3, it expands on the discussion in GHK concerning the effect of trade on earnings inequality, by studying numerically the comparative statics of the model with respect to output prices under a range of parameter values.

1 Solution Approach and Numerical Algorithms

In this section, we summarize the equations defining an equilibrium allocation in the GHK model and then discuss how to solve these equations numerically.

Recall that in the model, there are two factors of production - workers and managers - that are both heterogeneous in terms of a one-dimensional type, referred to as "ability" for concreteness. The inelastic supply of workers with ability q_L is $\bar{L}\phi_L(q_L)$, where $\bar{L}$ is the aggregate measure of workers in the economy and ϕ_L is a probability density function over worker abilities with support $S_L = [q_{Lmin}, q_{Lmax}]$. Similarly, the inelastic supply of managers with ability q_H is $\bar{H}\phi_H(q_H)$, where $\bar{H}$ denotes the aggregate measure of managers and ϕ_H is a probability density function over manager abilities with support $S_H = [q_{Hmin}, q_{Hmax}]$. Workers and managers can be employed in two sectors $i \in \{1, 2\}$, where the production technology of sector i is such that if a manager of ability q_H hires l workers of ability q_L, output is given by $x_i = \psi_i(q_H, q_L)\, l^{\gamma_i}$. The productivity function ψ_i for $i \in \{1, 2\}$ is assumed to be strictly increasing and continuously differentiable in both arguments, and also to be log supermodular.

As discussed in section 3.1 of GHK, when the productivity functions ψ_i take a Cobb-Douglas form (such that each ψ_i is log supermodular but not strictly so), the model admits closed-form solutions for the equilibrium wage and salary functions, but leaves matching between workers and managers of different abilities as an indeterminate outcome. In what follows, we therefore focus on the case in which each ψ_i is strictly log supermodular. As discussed in section 3.2 of the main text, the solutions for the matching, wage, and salary functions must then satisfy the following conditions:

$$r\left[\mu\left(q_L\right)\right] = \bar{\gamma}_i p_i^{\frac{1}{1-\gamma_i}} \psi_i\left[\mu\left(q_L\right), q_L\right]^{\frac{1}{1-\gamma_i}} w\left(q_L\right)^{-\frac{\gamma_i}{1-\gamma_i}}, \quad \forall q_L \in Q_{Li},\ i = 1, 2; \tag{1.1}$$

$$\frac{w'\left(q_L\right)}{w\left(q_L\right)} = \frac{\psi_{iL}\left[\mu\left(q_L\right), q_L\right]}{\gamma_i \psi_i\left[\mu\left(q_L\right), q_L\right]}, \quad \forall \left\{\mu\left(q_L\right), q_L\right\} \in M_i^{n,int},\ n \in N_i,\ i = 1, 2; \tag{1.2}$$

$$\mu'\left(q_L\right) = \frac{\left(1-\gamma_i\right) \bar{L} \phi_L\left(q_L\right) w\left(q_L\right)}{\gamma_i \bar{H} \phi_H\left[\mu\left(q_L\right)\right] r\left[\mu\left(q_L\right)\right]}, \quad \forall \left\{\mu\left(q_L\right), q_L\right\} \in M_i^{n,int},\ n \in N_i,\ i = 1, 2. \tag{1.3}$$

As in the main text, $w(\cdot)$ and $r(\cdot)$ denote the wage and salary functions respectively, Q_{Li} denotes the set of workers hired in sector i, and $\left\{M_i^{n,int}\right\}_{n=1}^{N_i}$ denotes the interiors of the sets $\{M_i^n\}_{i=1}^{N_i}$, the union of which comprise the graph $M_i = \left[\{q_H, q_L\} \,|\, q_H = \mu(q_L)\, \forall q_L \in Q_{Li}\right]$. Equation (1.1) follows from the zero-profit condition, equation (1.2) from the first-order condition with respect to worker ability, and equation (1.3) from labor market clearing.

Note that here we choose to work with the inverse matching function $\mu(\cdot)$, where $\mu(q_L) = \{q_H | m(q_H) = q_L\}$ is the ability of managers that match to workers with ability q_L, instead of the matching function $m(\cdot)$ used in the main text. This allows us to solve jointly for $\mu(\cdot)$ and $w(\cdot)$ with both as functions of worker ability q_L. An alternative and equivalent approach would be to solve jointly for $m(\cdot)$ and $r(\cdot)$ with both as functions of manager ability q_H.

To solve the system of equations (1.1)-(1.3), we first substitute (1.1) into (1.3) to eliminate the salary function $r(\cdot)$, obtaining:

$$\mu'\left(q_L\right) = \left[\frac{\bar{L}\phi_L\left(q_L\right)}{\bar{H}\phi_H\left[\mu\left(q_L\right)\right]}\right] \left[\frac{w\left(q_L\right)}{\gamma_i p_i \psi_i\left[\mu\left(q_L\right), q_L\right]}\right]^{\frac{1}{1-\gamma_i}}, \quad \forall \left\{\mu\left(q_L\right), q_L\right\} \in M_i^{n,int},\ n \in N_i,\ i = 1, 2 \tag{1.4}$$

Equations (1.2) and (1.4) give a system of two differential equations in the unknown functions $w(\cdot)$ and $\mu(\cdot)$. With the appropriate boundary conditions, we can solve these equations numerically, and then use equation (1.1) to recover the salary function.

However, numerical solution of the model is complicated by the fact that the appropriate boundary conditions for equations (1.2) and (1.4) depend on the sorting pattern of workers and managers to sectors (specifically, the form of the graphs $\{M_i\}_{i\in\{1,2\}}$), which is itself an equilibrium outcome. Therefore, the approach that we adopt to solve the model is to first fix the sorting pattern of interest, and then try to determine whether a given set of parameter values is consistent with an equilibrium of that form.

1.1 Two Regions of Sorting

In this section, we discuss the solution approach for the case in which each of the sets Q_{Li} and Q_{Hi} is an interval, such that each graph M_i consists of a single connected set (section 1.2 discusses the solution approach for more complicated sorting patterns). In this case, there exist cutoff ability levels $q_L^* \in S_L$ and $q_H^* \in S_H$, with workers of ability $q_L \geq q_L^*$ sorting into one sector and workers of ability $q_L < q_L^*$ sorting into the other sector, and similarly for managers. Within this class of threshold equilibria, there are two qualitatively distinguishable patterns of sorting (we label the sectors such that the best workers always sort to sector 1, without loss of generality).

First, a threshold equilibrium could have the best workers and best managers sorting to the same sector, which we refer to as a high-high/low-low (HH/LL) equilibrium. As stated in Proposition 4 of GHK, sufficient conditions for an HH/LL equilibrium are

$$\frac{\psi_{1H}(q_H, q_L)}{(1-\gamma_1)\psi_1(q_H, q_L)} > \frac{\psi_{2H}(q_H, q_L)}{(1-\gamma_2)\psi_2(q_H, q_L)}, \ \ \forall q_H \in S_H, \, q_L \in S_L \tag{1.5}$$

$$\frac{\psi_{1L}(q_H, q_L)}{\gamma_1\psi_1(q_H, q_L)} > \frac{\psi_{2L}(q_H, q_L)}{\gamma_2\psi_2(q_H, q_L)}, \ \ \forall q_H \in S_H, \, q_L \in S_L \tag{1.6}$$

The boundary conditions accompanying equations (1.2) and (1.4) are then as follows:

1. continuity of $w(\cdot)$ at q_L^*,
2. continuity of $\mu(\cdot)$ at q_L^*,
3. $\mu(q_{Lmin}) = q_{Hmin}$, and
4. $\mu(q_{Lmax}) = q_{Hmax}$.

Second, the best workers and the worst managers could sort to the same sector, which we refer to as a high-low/low-high (HL/LH) equilibrium. As stated in Propositions 2 and 3 of GHK, sufficient conditions for an HL/LH equilibrium are

$$\frac{\psi_{2H}(q_H, q_{Lmin})}{(1-\gamma_2)\psi_2(q_H, q_{Lmin})} > \frac{\psi_{1H}(q_H, q_{Lmax})}{(1-\gamma_1)\psi_1(q_H, q_{Lmax})}, \ \ \forall q_H \in S_H \tag{1.7}$$

$$\frac{\psi_{1L}(q_{Hmin}, q_L)}{\gamma_1\psi_1(q_{Hmin}, q_L)} > \frac{\psi_{2L}(q_{Hmax}, q_L)}{\gamma_2\psi_2(q_{Hmax}, q_L)}, \ \ \forall q_L \in S_L \tag{1.8}$$

The boundary conditions accompanying equations (1.2) and (1.4) are then:

1. continuity of $w(\cdot)$ at q_L^*,

2. $\mu(q_{Lmin}) = q_H^*$,

3. $\mu(q_{Lmax}) = q_H^*$, and

4. $\mu\left(q_L^{*-}\right) = q_{Hmax}$ and $\mu\left(q_L^{*+}\right) = q_{Hmin}$, where $q_L^{*-} = \lim_{q \nearrow q_L^*} q$ and $q_L^{*+} = \lim_{q \searrow q_L^*} q$.

Regardless of whether the equilibrium is of the HH/LL or the HL/LH form, the boundary conditions specified above allow us to solve equations (1.2) and (1.4) numerically for a given value of q_L^*.

In the Matlab file **GHK_algorithm.m**, this computation is performed using the **bvp4c** solver, which is capable of solving multipoint boundary value problems such as the one described above. The solver requires separate functions that specify (i) the differential equations, (ii) the boundary conditions, and (iii) initial guesses for the wage and matching functions. In the Matlab file, the differential equations are specified in the function **odefun_2sec**, while the boundary conditions and initial guesses are specified in the functions **bcfun_2sec_HHLL** and **yinit_2sec_HHLL** respectively for the HH/LL equilibrium case, and **bcfun_2sec_HLLH** and **yinit_2sec_HLLH** for the HL/LH case.[1]

For any given value of q_L^*, the **bvp4c** solver yields solutions for the matching, wage, and salary functions that are consistent with equations (1.1)-(1.3) and the boundary conditions. However, the zero-profit condition (1.1) only ensures that a manager of a given ability $q_H \in Q_{Hi}$ employed in a sector i cannot earn positive profits by hiring workers of any ability, *if that manager remains in sector i*. That is, $\Pi_i(q_H) = 0$ for all $q_H \in Q_{Hi}$ but not necessarily for all $q_H \in Q_{Hj}$ with $j \neq i$, where the profit functions are defined by:

$$\Pi_i(q_H) \equiv \max_{q_L \in S_L} \pi_i(q_H, q_L) \tag{1.9}$$

$$\pi_i(q_H, q_L) \equiv \bar{\gamma}_i p_i^{\frac{1}{1-\gamma_i}} \psi_i(q_H, q_L)^{\frac{1}{1-\gamma_i}} w(q_L)^{-\frac{\gamma_i}{1-\gamma_i}} - r(q_H) \tag{1.10}$$

Therefore, in solving for the matching, wage, and salary functions, we must adjust the worker ability cutoff q_L^* until the solutions obtained do not enable managers to make positive profits by hiring workers of any ability, even after allowing managers to switch the sector in which they operate. The outline of this algorithm is summarized below:

1. Guess a value for the worker ability cutoff $q_L^* \in S_L$.

[1] See the Matlab help file on the **bvp4c** function for more details about the syntax and implementation of the solver.

2. Given this value of q_L^*, solve the system of differential equations (1.2) and (1.4) using the appropriate boundary conditions, and compute the implied salary function using equation (1.1).

3. Using the solutions for $\mu(\cdot)$, $w(\cdot)$, and $r(\cdot)$, compute the profit differentials for managers from switching sectors, $\Delta\Pi_i[\mu(q_L)] = \pi_i[\mu(q_L), q_L] - \Pi_j[\mu(q_L)]$, $j \neq i$, and check that these differentials are non-positive within some tolerance $\varepsilon > 0$.[2]

   (a) If $\Delta\Pi_1[\mu(q_L)] \leq \varepsilon$ for all $q_L \in Q_{L1}$ but $\Delta\Pi_2[\mu(q_L)] > \varepsilon$ for some $q_L \in Q_{L2}$, adjust q_L^* upwards and repeat from step 1.

   (b) If $\Delta\Pi_2[\mu(q_L)] \leq \varepsilon$ for all $q_L \in Q_{L2}$ but $\Delta\Pi_1[\mu(q_L)] > \varepsilon$ for some $q_L \in Q_{L1}$, adjust q_L^* downwards and repeat from step 1.

4. Once $\Delta\Pi_i[\mu(q_L)] \leq \varepsilon$ for all $q_L \in Q_{Li}$ for both $i = 1, 2$, check that $\Pi_i(\mu(q_L)) = 0$ for all $q_L \in Q_{Li}$ for both $i = 1, 2$.

Note that, in determining the direction of adjustment for q_L^* in step 3 of the algorithm, it is possible in principle that there exists some $q_L \in Q_{Li}$ such that$\Delta\Pi_i[\mu(q_L)] > \varepsilon$, for *both* $i = 1, 2$. In this case, the algorithm breaks down. However, we find that whenever the sufficient conditions (1.5)-(1.6) or (1.7)-(1.8) are satisfied and we search for an equilibrium with the appropriate sorting pattern, this problem is never encountered in practice.

Also, note that the final check on the zero-profit condition in step 4 is needed because equation (1.2) is a first-order condition that is necessary but not sufficient to ensure zero profits for any manager (the typical second order condition depends on $w(\cdot)$ and $r(\cdot)$, which are endogenous). Therefore, while equations (1.1) and (1.2) guarantee that $\pi_i[q_H, \mu^{-1}(q_H)] = 0$ for all $q_H \in Q_{Hi}$, they do not rule out the possibility that $\mu^{-1}(q_H)$ is a local but not global maximizer of (1.9), so that $\Pi_i(q_H) > \pi_i(q_H, \mu^{-1}(q_H))$ for some $q_H \in Q_{Hi}$. Nonetheless, any solution for the matching, wage, and salary functions obtained via the algorithm described above is by design consistent with equations (1.1)-(1.3), the appropriate boundary conditions, as well as zero maximal profits for all firms, and therefore accurately characterizes an equilibrium of the model.

The Matlab file **GHK_algorithm.m** implements the above algorithm using the following search routine on q_L^*. First, it sets the bounds for the cutoff worker ability to be $[q_{Lmin}^*, q_{Lmax}^*] = [q_{Lmin}, q_{Lmax}]$. Then, it sets the initial guess to be $q_L^* = \frac{q_{Lmin}^* + q_{Lmax}^*}{2}$. To adjust the guess upwards in step 3 of the algorithm, it sets q_{Lmin}^* equal to the current value

[2] Note that when a manager with ability $\mu(q_L)$ switches sectors, he does not necessarily employ workers of ability q_L, but rather the best workers given his type.

of the guess for q_L^*; to adjust the guess downwards, it sets q_{Lmax}^* equal to the current value of the guess for q_L^*. This routine halves the search region for q_L^* with every iteration.

1.2 More than Two Regions of Sorting

When neither conditions (1.5) and (1.6) nor (1.7) and (1.8) are satisfied, we can no longer be sure *a priori* about the sorting pattern of managers and workers in equilibrium, which makes numerical solution of the model a more challenging problem. Specifically, the difficulty arises from the fact that implementation of the **bvp4c** solver requires identification of the number of distinct regions that characterize the differential equation system, as well as specification of the boundary conditions that automatically fix the sorting pattern being considered. Nonetheless, the approach to solving the model numerically for the case in which there are more than two regions of sorting is qualitatively similar to the case with only two regions of sorting.

First, for a given number of sorting regions, we identify all possible types of sorting patterns that could obtain in equilibrium. For example, with three regions of sorting, there are two pairs of ability cutoffs $\{q_L^*, q_H^*\}$ and $\{q_L^{**}, q_H^{**}\}$, with $q_{Lmin} \leq q_L^* < q_L^{**} \leq q_{Lmax}$ and $q_{Hmin} \leq q_H^* < q_H^{**} \leq q_{Hmax}$. The fact that any equilibrium must exhibit positive assortative matching within each sector then implies that there are three possible patterns of sorting:

1. Workers of ability $q_L > q_L^{**}$ sort to sector 1 and match with managers of ability $q_H > q_H^{**}$; workers of ability $q_L \in (q_L^*, q_L^{**}]$ sort to sector 2 and match with managers of ability $q_H \in (q_H^*, q_H^{**}]$; workers of ability $q_L \leq q_L^*$ sort to sector 1 and match with managers of ability $q_H \leq q_H^*$. (high-high/mid-mid/low-low equilibrium, HH/MM/LL)

2. Workers of ability $q_L > q_L^{**}$ sort to sector 1 and match with managers of ability $q_H \in (q_H^*, q_H^{**}]$; workers of ability $q_L \in (q_L^*, q_L^{**}]$ sort to sector 2 and match with managers of ability $q_H > q_H^{**}$; workers of ability $q_L \leq q_L^*$ sort to sector 1 and match with managers of ability $q_H \leq q_H^*$. (high-mid/mid-high/low-low equilibrium, HM/MH/LL)

3. Workers of ability $q_L > q_L^{**}$ sort to sector 1 and match with managers of ability $q_H > q_H^{**}$; workers of ability $q_L \in (q_L^*, q_L^{**}]$ sort to sector 2 and match with managers of ability $q_H \leq q_H^*$; workers of ability $q_L \leq q_L^*$ sort to sector 1 and match with managers of ability $q_H \in (q_H^*, q_H^{**}]$. (high-high/mid-low/low-mid equilibrium, HH/ML/LM)

Next, for each possible sorting pattern, we specify the boundary conditions for the numerical solver. For example, for an HH/MM/LL equilibrium, the six boundary conditions would be:

1. continuity of $w(\cdot)$ at q_L^*,

2. continuity of $w(\cdot)$ at q_L^{**},

3. continuity of $\mu(\cdot)$ at q_L^{*},

4. continuity of $\mu(\cdot)$ at q_L^{**},

5. $\mu(q_{Lmin}) = q_{Hmin}$, and

6. $\mu(q_{Lmax}) = q_{Hmax}$.

We can then proceed using the same algorithm as in the previous section, guessing values for the cutoff worker ability levels q_L^* and q_L^{**}, and adjusting these guesses until profitable deviations are ruled out for all managers.

In contrast to the problem of solving for threshold equilibria with only two regions of sorting, however, an additional complication that arises here is that the algorithm requires a search routine on two cutoff values, q_L^* and q_L^{**}, instead of only one. Therefore, the search routine described in section 1.1 that halves the search region with every iteration can no longer be employed. Since the goal of the numerical analysis in this section is simply to show that the GHK model can admit sorting patterns more complicated than those described in section 1.1, we refrain from tackling the more challenging problem of implementing efficient search routines on a two-dimensional space. Instead, we present an example of a non-threshold equilibrium.

Suppose that the distributions of worker and manager abilities are truncated Pareto with shape parameters k_L and k_H respectively:

$$\phi_L(q_L) = \frac{k_L (q_{Lmin})^{k_L} (q_L)^{-k_L-1}}{1-\left(\frac{q_{Lmin}}{q_{Lmax}}\right)^{k_L}} \tag{1.11}$$

$$\phi_H(q_H) = \frac{k_H (q_{Hmin})^{k_H} (q_H)^{-k_H-1}}{1-\left(\frac{q_{Hmin}}{q_{Hmax}}\right)^{k_H}} \tag{1.12}$$

and that the productivity function ψ_i in sector i is given by:

$$\psi_i(q_H, q_L) = (\alpha_i q_L^{\rho_i} + \beta_i q_H^{\rho_i})^{\frac{\alpha_i+\beta_i}{\rho_i}} \tag{1.13}$$

with $\rho_i < 0$. (Note that this specification of the productivity function is strictly log supermodular for any $\rho_i < 0$, and approaches the Cobb-Douglas specification discussed in sections 3.1 and 5.1 of GHK as ρ_i approaches 0). By manually adjusting the cutoff values q_L^* and q_L^{**} and checking for consistency with equilibrium, we then find that an example of parameter values generating an equilibrium with three regions of sorting (specifically, one of the HH/ML/LM form) is listed in Table 1. The cutoff values for the manager and worker qualities are

$$q_H^* = 1.0584,$$
$$q_H^{**} = 1.0853,$$
$$q_L^* = 1.1577,$$
$$q_L^{**} = 1.5115,$$

and the resulting matching, wage, and salary functions are shown in Figure 1.

[Table 1 about here.]

[Figure 1 about here.]

2 Matlab Code for Reproducing Figures

The Matlab file **GHK_figures.m** contains code to reproduce all figures in this appendix, and in particular figures 4, 5, and 7 in the main text of GHK (shown below).[3] The code uses the algorithm described in section 1.1 (implemented in **GHK_algorithm.m**) to solve for the equilibrium matching, wage, and salary functions under different sets of parameter values, where the factor ability distributions and productivity functions are again assumed to be given by (1.11)-(1.13). When used to study the effects of trade on earnings inequality, the script first solves for a baseline equilibrium and then for a comparative static scenario in which the sector 2 goods price, p_2, is increased. The code then uses the function **GHK_intpol.m** to interpolate the wage and salary functions for the baseline and comparative static cases over common grids, which allows computation of the percentage change in wages and salaries.

[Figure 2 about here.]

[Figure 3 about here.]

[Figure 4 about here.]

[3] To reproduce a given figure, simply uncomment the respective line of code assigning the variable "figure_mode" at the top of the file.

3 The Effects of Trade on Earnings Inequality: Numerical Analysis

In this section, we expand on the discussion in section 5 of GHK concerning the model's predictions for how trade affects earnings inequality, by employing the computational algorithm discussed in section 1 of this appendix to study the model's comparative statics with respect to output prices under a range of parameter values (again assuming that the factor ability distributions and productivity functions are given by (1.11)-(1.13)). The approach taken is to first identify all qualitatively distinct cases of parameter values that might be of interest and then to characterize the comparative static properties of the model for each case. We also restrict attention here to equilibria with two regions of sorting, and are particularly interested in determining how a change in the relative goods price p_2/p_1 affects the following four key characteristics of the equilibrium:

1. Sorting: do more workers and managers sort to a particular sector following the change in parameters?

2. Matching: does the quality of the match for a given worker or manager improve or worsen?

3. Inter-sector inequality: do real wages and salaries of workers and managers in one sector increase more than real wages and salaries of workers and managers in the other sector?

4. Intra-sector inequality: do real wages and salaries of high ability workers and managers increase more than real wages and salaries of low ability workers and managers within the same sector?

Section 3.1 considers comparative statics under equilibria in which the best workers and managers sort to the same sector (HH/LL equilibria), while section 3.2 considers comparative statics under equilibria in which the best workers and the worst managers sort to the same sector (HL/LH equilibria). Again, we label the sectors without loss of generality such that the best workers always sort to sector 1.

3.1 Inequality in HH/LL Equilibria

In this section, we use parameter values listed in Table 2. The values for $\{\gamma_i, \alpha_i, \beta_i\}$, $i \in \{1, 2\}$ are varied to explore a range of qualitatively distinct cases, but always ensuring that conditions (1.5) and (1.6) (guaranteeing sorting of the best workers and managers to sector 1) are satisfied. Since these inequalities require $\alpha_1 + \beta_1 > \alpha_2 + \beta_2$ when $\gamma_1 = \gamma_2$, we fix $\alpha_1 + \beta_1 = 2$ and $\alpha_2 + \beta_2 = 1$.

[Table 2 about here.]

To summarize the results of this section, an increase in p_2 always leads more workers and managers to sort to sector 2, but in terms of the implications for (i) the quality of matches, (ii) inter-sector inequality, and (iii) intra-sector inequality, there are 5 qualitatively distinguishable sets of matching-wage-salary responses. These cases are described in Table 3. We now examine under what kinds of parameter values each case is more likely to obtain.

[Table 3 about here.]

First, case 1 is a knife-edge case that results only when $\gamma_1 = \gamma_2 = 0.5$ and $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = 1$. Figure 5 shows the matching, wage, and salary function responses for this case.[4] Here, we see that more workers and managers sort to sector 2, but the quality of the match for a given worker or manager does not change. Regarding inter-sector inequality, workers and managers remaining in sector 2 enjoy wage and salary increases that are exactly proportional to the price increase, whereas workers and managers remaining in sector 1 see no change in their wages or salaries. Hence, real wages and salaries increase for workers and managers remaining in sector 2, but decrease for workers and managers remaining in sector 1, and change ambiguously for workers and managers that switch sectors. Furthermore, there is no change in intra-sector wage or salary inequality.

[Figure 5 about here.]

Second, case 2 is more likely to obtain whenever $|\gamma_1 - \gamma_2| = \varepsilon$ for ε sufficiently small, and at least one of the following is true: (i) γ_1 and γ_2 are both small, (ii) $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2}$ and both ratios are large, or (iii) $\frac{\alpha_2}{\beta_2}$ is low. Examples for this case are listed in Table 4, and Figure 6 shows a typical example of the matching, wage, and salary function responses.[5] As in case 1, more workers and managers sort to sector 2, but now the quality of the match for any given worker increases and the quality of the match for any given manager decreases after the price change. Regarding inter-sector inequality, real wages increase for workers remaining in sector 2, but decrease for workers remaining in sector 1; real salaries of managers change ambiguously. Furthermore, we see that now intra-sector wage inequality increases in both

[4] Specific parameter values for this figure are $\{\gamma_1, \alpha_1, \beta_1\} = \{0.5, 1, 1\}$, $\{\gamma_2, \alpha_2, \beta_2\} = \{0.5, 0.5, 0.5\}$, and $\Delta p_2 = 20\%$.

[5] Specific parameter values for this figure are $\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 1, 1\}$, $\{\gamma_2, \alpha_2, \beta_2\} = \{0.1, 0.5, 0.5\}$, and $\Delta p_2 = 20\%$.

sectors, whereas intra-sector salary inequality decreases in both sectors. Note that it is possible to have $\gamma_1 \neq \gamma_2$ and still have the matching-wage-salary responses characterized by case 2. For example, when $\{\gamma_1, \alpha_{1,}\beta_1\} = \{0.4, 1, 1\}$ and $\{\gamma_2, \alpha_2, \beta_2\} = \{0.41, 0.5, 0.5\}$, the responses are characterized by case 2.

[Table 4 about here.]

[Figure 6 about here.]

Third, case 3 is more likely to obtain whenever $|\gamma_1 - \gamma_2| = \varepsilon$ for ε sufficiently small, and at least one of the following is true: (i) γ_1 and γ_2 are both large, (ii) $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2}$ and both ratios are small, or (iii) $\frac{\alpha_1}{\beta_1}$ is low. Examples for this case are listed in Table 5, and Figure 7 shows a typical example of the matching, wage, and salary function responses.[6] Here, we see that the results are qualitatively the same as those for case 2, except that the roles of workers and managers are reversed. Specifically, the quality of the match for any given worker decreases and the quality of the match for any given manager increases after the price change. Regarding inter-sector wage inequality, real salaries increase for managers remaining in sector 2, but decrease for managers remaining in sector 1; real wages of workers change ambiguously. Furthermore, we see that now intra-sector wage inequality decreases in both sectors, whereas intra-sector salary inequality increases in both sectors. Note that it is possible to have $\gamma_1 \neq \gamma_2$ and still have the matching-wage-salary responses characterized by case 3. For example, when $\{\gamma_1, \alpha_{1,}\beta_1\} = \{0.6, 1, 1\}$ and $\{\gamma_2, \alpha_2, \beta_2\} = \{0.61, 0.5, 0.5\}$, the responses are characterized by case 3.

[Table 5 about here.]

[Figure 7 about here.]

Fourth, case 4 is more likely to obtain whenever $\gamma_2 - \gamma_1 = \varepsilon > 0$ and ε is large enough, regardless of $\frac{\alpha_1}{\beta_1}$ and $\frac{\alpha_2}{\beta_2}$. Examples for this case are listed in Table 6, and Figure 8 shows a typical example of the matching, wage, and salary function responses.[7] Here, we see that the quality of the match deteriorates for a given worker remaining in sector 1, but improves for a given worker remaining in sector 2. Conversely, the quality of the match improves for a given manager remaining in sector 1, but deteriorates for a given manager remaining in sector 2. Regarding inter-sector inequality, the real wages of workers remaining in sector 2 increase,

[6] Specific parameter values for this figure are $\{\gamma_1, \alpha_1, \beta_1\} = \{0.9, 1, 1\}$, $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.5, 0.5\}$, and $\Delta p_2 = 20\%$.

[7] Specific parameter values for this figure are $\{\gamma_1, \alpha_1, \beta_1\} = \{0.4, 1, 1\}$, $\{\gamma_2, \alpha_2, \beta_2\} = \{0.6, 0.5, 0.5\}$, and $\Delta p_2 = 20\%$.

and the real salaries of managers remaining in sector 1 decrease. Real wages for workers remaining in sector 1 could either change ambiguously (as in Figure 8) or could strictly increase (not shown). Real salaries for managers remaining in sector 2 could either strictly decrease (as in Figure 8) or could change ambiguously (not shown). Regarding intra-sector inequality, wage inequality decreases in sector 1 and increases in sector 2, whereas salary inequality increases in sector 1 and decreases in sector 2. Note that it is possible to have $\gamma_2 > \gamma_1$ and yet not have the matching-wage-salary responses characterized by case 4. For example, when $\{\gamma_1, \alpha_1, \beta_1\} = \{0.4, 1, 1\}$ and $\{\gamma_2, \alpha_2, \beta_2\} = \{0.41, 0.5, 0.5\}$, the responses are characterized by case 2, and when $\{\gamma_1, \alpha_1, \beta_1\} = \{0.6, 1, 1\}$ and $\{\gamma_2, \alpha_2, \beta_2\} = \{0.61, 0.5, 0.5\}$, the responses are characterized by case 3.

[Table 6 about here.]

[Figure 8 about here.]

Finally, Case 5 is more likely to obtain whenever $\gamma_1 - \gamma_2 = \varepsilon > 0$ and ε is large enough, regardless of $\frac{\alpha_1}{\beta_1}$ and $\frac{\alpha_2}{\beta_2}$. Examples for this case are listed in Table 7, and Figure 9 shows a typical example of the matching, wage, and salary function responses.[8] Here, we see that the results are qualitatively the same as those for case 4, except that the roles of workers and managers are reversed. Specifically the quality of the match improves for a given worker remaining in sector 1, but deteriorates for a given worker remaining in sector 2, and conversely, the quality of the match deteriorates for a given manager remaining in sector 1, but improves for a given manager remaining in sector 2. Regarding inter-sector inequality, the real salaries of managers remaining in sector 2 increase, and the real wages of workers remaining in sector 1 decrease. Real salaries for managers remaining in sector 1 could either change ambiguously (as in Figure 9) or could strictly increase (not shown). Real wages for workers remaining in sector 2 could either strictly decrease (as in Figure 9) or change ambiguously (not shown). Regarding intra-sector inequality, wage inequality increases in sector 1 and decreases in sector 2, whereas salary inequality decreases in sector 1 and increases in sector 2. Note that it is possible to have $\gamma_1 > \gamma_2$ and yet not have the matching-wage-salary responses characterized by case 4. For example, when $\{\gamma_1, \alpha_1, \beta_1\} = \{0.4, 1, 1\}$ and $\{\gamma_2, \alpha_2, \beta_2\} = \{0.39, 0.5, 0.5\}$, the responses are characterized by case 2, and when $\{\gamma_1, \alpha_1, \beta_1\} = \{0.6, 1, 1\}$ and $\{\gamma_2, \alpha_2, \beta_2\} = \{0.59, 0.5, 0.5\}$, the responses are characterized by case 3.

[Table 7 about here.]

[Figure 9 about here.]

[8] Specific parameter values for this figure are $\{\gamma_1, \alpha_1, \beta_1\} = \{0.6, 1, 1\}$, $\{\gamma_2, \alpha_2, \beta_2\} = \{0.4, 0.5, 0.5\}$, and $\Delta p_2 = 20\%$.

3.2 Inequality in HL/LH Equilibria

In this section, we use parameter values listed in Table 8. The values for $\{\gamma_i, \alpha_i, \beta_i\}$, $i \in \{1, 2\}$ are varied to explore a range of qualitatively distinct cases, but always ensuring that conditions (1.7) and (1.8) (guaranteeing sorting of the best workers and the worst managers to sector 1) are satisfied. Since these inequalities do not require $\alpha_1 + \beta_1 \neq \alpha_2 + \beta_2$ for particular values of γ_1 and γ_2, we fix $\alpha_1 + \beta_1 = \alpha_2 + \beta_2 = 1$ to keep things simple.

[Table 8 about here.]

To summarize the results of this subsection, an increase in p_2 always leads more workers and more managers to sort to sector 2. Furthermore, the change in the matching function is always characterized as follows: the quality of the match deteriorates for all workers that remain in their original sector, but improves for workers that switch sectors; conversely, the quality of the match improves for all managers remaining in their original sector, but deteriorates for managers that switch sectors. The implications for intra-sector inequality are also always the same: wage inequality decreases in both sectors and salary inequality increases in both sectors following the price change. The only difference in the comparative static results for this sorting pattern concerns the implications of the price change for inter-sector inequality. Here, there are 5 qualitatively distinguishable sets of responses, as described in Table 9. We now examine under what kinds of parameter values each case is more likely to obtain.

[Table 9 about here.]

First, case 1 is more likely to obtain when $|\gamma_1 - \gamma_2| = \varepsilon$ for ε sufficiently small, and both γ_1 and γ_2 are close to 0.5, regardless of $\frac{\alpha_1}{\beta_1}$ and $\frac{\alpha_2}{\beta_2}$. Note, however, that when $\gamma_1 = \gamma_2$, the inequalities in Proposition 10 require that $\frac{\alpha_1}{\beta_1} > \frac{\alpha_2}{\beta_2}$. Examples for this case are listed in Table 10, and Figure 10 shows a typical example of the matching, wage, and salary function responses.[9] Here, we see that the matching function response and the implications for intra-sector inequality are as described above. With regard to inter-sector inequality, we see that real wages increase for the worst workers remaining in sector 2, change ambiguously for the best workers remaining in sector 2, and decrease for all workers remaining in sector 1. On the other hand, real salaries increase for the best managers remaining in sector 2, and change ambiguously for the worst managers remaining in sector 2 as well as for all managers remaining in sector 1. It is also possible, however, for real wages of the worst workers

[9]Specific parameter values for this figure are $\{\gamma_1, \alpha_1, \beta_1\} = \{0.5, 0.6, 0.4\}$, $\{\gamma_2, \alpha_2, \beta_2\} = \{0.5, 0.4, 0.6\}$, and $\Delta p_2 = 5\%$.

remaining in sector 1 to change ambiguously, and for real salaries of the worst managers managers in sector 2 to decrease instead.[10] Nonetheless, real wages of the worst workers remaining in sector 2 and real salaries of the best managers remaining in sector 2 always increase.

[Table 10 about here.]

[Figure 10 about here.]

Second, case 2 is more likely to obtain when $|\gamma_1 - \gamma_2| = \varepsilon$ for ε sufficiently small, and both γ_1 and γ_2 are small, regardless of $\frac{\alpha_1}{\beta_1}$ and $\frac{\alpha_2}{\beta_2}$. Note, however, that when $\gamma_1 = \gamma_2$, the inequalities in Proposition 10 require that $\frac{\alpha_1}{\beta_1} > \frac{\alpha_2}{\beta_2}$. Examples for this case are listed in Table 11, and Figure 11 shows a typical example of the matching, wage, and salary function responses.[11] Here, we see that the matching function response and the implications for intra-sector inequality are the same as in case 1. With regard to inter-sector inequality, we see that real wages increase for workers remaining in sector 2, but decrease for workers remaining in sector 1. Real salaries, on the other hand, change ambiguously for all managers.

[Table 11 about here.]

[Figure 11 about here.]

Third, case 3 is more likely to obtain when $|\gamma_1 - \gamma_2| = \varepsilon$ for ε sufficiently small, and both γ_1 and γ_2 are large, regardless of $\frac{\alpha_1}{\beta_1}$ and $\frac{\alpha_2}{\beta_2}$. Note, however, that when $\gamma_1 = \gamma_2$, the inequalities in Proposition 10 require that $\frac{\alpha_1}{\beta_1} > \frac{\alpha_2}{\beta_2}$. Examples for this case are listed in Table 12, and Figure 12 shows a typical example of the matching, wage, and salary function responses.[12] Here, we see that the matching function response and the implications for intra-sector inequality are the same as in case 1. With regard to inter-sector inequality, we see that real salaries increase for managers remaining in sector 2, but decrease for managers remaining in sector 1. Real wages, on the other hand, change ambiguously for all workers.

[Table 12 about here.]

[Figure 12 about here.]

[10] For example, this happens when parameter values are the same as in Figure 10, but p_2 increases by 1% instead of 5%.

[11] Specific parameter values for this figure are $\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.9, 0.1\}$, $\{\gamma_2, \alpha_2, \beta_2\} = \{0.1, 0.1, 0.9\}$, and $\Delta p_2 = 10\%$.

[12] Specific parameter values for this figure are $\{\gamma_1, \alpha_1, \beta_1\} = \{0.9, 0.9, 0.1\}$, $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.1, 0.9\}$, and $\Delta p_2 = 10\%$.

Fourth, case 4 is more likely to obtain when either (i) $\frac{\alpha_1}{\beta_1} \leq \frac{\alpha_2}{\beta_2}$ or (ii) $\frac{\alpha_1}{\beta_1} > \frac{\alpha_2}{\beta_2}$ and $\gamma_2 - \gamma_1 = \varepsilon > 0$ for ε sufficiently large. Note that when $\frac{\alpha_1}{\beta_1} \leq \frac{\alpha_2}{\beta_2}$, the inequalities in Proposition 10 require that $\gamma_1 < \gamma_2$ (even if we allow for $\alpha_1 + \beta_1 \neq \alpha_2 + \beta_2$). Examples for this case are listed in Table 13, and Figure 13 shows a typical example of the matching, wage, and salary function responses.[13] Here, we see that the matching function response and the implications for intra-sector inequality are the same as in case 1. With regard to inter-sector inequality, we see that real wages increase for all workers, while real salaries decrease for all managers. It is also possible, however, for real wages of workers remaining in sector 1 and real salaries of managers remaining in sector 2 to change ambiguously instead.[14] Nonetheless, real wages of workers remaining in sector 2 always increase, and real salaries of managers remaining in sector 1 always decrease.

[Table 13 about here.]

[Figure 13 about here.]

Finally, case 5 is more likely to obtain when $\gamma_1 - \gamma_2 = \varepsilon > 0$ for ε sufficiently large. Note that when $\gamma_1 > \gamma_2$, the inequalities in Proposition 10 also require $\frac{\alpha_1}{\beta_1} > \frac{\alpha_2}{\beta_2}$. Examples for this case are listed in Table 14, and Figure 14 shows a typical example the matching, wage, and salary function responses.[15] Here, we see that the matching function response and the implications for intra-sector inequality are the same as in case 1. With regard to inter-sector inequality, we see that real wages decrease for all workers, while real salaries increase for all managers. It is also possible, however, for real wages of workers remaining in sector 2 and real salaries of managers remaining in sector 1 to change ambiguously instead.[16] Nonetheless, real wages of workers remaining in sector 1 always decrease, and real salaries of managers remaining in sector 2 always increase.

[Table 14 about here.]

[Figure 14 about here.]

[13] Specific parameter values for this figure are $\{\gamma_1, \alpha_1, \beta_1\} = \{0.4, 0.5, 0.5\}$, $\{\gamma_2, \alpha_2, \beta_2\} = \{0.6, 0.5, 0.5\}$, and $\Delta p_2 = 10\%$.

[14] The following parameter values generate an example with these characteristics:$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.1, 0.9\}$ and $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.2, 0.8\}$.

[15] Specific parameter values for this figure are $\{\gamma_1, \alpha_1, \beta_1\} = \{0.7, 0.9, 0.1\}$, $\{\gamma_2, \alpha_2, \beta_2\} = \{0.3, 0.1, 0.9\}$, and $\Delta p_2 = 10\%$.

[16] The following parameter values generate an example with these characteristics:$\{\gamma_1, \alpha_1, \beta_1\} = \{0.55, 0.9, 0.1\}$ and $\{\gamma_2, \alpha_2, \beta_2\} = \{0.45, 0.6, 0.4\}$.

List of Figures

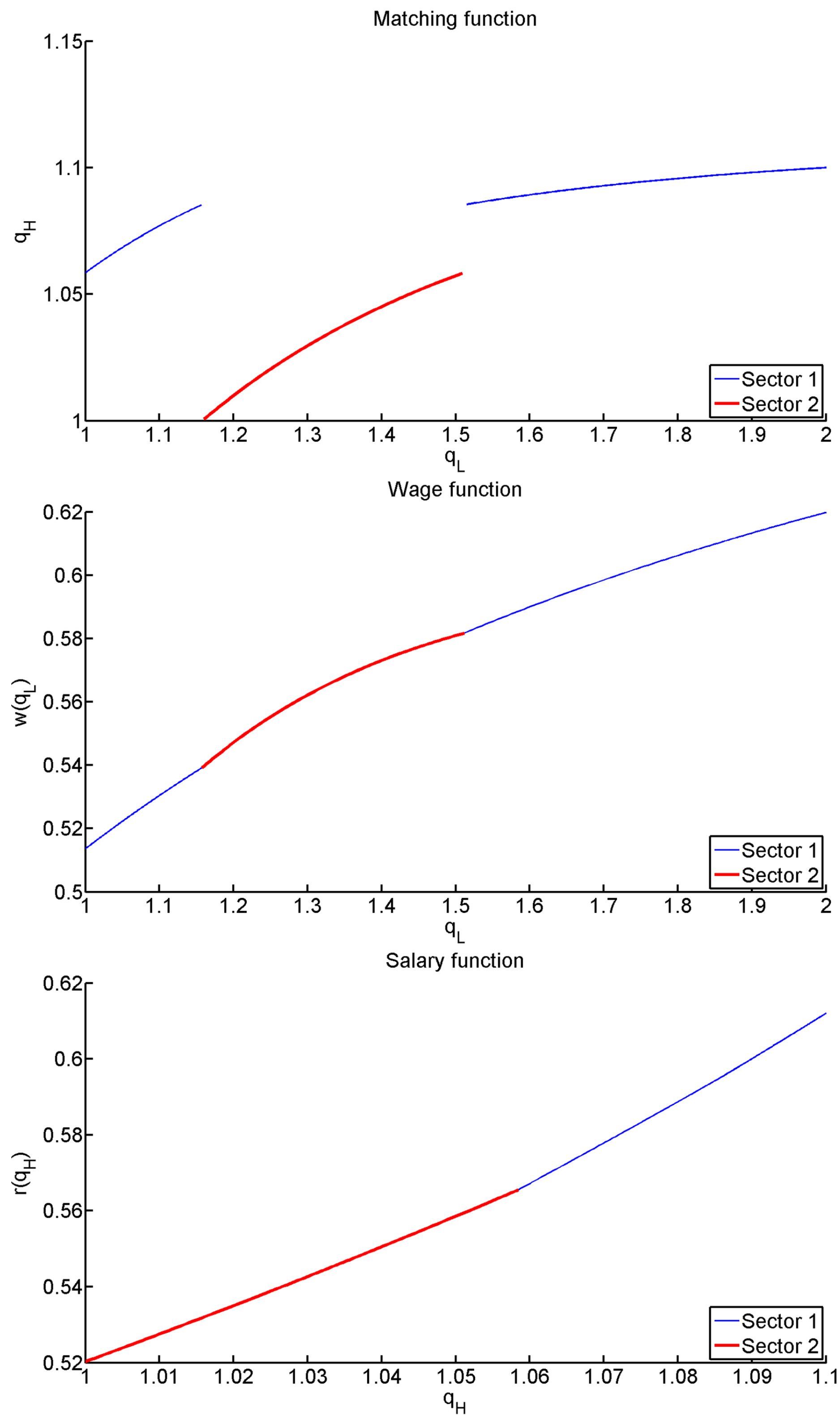

Figure 1: Example of matching, wage, and salary functions in an equilibrium with three regions of sorting

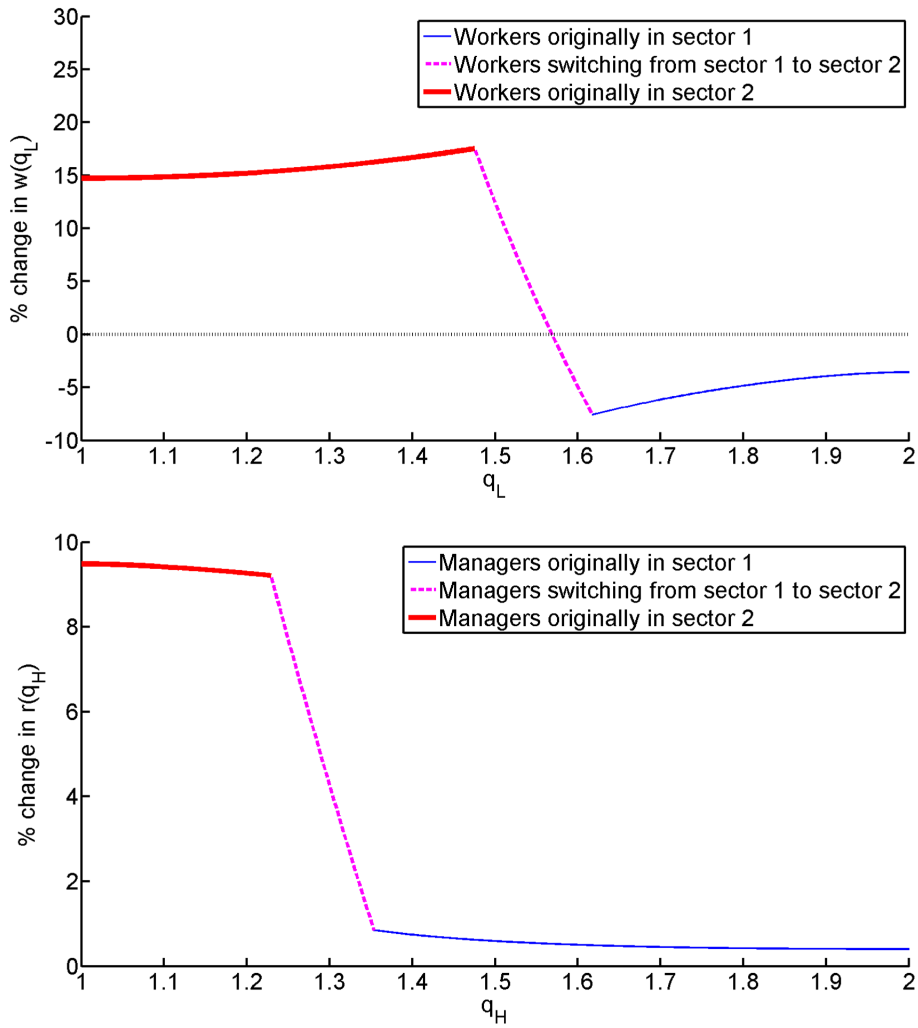

Figure 2: Figure 4 in GHK

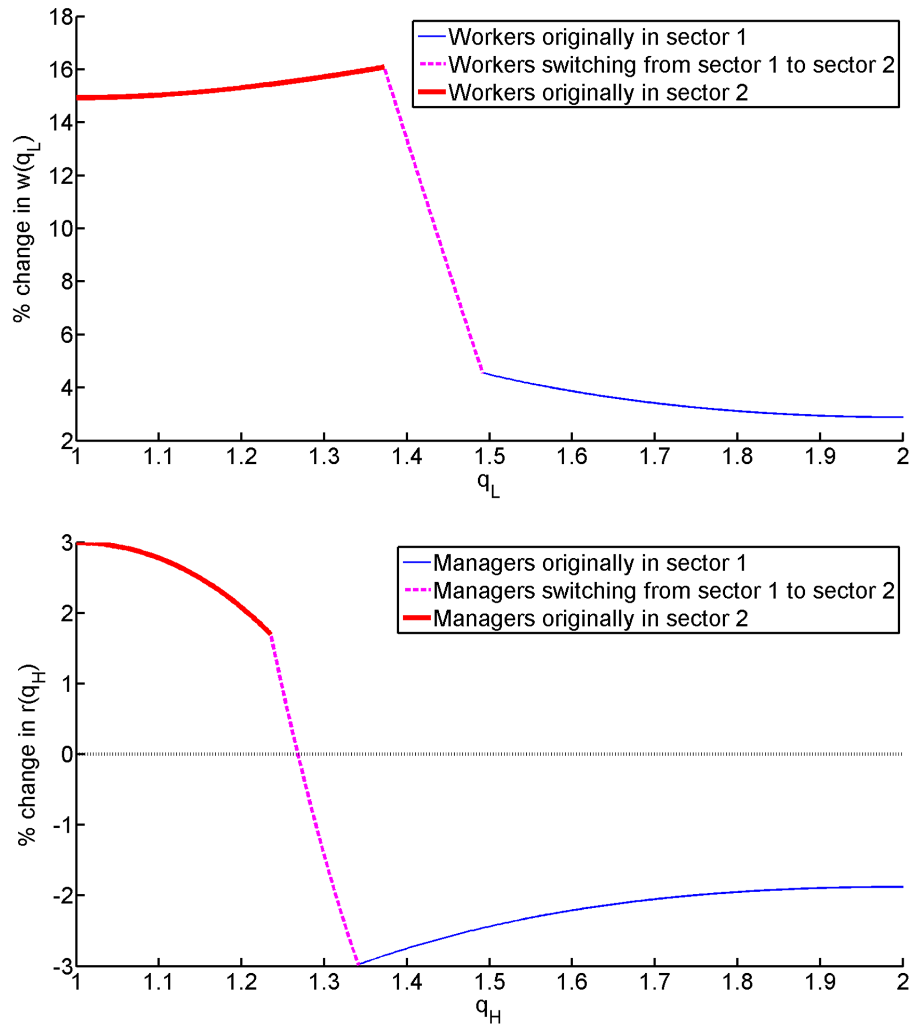

Figure 3: Figure 5 in GHK

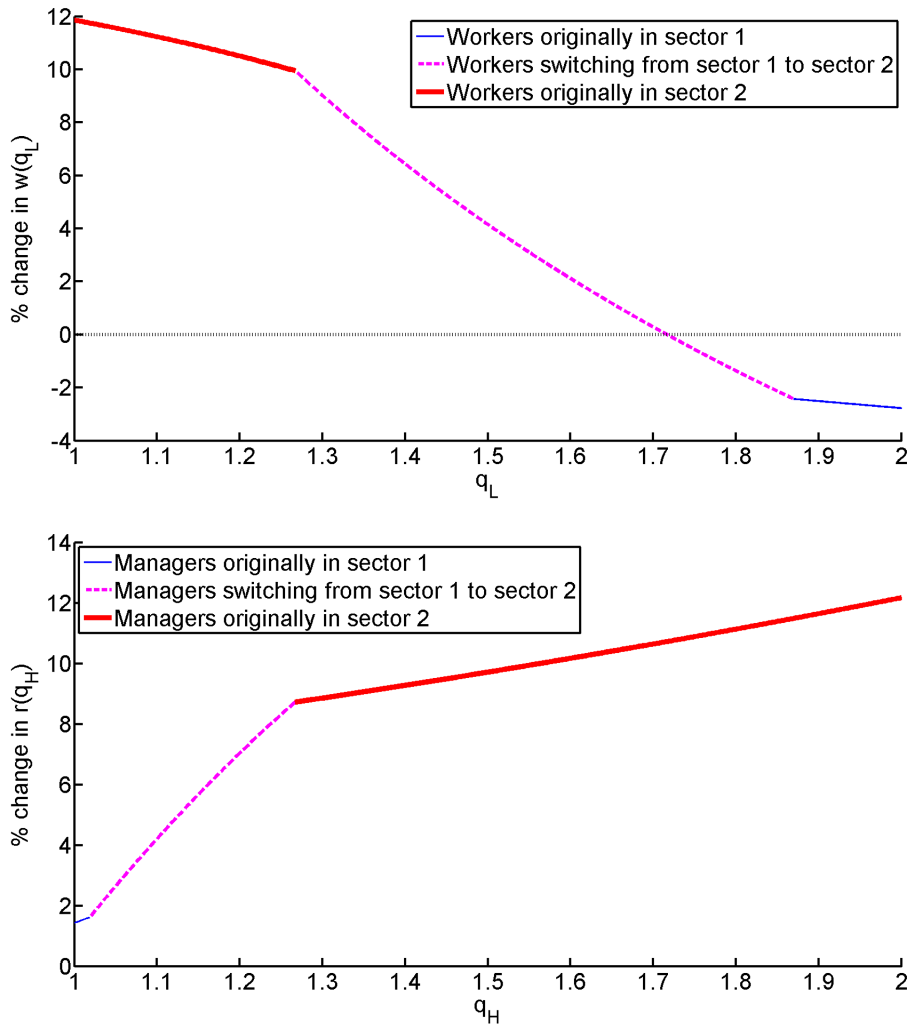

Figure 4: Figure 7 in GHK

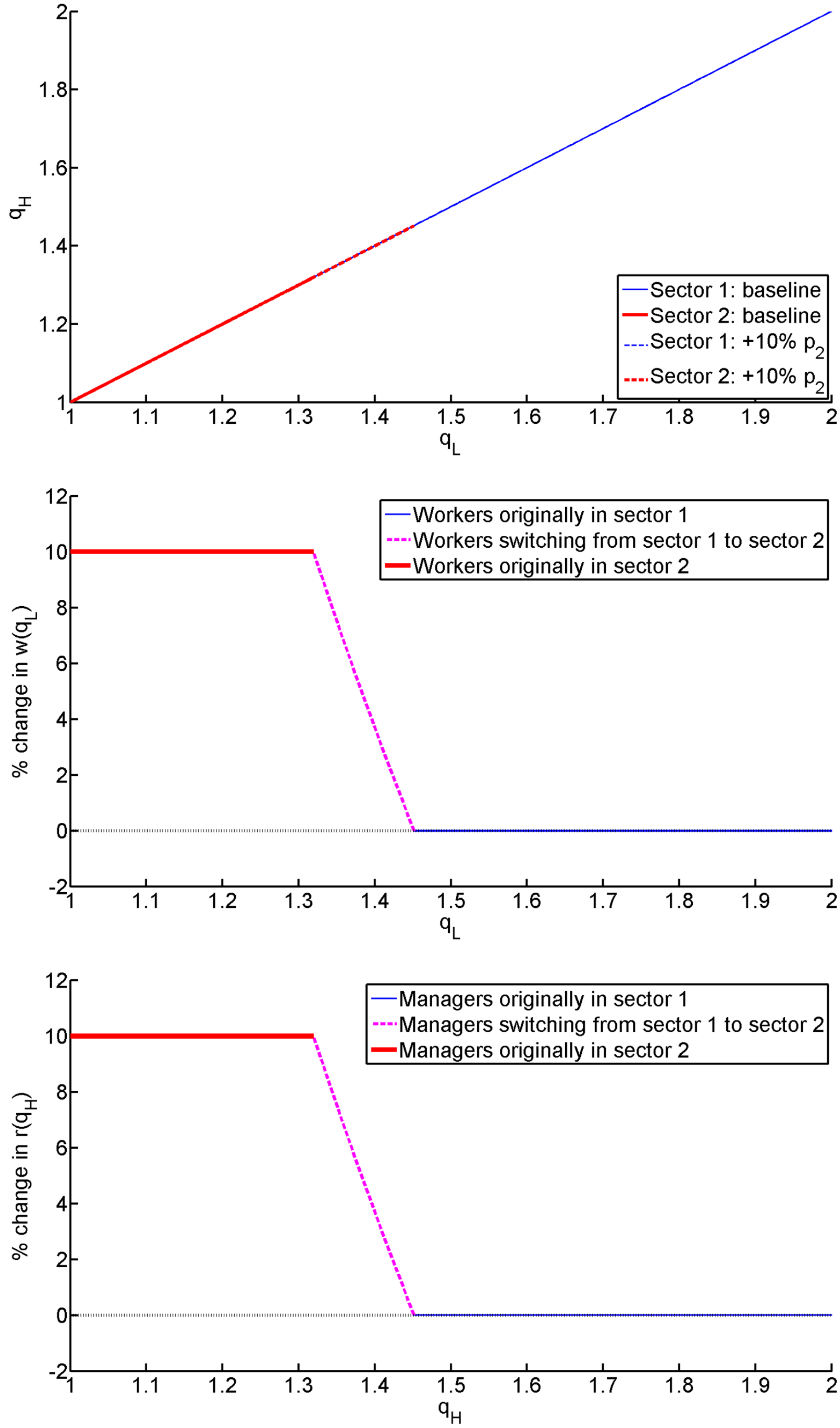

Figure 5: Response of matching, wage, and salary functions for case 1, HH/LL equilibria

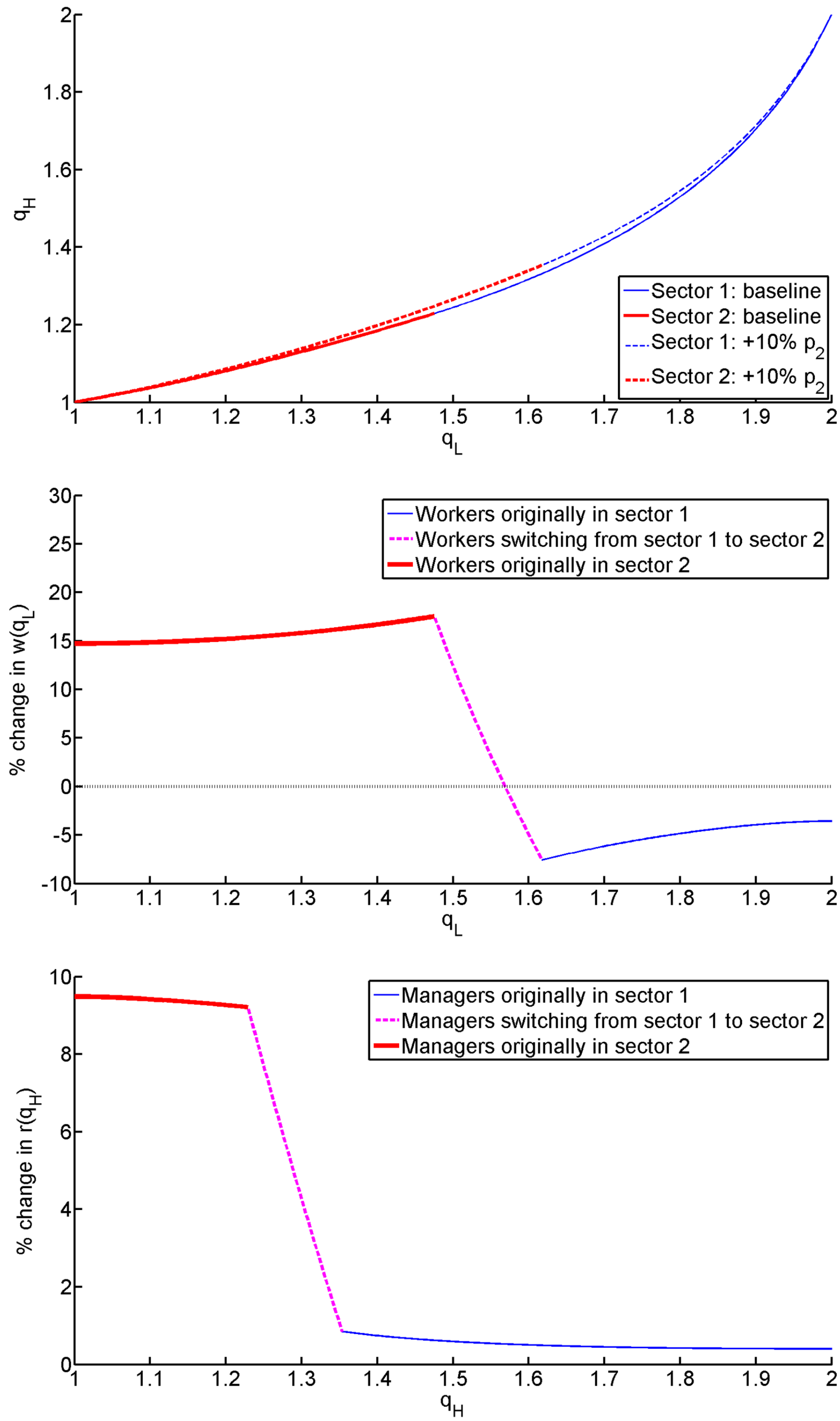

Figure 6: Response of matching, wage, and salary functions for case 2, HH/LL equilibria

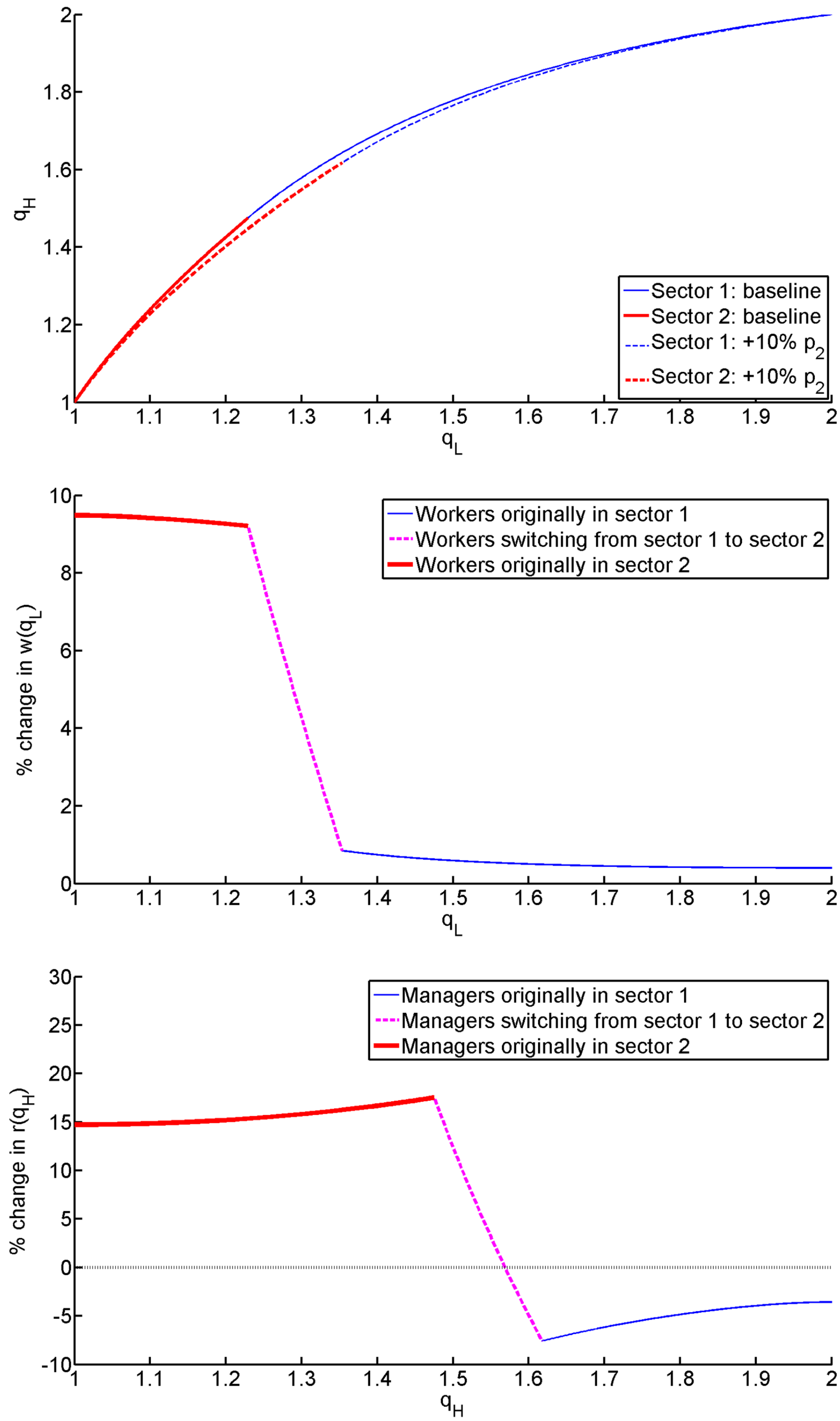

Figure 7: Response of matching, wage, and salary functions for case 3, HH/LL equilibria

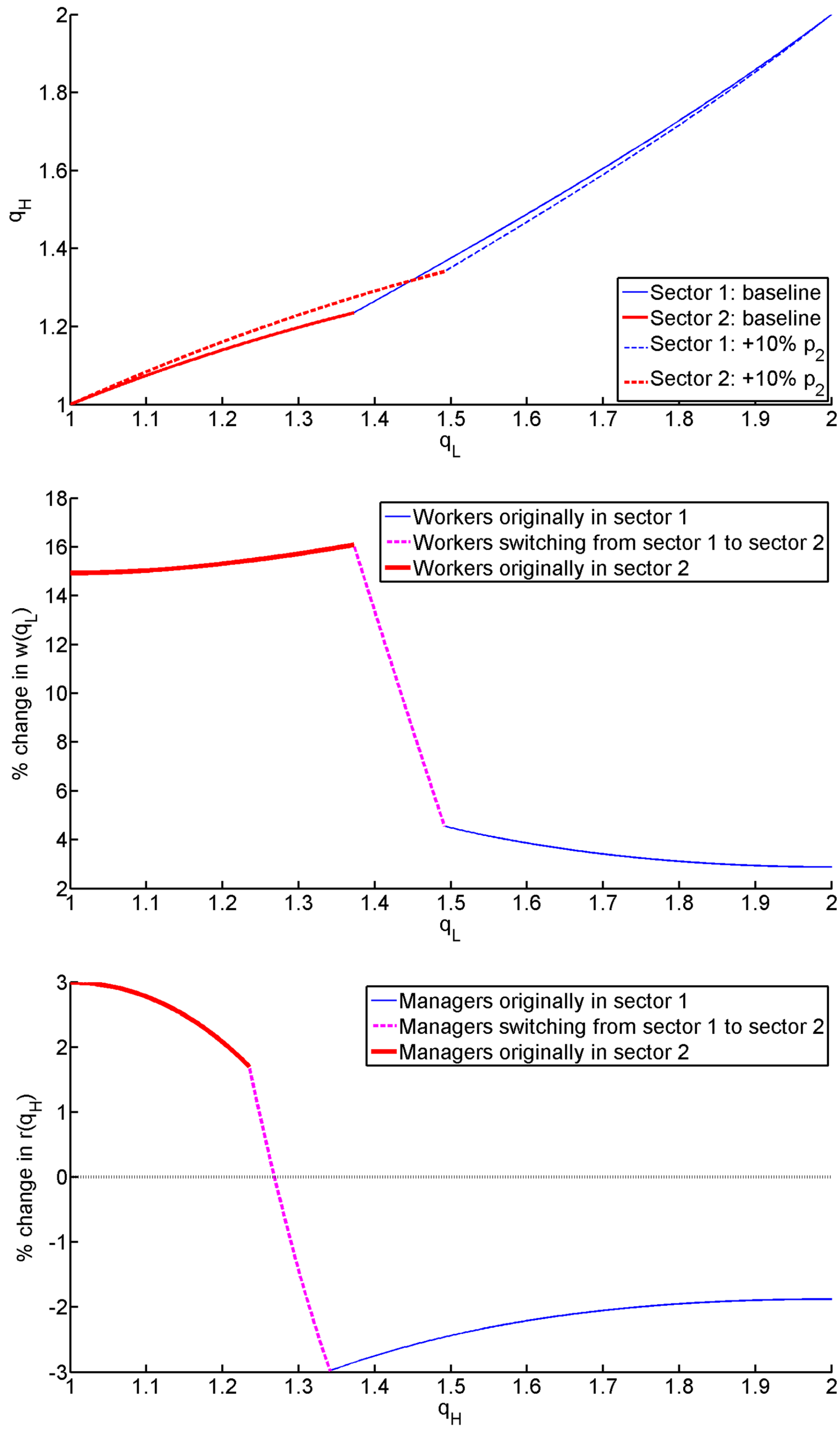

Figure 8: Response of matching, wage, and salary functions for case 4, HH/LL equilibria

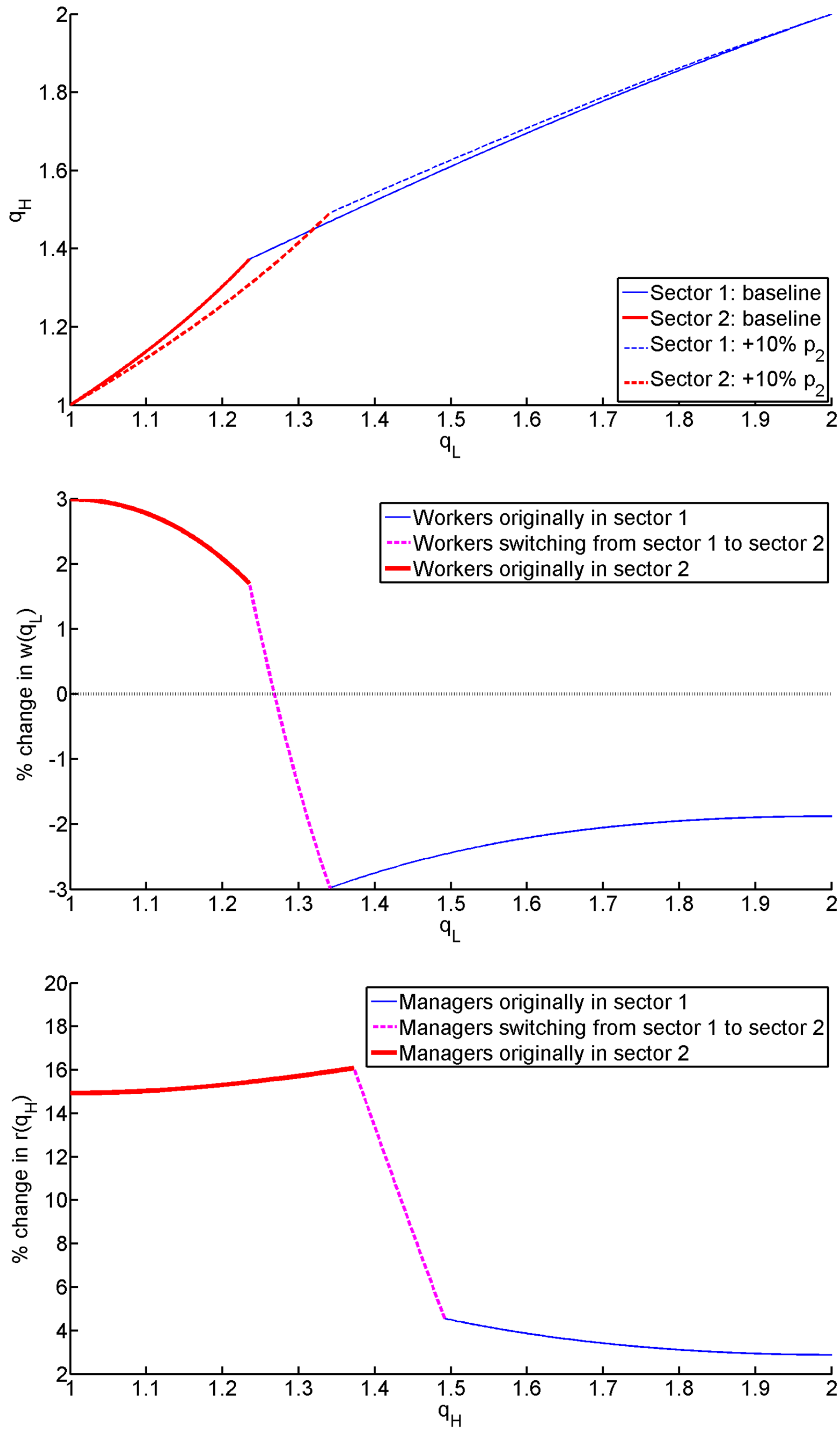

Figure 9: Response of matching, wage, and salary functions for case 5, HH/LL equilibria

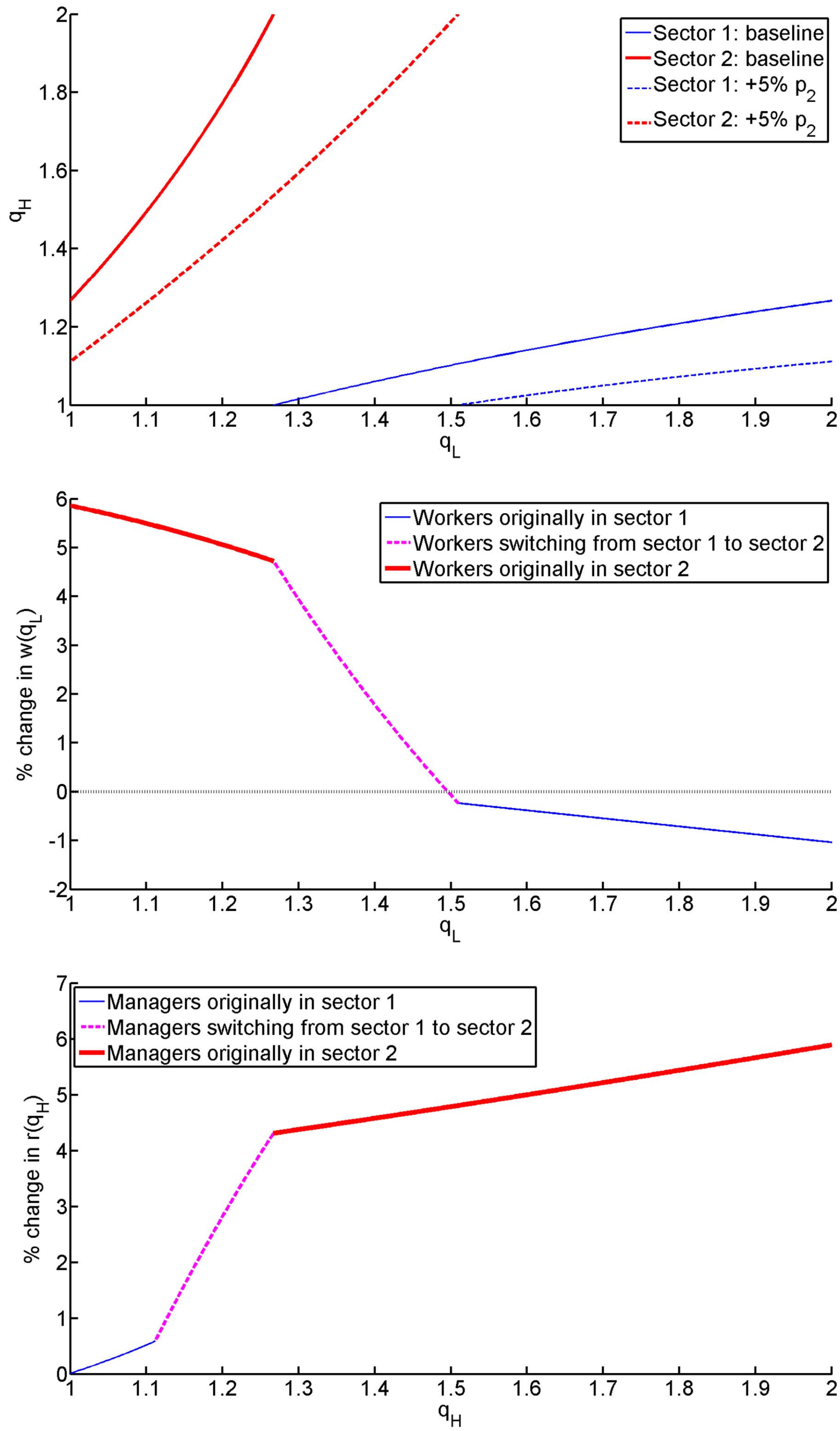

Figure 10: Response of matching, wage, and salary functions for case 1, HL/LH equilibria

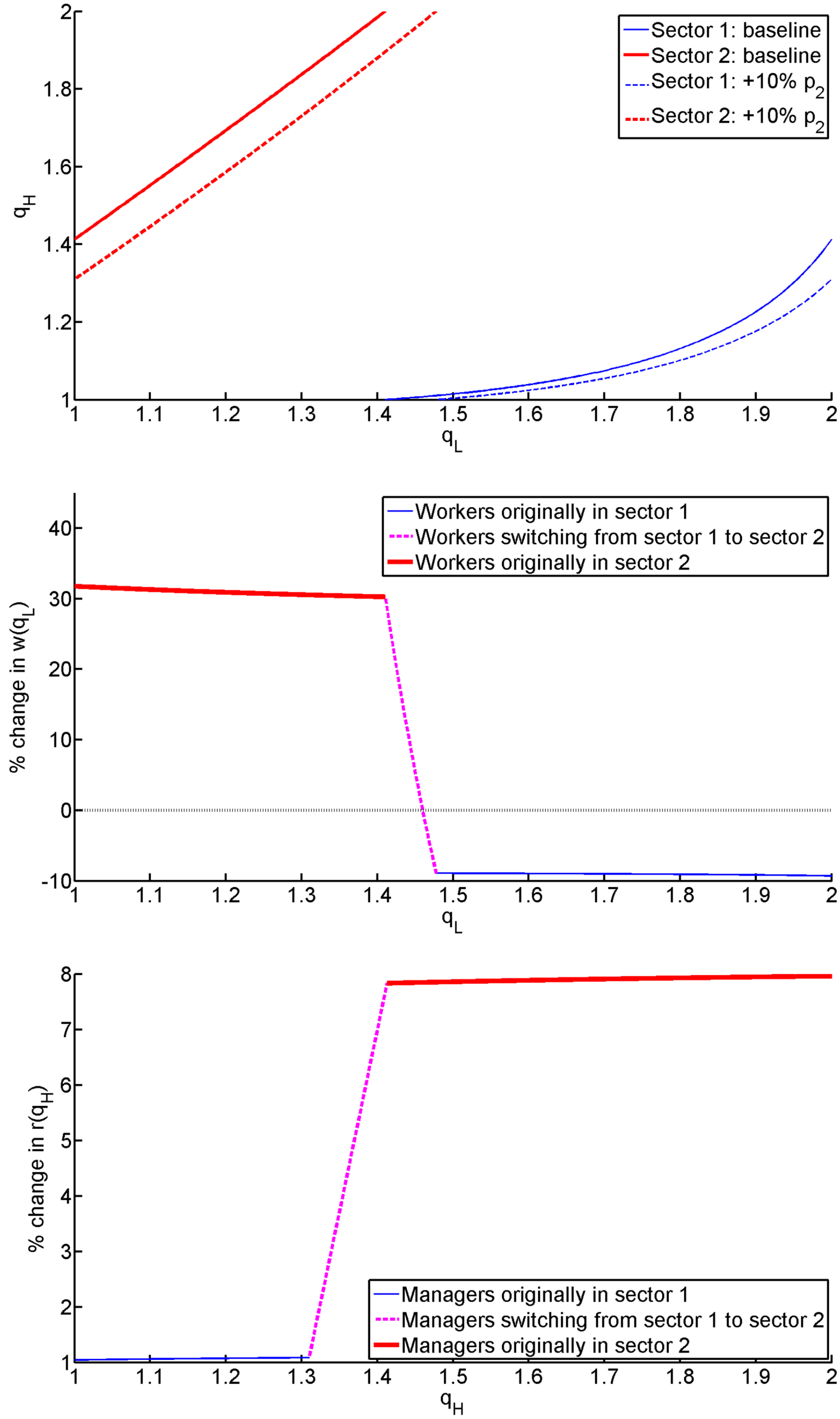

Figure 11: Response of matching, wage, and salary functions for case 2, HL/LH equilibria

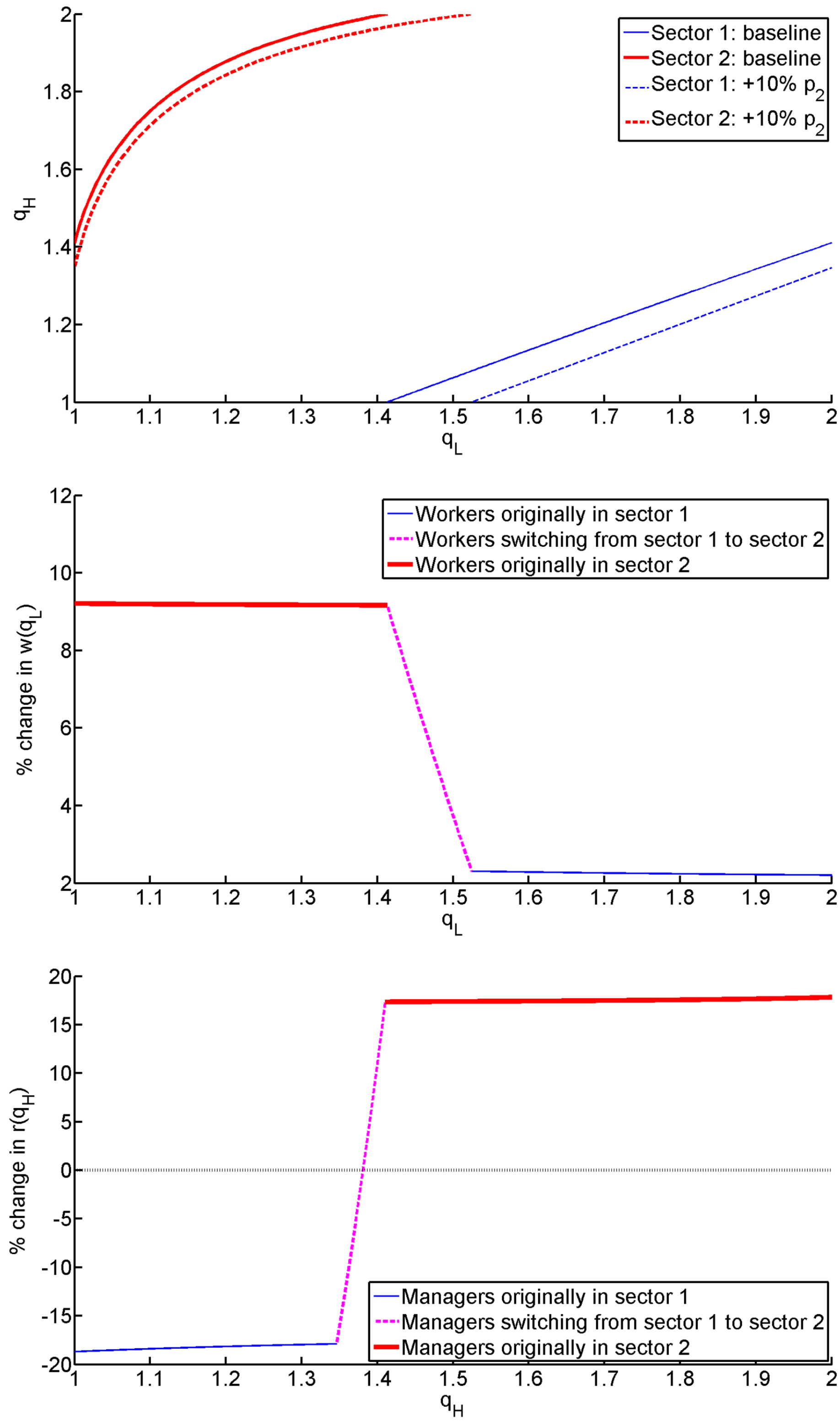

Figure 12: Response of matching, wage, and salary functions for case 3, HL/LH equilibria

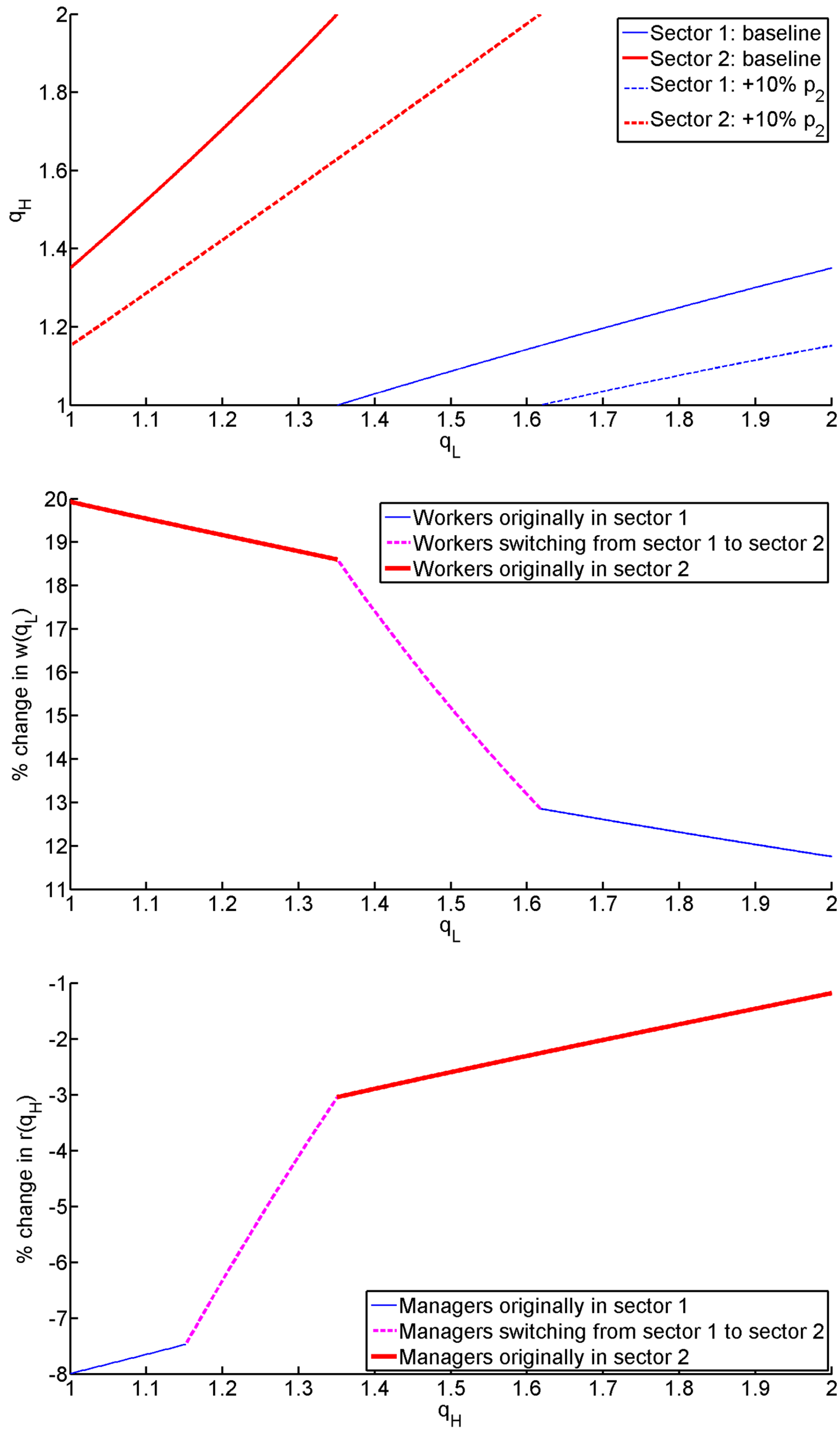

Figure 13: Response of matching, wage, and salary functions for case 4, HL/LH equilibria

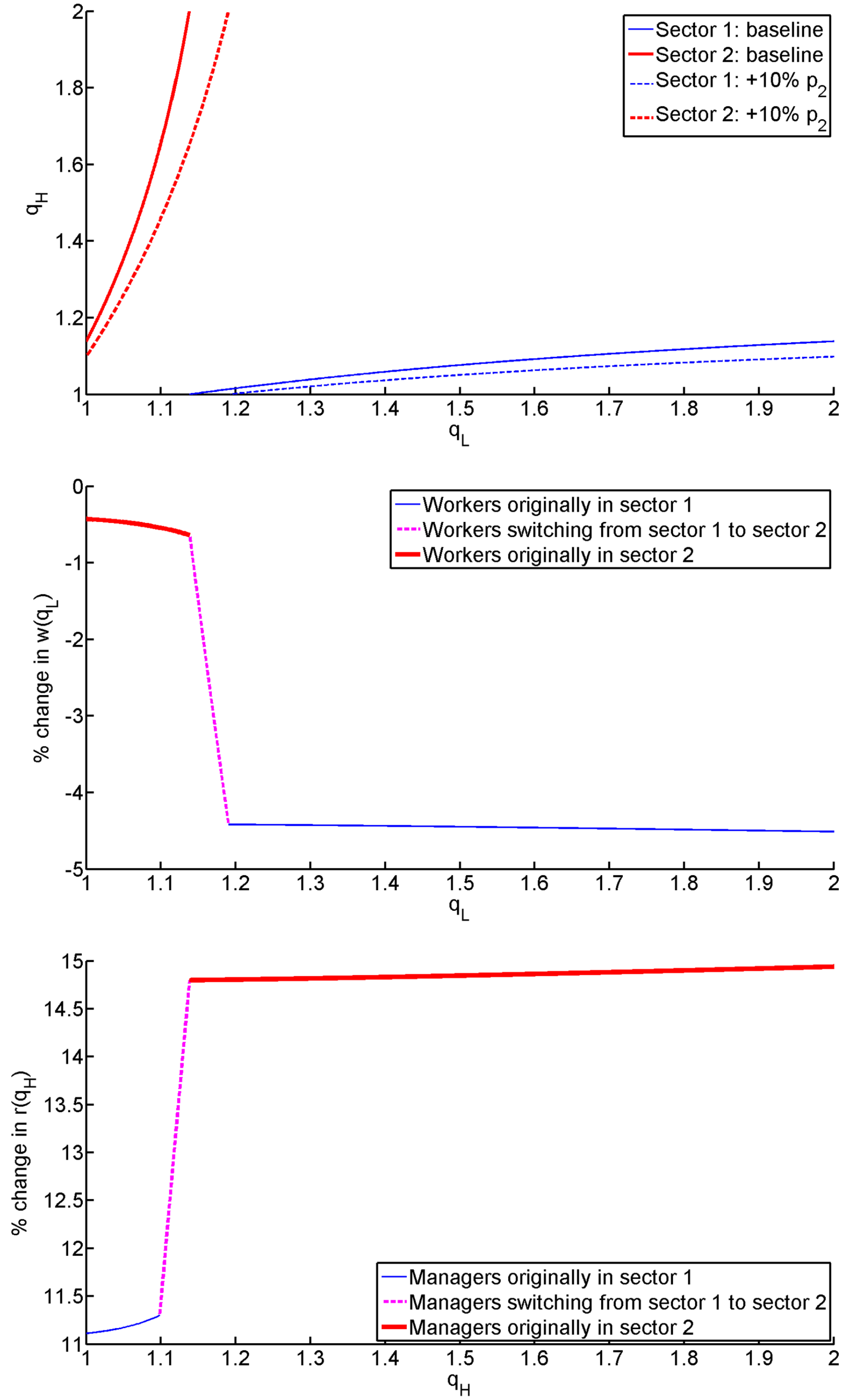

Figure 14: Response of matching, wage, and salary functions for case 5, HL/LH equilibria

List of Tables

H	1
L	1
S_H	$[1, 1.1]$
S_L	$[1, 2]$
k_H	3
k_L	3

p_1	1
γ_1	0.6
α_1	0.2
β_1	0.8
ρ_1	-1

p_2	1
γ_2	0.4
α_2	0.3
β_2	0.7
ρ_2	-5

Table 1: Example of parameter values generating an equilibrium with three regions of sorting

H	1
L	1
S_H	$[1, 2]$
S_L	$[1, 2]$
k_H	3
k_L	3

p_1	1
γ_1	varied
α_1	varied
β_1	varied
ρ_1	-5

p_2	1
γ_2	varied
α_2	varied
β_2	varied
ρ_2	-5

Table 2: Parameter values used for studying comparative statics with respect to p_2/p_1, HH/LL equilibria

Case	Sorting	Matching	Inter-sector Inequality	Intra-sector Inequality
(1)	more Ws and Ms sort to S2	no change in match quality for a given W or M	real w and r increase for Ws and Ms in S2, decrease for Ws and Ms in S1, and change ambiguously for Ws and Ms that switch sectors	no change in w or r inequality
(2)	same as (1)	quality of match for a given W increases	real w increases for Ws in S2, and decreases for Ws in S1; ambiguous change in real r for Ms	w inequality increases in both S1 and S2, r inequality decreases in both S1 and S2
(3)	same as (1)	quality of match for a given W decreases	real r increases for Ms in S2, and decreases for Ms in S1; ambiguous change in real w for Ws	w inequality decreases in both S1 and S2, r inequality increases in both S1 and S2
(4)	same as (1)	quality of match for a given W in S1 decreases, quality of match for a given W in S2 increases	real w increases for Ws in S2, and either increases or changes ambiguously for Ws in S1; real r decreases for Ms in S1, and either decreases or changes ambiguously for Ms in S2	w inequality decreases in S1 and increases in S2, r inequality increases in S1 and decreases in S2
(5)	same as (1)	quality of match for a given W in S1 increases, quality of match for a given W in S2 decreases	real r increases for Ms in S2, and either increases or changes ambiguously for Ms in S1; real w decreases for Ws in S1, and either decreases or changes ambiguously for Ws in S2	w inequality increases in S1 and decreases in S2, r inequality decreases in S1 and increases in S2

Table 3: Possible cases for p_2/p_1 comparative statics, HH/LL equilibria (W: worker, M: manager, S: sector)

Example	Qualitative Type	Specific values
2.1	$\gamma_1 = \gamma_2 < 0.5$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = 1$	$\{\gamma_1, \alpha_{1,}\beta_1\} = \{0.1, 1, 1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.1, 0.5, 0.5\}$
2.2	$\gamma_1 = \gamma_2 = 0.5$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} > 1$	$\{\gamma_1, \alpha_{1,}\beta_1\} = \{0.5, 1.8, 0.2\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.5, 0.9, 0.1\}$
2.3	$\gamma_1 = \gamma_2 = 0.5$ $\frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1} = 1$	$\{\gamma_1, \alpha_{1,}\beta_1\} = \{0.5, 1, 1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.5, 0.4, 0.6\}$

Table 4: Examples for case 2, HH/LL equilibria

Example	Qualitative Type	Specific values
3.1	$\gamma_1 = \gamma_2 > 0.5$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.9, 1, 1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.5, 0.5\}$
3.2	$\gamma_1 = \gamma_2 = 0.5$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.5, 0.2, 1.8\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.5, 0.1, 0.9\}$
3.3	$\gamma_1 = \gamma_2 = 0.5$ $\frac{\alpha_1}{\beta_1} < \frac{\alpha_2}{\beta_2} = 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.5, 0.8, 1.2\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.5, 0.5, 0.5\}$

Table 5: Examples for case 3, HH/LL equilibria

Example	Qualitative Type	Specific values
4.1	$\gamma_1 < \gamma_2 < 0.5$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = 1$	$\{\gamma_1, \alpha_{1,}\beta_1\} = \{0.2, 1, 1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.3, 0.5, 0.5\}$
4.2	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = 1$	$\{\gamma_1, \alpha_{1,}\beta_1\} = \{0.4, 1, 1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.6, 0.5, 0.5\}$
4.3	$0.5 < \gamma_1 < \gamma_2$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = 1$	$\{\gamma_1, \alpha_{1,}\beta_1\} = \{0.7, 1, 1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.8, 0.5, 0.5\}$
4.4	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} < 1$	$\{\gamma_1, \alpha_{1,}\beta_1\} = \{0.4, 0.2, 1.8\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.6, 0.1, 0.9\}$
4.5	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} > 1$	$\{\gamma_1, \alpha_{1,}\beta_1\} = \{0.4, 1.8, 0.2\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.6, 0.9, 0.1\}$
4.6	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_1}{\beta_1} < \frac{\alpha_2}{\beta_2} < 1$	$\{\gamma_1, \alpha_{1,}\beta_1\} = \{0.4, 0.3, 1.7\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.6, 0.2, 0.8\}$
4.7	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1} < 1$	$\{\gamma_1, \alpha_{1,}\beta_1\} = \{0.4, 0.5, 1.5\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.6, 0.2, 0.8\}$
4.8	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_1}{\beta_1} > \frac{\alpha_2}{\beta_2} > 1$	$\{\gamma_1, \alpha_{1,}\beta_1\} = \{0.4, 1.7, 0.3\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.6, 0.8, 0.2\}$
4.9	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_2}{\beta_2} > \frac{\alpha_1}{\beta_1} > 1$	$\{\gamma_1, \alpha_{1,}\beta_1\} = \{0.4, 1.5, 0.5\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.6, 0.8, 0.2\}$

Table 6: Examples for case 4, HH/LL equilibria

Example	Qualitative Type	Specific values
5.1	$\gamma_2 < \gamma_1 < 0.5$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.3, 1, 1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.2, 0.5, 0.5\}$
5.2	$\gamma_2 < 0.5 < \gamma_1$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.6, 1, 1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.4, 0.5, 0.5\}$
5.3	$0.5 < \gamma_2 < \gamma_1$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.8, 1, 1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.7, 0.5, 0.5\}$
5.4	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.6, 0.2, 1.8\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.4, 0.1, 0.9\}$
5.5	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} > 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.6, 1.8, 0.2\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.4, 0.9, 0.1\}$
5.6	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_1}{\beta_1} < \frac{\alpha_2}{\beta_2} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.6, 0.3, 1.7\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.4, 0.2, 0.8\}$
5.7	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.6, 0.5, 1.5\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.4, 0.2, 0.8\}$
5.8	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_1}{\beta_1} > \frac{\alpha_2}{\beta_2} > 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.6, 1.7, 0.3\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.4, 0.8, 0.2\}$
5.9	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_2}{\beta_2} > \frac{\alpha_1}{\beta_1} > 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.6, 1.5, 0.5\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.4, 0.8, 0.2\}$

Table 7: Examples for case 5, HH/LL equilibria

H	1
L	1
S_H	$[1, 2]$
S_L	$[1, 2]$
k_H	3
k_L	3

p_1	1
γ_1	varied
α_1	varied
β_1	varied
ρ_1	-0.5

p_2	1
γ_2	varied
α_2	varied
β_2	varied
ρ_2	-0.5

Table 8: Parameter values used for studying comparative statics with respect to p_2/p_1, HL/LH equilibria

Case	Sorting	Matching	Inter-sector Inequality	Intra-sector Inequality
(1)	more Ws and Ms sort to S2	quality of match for a given W decreases	real w increases for worst Ws in S2, changes ambiguously for best Ws in S2, and decreases or changes ambiguously for Ws in S1; real r increases for best Ms in S2, changes ambiguously for worst Ms in S2, and decreases or changes ambiguously for Ms in S1	w inequality decreases in both S1 and S2, r inequality increases in both S1 and S2
(2)	same as (1)	same as (1)	real w increases for Ws in S2, and either increases or changes ambiguously for Ws in S1; real r decreases for Ms in S1, and either decreases or changes ambiguously for Ms in S2	same as (1)
(3)	same as (1)	same as (1)	real r increases for Ms in S2, and either increases or changes ambiguously for Ms in S1; real w decreases for Ws in S1, and either decreases or changes ambiguously for Ws in S2	same as (1)
(4)	same as (1)	same as (1)	real w increases for Ws in S2, and decreases for Ws in S1; ambiguous change in real r for Ms	same as (1)
(5)	same as (1)	same as (1)	real r increases for Ms in S2, and decreases for Ms in S1; ambiguous change in real w for Ws	same as (1)

Table 9: Possible cases for p_2/p_1 comparative statics, HL/LH equilibria (W: worker, M: manager, S: sector)

Example	Qualitative Type	Specific values
1.1	$\gamma_1 = \gamma_2 = 0.5$ $\frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.5, 0.2, 0.8\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.5, 0.1, 0.9\}$
1.2	$\gamma_1 = \gamma_2 = 0.5$ $\frac{\alpha_2}{\beta_2} < 1 < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.5, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.5, 0.1, 0.9\}$
1.3	$\gamma_1 = \gamma_2 = 0.5$ $1 < \frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.5, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.5, 0.8, 0.2\}$
1.4	$\gamma_1 = \gamma_2 < 0.5$ $\frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.45, 0.6, 0.4\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.45, 0.4, 0.6\}$
1.5	$\gamma_1 = \gamma_2 > 0.5$ $\frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.55, 0.6, 0.4\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.55, 0.4, 0.6\}$
1.6	$\gamma_1 < \gamma_2$ $\frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.5, 0.6, 0.4\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.505, 0.4, 0.6\}$
1.7	$\gamma_1 > \gamma_2$ $\frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.505, 0.6, 0.4\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.5, 0.4, 0.6\}$

Table 10: Examples for case 1, HL/LH equilibria

Example	Qualitative Type	Specific values
2.1	$\gamma_1 = \gamma_2 < 0.5$ $\frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.2, 0.8\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.1, 0.1, 0.9\}$
2.2	$\gamma_1 = \gamma_2 < 0.5$ $\frac{\alpha_2}{\beta_2} < 1 < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.1, 0.1, 0.9\}$
2.3	$\gamma_1 = \gamma_2 < 0.5$ $1 < \frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.1, 0.8, 0.2\}$
2.4	$\gamma_1 < \gamma_2 < 0.5$ $\frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.15, 0.1, 0.9\}$
2.5	$\gamma_2 < \gamma_1 < 0.5$ $\frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.15, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.1, 0.1, 0.9\}$

Table 11: Examples for case 2, HL/LH equilibria

Example	Qualitative Type	Specific values
3.1	$\gamma_1 = \gamma_2 > 0.5$ $\frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.9, 0.2, 0.8\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.1, 0.9\}$
3.2	$\gamma_1 = \gamma_2 > 0.5$ $\frac{\alpha_2}{\beta_2} < 1 < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.9, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.1, 0.9\}$
3.3	$\gamma_1 = \gamma_2 > 0.5$ $1 < \frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.9, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.8, 0.2\}$
3.4	$\gamma_1 > \gamma_2 > 0.5$ $\frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.9, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.85, 0.1, 0.9\}$
3.5	$\gamma_2 > \gamma_1 > 0.5$ $\frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.85, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.1, 0.9\}$

Table 12: Examples for case 3, HL/LH equilibria

Example	Qualitative Type	Specific values
4.1	$\gamma_1 < \gamma_2 < 0.5$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.5, 0.5\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.3, 0.5, 0.5\}$
4.2	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.5, 0.5\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.5, 0.5\}$
4.3	$0.5 < \gamma_1 < \gamma_2$ $\frac{\alpha_2}{\beta_2} = \frac{\alpha_1}{\beta_1} = 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.7, 0.5, 0.5\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.5, 0.5\}$
4.4	$\gamma_1 < \gamma_2 < 0.5$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.1, 0.9\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.3, 0.1, 0.9\}$
4.5	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.1, 0.9\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.1, 0.9\}$
4.6	$0.5 < \gamma_1 < \gamma_2$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.6, 0.1, 0.9\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.1, 0.9\}$
4.7	$\gamma_1 < \gamma_2 < 0.5$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} > 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.4, 0.9, 0.1\}$
4.8	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} > 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.9, 0.1\}$
4.9	$0.5 < \gamma_1 < \gamma_2$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} > 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.8, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.9, 0.1\}$
4.10	$\gamma_1 < \gamma_2$ $\frac{\alpha_1}{\beta_1} < \frac{\alpha_2}{\beta_2} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.1, 0.9\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.3, 0.2, 0.8\}$
4.11	$\gamma_1 < \gamma_2$ $\frac{\alpha_1}{\beta_1} < 1 < \frac{\alpha_2}{\beta_2}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.3, 0.7\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.7, 0.3\}$
4.12	$\gamma_1 < \gamma_2$ $1 < \frac{\alpha_1}{\beta_1} < \frac{\alpha_2}{\beta_2}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.7, 0.8, 0.2\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.9, 0.1\}$
4.13	$\gamma_1 < \gamma_2$ $\frac{\alpha_1}{\beta_1} > \frac{\alpha_2}{\beta_2}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.1, 0.9\}$

Table 13: Examples for case 4, HL/LH equilibria

Example	Qualitative Type	Specific values
5.1	$\gamma_2 < \gamma_1 < 0.5$ $\frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.2, 0.3, 0.7\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.1, 0.1, 0.9\}$
5.2	$\gamma_2 < \gamma_1 < 0.5$ $\frac{\alpha_2}{\beta_2} < 1 < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.4, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.1, 0.1, 0.9\}$
5.3	$\gamma_2 < \gamma_1 < 0.5$ $1 < \frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.45, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.35, 0.6, 0.4\}$
5.4	$\gamma_2 < 0.5 < \gamma_1$ $\frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.55, 0.4, 0.6\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.45, 0.1, 0.9\}$
5.5	$\gamma_2 < 0.5 < \gamma_1$ $\frac{\alpha_2}{\beta_2} < 1 < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.7, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.3, 0.1, 0.9\}$
5.6	$\gamma_2 < 0.5 < \gamma_1$ $1 < \frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.55, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.45, 0.6, 0.4\}$
5.7	$0.5 < \gamma_2 < \gamma_1$ $\frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.65, 0.4, 0.6\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.55, 0.1, 0.9\}$
5.8	$0.5 < \gamma_2 < \gamma_1$ $\frac{\alpha_2}{\beta_2} < 1 < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.9, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.6, 0.1, 0.9\}$
5.9	$0.5 < \gamma_2 < \gamma_1$ $1 < \frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.9, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.8, 0.7, 0.3\}$

Table 14: Examples for case 5, HL/LH equilibria