Resilience in Vertical Supply Chains*

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Abstract

Forward-looking investments determine the resilience of firms' supply chains. Such investments confer externalities on other firms in the production network. We compare the equilibrium and optimal allocations in a general equilibrium model with an arbitrary number of vertical production tiers. Our model features endogenous investments in resilience, endogenous formation of supply links, and sequential bargaining over quantities and payments between firms in successive tiers. We derive policies that implement the first-best allocation, allowing for subsidies to input purchases, network formation, and investments in resilience. The first-best policies depend only on production function parameters of the pertinent tier. When subsidies to transactions are infeasible, the second-best subsidies for resilience and network formation depend on production function parameters throughout the network, and subsidies are larger upstream than downstream whenever the bargaining weights of buyers are non-increasing along the chain.

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1 Introduction

A spate of highly publicized supply chain disruptions—owing not only to the COVID-19 pandemic, but also to natural disasters, cyber-attacks, extreme weather events, logistics bottlenecks, and a host of other causes—has drawn policymakers' attention to the importance of supply chain resilience. International institutions such as the O.E.C.D (2021) and European Parliament (2021) have issued reports with "resilience" in their titles. Government publications, such as the U.K. Department of International Trade (2022) and the U.S. *Economic Report of the President* (Council of Economic Advisors, 2022, chapter 6), and international organizations such as the World Bank (2023), have also addressed these issues. Think tanks, such as McKinsey Global Institute (Lund et al., 2020) and the Brookings Institution (Iakovou and White, 2020), have offered guidance as well. Yet little formal economic analysis has addressed the topic of optimal government policy in the face of recurrent supply chain disturbances.

In this paper, we consider the market failures that may arise in vertical supply chains with multiple tiers, limited networks, arms-length transactions, and risks of disruption at every node. We develop a novel general-equilibrium model of network production with many realistic features. In our model, a finite measure of "lead" firms produce differentiated consumer goods that they sell to households in a setting of monopolistic competition. The lead firms, which we designate as active in tier S, produce their unique varieties using labor and a bundle of differentiated intermediate inputs that they purchase from firms in their network operating in tier $S-1$. The firms in tier $S-1$, in turn, fulfill their orders by combining labor and differentiated inputs procured from their suppliers in tier $S-2$. Firms in tier $S-2$ buy inputs from partners further upstream, and so on up the chain. The vertical chain ends with tier 0, where companies produce inputs from labor alone and sell them to firms in tier 1.

Every firm in the economy faces a non-zero probability of a catastrophic disruption. If a firm suffers such a disturbance, it will be unable to produce in the period captured by the model. The risk of disruption may vary across tiers of the supply chain. Moreover, we grant every firm an opportunity to invest resources to moderate its risk, so that each firm's "resilience" is endogenous in the model.

Firms also invest in the thickness of their networks. Each firm can form relationships with any fraction of the firms in the tier immediately above. By forming thicker supplier networks, firms purchase a different type of resilience; they protect themselves against the risk that some of their potential suppliers will be unable to deliver. However, creating extra supply links is costly.

Bilateral relationships play an important role in our model. The supply chains that we envision do not involve off-the-shelf inputs that might be available on anonymous markets. Rather, they are produced and sold to order. Each supplier negotiates the terms of a contract with each of its potential customers. The contracts specify the quantities that will be delivered by the upstream firms and the payments that will be made in return. Transactions can take place only between firms that have formed a prior relationship.

Since each supplier has many customers and each customer has many suppliers, and since firms have overlapping but not identical networks, it would be impractical for a grand negotiation to take place among all firms in the economy. Instead, we assume cooperative bargaining among isolated pairs. When the firms in some tier bargain with their suppliers upstream, the negotiations take place simultaneously. We assume a Nash-in-Nash equilibrium for the bargaining outcome (Horn and Wolinsky, 1988); that is, each member of a pair takes as given the outcomes of its negotiations with all of its other suppliers or buyers, as the case may be. We also impose a reasonable, sequential structure to the series of negotiations. First, the lead firms arrange a set of input purchases from their networks of suppliers. Then the suppliers, who are now contractually obligated to deliver specified quantities to each of their customers, turn to their own suppliers upstream to purchase the inputs they demand to fulfill their orders. The subsequent negotiations also take place in order, until finally the firms in tier 1 negotiate with the firms in tier 0. All pairs are forward looking, recognizing that their agreements have implications for their subsequent purchases and payments both on and off the equilibrium path.

Our model is meant to capture one of the canonical supply-chain forms described in Lund et al. (2020) and the *Economic Report of the President* (Council of Economic Advisors, 2022); see Panel B of Figure 6.1 in the latter.[1] In what the latter report calls "outsourcing with isolated industries," inputs travel downstream through several tiers until they are ultimately transformed into a consumer good by a lead firm. As they explain, the lead firms create product designs and oversee specifications, at least from their immediate suppliers if not further up the chain, but they typically do not own or control most of these suppliers. Lund et al. (2020) examined the lists of suppliers for 668 large manufacturing companies and report that most have hundreds of direct suppliers, who collectively have thousands of suppliers in the tier immediately upstream. For example, General Motors reports 856 direct suppliers and a total of more than 18,000 suppliers to those direct suppliers. For Apple, those numbers are 638 and more than 7,400, respectively, while for Nestlé they are 717 and more than 5,000. Also in keeping with our model, Carvalho and Tahbaz-Salehi (2019) observe that input suppliers typically sell to several or many lead firms. For example, Dell and Lenovo share 2,272 direct suppliers among the total of 7,033 serving the former company and the 6,240 serving the latter; see Lund et al. (2020, p.9).

The *Economic Report of the President* (Council of Economic Advisors, 2022, pp. 211-212) also describes several categories of investments that firms make to manage their supply risks. Under the heading of *Redundancy,* they note that firms "invest in developing relationships with additional suppliers. Finding alternative suppliers ... is time-consuming, and suppliers must often go through quality verification. If firms proactively invest in building relationships with several suppliers, the lead firm has ready alternatives. Even if one supplier is unable to produce, another one can step in as a replacement." Under the heading of *Agility*, they note that "[f]irms can invest in their workers'

[1] Baldwin and Venables (2013) coined the terms "snake" and "spider" to distinguish supply chains in which an input passes through multiple stages with sequencing dictated by engineering considerations from chains that involve the assembly of parts in no particular order. They focus on the effects of a reduction in international frictions on the location of production in these alternative types of global supply chains. Our model is something of a hybrid, with a spider structure at every tier and a snake structure that links the different tiers.

ability to solve problems, thus enabling them to pivot quickly to alternative products or processes or react to abnormal situations." They observe that such investments, and others that fall into this category, often require up-front spending. Our model features both up-front investments in relationships and up-front investments to mitigate the risk of internal disruptions. A key question that the report raises and that we address below is whether firms have adequate private incentives to make these investments in the light of the externalities that may be conferred on other actors in the supply chain.

Our analysis focuses on the wedges that exist between private and social incentives at different stages of the supply chain. To identify these wedges, we solve a planner's direct-control problem and then ask what instruments the government would need to implement the first-best allocation as a decentralized equilibrium. In general, the government would need three types of policy instruments in our setting: a set of subsidies or taxes on transactions between firms in adjacent tiers; a set of subsidies or taxes to promote or discourage investments in agility (own resilience) in different tiers; and a set of subsidies or taxes on investments in supplier relationships. We are particularly interested in how the optimal policies vary across tiers of the supply chain and whether the policies targeted to a particular tier depend only on technological conditions and bargaining weights in that tier and the next, or whether they reflect conditions that prevail throughout the supply chain.

We find that the outcome of each bargaining game yields an intuitive "markup factor" relating the price paid for inputs by firms in some tier to the production cost for the firms in the tier above. The endogenous markup reflects the relative bargaining weights of the upstream and downstream firms and the substitutability between the various inputs used by the latter. The optimal transaction subsidy counteracts the effect of the markup on marginal cost, much as in settings with imperfectly-competitive *markets* (rather than bilateral bargaining) for *standardized* inputs. The optimal policy to promote or discourage investments in resilience reflects two offsetting considerations. On the one hand, such investments confer a positive externality to the customers immediately downstream in a firm's network. On the other hand, the subsidy to transactions that is part of the first-best policy package inflates the private profitability of investments in resilience relative to their social value. If bargaining and technology parameters are common across tiers, then the first-best subsidies to resilience do not vary except for those at the extremes of the supply chains. In any case, the optimal "subsidy" for investments in resilience by firms in any middle tier may in fact be a *tax*, if the first-best subsidy for input purchases by those firms is large enough. Finally, we show that the optimal subsidies for network formation are the same as those for resilience, despite the fact that firms have a private incentive to use these investments to improve their bargaining position vis-à-vis their suppliers and buyers.

It is perhaps surprising that the positive and negative externalities generated by investments in resilience and supplier links do not figure in a more complicated way in the formulas for the optimal subsidies. After all, when a firm becomes more resilient or creates a larger network, the greater productivity that results from its presence or from its greater number of suppliers confers a *positive* externality to other companies upstream and downstream in the firm's own network, while conferring a *negative* externality on firms in other networks, including those in its own tier. We show, however, that in the presence of optimal subsidies to counteract the distorting effects of the negotiated markups, these positive and negative spillovers to firms that are not direct suppliers exactly cancel in the general equilibrium. What remain are only the benefits that accrue to the firm's immediate customers and the wedge between social and private returns to investment that results from the transaction subsidies.

As we have noted, the first-best policies for resilience and network formation reflect the fact that the government uses subsidies for input purchases to ensure the ideal sizes of tier-to-tier transactions. But such subsidies may be politically sensitive, if they are viewed as handouts to the corporate sector. Given the public focus on resilience, we feel it is interesting also to address a second-best setting in which policies to promote redundancy and agility are used in the absence of subsidies to transactions. We find that the second-best policies differ from the first-best policies not only in magnitude, but also in the information that enters into their design. Whereas the first-best subsidies to investments in resilience and network thickness depend only on technological parameters relevant to the tier being targeted, the second-best policies reflect technological parameters that describe the entire supply chain.

Although our main focus here is on the policy imperative that arises from the risk of supply disturbances, our paper also contributes a new model to the toolkit on supply chains. Our model is distinctive in its combination of vertical chains with multiple tiers, endogenous network formation, endogenous investments in resilience, bilateral and sequential bargaining, and general equilibrium. Models of endogenous networks such as Oberfield (2018), Acemoglu and Azar (2020) and Kopytov et al. (2022), typically assume roundabout production processes, whereas those with vertical chains such as Ostrovsky (2008), Antràs and Chor (2013) and Johnson and Moxnes (2023) often take the network as given. Like us, Dhyne et al. (2023) and Grossman et al. (2023) allow for costly investments in supplier relationships, but in both cases the probabilities of supply failures are completely exogenous and in both cases downstream firms subsequently purchase inputs from their suppliers at marginal cost.

Many of the supply chains modeled in the literature are fully efficient, either because a lead firm organizes all the transactions along the chain (Antràs and de Gortari, 2020), because the market structure is perfectly competitive (Kopytov et al., 2022; Johnson and Moxnes, 2023), or because a stability mechanism weeds out inefficient pairings (Oberfield, 2018). These models are not suitable for studying the externalities that arise from network formation or investments in resilience, which are the main focus of our analysis.[2]

Perhaps the closest paper to ours is Elliot et al. (2022), who also study supply chain disturbances with idiosyncratic risks of failure. In their decentralized equilibrium, firms source inputs from multiple suppliers and invest resources to strengthen their relationships. However, there are several differences between their setting and ours. In their model, each firm has a finite set of critical inputs

[2] Few models allow for negotiated prices and quantities along the chain. An exception is Alviarez et al. (2023), but they allow for only two production tiers and have no investments in resilience or network formation.

(much as in Grossman et al., 2023). Also, the microfoundations that they provide in their Appendix feature roundabout production, not vertical relationships. Their formulation does not allow for bilateral bargaining to determine quantities and prices. Finally, they address the determinants of resilience only in a single supply chain, because the complexity of their model precludes a general-equilibrium analysis.

It is worth noting that, in this paper, we treat only networks that form in a closed economy. In contrast, Antràs and Chor (2013), Antràs and de Gortari (2020), Grossman et al. (2023), Alviarez (2023), Johnson and Moxnes (2023) and Fontaine et al. (2023), among others, deal with issues of international specialization in *global* supply chains. We hope to study optimal policy in the open economy in our future research.

To reiterate, our main contribution in this paper is to provide a rich yet tractable framework that can be used to study complex investment decisions in supply chains. Our model features an arbitrary number of tiers, bilateral bargaining, costly supplier relationships, and investments in resilience. It captures several realistic externalities that arise in this setting and we provide a complete characterization of first-best and second-best policies for a closed economy.

The remainder of our paper is organized as follows. In the next section, we develop our model and describe the outcomes of the sequential bargaining and the equilibrium choices of investments in resilience and network formation. In Section 3, we study the first-best allocation, outlining first the solution to the planner's direct-control problem and then the policies that a benevolent government can use to implement the optimum as a decentralized equilibrium. We characterize in turn the optimal subsidies for input transactions, for investments in resilience, and for the formation of supplier relationships. Section 4 addresses the second-best policy problem that arises when the government cannot subsidize transactions, but can only promote (or discourage) investments in resilience and network formation. Section 5 concludes.

2 A Model of Vertical Supply Chains

In this section, we develop a general-equilibrium model of vertical supply chains with an arbitrary number $S+1$ of production tiers and risks of supply disruptions throughout. A firm in the uppermost tier 0 produces a differentiated intermediate input using labor alone. A firm in a middle tier $s \in \{1,\ldots,S-1\}$ produces an intermediate using labor and a bundle of inputs from tier $s-1$. It procures this bundle by bargaining over quantities and prices with the various suppliers in its production network. A firm in tier S produces a differentiated consumer good using labor and a bundle of tier $S-1$ inputs.

Each producer of a final good faces a constant elasticity of demand ε and takes an aggregate demand shifter, A, as given. Thus,

$$x_S = Ap^{-\varepsilon}, \tag{1}$$

where x_S denotes sales by a typical firm in tier S and p is the price that it sets.

There is an exogenous measure N_s of symmetric firms in tier $s \in \{0,1,...,S\}$. Each such firm faces an independent risk of a catastrophic disruption to its operations. When a disruption occurs, it renders a firm totally unable to produce. But firms can moderate their exposure to supply shocks by investing in "resilience." A firm in tier s that hires r_s units of labor for this purpose will avoid a subsequent disturbance with probability $\phi_s(r_s)$. We assume $\phi_s' > 0$ and $\phi_s'' < 0$.

Besides investing in their own survival, firms can protect themselves against disruptions to their input sources by forming thick supply networks. A firm in tier s forges supply relationships with the fraction η_s of the N_{s-1} firms in the tier above, for $s \in \{1, 2, \ldots, S\}$, at a cost of k units of labor per relationship. Thus, a network of size $\eta_s N_{s-1}$ comes at the cost of $k\eta_s N_{s-1}$ workers.

After firms have invested in resilience and formed their supplier links, the disruption shocks are realized. A firm in tier $s \in \{1, 2, ..., S\}$ that survives this shock purchases differentiated inputs from survivors with whom it has links in tier $s-1$. The inputs used by the firms in s combine with a constant elasticity of substitution $\sigma_s > 1$ to form a composite input bundle. Considering the "love of variety" implied by this formulation, the typical surviving firm in tier s would like to buy inputs from all $\eta_s N_{s-1} \phi_{s-1}(r_{s-1})$ of its potential suppliers.

Assuming symmetric outcomes, each firm in tier $s \in \{1, 2, ..., S\}$ negotiates a quantity m_{s-1} and a payment t_{s-1} with its typical supplier in tier $s-1$. Bargaining takes place sequentially. First, firms in tier S arrange to buy inputs from their surviving suppliers in tier $S-1$. Then firms in $S-1$, who have agreed to provide quantities of m_{S-1} to each of their downstream partners, contract to buy inputs from their own surviving suppliers in tier $S-2$. And so on up the supply chain, until finally firms in tier 1 bargain with their potential suppliers in tier 0.

When it comes time for a firm in tier s to bargain with its various suppliers in tier $s-1$, it does so *simultaneously*. That is, we seek bargaining solutions that are Nash-in-Nash equilibria,[3] with each downstream member of a pair taking as given the outcome of all negotiations between itself and its other suppliers and each upstream member taking as given the outcome of negotiations between itself and its other customers.[4] This seems appropriate in our setting in the light of the vast number of negotiations that take place, which makes it impractical to undertake an inclusive (and ultimately efficient) grand bargain.

In its negotiation with a supplier from tier $s-1$, a firm in tier s is committed to supply a total of $m_s n_s^d$ units of its wares to its n_s^d downstream clients. The outside option for the firm in tier s is to buy inputs only from its other suppliers and then to achieve its promised output of $m_s n_s^d$ using additional labor. The outside option for the upstream firm in any negotiation is to sell only to its other customers, recognizing that this will affect the quantities it later purchases from its own suppliers and the labor it ultimately hires. In other words, the upstream firm must forecast the outcome of its subsequent negotiations with firms that will be its own suppliers, both on and off the equilibrium path.[5]

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[3] The Nash-in-Nash equilibrium concept was first proposed by Horn and Wolinsky (1988). Collard-Wexler et al. (2019) provided non-cooperative microfoundations as an extension of Rubinstein (1982).

[4] In Section A1 of the appendix, where we derive the outcome of the various Nash bargains that are discussed in Section 2.3, we assume that each firm in tiers $s \in \{0, 1, ..., S-1\}$ controls a finite measure δ of inputs, so that the breakdown of any negotiation has a non-negligible impact on the firms involved. Then we take the limits as $\delta \to 0$.

[5] Notice that we have not specified whether the inputs produced by some firm in tier s need to be customized

After procuring its inputs, the firm in tier s, $s \in \{1, 2, ..., S\}$, hires labor l_s to produce output according to the Cobb-Douglas production function[6]

$$x_s = l_s^{\gamma_s} \left[\int_{z \in \Omega_{s-1}} m_{s-1}(z)^{\alpha_s} \, dz \right]^{\frac{1-\gamma_s}{\alpha_s}}, \; s = \{1, \dots, S\}, \tag{2}$$

where $m_{s-1}(z)$ is the quantity of inputs that a firm in tier s purchases from supplier z in tier $s-1$, Ω_{s-1} is the set of surviving suppliers within its network from whom it buys, and $\alpha_s \equiv (\sigma_s - 1)/\sigma_s$. The firms in the uppermost tier use only labor, so that

$$x_0 = \frac{l_0}{a}.$$

Figure 1 summarizes the timing in the model. First, firms invest in resilience and form their costly supply relationships. Then, supply disturbances are realized. Firms that survive these disturbances move on to the bargaining stage. Negotiations take place first between final producers and suppliers in tier $S-1$. After these negotiations have been concluded, firms in tier $S-1$ bargain with firms in tier $S-2$. This sequential bargaining continues until finally firms in tier 1 sign contracts with firms in tier 0. In the production stage that follows, firms hire labor to combine with the intermediate inputs they have purchased in order to fulfill their contracts with downstream buyers. Finally, the final producers hire labor to manufacture their differentiated outputs, which they sell to consumers at the prices that maximize profits.

We proceed in the following sections to analyze the stages of the model in reverse order. We describe a *symmetric equilibrium,* beginning with production of final goods, followed by production of inputs, sequential bargaining between suppliers and buyers, and finally investments in resilience and the formation of supply networks. In Section 2.6 we lay out the conditions for a general equilibrium in an economy with an inelastic labor supply, L. Throughout, we take the wage rate as numeraire.

2.1 Production of Consumer Goods

Producers of final goods engage in monopolistic competition, each facing the demand given in (1). By the time these producers hire their labor, they have contracted for the purchase of m_{S-1} units of inputs from each of their $n_S^u = \eta_S N_{S-1} \phi(r_{S-1})$ undisturbed suppliers at an agreed total cost of $n_S^u t_{S-1}$, and have invested r_S in resilience and $k \eta_S N_{S-1}$ in forming their supply network. The typical firm chooses l_S to maximize its operating profits,

$$\pi_S = A^{\frac{1}{\varepsilon}} l_S^{\frac{\gamma_S(\varepsilon-1)}{\varepsilon}} (m_{S-1})^{\frac{(1-\gamma_S)(\varepsilon-1)}{\varepsilon}} (n_S^u)^{\frac{(1-\gamma_S)(\varepsilon-1)}{\alpha_S \varepsilon}} - l_S - n_S^u t_{S-1}. \tag{3}$$

for use by a particular customer in tier $s+1$. Inasmuch as the firms conclude a deal for m_s units of input before any relationship-specific resources have been sunk, our analysis applies both when the differentiated inputs must be customized for each user and when they can be used interchangeably by different customers.

[6] We could add a multiplicative TFP term that varies across tiers without affecting any of our results. To avoid clutter, we normalize TFP to equal one in all tiers.

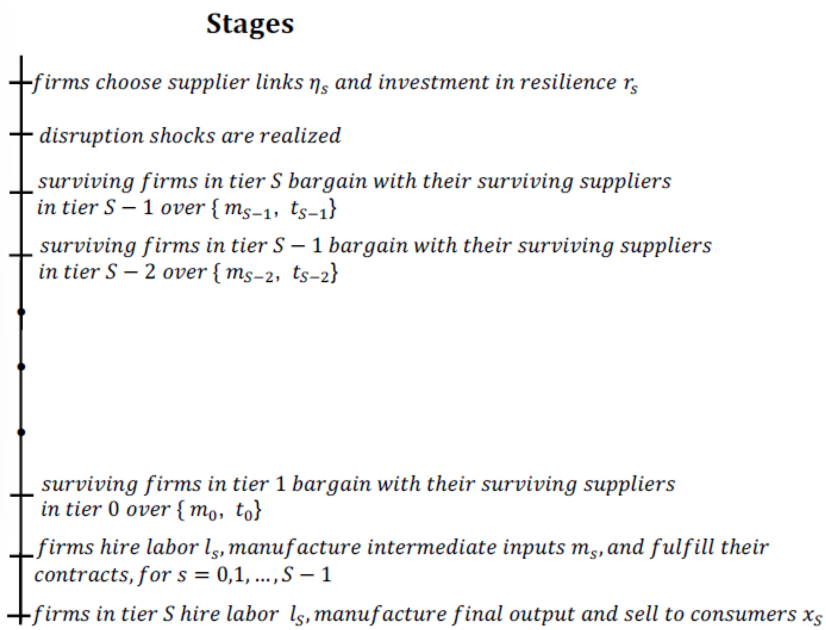

Figure 1: Sequence of Events and Decisions

2.2 Production of Inputs

Producers in tier $s \in \{0, 1, \ldots, S-1\}$ have agreed to supply m_s units of their output to each of n_s^d customers and to purchase m_{s-1} units of inputs from each of their n_s^u suppliers.[7] For those in tiers $s \in \{1, 2, \ldots, S-1\}$, this means hiring labor to satisfy

$$l_s^{\gamma_s} \left(m_{s-1}\right)^{1-\gamma_s} \left(n_s^u\right)^{\frac{1-\gamma_s}{a_s}} = n_s^d m_s,$$

so that

$$l_s = \left[\frac{n_s^d m_s}{\left(m_{s-1}\right)^{1-\gamma_s} \left(n_s^u\right)^{\frac{1-\gamma_s}{a_s}}}\right]^{\frac{1}{\gamma_s}}. \tag{4}$$

Firms in tier 0 must hire workers to produce $n_0^d m_0$ units of output, which implies

$$l_0 = n_0^d a m_0. \tag{5}$$

2.3 Bargaining between Suppliers and Buyers

We solve the various bargaining games "backwards," beginning with the last round of negotiations that takes place between far-upstream firms in tiers 0 and 1.

A typical firm in tier 1 has committed to supply m_1 units to each of its n_1^d customers. In a Nash-in-Nash negotiation, it takes as given its agreement to purchase m_0 units of inputs from each

[7] We show in Section 2.6 that the number of upstream suppliers of a firm in tier s is related in the general equilibrium to the number of downstream partners of a firm in tier $s-1$. For now, we treat n_s^u and n_{s-1}^d as arbitrary numbers.

of its suppliers other than the one with whom it now negotiates.[8] In the negotiation at hand, the firm might buy $\tilde{m}_0$ units from the particular supplier in tier 0 in exchange for a payment of $\tilde{t}_0$. If the negotiation ends successfully, the firm anticipates hiring $l_1(\tilde{m}_0)$ units of labor to produce $n_1^d m_1$ units of output, as dictated by inverting the production function in (2). If the negotiation fails, the firm will be forced to produce the same amount of output with one fewer input. To fulfill its commitments, it will need to hire more labor to make up the difference, say $\hat{l}_1 > l_1(\tilde{m}_0)$. We write the firm's surplus from an agreement, a function of the contract terms $\left(\tilde{m}_0, \tilde{t}_0\right)$, as

$$\psi_1^d\left(\tilde{m}_0, \tilde{t}_0\right) = \hat{l}_1 - l_1\left(\tilde{m}_0\right) - \tilde{t}_0.$$

Meanwhile, the tier 0 firm can produce output at a constant cost of a per unit. If the negotiation ends in failure, it receives no payment from this customer but saves the cost of producing for that firm. The tier 0 firm achieves a surplus from an agreement $\left(\tilde{m}_0, \tilde{t}_0\right)$ equal to

$$\psi_0^u\left(\tilde{m}_0, \tilde{t}_0\right) = \tilde{t}_0 - a\tilde{m}_0.$$

The Nash bargaining solution divides the surplus with exogenous shares β_s for the downstream firm in tier s and $1 - \beta_s$ for the upstream firm in tier $s-1$. It achieves

$$\left(m_0^{Nash}, t_0^{Nash}\right) = \arg\max_{\left(\tilde{m}_0, \tilde{t}_0\right)} \psi_1^d\left(\tilde{m}_0, \tilde{t}_0\right)^{\beta_1} \psi_0^u\left(\tilde{m}_0, \tilde{t}_0\right)^{1-\beta_1}.$$

Moreover, symmetry requires $\left(m_0^{Nash}, t_0^{Nash}\right) = (m_0, t_0)$.

Now consider the negotiation between a tier s firm and a tier $s+1$ firm, for $s \in \{1, \ldots, S-2\}$. Neither of these firms is at an extreme end of the supply chain. The downstream firm, which has committed to sell output of $n_{s+1}^d m_{s+1}$ to its various customers, evaluates a contract with terms $\left(\tilde{m}_s, \tilde{t}_s\right)$. It takes as given its successful negotiations to purchase m_s units at a cost of t_s per transaction from its other suppliers. If the deal at hand goes through, it will have to hire $l_{s+1}(\tilde{m}_s)$ units of labor to fulfill its sales contracts. If not, it will instead need to hire $\hat{l}_{s+1}$ workers. An agreement with terms $\left(\tilde{m}_s, \tilde{t}_s\right)$ offers the downstream firm a surplus of

$$\psi_{s+1}^d\left(\tilde{m}_s, \tilde{t}_s\right) = \hat{l}_{s+1} - l_{s+1}\left(\tilde{m}_s\right) - \tilde{t}_s.$$

The upstream firm in tier s stands to gain a payment of $\tilde{t}_s$, in addition to the proceeds from other sales, which it takes as given. But its cost calculation is more subtle. If it proceeds with the contract, it will anticipate purchasing $m_{s-1}^e(\tilde{m}_s, m_s)$ units from each of its suppliers at a cost of $t_{s-1}^e(\tilde{m}_s, m_s)$ per transaction, and then it will hire $l_s^e(\tilde{m}_s, m_s)$ units of labor to finish the job. Notice

[8] With a continuum of suppliers, each such firm is infinitessimally small. In Section A1 of the appendix, where we conduct the formal analysis, we assume a discrete number of upstream suppliers, say U_1, each of which sells a finite range δ of intermediate goods. In this setting, we can consider off-the-equilibrium outcomes with $U_1 - 1$ suppliers. Then we take the limit as $U_1 \to \infty$ and $\delta \to 0$ such that $U_1\delta = n_1^u$. We proceed similarly for the other negotiations described below.

that both the payment to suppliers and the labor cost depend on the size $\tilde{m}_s$ of the proposed deal with the negotiating partner in addition to the size m_s of its deals with all of its other suppliers, which it takes as given. If, instead, the negotiation breaks down, the upstream firm will expect to buy $\hat{m}_{s-1}(m_s)$ units from each of its suppliers at a cost of $\hat{t}_{s-1}(m_s)$ per transaction, and will employ $\hat{l}_s(m_s)$ workers. These, of course, do not depend on the proposed terms, $\tilde{m}_s$ or $\tilde{t}_s$. We calculate the firm's surplus from this particular relationship as

$$\psi_s^u\left(\tilde{m}_s, \tilde{t}_s, m_s\right) = \tilde{t}_s(m_s) + \hat{l}_s(m_s) - l_s^e(\tilde{m}_s, m_s) + n_s^u\left[\hat{t}_{s-1}(m_s) - t_{s-1}^e(\tilde{m}_s, m_s)\right], \tag{6}$$

and compute the Nash solution as

$$\left(m_s^{Nash}, t_s^{Nash}\right) = \arg\max_{\left(\tilde{m}_s, \tilde{t}_s\right)} \psi_{s+1}^d\left(\tilde{m}_s, \tilde{t}_s\right)^{\beta_{s+1}} \psi_s^u\left(\tilde{m}_s, \tilde{t}_s, m_s\right)^{1-\beta_{s+1}}.$$

By symmetry, we impose that $\left(m_s^{Nash}, t_s^{Nash}\right) = (m_s, t_s)$.

Finally, consider the negotiation between an input producer in tier $S-1$ and a final producer in tier S. Under a proposal of $\left(\tilde{m}_{S-1}, \tilde{t}_{S-1}\right)$, the final producer anticipates hiring labor of $l_S^e(\tilde{m}_{S-1}, m_{S-1})$ considering the contracts for m_{S-1} units of inputs that it has with each of its other suppliers. This hiring will yield maximal operating profits of $\pi_S^e(\tilde{m}_{S-1}, m_{S-1})$. The firm's outside option is to hire (profit-maximizing) labor of $\hat{l}_S(m_{S-1})$ and generate operating profits of $\hat{\pi}_S(m_{S-1})$. The final producer captures surplus from the relationship of

$$\psi_S^d\left(\tilde{m}_{S-1}, \tilde{t}_{S-1}, m_{S-1}\right) = \pi_S^e(\tilde{m}_{S-1}, m_{S-1}) - \tilde{t}_{S-1} - \hat{\pi}_S(m_{S-1}).$$

For the supplier, the situation is the same as for other suppliers with $s \in \{1, \ldots, S-2\}$, as described in the paragraph leading to (6). It yields surplus to the tier $S-1$ firm of

$$\begin{aligned}\psi_{S-1}^u\left(\tilde{m}_{S-1}, \tilde{t}_{S-1}, m_{S-1}\right) &= \tilde{t}_{S-1} + \hat{l}_{S-1}(m_{S-1}) - l_{S-1}^e(\tilde{m}_{S-1}, m_{S-1}) \\ &\quad + n_{S-1}^u\left[\hat{t}_{S-1}(m_{S-1}) - t_{S-1}^e(\tilde{m}_{S-1}, m_{S-1})\right].\end{aligned}$$

We compute the Nash bargaining solution as before.

In Section A1.1 of the appendix, we use the bargaining solutions to derive expressions that relate the volume of inputs m_{S-1} and the payment t_{S-1} in a typical transaction between a tier $S-1$ and a final producer to the numbers of active links at all stages in the supply chain. Then we derive recursive equations relating m_s and t_s to m_{s+1} and the numbers of suppliers and customers for each firm, namely

$$m_s = C_{m_s}\left(n_{s+1}^u\right)^{\frac{\gamma_{s+1}(1-\alpha_{s+1})-1}{\alpha_{s+1}}}\left[\prod_{j=1}^{s}\left(n_j^u\right)^{\frac{\gamma_{s+1}\Gamma_j^s(1-\alpha_j)}{\alpha_j}}\right] n_{s+1}^d m_{s+1}$$

and

$$t_s = C_{t_s} \left(n^u_{s+1}\right)^{\frac{\gamma_{s+1}(1-\alpha_{s+1})-1}{\alpha_{s+1}}} \left[\prod_{j=1}^{s} \left(n^u_j\right)^{\frac{-\Gamma^{s+1}_j(1-\alpha_j)}{\alpha_j}}\right] n^d_{s+1} m_{s+1}$$

where C_{m_s} and C_{t_s} are constants given in (A.33) and (A.35), and $\Gamma^s_j \equiv \Pi^s_{i=j}(1-\gamma_i)$, i.e., the product of the input shares for all stages between j and s. Finally, combining the solution for m_{S-1} with the recursive equations, we can express m_s and t_s as functions of network properties (i.e., the number of links between firms in different tiers) and aggregate demand. These expressions have the property that the quantity sold by a tier s firm to a tier $s+1$ firm increases in the number of links upstream from s (because these enhance the productivity of the firm in tier s) and decrease in the number of links between firms downstream from s (because these increase the competition for the firm's sales). The payment by a firm in tier $s+1$ to a firm in tier s also depends on upstream productivity and downstream competition, as well as on the bargaining outcome that divides the surplus between these two firms.

We need to impose a restriction on the technology for producing final goods and the demand for these goods to ensure that payoffs for final producers are positive. In the appendix, we establish a sufficient condition for this, which is $\sigma_S > \varepsilon$; that is, the elasticity of substitution between inputs in the final tier is strictly greater than the elasticity of substitution between final goods in the eyes of consumers (see (A.49) and the discussion that follows this equation). We shall further assume that inputs become (weakly) more differentiated and less substitutable as we proceed down the supply chain. Although nothing in our formulation requires this assumption, it seems a realistic one and it gives some sharper predictions about the relationship between policies addressed to different tiers.[9]

We henceforth adopt

Assumption 1 $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_S > \varepsilon$.

2.4 The Markup Factor

In this section, we report an intuitive relationship between costs and payments at successive stages of the supply chain. This relationship allows us to define a *markup factor* that relates payments for inputs to their production cost. The markup factor, in turn, will prove useful when it comes time to interpret the market distortions that arise in the absence of government intervention and to derive optimal policy responses.

First, we define q_s as the cost to a firm in tier s of supplying one unit of its input. This cost comprises input costs of $n^u_s t_{s-1}/m_s$ and labor costs of l_s/m_s, where l_s is related to m_{s-1} by the equilibrium relationship in (4). Combining, we derive in Section A6 of the appendix the recursive relationship between unit costs at successive tiers,

$$(1-\gamma_s)\, q_s n^d_s m_s = \left[\gamma_s + (1-\gamma_s)\frac{1-\beta_s+\alpha_s\beta_s}{\alpha_s}\right] q_{s-1} n^u_s m_{s-1}. \tag{7}$$

[9] We shall later find that the assumption that $\sigma_1 \geq \cdots \geq \sigma_S$ is sufficient, though not necessary, for the concavity of the value function of a firm in every tier s with respect to its investment η_s in creating supplier links.

The left-hand side of (7) represents the total spending on intermediate goods by a firm in tier s, considering its commitment to produce m_s and the cost-minimizing techniques implied by the Cobb-Douglas production structure. These expenditures must be equal to the tier $s-1$ firm's revenues from their sales to the tier s firm, which are its production costs multiplied by the term in square brackets. That term guides the mapping from tier $s-1$ marginal costs to tier s marginal costs; it represents a weighted average of the markup of labor costs (equal to one in a competitive labor market) and the markup on inputs, with the Cobb-Douglas exponents as weights. Evidently,

$$\mu_{s-1} \equiv \frac{1-\beta_s+\alpha_s\beta_s}{\alpha_s} = (1-\beta_s)\,\frac{\sigma_s}{\sigma_s-1}+\beta_s \tag{8}$$

is the markup realized by firms in tier $s-1$ in their sales to firms in tier s. This interpretation becomes even clearer when, after a bit more manipulation, we write

$$t_{s-1} = \mu_{s-1}q_{s-1}m_{s-1};$$

the payments made in a typical sale from tier $s-1$ to tier s amount to a multiple of cost. Notice that μ_{s-1} is a weighted average of the competitive markup of unity that emerges when the downstream firm has all of the bargaining power ($\beta_s = 1$) and the standard monopoly markup of $\sigma_s/(\sigma_s-1)$ that emerges when the upstream firm has all of the bargaining power ($\beta_s = 0$).

2.5 Choice of Resilience and Network Formation

Firms choose the resilience of their operations and the thickness of their production networks. An increase in resilience (higher r) directly boosts a firm's expected operating profits by raising its probability of survival. A thicker network (greater η) provides several benefits to a firm. First, even without any risk of supplier disruption, the firm's productivity increases with the variety of its input purchases, as in Ethier (1982) and subsequent models of differentiated inputs with CES bundles. Second, a thicker network provides a hedge against outages among a firm's potential suppliers. Third, the thicker is a firm's network, the stronger will be its bargaining position vis-à-vis its upstream suppliers (for all firms except those in tier 0), because an increase in n_s^u spells better outside options in relation to any one of them. Fourth, an increase in the number of a firm's suppliers also improves its outside options in the negotiations with every one of its downstream customers (for all firms except those in tier S). Thus, increasing η leaves a firm in some tier s paying less to each of its supplier (for $s \neq 0$) and collecting more from each of its customers (for $s \neq S$). We can write $\tilde{v}_s(\tilde{\eta})$ to capture all of these ways in which the thickness of a firm's own production network in tier s affects its operating profits conditional on its survival, taking as given the choices $\{r_s\}$ and $\{\eta_s\}$ of all other firms in the economy. Then a firm in tier s chooses $\tilde{\eta}$ and r to maximize $\phi_s(r)\,\tilde{v}_s(\tilde{\eta}) - k\tilde{\eta}N_{s-1} - r$ and symmetry implies $(\eta_s, r_s) = \arg\max_{(\tilde{\eta},r)} \phi_s(r)\,\tilde{v}_s(\tilde{\eta}) - k\tilde{\eta}N_{s-1} - r$.

The operating profits for a firm in an intermediate tier s are equal to the difference between the revenues it receives from all of its customers and the payments it makes for all of its inputs and labor. In the appendix, we show that, holding constant the links formed by other firms in its own tier, we can write $\tilde{v}_s$ as a power function of a firm's choice of $\tilde{\eta}$, i.e.,

$$\tilde{v}_s(\tilde{\eta}) = \begin{cases} n_s^d t_s - n_s^u t_{s-1} - l_s = Q_{v_s} \tilde{\eta}^{\frac{(1-\gamma_s)(\sigma_{s+1}-1)}{\sigma_s - 1}} & \text{for } s \in \{1, \ldots, S-1\} \\ Ap^{1-\varepsilon} - n_S^u t_{S-1} - l_S = Q_{v_S} \tilde{\eta}^{\frac{(1-\gamma_S)(\varepsilon-1)}{\sigma_S - 1}} & \text{for } s = S \end{cases} \tag{9}$$

where Q_{v_s} for $s \in \{1, \ldots, S\}$ are constants from the firm's point of view; see (A.56), (A.57) and (A.59) in the appendix. The elasticity of expected profits with respect to a firm's number of suppliers is greater when having a more diverse set of inputs contributes more to productivity, i.e., when inputs are a greater share of production costs for firms in tier s (higher $1-\gamma_s$) and when the inputs used by these firms are more differentiated (smaller σ_s). A given productivity gain is more beneficial to a firm in tier s when its competitors produce inputs that are closer substitutes for its own (higher σ_{s+1}). Assumption 1 ensures that the powers on $\tilde{\eta}$ are between zero and one for all input cost shares, and thus that $\tilde{v}_s(\tilde{\eta})$ is concave. The expressions for $\tilde{v}_s(\tilde{\eta})$ take into account that the measure of a firm's suppliers affects quantities and payoffs not only on the equilibrium path, but also off that path, i.e., when evaluating firms' outside options in the event of a breakdown of any negotiation.

2.6 General Equilibrium

Finally, to close the model, we need the total number of purchase transactions to match the total number of sales transactions at every tier. Firms in tier s conduct a total of $n_s^u \phi_s(r_s) N_s$ transactions with their upstream suppliers. Firms in tier $s-1$ conduct a total of $n_{s-1}^d \phi_{s-1}(r_{s-1}) N_{s-1}$ transactions with their downstream customers. Equating these two, and recalling that $n_s^u = \eta_s \phi_{s-1}(r_{s-1}) N_{s-1}$, we find[10]

$$n_{s-1}^d = \eta_s \phi_s(r_s) N_s \text{ for } s \in \{1, \ldots, S\}.$$

We also need the labor market to clear. Labor is used to produce intermediate inputs, to produce final goods, to form supply networks, and to foster resilience at every level in the supply chain. Production labor in a typical firm in tier s must satisfy (4) for $s \in \{1, \ldots, S-1\}$ and (5) for $s = 0$. Final producers hire labor l_S to maximize operating profits in (3). In addition, each firm in tier s employs r_s workers to promote resilience and each firm in tier $s \neq 0$ employs $k\eta_s N_{s-1}$ workers to form supply relationships. There are $\phi_s(r_s) N_s$ active firms in tier s after the resolution of the supply disturbances. Therefore, the general equilibrium requires

$$\sum_{s=0}^{S} N_s r_s + \sum_{s=1}^{S} N_s k \eta_s N_{s-1} + \sum_{s=0}^{S} \phi_s(r_s) N_s l_s = L.$$

The demand shifter A in (1) is determined by this condition. See (A.70) in the appendix and the

[10] Recall that η_s is chosen by the downstream firm that seeks links with potential suppliers.

discussion that follows this equation.

3 First-Best Allocation and Optimal Policy

We turn now to the planner's maximization problem. We wish to characterize the first-best allocation of resources and the policies that would be needed to decentralize it. The constant-elasticity demand function in (1) derives, as usual, from a CES utility function for the representative consumer that takes the form

$$W = \left[\int_{z \in \Omega_S} x_S(z)^{\frac{\varepsilon - 1}{\varepsilon}} dz \right]^{\frac{\varepsilon}{\varepsilon - 1}}, \tag{10}$$

where Ω_S is the set of differentiated products available to consumers. We take W as the planner's objective.

Inasmuch as the differentiated products enter the utility function symmetrically, the first-best allocation entails equal quantities $x_S(z) = x_S$ of each consumer good. Then we can rewrite the welfare objective as $W = (n_S)^{\frac{\varepsilon}{\varepsilon - 1}} x_S$, where $n_S = \phi_S(r_S) N_S$ is the measure of final producers that avoid supply disturbances. Also, the symmetry of the production function (2) implies that the planner uses equal amounts of the inputs available at a given stage; i.e., $m_s(z) = m_s$. Using $n_S^u = \eta_S \phi_{S-1}(r_{S-1}) N_{S-1}$, we can write the planner's problem as

$$\max_{\{r_s\},\{\eta_s\},\{m_s\},\{l_s\}} W = [\phi_S(r_S) N_S]^{\frac{\varepsilon}{\varepsilon-1}} l_S^{\gamma_S} (m_{S-1})^{1-\gamma_S} \left[\eta_S \phi_{S-1}(r_{S-1}) N_{S-1}\right]^{\frac{1-\gamma_S}{\alpha_S}} \tag{11}$$

subject to

$$\sum_{s=0}^{S} N_s r_s + \sum_{s=1}^{S} N_s k \eta_s N_{s-1} + \sum_{s=0}^{S} \phi_s(r_s) N_s l_s \leq L, \tag{12}$$

$$\left[\phi_{s+1}(r_{s+1}) N_{s+1}\right] \left[\eta_{s+1} \phi_s(r_s) N_s\right] m_s \leq \phi_s(r_s) N_s l_s^{\gamma_s} (m_{s-1})^{1-\gamma_s} \left[\eta_s \phi_{s-1}(r_{s-1}) N_{s-1}\right]^{\frac{1-\gamma_s}{\alpha_s}}, \quad \text{for } s \in \{1, \ldots, S-1\} \tag{13}$$

and

$$[\phi_1(r_1) N_1] [\eta_1 \phi_0(r_0) N_0] m_0 \leq \frac{\phi_0(r_0) N_0 l_0}{a}. \tag{14}$$

The planner chooses investments in resilience, $\{r_s\}$, the structure of supply networks, $\{\eta_s\}$, the input quantities, $\{m_s\}$, and the allocations of labor to production in every tier, $\{l_s\}$, to maximize welfare of the representative agent. The constraint (12) stipulates that labor in all uses should not exceed the available supply. The left-hand side of (13) is the total quantity of tier s inputs used by the $\left[\phi_{s+1}(r_{s+1}) N_{s+1}\right]$ producers in tier $s+1$; each one purchases m_s units from each of its $\eta_{s+1} \phi_s(r_s) N_s$ upstream suppliers. This total quantity demanded should not exceed what is produced of this input by the $\phi_s(r_s) N_s$ producers, considering the technology described by (2). Similarly, (14) restricts the uses of the tier 0 inputs not to exceed its aggregate supply, considering the linear technology specified in (5).

In the optimal allocation, the constraints are satisfied with equality. Using the first-order conditions with respect to labor l_s for all $s \in \{0, \ldots, S\}$ and input quantities m_s for all $s \in \{0, \ldots, S-1\}$, we can solve for the optimal allocations of labor, $\{l_s^*\}_{s=0}^{S}$ and the optimal transaction quantities, $\{m_s^*\}_{s=0}^{S-1}$, for any given numbers of upstream and downstream links, $\{n_s^d\}_{s=0}^{S-1}$ and $\{n_s^u\}_{s=1}^{S}$.[11] These first-order conditions imply that the ratio $l_s^*/n_s^u m_{s-1}^*$ of labor to aggregate inputs used in production by a firm in tier s, $s \in \{1, \ldots S\}$, should equal $\frac{\gamma_s}{1-\gamma_s}\frac{\rho_{s-1}}{\omega}$, where ρ_s denotes the shadow value of a tier s input (the Lagrange multiplier on constraint (13) or (14), as the case may be), and ω denotes the shadow value of labor (the Lagrange multiplier on constraint (12)); see Section A5.3 in the appendix. So, we have

$$\frac{l_1^*}{n_1^u m_0^*} = \frac{a\gamma_1}{1-\gamma_1}, \tag{15}$$

considering that $\rho_0 = a\omega$. Then we compute

$$\frac{l_2^*}{n_2^u m_1^*} = \kappa_1 \frac{\gamma_2}{1-\gamma_2} \left(n_1^u\right)^{-\frac{1-\gamma_1}{\sigma_1 - 1}}, \tag{16}$$

where $\kappa_1 \equiv a^{1-\gamma_1}\gamma_1^{-\gamma_1}\left(1-\gamma_1\right)^{-(1-\gamma_1)}$. This equation follows from the fact that $\left(1-\gamma_1\right)\rho_1 n_1^d m_1^* = \rho_0 n_1^u m_0^* = a\omega n_1^u m_0^*$,[12] that $n_1^d m_1^*$ is related to m_0^* and l_1^* by the production function (4), and that $l_1^*/n_1^u m_0^*$ has been solved in (15). The right-hand side of (16) represents the ratio of the Cobb-Douglas exponents in the production of tier 2 goods, adjusted for the productivity of the tier 1 inputs that reflects their variety. We proceed similarly and recursively to compute the optimal input ratios $l_s^*/n_s^u m_{s-1}^*$ for $s \in \{3, \ldots, S\}$ using $\left(1-\gamma_s\right)\rho_s n_s^d m_s^* = \rho_{s-1} n_s^u m_{s-1}^*$ and the relationship between output $n_s^d m_s^*$ and inputs l_s^* and m_{s-1}^* that is implied by (4).

Next we optimize the structure of the networks and firms' resiliency, which together determine the first-best number of upstream and downstream links for the typical firm in each tier. In the appendix, we show that the first-order conditions with respect to η_s, l_s and m_{s-1} together imply (see (A.78) and (A.79) in the appendix)

$$\frac{kN_sN_{s-1}\eta_s^*}{L - \sum_{j=0}^{S} N_j r_j^* - \sum_{j=1}^{S} kN_{j-1}N_j\eta_j^*} = \frac{\Gamma_s^S}{\sigma_s - 1} \qquad \text{for } s = \{1, \ldots, S\}. \tag{17}$$

The left-hand side of (17) is the ratio of the aggregate amount of labor optimally used for forming supplier links to the aggregate labor optimally used for manufacturing. The right-hand side of (17) reflects the cost share of intermediate inputs in tier s and all tiers further downstream, and the elasticity of substitution between inputs produced in tier s. The greater are the input shares downstream and the less substitutable are the inputs used in tier s, the more socially valuable are

[11] Using the solutions for l_S^* and m_{S-1}^*, we can then recover the optimal sales of a typical final good, x_S^*, from the production function.

[12] That is, the fraction $1-\gamma_1$ of the shadow value of the $n_1^d m_1^*$ units of output generated by a typical tier 1 producer is devoted to spending on tier 0 inputs.

supply links for the firms in that tier.

Similarly, we combine the first-order conditions with respect to r_s with the conditions for the optimal quantities, and find (see (A.76) and (A.77) in the appendix)

$$\frac{N_s r_s^*}{L-\sum_{j=0}^{S} N_j r_j^* - \sum_{j=1}^{S} kN_{j-1}N_j\eta_j^*} = \frac{\Gamma_{s+1}^S}{\sigma_{s+1}-1}\frac{\phi_s'\left(r_s^*\right)r_s^*}{\phi_s\left(r_s^*\right)} \qquad \text{for } s=\{0,1,\ldots,S-1\} \tag{18}$$

and

$$\frac{N_S r_S^*}{L-\sum_{j=0}^{S} N_j r_j^* - \sum_{j=1}^{S} kN_{j-1}N_j\eta_j^*} = \frac{1}{\varepsilon-1}\frac{\phi_S'\left(r_S^*\right)r_S^*}{\phi_S\left(r_S^*\right)}. \tag{19}$$

In both (18) and (19), the left-hand side is the ratio of the aggregate labor optimally used to promote resilience in some tier to the aggregate labor optimally used for manufacturing, while the right-hand side reflects the social benefits of resilience at that tier. In all tiers, the benefits increase with the elasticity of survival probability with respect to investment For intermediate goods, they also increase with the cost shares of intermediates in all tiers downstream from s and decrease with the elasticity of substitution between tier s inputs when used in tier $s+1$. For final goods, resilience is more valuable when the products are less substitutable in the eyes of consumers.

Finally, we compare the equilibrium allocation described in Section 2 with the first-best allocation described immediately above. To do so, we introduce three sets of policies that would allow the planner to implement the first-best allocation as a decentralized equilibrium. These policies represent "wedges" between private and social incentives for each use of resources. We let $\{\tau_s\}_{s=0}^{S-1}$ be the sequence of sales policies along the supply chain, where τ_s denotes the fraction of the cost of a tier s input optimally paid by the downstream firm in tier $s+1$. Clearly, $\tau_s<1$ represents a *subsidy* to promote sales from tier s to tier $s+1$, whereas $\tau_s>1$ represents a *tax* to discourage such sales. Similarly, we let $\{\theta_s\}_{s=0}^{S}$ be the sequence of investment policies, where θ_s is the fraction (or multiple) of any investment aimed at avoiding supply disruptions that is paid by the firms in tier s. Finally, we let $\{\vartheta_s\}_{s=1}^{S}$ denote the sequence of policies directed at network formation, where ϑ_s denotes the fraction (or multiple) of the cost of search paid by a typical tier s producer when forming links to potential suppliers in tier $s-1$. We assume that all subsidies are financed by lump-sum taxation, while tax revenues are rebated similarly. We discuss each of the wedges in turn.

3.1 Optimal Policies to Promote First-Best Input Transactions

Consider first the social versus private incentives for transactions between the most upstream firms, those in tier 0 and those in tier 1. In the bargaining equilibrium, the pair of firms in tiers 0 and 1 choose m_0 to maximize their joint surplus. When the downstream firms pays only the fraction τ_0

of what the upstream firm receives, the Nash bargain calls for (see (A.5) in the appendix)

$$m_0 = \left(\frac{1-\gamma_1}{a\gamma_1\tau_0}\right)^{\gamma_1} [n_1^u]^{\frac{\gamma_1-\sigma_1}{\sigma_1-1}} n_1^d m_1.$$

Then, using the technological constraints in (4) and (5), this implies

$$\frac{l_1}{n_1^u m_0} = \frac{\tau_0 a \gamma_1}{1-\gamma_1}. \tag{20}$$

Now comparing the left-hand side of (20), which is the equilibrium ratio of labor to intermediate inputs in a tier 1 firm, to the optimal ratio expressed in (15), we see that the social planner can implement the first-best transactions between these firms with $\tau_0^* = 1$, i.e., with no tax or subsidy whatsoever.

Why are private and social incentives aligned for these transactions between far-upstream firms? In our model with sequential bargaining, the negotiations between tier 0 firms and tier 1 firms are the last to occur. The deals that emerge at this stage do not affect any prior negotiations, nor do they affect the simultaneous negotiations between other tier 0 suppliers and tier 1 buyers, due to the Nash-in-Nash structure of the bargaining game. Without any externalities, what remains is only a desire for joint efficiency in production, which the two firms share with the social planner. Put differently, when the most upstream firms bargain, the potential surplus for the pair reflects the private marginal cost of producing the tier 0 input. But the private marginal cost mirrors the social marginal cost, because only labor is used in its production. It follows that the planner need not intervene in these negotiations.

Next, consider the incentives for the transaction between a tier 1 firm and a tier 2 firm. The joint-surplus maximization in the Nash bargaining implies (see (A.6) and (A.95) in the appendix)

$$\frac{l_2}{n_2^u m_1} = \kappa_1 \frac{\gamma_2}{1-\gamma_2} (n_1^u)^{-\frac{1-\gamma_1}{\sigma_1-1}} \tau_1 \left[\gamma_1 + (1-\gamma_1)\mu_0\right],$$

where we recall from (8) that $\mu_0 = (1-\beta_1)\frac{\sigma_1}{\sigma_1-1} + \beta_1$ and thus the term in square brackets is proportional to the marginal cost of inputs to the firm in tier 1, considering the markup that will accrue to the tier 0 firms and the cost shares of labor and inputs. Evidently, in order to induce the first-best input ratios in tier 2 firms as prescribed by (16), the planner needs to implement a subsidy on input purchases such that

$$\tau_1^* = \frac{1}{\gamma_1 + (1-\gamma_1)\mu_0} < 1.$$

The optimal subsidy on sales of tier 1 inputs to tier 2 producers reflects a divergence between private and social incentives. In the absence of any policy, the pair of firms will negotiate based on an anticipated *private* marginal cost of producing the tier 1 input that reflects the markup that will later result when the tier 1 firm negotiates with its tier 0 suppliers. As we have noted, $\gamma_1 + (1-\gamma_1)\mu_0$ measures how much this anticipated markup distorts the cost of producing tier 1 inputs, considering the Cobb-Douglas production technology in (2). The elevated private cost will lead them to transact too little. The optimal subsidy counteracts this distortion, ensuring that the parties consider the *social* cost of producing tier 1 inputs when they decide the size of the transaction.

The qualitative properties of the optimal subsidy rate are readily understood.[13] First, the markup on the tier 0 input depends on the bargaining weights in the negotiation between the suppliers and the tier 1 buyers. The optimal subsidy to sales by a tier 1 firm decreases monotonically with its bargaining weight in its subsequent negotiations with its suppliers. If $\beta_1 = 1$, all of the bargaining power in the negotiation between firms in tier 0 and tier 1 resides with the downstream firm, and then $\mu_0 = 1$. In this case, $\tau_1^* = 1$, i.e., there is no subsidy. The optimal subsidy declines with the elasticity of substitution between tier 0 inputs in producing tier 1 goods, because substitutability between these inputs weakens the bargaining position of the suppliers and so reduces the markup. The optimal subsidy falls with the labor share of cost in producing the tier 1 inputs, because a higher γ_1 implies that a given markup of input prices has a smaller impact on the distortion in marginal cost.

In Section A5.3 of the appendix, we show that

$$\tau_s^* = \frac{1}{\gamma_s + (1-\gamma_s)\,\mu_{s-1}} < 1, \text{ for all } s \in \{1, \ldots, S-1\}\,. \tag{21}$$

The logic for all of the subsidies is similar; in each negotiation, the private parties in tiers s and $s+1$ face a distorted marginal cost of the good they are transacting, because the producer of this good anticipates paying an elevated price for its own inputs in its subsequent negotiations. At each stage, the planner offsets the anticipated markup that firms in tier s anticipate from their negotiations with the upstream firms in tier $s-1$, thereby ensuring that the firms in s and $s+1$ choose the efficient quantities.

If all negotiations give similar weight to the relatively upstream firm and all inputs have similar production technologies, then all subsidies for tiers $s \geq 1$ will be the same. Alternatively, if inputs become more specialized (and thus strictly less substitutable) as we proceed down the supply chain (so that μ_{s-1} rises with s), and if bargaining weights and labor shares are similar all along the chain, then the optimal purchase subsidies rise monotonically as we move downstream.

Finally, the planner eschews any subsidy or tax on sales of the final good; $\tau_S = 1$. Although the producers charge prices for these goods in excess of marginal costs, the markups are common to all goods and so do not distort any consumption decisions.

We summarize in

Proposition 1 *To achieve the first best, the planner subsidizes sales by all firms in intermediate tiers $s = \{1, \ldots, S-1\}$. The optimal subsidy for any good depends only on parameters describing the technology for producing that good and on the bargaining weight of the producer when it negotiates with its suppliers. The planner neither subsidizes nor taxes sales by firms in the extreme ends*

[13] Note that the ad valorem subsidy rate is $1 - \tau_1^*$.

of the supply chains.

3.2 Optimal Policies to Promote First-Best Resilience

Next we compare the private and social incentives for investments in resilience. We identify two conflicting forces that drive a wedge between the two. On the one hand, the firm in tier s garners only the fraction $1 - \beta_{s+1}$ of the joint surplus in its relationship with firms in tier $s + 1$. The smaller is this share, the smaller is the firm's incentive to invest in resilience, which represents a relationship-specific investment as far as the firm and its customers are concerned. The planner, on the other hand, is concerned with the total surplus, not the division between the parties. Thus, the bargaining over surplus tends to cause underinvestment in resilience by the firms in every tier s. On the other hand, the planner uses optimal subsidies for sales by the tier s firm to its customers in tier $s+1$ in order to promote sales that would otherwise be suboptimally small. These transaction subsidies raise the profitability for the firms in tier s in the service of encouraging a larger m_s, which tends to incentivize investments in resilience beyond their social value.

In keeping with this intuition, we report in equation (A.82)) in the appendix a very simple expression for the optimal policy regarding investments in resilience by firms producing intermediate inputs, namely

$$\theta_s^* = \frac{1 - \beta_{s+1}}{\tau_s^*} \text{ for all } s \in \{0, 1, \ldots, S-1\}.$$

First, notice that the optimal policy does not depend on properties of the function $\phi(r)$ that relates the probability of a disruption to the size of the investment in resilience. Although the elasticity of $\phi(r)$ affects the planner's preferred resilience (see (18)), that same elasticity also affects the firms' private incentives to avoid a disturbances, and in much the same way. Second, the optimal policy depends only on the bargaining weight for the firm in its negotiations with its downstream customers, and on the optimal subsidy on its purchases from its upstream suppliers. Since there is no subsidy for purchases of tier 0 inputs ($\tau_0^* = 1$), the planner always wishes to promote resilience in the most upstream tier of the supply chain ($\theta_0^* = 1 - \beta_1 < 1$). It might be that other far-upstream inputs are highly substitutable, in which case the transaction subsidies for these tiers will be small. Then, with τ_s^* close to one, the optimal policy promotes resilience. Further downstream, inputs may become more specialized. If the elasticity of substitution between inputs falls monotonically as the good proceeds down the supply chain, and if bargaining weights and labor shares are similar along the chain, then the optimal subsidies for investment in resilience will decline monotonically and may eventually turn from subsidy to tax.

One might ask why the policy terms τ_j, for $j \neq s$, do not enter the formula for θ_s^*. After all, these subsidy rates would seem to affect the potential profitability of a firm in tier s and so alter the incentives for it to invest in resilience. Indeed, an arbitrary subsidy downstream from a firm in tier s alters the firm's prospects both positively, by boosting operating profits within its network, and negatively, by boosting the demand for labor in rival networks. However, when we examine the expression for v_s and v_S in (A.80) and (A.81) in the appendix, and the discussion that follows these equations, we see that these forces exactly offset when $\tau_j = \tau_j^*$ for $j \neq s$. That is, the planner's optimal subsidy makes the value of a firm in any tier independent of joint surplus in sales that occur between firms in tiers different from its own.[14] Since an optimal subsidy to sales in tiers $j \neq s$ neither encourages nor discourages resilience investments by firms in tier s, the planner need not make any adjustments to θ_s^* on account of such effects.

Turning to the resilience of final producers, we find (see (A.83) in the appendix)

$$\theta_S^* = 1 - \frac{(1-\beta_S)(1-\gamma_S)(\varepsilon - 1)}{\sigma_S - 1} < 1. \tag{22}$$

Since the sales by final producers are not subsidized (or taxed) in the first best, all that remains for the planner is to induce the producers to internalize the positive externalities from their survival for consumers and upstream firms. We have established

Proposition 2 *To achieve the first best, the planner subsidizes resilience at both extreme ends of the supply chain. For intermediate stages, the optimal policy depends only on parameters describing the technology for producing that good and on the bargaining weight of the producer when it negotiates with its customers. Under Assumption 1, if bargaining weights and labor shares are similar along the chain, then the optimal subsidies for investment in resilience decline monotonically. If γ_s is large and σ_s is small, it may be optimal to tax resilience to offset the private investment incentive induced by a large transaction subsidy.*

3.3 Optimal Policies to Promote First-Best Linkages

Similar considerations come into play when we consider the optimal policy toward network formation. On the one hand, firms in intermediate tier s tend to have insufficient incentive to form links with upstream suppliers, because they capture only a fraction $1 - \beta_{s+1}$ of the surplus created by such investments when they sell their wares. On the other hand, the sales by firms in tier s are subsidized at rate $1 - \tau_s^*$, generating private profits that are not part of social surplus. These extra profits tend to incentivize excess investments in network formation.

It might seem that the planner would wish to subsidize network formation differently from resilience. After all, when a firm invests in resilience, it anticipates its potential payoff conditional on survival, $\tilde{v}_s(\tilde{\eta})$, which does not depend on its own resilience, r. In contrast—and as we discussed in Section 2.5—when a firm chooses the number of its upstream suppliers, it takes into account the effect that this choice will have on negotiations with its customers and its suppliers, as well as its prospective labor costs. Comparing the first-order conditions for the two types of investment, we note that a firm's choice of r_s requires (see (A.60) and (A.61) in the appendix)

$$\tilde{v}_s(\tilde{\eta}_s)\,\phi'(r_s) = \theta_s \text{ for } s \in \{0, 1, \ldots, S\} \ , \tag{23}$$

[14] If all inputs are priced at their shadow value, a small change in some price has no first-order effect on the social surplus (profits evaluated at shadow prices) in any other tier of the supply chain.

considering that the firm pays only the fraction θ_s of the cost of the investment and defining $\tilde{v}_0(\tilde{\eta}_0) \equiv v_0$, while its choice of links requires (see (A.62) in the appendix)

$$(1-\gamma_s)\frac{\sigma_{s+1}-1}{\sigma_s-1}\phi(r_s)\tilde{v}_s(\tilde{\eta}_s) = \vartheta_s k \tilde{\eta}_s N_{s-1} \text{ for } s \in \{1,2,\ldots,S-1\} \ ,$$

considering that it pays only the fraction ϑ_s of these costs, where we have calculated the marginal private benefit from a link using (9). Combining these two first-order conditions and noting that $\tilde{\eta}_s = \eta_s$ in a symmetric equilibrium, we find that the firm chooses its up-front investments so that

$$\frac{r_s\phi'(r_s)}{\phi(r_s)} = (1-\gamma_s)\frac{\sigma_{s+1}-1}{\sigma_s-1}\frac{\theta_s r_s}{\vartheta_s k \eta_s N_{s-1}} \text{ for } s \in \{1,2,\ldots,S-1\}. \tag{24}$$

Meanwhile, dividing (17) by (18), we see that the planner seeks

$$\frac{r_s^*\phi'(r_s^*)}{\phi(r_s^*)} = (1-\gamma_s)\frac{\sigma_{s+1}-1}{\sigma_s-1}\frac{r_s^*}{k\eta_s^* N_{s-1}} \text{ for } s \in \{1,2,\ldots,S-1\}. \tag{25}$$

Evidently, to achieve the optimal investments in resilience and supplier links as a decentralized equilibrium, the government needs to set the same subsidy (or tax) for both; i.e., $\theta_s^* = \vartheta_s^*$ for $s = \{1,\ldots,S-1\}$.

A similar argument applies to network formation by final producers. Using (23) and (9) for $s = S$, we can derive a relationship between a firm's choices of r_S and η_S similar to (24). Meanwhile, dividing (17) by (19) gives the planner's desired relationship between r_S^* and η_S^*. Together, these imply $\vartheta_S^* = \theta_S^* < 1$; i.e., optimal network formation by final producers requires a subsidy to link formation, and the optimal subsidy rate mirrors that for investments in resilience.

We record the following

Proposition 3 *To achieve the first best, the planner levies a subsidy (or tax) on network formation at intermediate tier $s \in \{1,\ldots,S-1\}$ at the same rate as the optimal subsidy (or tax) on investments in resilience. The planner subsidizes investments in network formation by final producers at the same rate as investments in resilience.*

4 Second-Best Policies for Resilience and Network Formation

The salience of recent supply-chain disruptions has directed attention to what the government might do to promote greater resilience of these chains. In the current environment, policies that encourage firms to become more resilient or to diversify their sourcing might be politically palatable even when direct subsidies to their sales are not so. To address this apparent political reality, we consider in this section a second-best setting in which the government can subsidize investments in resilience and network formation, but cannot bankroll firm-to-firm transactions along the supply chain.

The government's problem is the same as before, except that we impose $\tau_s = 1$ for all s. We denote by θ_s° the fraction of the cost of investing in resilience paid by a firm in tier s, $s \in \{0, 1, \ldots, S\}$, with the optimal second-best subsidy (or tax) in place. Similarly, ϑ_s° is the share of the cost of network formation borne by firms in tier s, $s \in \{1, 2, \ldots, S\}$, in the second-best equilibrium.

In the appendix, we show that (see (A.85) and (A.86) in the appendix)

$$\theta_s^\circ = \frac{1}{J}\left\{\frac{1-\beta_{s+1}}{\Pi_{j=s+1}^{S-1}\left[\gamma_j + \left(1-\gamma_j\right)\mu_{j-1}\right]}\right\} \qquad \text{for } s \in \{0, 1, \ldots, S-1\} \tag{26}$$

and

$$\theta_S^\circ = \frac{1}{J}\left[1 - \frac{\left(1-\beta_S\right)\left(1-\gamma_S\right)\left(\varepsilon-1\right)}{\sigma_S - 1}\right], \tag{27}$$

where $J \leq 1$ is a term that captures the aggregate labor-market effects of all the input markups.[15] The term $1-\beta_{s+1}$ in (26) reflects, as before, that the firm paying for resilience captures only a fraction of the return to that investment, the remainder accruing as a positive spillover to its customers. The denominator of the term in curly brackets represents the product of all the pricing distortions in tiers downstream from s. Inasmuch as this term exceeds one, the denominator as a whole may be less than or greater than one; i.e., the pricing distortions downstream from tier s directly reduce their sales and profits and thus their incentives to invest in resilience, whereas collectively the distortions reduce labor demand and thereby depress wages relative to prices, which tends to cause overinvestment.

Notice that the first-best investment policy, $\theta_s^* = \left(1-\beta_{s+1}\right)\left[\gamma_s + \left(1-\gamma_s\right)\mu_{s-1}\right]$, depends only on the markups faced by firms in tier s and the input cost share in that tier. Moreover, the first-best subsidy shrinks with μ_{s-1} and $1-\gamma_s$, because the government's contribution to input sales corrects for this distortion but artificially boosts the private profitability of firms' investments in resilience. In contrast, the second-best subsidies for resilience *increase* with markups and input shares *downstream* from s (for given J). These pricing distortions—which are not corrected in the second best—lead downstream firms to contract their demand for inputs, which in turn reduces the sales and profits for firms in tier s. The second-best policy partially reflects that the shortfall in profits for firms in tier s tends to reduce their investments in resilience below the first-best level.

The first-best and second-best subsidies both address the externality that results from rent sharing, as reflected in the bargaining weight, $1-\beta_{s+1}$. Beyond that, they address different distortions: excess private profitability created by input subsidies on the one hand, and contraction of downstream input demand caused by uncorrected markups on the other. Therefore, these subsidies are not directly comparable in size. If the denominator of (26) exceeds one, as is mostly likely for firms that are far upstream, then $\theta_s^\circ < \theta_s^*$; i.e., the optimal second-best subsidy to resilience must exceed the first-best subsidy at tier s. This is a situation in which the downstream contraction of

[15] Specifically,

$$J := \frac{\Gamma_1^S}{\prod_{j=1}^{S-1}\left[\gamma_j + \left(1-\gamma_j\right)\mu_{j-1}\right]} + \sum_{j=1}^{S-1}\frac{\gamma_j}{1-\gamma_j}\Gamma_j^S \prod_{z=j}^{S-1}\frac{1}{\left[\gamma_z + \left(1-\gamma_z\right)\mu_{z-1}\right]} + \gamma_S \ .$$

input demand caused by the successive markups leads to a substantial underinvestment in resilience in the absence of policy. If, however, the product in the denominator is sufficiently less than one, as it may be for firms far downstream, then the second-best subsidy to investments in resilience may be smaller than the first best. Comparing (27) with (22), we see that $\theta_S^\circ > \theta_S^*$; i.e., the government always shaves the second-best subsidy to investments in resilience by final producers relative to the first best; for these firms, there are no downstream distortions, but the markups upstream boost their overall profitability, which tends to lead them to overinvest in resilience compared to the incentives they see in the first best.

We can readily compare the second-best subsidies at different tiers in the supply chain. Note that the general-equilibrium term J is common to all tiers. Using (26) for s and $s-1$, we have

$$\frac{\theta_{s-1}^\circ}{\theta_s^\circ} = \frac{1-\beta_s}{1-\beta_{s+1}} \left[\frac{1}{\gamma_s + (1-\gamma_s)\,\mu_{s-1}} \right].$$

Thus, if $\beta_{s+1} \le \beta_s$, then $\theta_{s-1}^\circ < \theta_s^\circ$; i.e., the social imperative for resilience (in the absence of transaction subsidies) is greater for the upstream firm in the supplier-buyer relationship.

Turning to network formation, we find once again—and by an argument completely analogous to that used above when comparing the equilibrium investments in (24) with the first-best investments in (25)—that the second-best policies for investment in links mirror their second-best counterparts for resilience; i.e., $\vartheta_s^\circ = \theta_s^\circ$ for all $s \in \{1, 2, \ldots, S\}$.

We summarize our findings about the second-best subsidies to investments in resilience and network formation in

Proposition 4 *The second-best subsidies to link formation are equal to the second-best subsidies to investments in resilience for all tiers of the supply chain. The second-best subsidies in tier $s \in \{0, 1, \ldots, S-1\}$ reflect the cumulative pricing distortions downstream from s that result from positive markups. The second-best subsidy at tier s might be bigger or smaller than the first-best subsidy, but is more likely to be bigger the further upstream is s. If $\beta_{s+1} \le \beta_s$ for all $s \in \{0, 1, \ldots, S-1\}$, then the second-best subsidies for investments in both resilience and network thickness fall with s. The second-best subsidies to final producers are always smaller than the first-best subsidies.*

5 Concluding Remarks

We have identified several sources of inefficiency in the market equilibrium of an economy with vertical supply chains and endogenous determination of firms' resilience to supply disturbances. First, in the absence of government policy, firms in adjacent tiers of the supply chain will not choose the socially-optimal volume of input sales. Instead, they will negotiate a contract that calls for more limited sales, in anticipation that the supplier will face a marked-up cost of its own inputs when it subsequently bargains with its own suppliers. The wedge between the private and social incentives for input transactions dictates an optimal subsidy on input sales in all transactions other than between the firms that are most upstream. Second, firms in every tier will not on their own choose the socially-optimal investments to avoid their own supply disturbances. On the one hand, these investments tend to be socially insufficient because firms to do not take account that their own resilience affects the profitability of their downstream customers. On the other hand, these investments may be socially excessive, if the optimal subsidy for sales creates a large profit boost that comes at the expense of the public finances. If the bargaining weights and the labor shares are similar across input tiers but goods become less substitutable as we move down the supply chain, then the optimal subsidies for resilience will be largest upstream and decline monotonically, possible turning to an optimal tax at some point in the chain. Neither the optimal subsidies on sales nor the optimal subsidies for investments in resilience depend on the number of backward links formed by suppliers, and thus the same subsidies apply for arbitrary networks. Finally, we find a wedge between private and social incentives for firms to form thick supply networks as a hedge against disturbances that might befall their suppliers. As with investments in resilience, firms do not take account that their relationships generate surplus for downstream partners. Despite the fact that firms have an incentive to manipulate the number of their upstream suppliers in order to improve their bargaining position vis-à-vis these suppliers and their downstream customers, the net effect of this strategic behavior just balances the offsetting general-equilibrium effects that result from their investments, so that the government's optimal policy toward network formation coincides with the optimal policy to promote or discourage resilience.

Political realities may limit the scope for subsidies to firm-to-firm transactions. If so, the government's choice of whether and how to promote resilience and thick networks takes on a second-best flavor. We considered optimal policies for investments in own resilience and for the formation of supplier relationships when a government lacks the ability to use subsidies to counteract the distortionary effects of negotiated input prices. In this setting, optimal policies reflect markups and input shares in all transactions downstream from a targeted tier. Resilience and supplier relationships are more socially valuable at upstream stages than at downstream stages due to the cumulative effects of double marginalization. If bargaining weights are common across tiers, then the second-best subsidies are larger for producers further upstream. Again, the optimal policies for investments in supplier relationships mirror those for investments in own resilience.

We have modeled vertical supply chains in a stylized but realistic way that captures many of the features described in the more descriptive literature. Each firm has multiple suppliers and multiple customers. Bargaining happens sequentially, beginning with final producers that purchase intermediate goods to use in their production processes and proceeding upstream to suppliers that seek inputs to fulfill their procurement contracts. Our bilateral negotiations involve a single buyer and a single seller, not grand coalitions of producers at various stages. Firms form their networks of potential suppliers by investing in bilateral relationships. Resilience reflects deliberate investment. Yet, as with all models of firm-to-firm dealings, the details matter and we recognize that a variety of alternative assumptions may be worthy of further consideration.

First, we have assumed a particular timing and a particular form of contracts. In our model, bargaining between upstream and downstream firms takes place *after* the realization of the supply shocks and firms negotiate only with partners that escape these disturbances. If negotiations were to occur *before* any disruptions, this would open a role for contingent contracts. Payments might be contingent on contract fulfillment, with penalties for failure to deliver. Payments might also be contingent on the size of an upstream firm's investment in resilience (which must be observable if they can be the target of subsidies). Even more sophisticated contracts might allow payments contingent on the resilience of a supplier's own upstream suppliers, or on a firm's realized production costs. Richer contracts would allow firms to mitigate the inefficiencies of double marginalization and to internalize to some extent the externalities that their resilience confers on downstream customers. However, complex contracts that allow for payments based on decisions throughout the network might be needed to achieve full efficiency, especially in a second-best setting in which the government cannot subsidize firm-to-firm transactions. So, the externalities that we highlight would likely still be relevant even in a world with a wider menu of contracts.

Second, if downstream firms could observe investments in resilience before they form their supply networks, they might seek out partners that are more likely to deliver. This would give upstream firms greater incentive to make such investments, thereby mitigating the externality associated with shared benefits. Even if firms could not observe investments before creating their supply chains, they might infer something about such investments if potential suppliers differed in some observable primitives that would affect their incentives to invest. In our model, the symmetry across firms in a given tier eliminates any reason to target the search for partners.

Finally, our model currently features only idiosyncratic supply shocks and only one place of production. An obvious extension would be to consider correlated shocks, based for example on geography. These would seem particularly important if combined with an extension to *global* supply chains; see, e.g., Grossman et al. (2023) for an analysis of country-wide shocks to input supplies in a two-country model, albeit one with only two tiers of production. The presence of correlated shocks would interact with the possibilities for contract contingencies, as penalties for breach might differ for failures that are specific to a firm versus those that result from more widespread disturbances that are outside a single firm's control. Analyzing optimal unilateral policy and optimal cooperative policy toward resilience in global supply chains will require that cross-country differences in wages, production technologies, and risks of disturbances be taken into account. We regard the modeling of global supply chains with endogenous networks and resilience as an important direction for future research.

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Appendix

A1 Bargaining

In this section, we characterize the solutions to the bargaining game between a supplier from tier s and its buyer from tier $s+1$, for all $s \in \{0, 1, ..., S-1\}$. We focus on symmetric equilibria. At the bargaining stage, resilience levels $\{r_s\}_{s=0}^{S}$ and the choice of links by buyers of intermediate inputs $\{\eta_s\}_{s=1}^{S}$ are given. Therefore the number of suppliers of every buyer, $\{n_s^u\}_{s=1}^{S}$, and the number of buyers of every seller, $\left\{n_s^d\right\}_{s=0}^{S-1}$, are also given, where $n_s^u = \eta_s \phi_{s-1}(r_{s-1}) N_{s-1}$ and $n_s^u \phi_s(r_s) N_s = n_{s-1}^d \phi_{s-1}(r_{s-1}) N_{s-1}$, which implies $n_{s-1}^d = \eta_s \phi_s(r_s) N_s$, $s \in \{0, 1, ..., S-1\}$. Since bargaining is sequential, starting with firms in tiers S and $S-1$ and ending with firms in tiers 1 and 0, we solve the bargaining games in the reverse order. Importantly, when a firms in tier s bargains with a firm in tier $s+1$, they take the commitment of the downstream firm to supply m_{s+1} units of intermediate inputs to each of its n_{s+1}^d customers as given. Moreover, due to Nash-in-Nash bargaining, the downstream firm takes as given its bargaining outcomes with suppliers other than the one with whom it bargains, and the upstream firm takes as given its bargaining outcomes with buyers other than the one with whom it bargains.

To solve the bargaining game, we assume that every firm controls a positive measure δ of inputs. We then take the limit as $\delta \to 0$ to solve for the equilibrium. Let D_s denote the number of downstream tier $s+1$ links for a tier s producer, $s \in \{0, 1, ..., S-2\}$. Let U_s denote the number of upstream tier $s-1$ links for a tier s producer, $s \in \{1, 2, ..., S\}$.

Output in each tier s, $s \in \{1, 2, ..., S\}$, is produced by combining labor with a CES bundle of tier $s-1$ inputs according to a Cobb-Douglas technology. For the bargaining game, we postulate

$$x_s = l_s^{\gamma_s} \left[(U_s - 1)\delta m_{s-1}^{\alpha_s} + \delta \tilde{m}_{s-1}^{\alpha_s} \right]^{\frac{1-\gamma_s}{\alpha_s}}, \tag{A.1}$$

where x_s is the output of the bargaining firm, l_s is its labor employment, m_{s-1} is its commitment per product to all the upstream firms other then the one with whom it bargains, and $\tilde{m}_{s-1}$ the potential volume of purchases per input from the firm with whom it bargains. In tier 0 output is produced with labor only, and

$$x_0 = \frac{l_0}{a}.$$

We solve the bargaining game for an arbitrary set of policies for purchases of intermediate inputs $\{\tau_s\}_{s=0}^{S-1}$, where τ_{s-1} is the the fraction of costs of intermediate inputs paid by a firm in tier s and $1-\tau_{s-1}$ is the cost share borne by the government. The firms bargain over $\tilde{m}_{s-1}$ per product and a transfer $\tilde{t}_{s-1}$ per product that the tier s firm will make to the tier $s-1$ firm, taking as given (m_{s-1}, t_{s-1}) to which the downstream firm is committed vis-à-vis its other $U_s - 1$ suppliers, and taking as given (m_s, t_s) to which the downstream firm is committed vis-à-vis all its D_s buyers, $s \in \{1, ..., S-1\}$ (we will deal separately with the case $s = S$).

First, consider bargaining between a tier 1 buyer and a tier 0 supplier. The tier 1 buyer receives transfers $D_1 \delta t_1$ from each of its tier 2 buyers, to whom it has to to supply m_1 units of each of the $D_1 \delta$ inputs. When negotiating with the upstream tier 0 firm, (m_1, t_1) is taken as given. To fulfil this commitment, the tier 1 firm has to produce an output $x_1 = D_1 \delta m_1$, which, using (A.1), leaves the tier 1 buyer with labor costs (the wage rate is normalized to equal one)

$$[D_1 \delta m_1]^{\frac{1}{\gamma_1}} \left[(U_1 - 1)\delta m_0^{\alpha_1} + \delta \tilde{m}_0^{\alpha}\right]^{\frac{\gamma_1 - 1}{\alpha_1 \gamma_1}}.$$

The tier 1 firm pays each of the tier 0 suppliers other than the one it bargains with a transfer δt_0. Thus, the payoff of a tier 1 supplier minus its outside option is

$$\psi_1^d\left(\tilde{m}_0, \tilde{t}_0\right) := D_1 \delta t_1 - [D_1 \delta m_1]^{\frac{1}{\gamma_1}} \left[(U_1 - 1)\delta m_0^{\alpha_1} + \delta \tilde{m}_0^{\alpha_1}\right]^{\frac{\gamma_1 - 1}{\alpha_1 \gamma_1}} - (U_1 - 1)\tau_0 \delta t_0 - \delta \tau_0 \tilde{t}_0 - O_1,$$

where O_1 is its outside option, given by

$$O_1 = D_1 \delta t_1 - [D_1 \delta m_1]^{\frac{1}{\gamma_1}} \left[(U_1 - 1)\delta m_0^{\alpha_1}\right]^{\frac{\gamma_1 - 1}{\alpha_1 \gamma_1}} - (U_1 - 1)\delta \tau_0 t_0.$$

Evidently, should the bargaining fail, the tier 1 firm will have to incur higher labor costs to satisfy commitments to its downstream buyers.

The tier 0 supplier receives transfers δt_0 for δm_0 units of the tier 0 input from each of $D_0 - 1$ buyers other than the one with whom it bargains. This provides a payoff net of the outside option

$$\psi_0^u\left(\tilde{m}_0, \tilde{t}_0\right) := \delta \tilde{t}_0 + (D_0 - 1)\delta t_0 - (D_0 - 1)\delta m_0 a - \delta a \tilde{m}_0 - O_0,$$

where O_0 is the outside option

$$O_0 = (D_0 - 1)\delta t_0 - (D_0 - 1)\delta a m_0.$$

Using bargaining weights β_1 and $1 - \beta_1$ for the buyer and seller, respectively, the solution to the bargaining game is obtained from

$$\left(\tilde{m}_0, \tilde{t}_0\right) = \arg\max_{t,m} \left[\beta_1 \log \psi_1^d(m, t) + (1 - \beta_1) \log \psi_0^u(m, t)\right]. \tag{A.2}$$

The first-order conditions with respect to m and t, respectively, yield (see Section A1.2 for second-order conditions)

$$-\frac{\beta_1}{\psi_1^d\left(\tilde{m}_0, \tilde{t}_0\right)} \frac{\partial \psi_1^d\left(\tilde{m}_0, \tilde{t}_0\right)}{\partial \tilde{m}_0} = \frac{1 - \beta_1}{\psi_0^u\left(\tilde{m}_0, \tilde{t}_0\right)} \delta a,$$

$$\frac{\beta_1}{\psi_1^d\left(\tilde{m}_0, \tilde{t}_0\right)} \tau_0 = \frac{1 - \beta_1}{\psi_0^u\left(\tilde{m}_0, \tilde{t}_0\right)}. \tag{A.3}$$

Dividing the first of these equations by the second equation and imposing symmetry of the bargaining solutions, i.e., $\left(\tilde{m}_0, \tilde{t}_0\right) = (m_0, t_0)$, yield

$$\frac{1-\gamma_1}{\gamma_1}\left[D_1\delta m_1\right]^{\frac{1}{\gamma_1}}\left[U_1\delta\right]^{\frac{\gamma_1-1}{\alpha_1\gamma_1}-1} m_0^{\frac{\gamma_1-1}{\gamma_1}-1}=\tau_0 a. \tag{A.4}$$

Note that this equation does not depend on $\tilde{t}_0$, because $\frac{\partial\psi_1^d(\tilde{m}_0,\tilde{t}_0)}{\partial\tilde{m}_0}$ does not depend on $\tilde{t}_0$. In other words, this gives us an equation from which we can solve m_0 as a function of m_1. This procedure works for bargaining at other tiers as well, because these types of cross-derivatives concerning $\tilde{m}_0$ and $\tilde{t}_0$ all equal zero. Then, using $n_s^u = U_s\delta$ and $n_s^d := D_s\delta$, we obtain

$$m_0 = C_{m_0}\left[n_1^u\right]^{\frac{\gamma_1(1-\alpha_1)-1}{\alpha_1}} n_1^d m_1, \tag{A.5}$$

where

$$C_{m_0}=\left(\frac{1-\gamma_1}{\tau_0 a\gamma_1}\right)^{\gamma_1}. \tag{A.6}$$

This provides a recursive equation that determines m_0 as a function of m_1 for given values of n_1^u and n_1^d.

Next, combining (A.4) with the first-order condition with respect to t, (A.3), using $\left(\tilde{m}_0,\tilde{t}_0\right)=(m_0,t_0)$ and the outside options, we obtain

$$t_0=\frac{1}{\tau_0}(1-\beta_1)\left[D_1\delta m_1\right]^{\frac{1}{\gamma_1}}\left[U_1\delta m_0^{\alpha_1}\right]^{\frac{\gamma_1-1}{\alpha_1\gamma_1}}\frac{\left(\frac{U_1-1}{U_1}\right)^{\frac{\gamma_1-1}{\alpha_1\gamma_1}}-1}{\frac{n_1^u}{U_1}}+\beta_1 a m_0.$$

Taking limits as $\delta\to 0$, $U_1\to\infty$, $D_1\to\infty$, $\delta U_1\to n_1^u$, $\delta D_1\to n_1^d$, using L'Hôpital's rule, then delivers

$$t_0=\frac{1}{\tau_0}(1-\beta_1)\left[n_1^d m_1\right]^{\frac{1}{\gamma_1}}\left[n_1^u m_0^{\alpha_1}\right]^{\frac{\gamma_1-1}{\alpha_1\gamma_1}}\frac{1-\gamma_1}{\alpha_1\gamma_1}\frac{1}{n_1^u}+\beta_1 a m_0.$$

Combining with (A.5) and (A.6),

$$t_0 = aC_{m_0}\left[n_1^u\right]^{\frac{\gamma_1(1-\alpha_1)-1}{\alpha_1}}\left[(1-\beta_1)\frac{1}{\alpha_1}+\beta_1\right]n_1^d m_1,$$

and therefore,

$$t_0 = C_{t_0}\left[n_1^u\right]^{\frac{\gamma_1(1-\alpha_1)-1}{\alpha_1}} n_1^d m_1 \tag{A.7}$$

with

$$C_{t_0}=C_{m_0}a\mu_1,\quad \mu_1=\frac{1-\beta_1+\beta_1\alpha_1}{\alpha_1}. \tag{A.8}$$

Using (A.5), (A.7) and (A.8), an alternative way to represent the transfer is

$$t_0 = a\mu_1 m_0. \tag{A.9}$$

Next, consider negotiations between a firm in tier $s+1$ with a firm in tier s, for $s\in\{1,2,...,S-2\}$. The payoff of the downstream firm, net of its outside option, is

$$\begin{aligned}\psi^d_{s+1}\left(\tilde{m}_s,\tilde{t}_s\right) \quad : \quad & = D_{s+1}\delta t_{s+1} - \left[D_{s+1}\delta m_{s+1}\right]^{\frac{1}{\gamma_{s+1}}}\left[(U_{s+1}-1)\delta m_s^{\alpha_{s+1}} + \delta\tilde{m}_s^{\alpha_{s+1}}\right]^{\frac{\gamma_{s+1}-1}{\alpha_{s+1}\gamma_{s+1}}} \\ & -(U_{s+1}-1)\delta\tau_s t_s - \delta\tau_s\tilde{t}_s - O_{s+1},\end{aligned}$$

and its outside option is

$$O_{s+1} = D_{s+1}\delta t_{s+1} - \left[D_{s+1}\delta m_{s+1}\right]^{\frac{1}{\gamma_{s+1}}}\left[(U_{s+1}-1)\delta m_s^{\alpha_{s+1}}\right]^{\frac{\gamma_{s+1}-1}{\alpha_{s+1}\gamma_{s+1}}} - (U_{s+1}-1)\delta\tau_s t_s.$$

That is, the downstream firm obtains transfers from its own downstream buyers, pays labor costs dependent on $\tilde{m}_s$, and takes as given all negotiated quantities and transfers with its other $U_{s+1}-1$ upstream suppliers when negotiating $\tilde{m}_s$ and $\tilde{t}_s$ with a given tier s firm (due to Nash-in-Nash bargaining). O_{s+1} shows that if the negotiation breaks down with the tier s firm, the tier $s+1$ firm will incur higher labor costs to satisfy the commitments to its D_{s+1} downstream buyers.

The payoff of the tier s firm net of the outside option is

$$\psi^u_s\left(\tilde{m}_s,\tilde{t}_s\right) := \delta\tilde{t}_s + (D_s-1)\delta t_s - \left[(D_s-1)\delta m_s + \delta\tilde{m}_s\right]^{\frac{1}{\gamma_s}}\left[U_s\delta m_{s-1}^{\alpha_s}\right]^{\frac{\gamma_s-1}{\alpha_s\gamma_s}} - U_s\delta\tau_{s-1}t_{s-1} - O_s,$$

where the outside option is

$$O_s = (D_s-1)\delta t_s - \left[(D_s-1)\delta m_s\right]^{\frac{1}{\gamma_s}}\left[U_s\delta(m^o_{s-1})^{\alpha_s}\right]^{\frac{\gamma_s-1}{\alpha_s\gamma_s}} - U_s\delta\tau_{s-1}t^o_{s-1}.$$

Here the outside option depends on $\left(m^o_{s-1},t^o_{s-1}\right)$, which represents what the tier s firm expects to negotiate with its upstream suppliers in case the negotiation with the downstream firm in tier $s+1$ fails. This condition is required for subgame perfection of the solutions of the bargaining games. We therefore need to include here the off-equilibrium values m^o_{s-1} and t^o_{s-1}. To do this, suppose that for a given commitment (m_s,t_s) the solution to the bargaining game between a tier s firm and its tier $s-1$ supplier yields

$$m_{s-1} = C_{m_{s-1}}\left[n^u_s\right]^{\frac{\gamma_s(1-\alpha_s)-1}{\alpha_s}}\left[\prod_{j=1}^{s-1}\left[n^u_j\right]^{\frac{\gamma_s\Gamma_j^{s-1}(1-\alpha_j)}{\alpha_j}}\right]n^d_s m_s, \tag{A.10}$$

$$t_{s-1} = C_{t_{s-1}}\left[n^u_s\right]^{\frac{\gamma_s(1-\alpha_s)-1}{\alpha_s}}\left[\prod_{j=1}^{s-1}\left[n^u_j\right]^{\frac{-\Gamma_j^{s}(1-\alpha_j)}{\alpha_j}}\right]n^d_s m_s, \tag{A.11}$$

where $C_{m_{s-1}}$ and $C_{t_{s-1}}$ are constants that only depend on production function parameters $\{\alpha_s,\gamma_s\}_{s=1}^S$,[redacted] a, and on $\{\tau_s\}_{s=0}^{S-1}$ (see Section A1.1 for details). Then $\left(m^o_{s-1},t^o_{s-1}\right)$ can be expressed as:

$$m^o_{s-1} = C_{m_{s-1}}\left[n^u_s\right]^{\frac{\gamma_s(1-\alpha_s)-1}{\alpha_s}}\left[\prod_{j=1}^{s-1}\left[n^u_j\right]^{\frac{\gamma_s\Gamma_j^{s-1}(1-\alpha_j)}{\alpha_j}}\right]\left[(D_s-1)\,\delta m_s\right], \tag{A.12}$$

$$t^o_{s-1} = C_{t_{s-1}} \left[n^u_s\right]^{\frac{\gamma_s(1-\alpha_s)-1}{\alpha_s}} \left[\prod_{j=1}^{s-1} \left[n^u_j\right]^{\frac{-\Gamma^s_j(1-\alpha_j)}{\alpha_j}}\right] \left[\left(D_s - 1\right)\delta m_s\right]. \tag{A.13}$$

The solution to the bargaining game is therefore

$$\left(\tilde{m}_s, \tilde{t}_s\right) = \arg\max_{(m,t)} \left[\beta_{s+1} \log \psi^d_{s+1}(m,t) + \left(1-\beta_{s+1}\right) \log \psi^u_s(m,t)\right] \tag{A.14}$$

and the first-order conditions for m and t, respectively, are

$$-\frac{\beta_{s+1}}{\psi^d_1\left(\tilde{m}_s, \tilde{t}_s\right)} \frac{\partial \psi^d_{s+1}\left(\tilde{m}_s, \tilde{t}_s\right)}{\partial \tilde{m}_s} = \frac{1-\beta_{s+1}}{\psi^u_s\left(\tilde{m}_s, \tilde{t}_s\right)} \frac{\partial \psi^u_s\left(\tilde{m}_s, \tilde{t}_s\right)}{\partial \tilde{m}_s}, \tag{A.15}$$

$$\frac{\beta_{s+1}}{\psi^d_{s+1}\left(\tilde{m}_s, \tilde{t}_s\right)} \tau_s = \frac{1-\beta_{s+1}}{\psi^u_s\left(\tilde{m}_s, \tilde{t}_s\right)}. \tag{A.16}$$

Dividing the first of these equations by the second equation yields

$$-\frac{\partial \psi^d_{s+1}\left(\tilde{m}_s, \tilde{t}_s\right)}{\partial \tilde{m}_s} = \tau_s \frac{\partial \psi^u_s\left(\tilde{m}_s, \tilde{t}_s\right)}{\partial \tilde{m}_s}. \tag{A.17}$$

Now note that this equation does not depend on $\tilde{t}_s$, because the partial derivatives on both sides depend only on $\tilde{m}_s$. That is,

$$\frac{\partial^2 \psi^d_{s+1}\left(\tilde{m}_s, \tilde{t}_s\right)}{\partial \tilde{m}_s \partial \tilde{t}_s} = \frac{\partial^2 \psi^u_s\left(\tilde{m}_s, \tilde{t}_s\right)}{\partial \tilde{m}_s \partial \tilde{t}_s} = 0.$$

In this case we can solve $\tilde{m}_s$ from this equation. Using $\delta U_j = n^u_j$ and $\delta D_j = n^d_j$, as well as the symmetry $\left(\tilde{m}_s, \tilde{t}_s\right) = (m_s, t_s)$, plus (A.10)-(A.13), yields

$$m_s = C_{m_s} \left[n^u_{s+1}\right]^{\frac{\gamma_{s+1}(1-\alpha_{s+1})-1}{\alpha_{s+1}}} \left[\prod_{j=1}^{s} \left[n^u_j\right]^{\frac{\gamma_{s+1}\Gamma^s_j(1-\alpha_j)}{\alpha_j}}\right] n^d_{s+1} m_{s+1}, \quad s \in \{1, 2, ..., S-2\}, \tag{A.18}$$

where the constant C_{m_s} is specified in Section A1.1. Finally, substituting the solution of $\tilde{m}_s$ from (A.17) into (A.16), we obtain an equation for $\tilde{t}_s$. Taking limits of this equation as $\delta \to 0$, $U_j \to \infty$, $D_j \to \infty$, $\delta U_j \to n^u_j$ and $\delta D_j \to n^d_j$, using L'Hôpital's rule, the symmetric solution $\left(\tilde{m}_s, \tilde{t}_s\right) = (m_s, t_s)$ yields

$$t_s = C_{t_s} \left[n^u_{s+1}\right]^{\frac{\gamma_{s+1}(1-\alpha_{s+1})-1}{\alpha_{s+1}}} \left[\prod_{j=1}^{s} \left[n^u_j\right]^{\frac{-\Gamma^{s+1}_j(1-\alpha_j)}{\alpha_j}}\right] n^d_{s+1} m_{s+1}, \quad s \in \{1, 2, ..., S-2\}, \tag{A.19}$$

where the constant C_{t_s} is specified in Section A1.1. Notice that the solutions for m_s and t_s generate a recursive system.

The final step is to solve the bargaining game between a final good producer in tier S and a tier $S-1$ intermediate input supplier. In this case, the upstream firm has a payoff net of the outside option equal to

$$\begin{aligned}\psi_{S-1}^{u}\left(\tilde{m}_{S-1},\tilde{t}_{S-1}\right) \quad : \quad &= \delta\tilde{t}_{S-1}+(D_{S-1}-1)\delta t_{S-1} \\ &-\left[(D_{S-1}-1)\delta m_{S-1}+\delta\tilde{m}_{S-1}\right]^{\frac{1}{\gamma_{S-1}}}\left[U_{S-1}\delta m_{S-2}^{\alpha_{S-1}}\right]^{\frac{\gamma_{S-1}-1}{\alpha_{S-1}\gamma_{S-1}}} \\ &-U_{S-1}\delta\tau_{S-2}t_{S-2}-O_{S-1},\end{aligned}$$

where

$$O_{S-1}=(D_{S-1}-1)\delta t_{S-1}-\left[(D_{S-1}-1)\delta m_{S-1}\right]^{\frac{1}{\gamma_{S-1}}}\left[U_{S-1}\delta(m_{S-2}^{o})^{\alpha_{S-1}}\right]^{\frac{\gamma_{S-1}-1}{\alpha_{S-1}\gamma_{S-1}}}-U_{S-1}\delta\tau_{S-2}t_{S-2}^{o}.$$

The value of m_{S-2}^{o} is given by (A.12) for $s=S-1$ and the value of t_{S-2}^{o} is given by (A.13) for $s=S-1$. This firm anticipates how a failure of its negotiation with the final producer will affect the transfers and quantities it will later negotiate with its upstream suppliers. This is an off-equilibrium consideration that affects the bargaining outcome.

To obtain the payoff of a final good producer, we assume that it does not commit to a given amount of labor prior to negotiations with suppliers. That is, labor can be freely adjusted after the negotiations, in line with the assumption we made for intermediate good producers. For given use of intermediate inputs, the amount of labor is chosen to maximize profits, subject to the demand function $x=Ap^{-\varepsilon}$, where (see (A.1))

$$x=l_{S}^{\alpha_{S}}\left[\left(U_{S}-1\right)\delta m_{S-1}^{\alpha_{S}}+\delta\tilde{m}_{S-1}^{\alpha_{S}}\right]^{\frac{1-\gamma_{S}}{\alpha_{S}}},$$

$$p=\left\{l_{S}^{\alpha_{S}}\left[\left(U_{S}-1\right)\delta m_{S-1}^{\alpha_{S}}+\delta\tilde{m}_{S-1}^{\alpha_{S}}\right]^{\frac{1-\gamma_{S}}{\alpha_{S}}}\right\}^{-\frac{1}{\varepsilon}}A^{\frac{1}{\varepsilon}}.$$

Therefore,

$$l_{S}=\arg\max_{l}l_{S}^{\frac{\varepsilon-1}{\varepsilon\alpha_{S}}}\left[\left(U_{S}-1\right)\delta m_{S-1}^{\alpha_{S}}+\delta\tilde{m}_{S-1}^{\alpha_{S}}\right]^{\frac{(1-\gamma_{S})(\varepsilon-1)}{\alpha_{S}\varepsilon}}A^{\frac{1}{\varepsilon}}-l.$$

This implies that the payoff of the final good producer from a deal with a tier $S-1$ supplier of intermediate inputs, net of the outside option, is

$$\begin{aligned}\psi_{S}^{d}\left(\tilde{m}_{S-1},\tilde{t}_{S-1}\right) \quad : \quad &= C_{\pi}\left[\left(U_{S}-1\right)\delta m_{S-1}^{\alpha_{S}}+\delta\tilde{m}_{S-1}^{\alpha_{S}}\right]^{\frac{(1-\gamma_{S})(\varepsilon-1)}{\alpha_{S}[\varepsilon-\gamma_{S}(\varepsilon-1)]}} \\ &-\left(U_{S}-1\right)\delta\tau_{S-1}t_{S-1}-\delta\tau_{S-1}\tilde{t}_{S-1}-O_{S},\end{aligned}$$

where

$$C_{\pi}=\left[\frac{\gamma_{S}\left(\varepsilon-1\right)}{\varepsilon}\right]^{\frac{\gamma_{S}(\varepsilon-1)}{\varepsilon-\gamma_{S}(\varepsilon-1)}}\frac{\varepsilon-\gamma_{S}\left(\varepsilon-1\right)}{\varepsilon}A^{\frac{1}{\varepsilon-\gamma_{S}(\varepsilon-1)}}, \tag{A.20}$$

and the outside option is

$$O_S = C_\pi \left[(U_S - 1)\, \delta m_{S-1}^{\alpha_S} \right]^{\frac{(1-\gamma_S)(\varepsilon-1)}{\alpha_S[\varepsilon-\gamma_S(\varepsilon-1)]}} - (U_S - 1)\, \delta \tau_{S-1} t_{S-1}.$$

The solution to this bargaining game is

$$\left(\tilde{m}_{S-1}, \tilde{t}_{S-1}\right) = \arg\max_{(m,t)} \beta_S \log \psi_S^d (m,t) + (1 - \beta_S) \log \psi_{S-1}^u (m,t)\,. \tag{A.21}$$

Following the same procedure as above, taking limits as $\delta \to 0$, $U_j \to \infty$, $D_j \to \infty$, $\delta U_j \to n_j^u$ and $\delta D_j \to n_j^d$, the symmetric solution $\left(\tilde{m}_{S-1}, \tilde{t}_{S-1}\right) = (m_{S-1}, t_{S-1})$ yields (see Section A1.1)

$$m_{S-1} = C_{m_{S-1}} \left[n_S^u\right]^{\frac{(1-\alpha_S)(1-\gamma_S)(\varepsilon-1)}{\alpha_S} - 1} \prod_{j=1}^{S-1} \left[n_j^u\right]^{\frac{\Gamma_j^{S-1}(1-\alpha_j)[\varepsilon-\gamma_S(\varepsilon-1)]}{\alpha_j}}, \tag{A.22}$$

$$t_{S-1} = C_{t_{S-1}} \left[n_S^u\right]^{\frac{(1-\alpha_S)(1-\gamma_S)(\varepsilon-1)}{\alpha_S} - 1} \prod_{j=1}^{S-1} \left[n_j^u\right]^{\frac{\Gamma_j^{S}(1-\alpha_j)(\varepsilon-1)}{\alpha_j}}. \tag{A.23}$$

We now have a recursive system from which we can solve $\{m_s, t_s\}_{s=0}^{S-1}$, given by equations (A.5), (A.7), (A.18), (A.19), (A.22) and (A.23). We also show in Section A1.1 that the coefficients $\{C_{m_s}, C_{t_s}\}_{s=1}^{S-1}$ satisfy the recursive structure

$$C_{m_s} = \left[\frac{\left(1 - \gamma_{s+1}\right) \gamma_s C_{m_{s-1}}^{\frac{1-\gamma_s}{\gamma_s}}}{\gamma_{s+1} \tau_s B_s} \right]^{\gamma_{s+1}}, \quad s \in \{1, 2, ..., S-2\}, \tag{A.24}$$

$$C_{m_{S-1}} = \left(\frac{\gamma_{S-1} C_{m_{S-2}}^{\frac{1-\gamma_{S-1}}{\gamma_{S-1}}}}{\tau_{S-1} B_{S-1}} \right)^{\varepsilon - \gamma_S(\varepsilon-1)} \left(\frac{\varepsilon - 1}{\varepsilon} \right)^{\varepsilon} (\gamma_S)^{\gamma_S(\varepsilon-1)} (1 - \gamma_S)^{\varepsilon - \gamma_S(\varepsilon-1)} A, \tag{A.25}$$

$$C_{t_s} = C_{m_s} \frac{1}{\gamma_s} C_{m_{s-1}}^{\frac{\gamma_s - 1}{\gamma_s}} B_s \mu_{s+1}, \quad s \in \{1, 2, ..., S-1\}, \tag{A.26}$$

where

$$\mu_{s+1} = \frac{1 - \beta_{s+1} + \beta_{s+1}\alpha_{s+1}}{\alpha_{s+1}} > 1,$$

$$B_s = \gamma_s + (1 - \gamma_s)\mu_s > 1.$$

Together with (A.6) and (A.8), they provide a recursive system from which we obtain solutions to $\{C_{m_s}, C_{t_s}\}_{s=0}^{S-1}$. This implies that only C_{S-1} and t_{S-1} depend on the demand shifter A, while the other constants depend only on technology parameters and the subsidies to purchases of intermediate inputs. Naturally, while A is exogenous to every firm, it is endogenous to the economy. We explain in Section A3 how A is determined in general equilibrium via the labor market clearing conditions.

Finally, note that (A.5), (A.18) and (A.22) provide a recursive system of equations in m_s that uses the constants C_{m_s}. The value of m_{S-1} is determined by (A.22) as a function of $\left\{n_j^u\right\}_{j=1}^S$. This value can then be used in the recursive equations (A.5) and (A.18) to solve all other values of m_s, $s \in \{0, 1, ..., S-2\}$, as functions of $\left\{n_j^u\right\}_{j=1}^S$, by recalling that $N_s n_s^d = N_{s+1} n_{s+1}^u$. Also note that (A.18) and (A.19) imply

$$\frac{t_s}{m_s} = \frac{C_{t_s}}{C_{m_s}} \prod_{j=1}^{s} \left[n_j^u\right]^{\frac{-\Gamma_j^s\left(1-\alpha_j\right)}{\alpha_j}}, \quad s \in \{1, 2, ..., S-1\},$$

while (A.5) and (A.7) imply

$$\frac{t_0}{m_0} = \frac{C_{t_0}}{C_{m_0}}.$$

That is, the terms of trade between a tier 0 firm and a tier 1 firm do not depend on $\left\{n_j^u\right\}_{j=1}^S$ while the terms of trade between a tier s firm and a tier $s+1$ firm for $s \geq 1$ depend on the number of upstream links of s and the further links of firms upstream from s; i.e., on $\left\{n_j^u\right\}_{j=1}^s$. In particular, the more links there are upstream the better the terms of trade of a firm in s.

A1.1 Proof of m_s and t_s Formulas

First, we show that (A.18), (A.19), (A.22)-(A.26) hold for $S = 3$.

Bargaining between tier 0 and tier 1 firms does not dependent on S, and the solution to this game is described by (A.5)-(A.8). We next consider bargaining between a tier 1 producer and a tier 2 buyer. The payoff of the tier 1 firm is

$$\delta\tilde{t}_1 + (D_1 - 1)\delta t_1 - \left[(D_1 - 1)\delta m_1 + \delta\tilde{m}_1\right]^{\frac{1}{\gamma_1}} \left[U_1 \delta m_0^{\alpha_1}\right]^{\frac{\gamma_1 - 1}{\alpha_1\gamma_1}} - U_1\delta\tau_0 t_0 - O_1,$$

where the outside option is

$$O_1 = (D_1 - 1)\delta t_1 - \left[(D_1 - 1)\delta m_1\right]^{\frac{1}{\gamma_1}} \left[U_1 \delta (m_0^o)^{\alpha_1}\right]^{\frac{\gamma_1 - 1}{\alpha_1\gamma_1}} - U_1\delta\tau_0 t_0^o.$$

Because the firms bargain sequentially, the firm in tier 1 anticipates an equilibrium outcome (m_0, t_0) in its subsequent negotiations with firms in tier 0, given by (A.5) and (A.7), or

$$m_0 = C_{m_0}[n_1^u]^{\frac{\gamma_1(1-\alpha_1)-1}{\alpha_1}}[(D_1 - 1)\delta m_1 + \delta\tilde{m}_1],$$
$$t_0 = C_{t_0}[n_1^u]^{\frac{\gamma_1(1-\alpha_1)-1}{\alpha_1}}[(D_1 - 1)\delta m_1 + \delta\tilde{m}_1].$$

If, however, its current negotiations fail, the firm expects the off-equilibrium solution (m_0^o, t_0^o), given by

$$m_0^o = C_{m_0}[n_1^u]^{\frac{\gamma_1(1-\alpha_1)-1}{\alpha_1}}(D_1-1)\delta m_1]$$
$$t_0^o = C_{t_0}[n_1^u]^{\frac{\gamma_1(1-\alpha_1)-1}{\alpha_1}}(D_1-1)\delta m_1].$$

Combining these terms, we obtain the net payoff of the firm in tier 1 as a function of $\left(\tilde{m}_1, \tilde{t}_1\right)$,

$$\psi_1^u\left(\tilde{m}_1, \tilde{t}_1\right) := \delta\tilde{t}_1 + (D_1-1)\delta t_1 - [n_1^u]^{\frac{(\gamma_1-1)(1-\alpha_1)}{\alpha_1}} C_{m_0}^{\frac{\gamma_1-1}{\gamma_1}} \frac{1}{\gamma_1} B_1\left[(D_1-1)\delta m_1 + \delta\tilde{m}_1\right] - O_1.$$

The net payoff of the tier 2 firm is in this case

$$\psi_2^d\left(\tilde{m}_1, \tilde{t}_1\right) := D_2\delta t_2 - [D_2\delta m_2]^{\frac{1}{\gamma_2}}\left[(U_2-1)\delta m_1^{\alpha_2} + \delta\tilde{m}_1^{\alpha_2}\right]^{\frac{\gamma_2-1}{\alpha_2\gamma_2}} - (U_2-1)\delta\tau_1 t_1 - \delta\tau_1\tilde{t}_1 - O_2,$$

where

$$O_2 = D_2\delta t_2 - [D_2\delta m_2]^{\frac{1}{\gamma_2}}\left[(U_2-1)\delta m_1^{\alpha_2}\right]^{\frac{\gamma_2-1}{\alpha_2\gamma_2}} - (U_2-1)\delta\tau_1 t_1.$$

Therefore the solution to this bargaining game is

$$\left(\tilde{m}_1, \tilde{t}_1\right) = \arg\max_{(m,t)} \beta_2 \log\psi_2^d(m,t) + (1-\beta_2)\log\psi_1^u(m,t).$$

Following the procedure we used above, for $\delta \to 0$, $U_j \to \infty$, $D_j \to \infty$, $\delta U_j \to n_j^u$ and $\delta D_j \to n_j^d$, the first-order conditions for this problem yield the symmetric solution $\left(\tilde{m}_1, \tilde{t}_1\right) = (m_1, t_1)$, given by

$$m_1 = C_{m_1}\,[n_2^u]^{\frac{\gamma_2(1-\alpha_2)-1}{\alpha_2}}\,[n_1^u]^{\frac{\gamma_2(1-\gamma_1)(1-\alpha_1)}{\alpha_1}}\,n_2^d m_2,$$

where

$$C_{m_1} = \left[\frac{(1-\gamma_2)\gamma_1 C_{m_0}^{\frac{1-\gamma_1}{\gamma_1}}}{\gamma_2\tau_1 B_1}\right]^{\gamma_2},$$

and

$$t_1 = C_{t_1}\,[n_1^u]^{-\frac{(1-\gamma_1)(1-\gamma_2)(1-\alpha_1)}{\alpha_1}}\,[n_2^u]^{\frac{\gamma_2(1-\alpha_2)-1}{\alpha_2}}\,n_2^d m_2,$$

where

$$C_{t_1} = C_{m_1}\frac{1}{\gamma_1} C_{m_0}^{\frac{\gamma_1-1}{\gamma_1}} B_1\mu_2.$$

Next consider bargaining between a tier 2 supplier and a tier 3 buyer. Output of the final good producer is (see (A.1))

$$x_3 = l_3^{\gamma_3} \left[(U_3 - 1) \, \delta m_2^{\alpha_3} + \delta \tilde{m}_2^{\alpha_3} \right]^{(1-\gamma_3)/\alpha_3} .$$

The inverse demand function is

$$p_3 = (A/x_3)^{1/\varepsilon} ,$$

and therefore revenue net of costs is

$$\pi_3 = A^{1/\varepsilon} l_3^{\gamma_3 (\varepsilon - 1)/\varepsilon} \left[(U_3 - 1) \, \delta m_2^{\alpha_3} + \delta \tilde{m}_2^{\alpha_3} \right]^{\frac{(1-\gamma_3)(\varepsilon-1)}{\alpha_3 \varepsilon}} - (U_3 - 1) \, \delta \tau_2 t_2 - \delta \tau_2 \tilde{t}_2 - l_3.$$

This firm chooses l_3 to maximize profits π_3 for given quantities of intermediate inputs and transfers. Therefore

$$l_3 = \left[\frac{\gamma_3 (\varepsilon - 1)}{\varepsilon} \right]^{\frac{\varepsilon}{\varepsilon - \gamma_3(\varepsilon - 1)}} A^{\frac{1}{\varepsilon - \gamma_3(\varepsilon-1)}} \left[(U_3 - 1) \, \delta m_2^{\alpha_3} + \delta \tilde{m}_2^{\alpha_3} \right]^{\frac{(1-\gamma_3)(\varepsilon-1)}{\alpha_3 [\varepsilon - \gamma_3(\varepsilon-1)]}} ,$$

which yields

$$\pi_3 \left(\tilde{m}_2, \tilde{t}_2 \right) := C_\pi \left[(U_3 - 1) \, \delta m_2^{\alpha} + \delta \tilde{m}_2^{\alpha_3} \right]^{\frac{(1-\gamma_3)(\varepsilon-1)}{\alpha_3 [\varepsilon - \gamma_3(\varepsilon-1)]}} - (U_3 - 1) \, \delta \tau_2 t_2 - \delta \tau_2 \tilde{t}_2,$$

where C_π is defined in (A.20). The outside option of this firm is

$$O_3 = C_\pi \left[(U_3 - 1) \, \delta m_2^{\alpha_3} \right]^{\frac{(1-\gamma_3)(\varepsilon-1)}{\alpha_3 [\varepsilon - \gamma_3(\varepsilon-1)]}} - (U_3 - 1) \, \delta \tau_2 t_2,$$

and its net payoff is

$$\psi_3^d \left(\tilde{m}_2, \tilde{t}_2 \right) := \pi_3 \left(\tilde{m}_2, \tilde{t}_2 \right) - O_3.$$

The payoff of a tier 2 firm from its negotiations with a tier 3 firm is

$$\delta \tilde{t}_2 + (D_2 - 1) \delta t_2 - \left[(D_2 - 1) \delta m_2 + \delta \tilde{m}_2 \right]^{\frac{1}{\gamma_2}} \left[U_2 \delta m_1^{\alpha_2} \right]^{\frac{\gamma_2 - 1}{\alpha_2 \gamma_2}} - U_2 \delta \tau_1 t_1 - O_2$$

where the outside option is

$$O_2 = (D_2 - 1) \delta t_2 - \left[(D_2 - 1) \delta m_2 \right]^{\frac{1}{\gamma_2}} \left[U_2 \delta (m_1^o)^{\alpha_2} \right]^{\frac{\gamma_2 - 1}{\alpha_2 \gamma_2}} - U_2 \delta \tau_1 t_1^o.$$

This firm anticipates the equilibrium outcome (m_1, t_1) in its subsequent negotiations with firms in tier 1, which from above is

$$\begin{aligned} m_1 &= C_{m_1} \left[n_2^u \right]^{\frac{\gamma_2 (1-\alpha_2) - 1}{\alpha_2}} \left[n_1^u \right]^{\frac{\gamma_2 (1-\gamma_1)(1-\alpha_1)}{\alpha_1}} \left[(D_2 - 1) \delta m_2 + \delta \tilde{m}_2 \right] , \\ t_1 &= C_{t_1} \left[n_1^u \right]^{-\frac{(1-\gamma_1)(1-\gamma_2)(1-\alpha_1)}{\alpha_1}} \left[n_2^u \right]^{\frac{\gamma_2 (1-\alpha_2) - 1}{\alpha_2}} \left[(D_2 - 1) \delta m_2 + \delta \tilde{m}_2 \right] , \end{aligned}$$

and it anticipates the outcome (off-equilibrium)

$$m_1^o = C_{m_1}\left[n_2^u\right]^{\frac{\gamma_2(1-\alpha_2)-1}{\alpha_2}}\left[n_1^u\right]^{\frac{\gamma_2(1-\gamma_1)(1-\alpha_1)}{\alpha_1}}(D_2-1)\delta m_2,$$
$$t_1^o = C_{t_1}\left[n_1^u\right]^{-\frac{(1-\gamma_1)(1-\gamma_2)(1-\alpha_1)}{\alpha_1}}\left[n_2^u\right]^{\frac{\gamma_2(1-\alpha_2)-1}{\alpha_2}}(D_2-1)\delta m_2,$$

in case its negotiations fail. Therefore its net payoff is

$$\begin{aligned}\psi_2^d\left(\tilde{m}_2,\tilde{t}_2\right) \quad : \quad & = \delta\tilde{t}_2 + (D_2-1)\delta t_2 \\ & - \left[n_2^u\right]^{-\frac{(1-\gamma_2)(1-\alpha_2)}{\alpha_2}}\left[n_1^u\right]^{-\frac{(1-\gamma_2)(1-\gamma_1)(1-\alpha_1)}{\alpha_1}} C_{m_1}^{\frac{\gamma_2-1}{\gamma_2}}\frac{1}{\gamma_2}B_2\left[(D_2-1)\delta m_2+\delta\tilde{m}_2\right]-O_2,\end{aligned}$$

and the solution to this bargaining game is

$$\left(\tilde{m}_2,\tilde{t}_2\right) = \arg\max_{(m,t)}\beta_3\log\psi_3^d(m,t)+(1-\beta_3)\log\psi_2^u(m,t)\,.$$

Using again the above described procedure, for $\delta\to 0$, $U_j\to\infty$, $D_j\to\infty$, $\delta U_j\to n_j^u$ and $\delta D_j\to n_j^d$, the first-order conditions for this problem yield the symmetric solution $\left(\tilde{m}_2,\tilde{t}_2\right)=(m_2,t_2)$:

$$m_2 = C_{m_2}[n_3^u]^{\frac{(1-\alpha_3)(1-\gamma_3)(\varepsilon-1)}{\alpha_3}-1}[n_2^u]^{\frac{(1-\gamma_2)(1-\alpha_2)[\varepsilon-\gamma_3(\varepsilon-1)]}{\alpha_2}}[n_1^u]^{\frac{(1-\gamma_1)(1-\gamma_2)(1-\alpha_1)[\varepsilon-\gamma_3(\varepsilon-1)]}{\alpha_1}},$$

$$C_{m_2} = \left(\frac{\gamma_2 C_{m_1}^{\frac{1-\gamma_2}{\gamma_2}}}{\tau_2 B_2}\right)^{\varepsilon-\gamma_3(\varepsilon-1)}\left(\frac{\varepsilon-1}{\varepsilon}\right)^{\varepsilon}(\gamma_3)^{\gamma_3(\varepsilon-1)}(1-\gamma_3)^{\varepsilon-\gamma_3(\varepsilon-1)}A,$$

and

$$t_2 = C_{t_2}\left[n_1^u\right]^{\frac{(1-\gamma_1)(1-\gamma_2)(1-\gamma_3)(1-\alpha_1)(\varepsilon-1)}{\alpha_1}}\left[n_2^u\right]^{\frac{(1-\gamma_2)(1-\gamma_3)(1-\alpha_2)(\varepsilon-1)}{\alpha_2}}\left[n_3^u\right]^{\frac{(1-\gamma_3)(\varepsilon-1)(1-\alpha_3)}{\alpha_3}-1},$$

$$C_{t_2} = C_{m_2}\frac{1}{\gamma_2}C_{m_1}^{\frac{\gamma_2-1}{\gamma_2}}B_2\mu_3.$$

We have therefore shown that equations (A.18), (A.19), (A.22)-(A.26) hold for $S=3$.

To extend the proof to an arbitrary number of tiers, suppose that the above recursive formulas hold for $S>3$. We will then show that they also hold when the final producers are in tier $S+1$. Recall that our solution for (m_0,t_0) holds for any S, including for $S+1$. So first consider bargaining between a tier s supplier and a tier $s+1$ buyer. The payoff of the tier s firm, which supplies inputs to the tier $s+1$ firm, net of its outside option is

$$\delta\tilde{t}_s+(D_s-1)\delta t_s-\left[(D_s-1)\delta m_s+\delta\tilde{m}_s\right]^{\frac{1}{\gamma_s}}\left[U_s\delta m_{s-1}^{\alpha_s}\right]^{\frac{\gamma_s-1}{\alpha_s\gamma_s}}-U_s\delta\tau_{s-1}t_{s-1}-O_s,$$

and the outside option is

$$O_s = (D_s - 1)\delta t_s - [(D_s - 1)\delta m_s]^{\frac{1}{\gamma_s}} \left[U_s \delta (m^o_{s-1})^{\alpha_s}\right]^{\frac{\gamma_s - 1}{\alpha_s \gamma_s}} - U_s \delta \tau_{s-1} t^o_{s-1}.$$

Under the inductive hypothesis

$$m_s = C_{m_s} \left[n^u_{s+1}\right]^{\frac{\gamma_{s+1}(1-\alpha_{s+1})-1}{\alpha_{s+1}}} \left[\prod_{j=1}^{s} \left[n^u_j\right]^{\frac{\gamma_{s+1}\Gamma^s_j(1-\alpha_j)}{\alpha_j}}\right] \left[(D_s - 1)\delta m_s + \delta \tilde{m}_s\right],$$

$$t_s = C_{t_s} \left[n^u_{s+1}\right]^{\frac{\gamma_{s+1}(1-\alpha_{s+1})-1}{\alpha_{s+1}}} \left[\prod_{j=1}^{s} \left[n^u_j\right]^{\frac{-\Gamma^{s+1}_j(1-\alpha_j)}{\alpha_j}}\right] \left[(D_s - 1)\delta m_s + \delta \tilde{m}_s\right],$$

which is independent of the tier of the final good producer, where

$$C_{m_s} = \left[\frac{(1-\gamma_{s+1})\gamma_s C_{m_{s-1}}^{\frac{1-\gamma_s}{\gamma_s}}}{\gamma_{s+1}\tau_s B_s}\right]^{\gamma_{s+1}}, \tag{A.27}$$

$$C_{t_s} = C_{m_s} \frac{1}{\gamma_s} C_{m_{s-1}}^{\frac{\gamma_s - 1}{\gamma_s}} B_s \mu_{s+1}. \tag{A.28}$$

Therefore the net payoff of the tier s producer can be expressed as

$$\begin{aligned} \psi^u_s\left(\tilde{m}_s, \tilde{t}_s\right) \quad : \quad &= \delta \tilde{t}_s + (D_s - 1)\delta t_s \qquad\qquad (A.29) \\ &- \left[\prod_{j=1}^{s} \left[n^u_j\right]^{\frac{-\Gamma^s_j(1-\alpha_j)}{\alpha_j}}\right] \left[C_{m_{s-1}}^{\frac{\gamma_s - 1}{\gamma_s}} + \tau_{s-1} C_{t_{s-1}}\right] \left[(D_s - 1)\delta m_s + \delta \tilde{m}_s\right] - O_s, \end{aligned}$$

where

$$m^o_s = C_{m_s} \left[n^u_{s+1}\right]^{\frac{\gamma_{s+1}(1-\alpha_{s+1})-1}{\alpha_{s+1}}} \left[\prod_{j=1}^{s} \left[n^u_j\right]^{\frac{\gamma_{s+1}\Gamma^s_j(1-\alpha_j)}{\alpha_j}}\right] (D_s - 1)\delta m_s,$$

$$t^o_s = C_{t_s} \left[n^u_{s+1}\right]^{\frac{\gamma_{s+1}(1-\alpha_{s+1})-1}{\alpha_{s+1}}} \left[\prod_{j=1}^{s} \left[n^u_j\right]^{\frac{-\Gamma^{s+1}_j(1-\alpha_j)}{\alpha_j}}\right] (D_s - 1)\delta m_s,$$

For the downstream firm there are two possibilities: either $s+1 < S+1$ and it manufactures intermediate inputs, or $s = S+1$ and it produces final goods. In the first case, the payoff of this firm net of the outside option is

$$\begin{aligned} \psi^d_{s+1}\left(\tilde{m}_s, \tilde{t}_s\right) \quad : \quad &= D_{s+1}\delta t_{s+1} - \left[D_{s+1}\delta m_{s+1}\right]^{\frac{1}{\gamma_{s+1}}} \left[(U_{s+1} - 1)\delta m_s^{\alpha_{s+1}} + \delta \tilde{m}_s^{\alpha_{s+1}}\right]^{\frac{\gamma_{s+1} - 1}{\alpha_{s+1}\gamma_{s+1}}} \qquad (A.30) \\ &- (U_{s+1} - 1)\delta \tau_s t_s - \delta \tau_s \tilde{t}_s - O_{s+1}, \end{aligned}$$

where

$$O_{s+1} = D_{s+1}\delta t_{s+1} - [D_{s+1}\delta m_{s+1}]^{\frac{1}{\gamma_{s+1}}} [(U_{s+1}-1)\delta m_s^{\alpha_{s+1}}]^{\frac{\gamma_{s+1}-1}{\alpha_{s+1}\gamma_{s+1}}} - (U_{s+1}-1)\delta\tau_s t_s.$$

In the latter case its net payoff is

$$\begin{aligned} \psi_{S+1}^d\left(\tilde{m}_S, \tilde{t}_S\right) \quad : \quad &= C_\pi \left[(U_{S+1}-1)\,\delta m_S^{\alpha_{S+1}} + \delta \tilde{m}_S^{\alpha_{S+1}}\right]^{\frac{(1-\gamma_{S+1})(\varepsilon-1)}{\alpha_{S+1}[\varepsilon-\gamma_{S+1}(\varepsilon-1)]}} \\ &- (U_{S+1}-1)\,\delta\tau_S t_S - \delta\tau_S\tilde{t}_S - O_{S+1}, \end{aligned} \tag{A.31}$$

and

$$O_{S+1} = C_\pi \left[(U_{S+1}-1)\,\delta m_S^{\alpha_{S+1}}\right]^{\frac{(1-\gamma_{S+1})(\varepsilon-1)}{\alpha_{S+1}[\varepsilon-\gamma_{S+1}(\varepsilon-1)]}} - (U_{S+1}-1)\,\delta\tau_S t_S.$$

In either case the solution to the bargaining game is

$$\left(\tilde{m}_s, \tilde{t}_s\right) = \arg\max_{(m,t)} \beta_{s+1}\log\psi_{s+1}^d(m,t) + \left(1-\beta_{s+1}\right)\log\psi_s^u(m,t).$$

In the first case, the first-order conditions of this maximization problem are (A.4) and (A.16), and as described above, for $\delta U_j = n_j^u$ and $\delta D_j = n_j^d$ we get the solution

$$m_s = C_{m_s}\left[n_{s+1}^u\right]^{\frac{\gamma_{s+1}(1-\alpha_{s+1})-1}{\alpha_{s+1}}} \left[\prod_{j=1}^{s}\left[n_j^u\right]^{\frac{\gamma_{s+1}\Gamma_j^s(1-\alpha_j)}{\alpha_j}}\right] n_{s+1}^d m_{s+1}. \tag{A.32}$$

Following a similar procedure, in the second case we get the solution

$$m_S = C_{m_S}\left[n_{S+1}^u\right]^{\frac{(1-\alpha_{S+1})(1-\gamma_{S+1})(\varepsilon-1)}{\alpha_{S+1}}-1} \prod_{j=1}^{S}\left[n_s^u\right]^{\frac{\Gamma_j^S(1-\alpha_j)[\varepsilon-\gamma_{S+1}(\varepsilon-1)]}{\alpha_j}}.$$

The constants C_{m_s} and C_{m_S} are

$$C_{m_s} = \left[\frac{(1-\gamma_{s+1})\gamma_s C_{m_{s-1}}^{\frac{1-\gamma_s}{\gamma_s}}}{\gamma_{s+1}\tau_s B_s}\right]^{\gamma_{s+1}}, \tag{A.33}$$

$$C_{m_S} = \left[C_\pi \frac{\gamma_S C_{m_{S-1}}^{\frac{1-\gamma_S}{\gamma_S}}(1-\gamma_{S+1})(\varepsilon-1)}{\tau_S B_S\left[\varepsilon-\gamma_{S+1}(\varepsilon-1)\right]}\right]^{\varepsilon-\gamma_{S+1}(\varepsilon-1)}.$$

Moreover, the first-order conditions with respect to $\tilde{m}_s$ and $\tilde{t}_s$, together with the postulated recursive equations, yield in the limit of $\delta \to 0$, $U_j \to \infty$, $D_j \to \infty$, $\delta U_j \to n_j^u$ and $\delta D_j \to n_j^d$

$$t_s = C_{t_s}\left[n_{s+1}^u\right]^{\frac{\gamma_{s+1}(1-\alpha_{s+1})-1}{\alpha_{s+1}}} \left[\prod_{j=1}^{s}\left[n_j^u\right]^{\frac{-\Gamma_j^{s+1}(1-\alpha_j)}{\alpha_j}}\right] n_{s+1}^d m_{s+1} \tag{A.34}$$

in the first case and

$$t_S = C_{t_S} \left[n^u_{S+1}\right]^{\frac{(1-\alpha_{S+1})(1-\gamma_{S+1})(\varepsilon-1)}{\alpha_{S+1}}-1} \prod_{j=1}^{S} \left[n^u_j\right]^{\frac{\Gamma_j^{S+1}(1-\alpha_j)(\varepsilon-1)}{\alpha_j}}$$

in the second case, where

$$C_{t_s} = C_{m_s} \frac{1}{\gamma_s} C_{m_{s-1}}^{\frac{\gamma_s-1}{\gamma_s}} B_s \mu_{s+1}, \quad s \in \{0, 1, ..., S\}. \tag{A.35}$$

This shows that our recursive formulas (A.18), (A.19), (A.22)-(A.26) hold for arbitrary values of $S \geq 3$.

A1.2 Second-Order Conditions of the Bargaining Game

First, we verify the second-order conditions for the bargaining game between a tier 0 and a tier 1 firm. Let

$$f^0(m,t) := \beta_1 \log \psi^d_1(m,t) + (1-\beta_1) \log \psi^u_0(m,t).$$

Then the first-order condition of (A.2) with respect to $\tilde{m}_0$ can be expressed as

$$f^0_m(m,t) = \beta_1 \frac{\varphi^d_1(m)}{\psi^d_1(m,t)} + (1-\beta_1) \frac{\varphi^u_0(m)}{\psi^u_0(m,t)} = 0,$$

where $f^0_m(m,t)$ is the partial derivative of $f^0(m,t)$ with respect to m and

$$\begin{aligned}
\varphi^d_1(m) &:= \frac{\partial \psi^d_1(m,t)}{\partial m} = \frac{1-\gamma_1}{\gamma_1} \left[D_1 \delta m_1\right]^{\frac{1}{\gamma_1}} \left[(U_1-1)\delta m_0^{\alpha_1} + \delta m^{\alpha_1}\right]^{\frac{\gamma_1-1}{\alpha_1\gamma_1}-1} \delta m^{\alpha_1-1} > 0, \\
\varphi^u_0(m) &:= \frac{\partial \psi^u_0(m,t)}{\partial m} = -\delta a < 0.
\end{aligned}$$

Since $\gamma_1 \in (0,1)$ and $\alpha_1 \in (0,1)$,

$$\frac{d\varphi^d_1(m)}{dm} < 0, \ \frac{d\varphi^u_0(m)}{dm} = 0, \ \frac{\partial \psi^d_1(m,t)}{\partial m} > 0, \ \frac{\partial \psi^u_0(m,t)}{\partial m} < 0.$$

Therefore, the partial derivative of $f^0_m(m,t)$ with respect to m satisfies

$$f^0_{mm}(m,t) < 0.$$

Moreover,

$$\frac{\partial \psi^d_1(m,t)}{\partial t} = -\delta \tau_0 < 0, \ \frac{\partial \psi^u_0(m,t)}{\partial t} = \delta > 0,$$

and therefore

$$f^0_{mt}(m,t) > 0.$$

Finally, the first-order condition of (A.2) with respect to t is

$$f_t^0(m,t) = -\beta_1 \frac{\delta \tau_0}{\psi_1^d(m,t)} + (1-\beta_1)\frac{\delta}{\psi_0^u(m,t)} = 0.$$

Since

$$\frac{\partial \psi_1^d(m,t)}{\partial t} < 0, \;\; \frac{\partial \psi_0^u(m,t)}{\partial t} > 0,$$

this implies

$$f_{tt}^0(m,t) < 0.$$

Clearly, the Hessian of $f^0(m,t)$ has negative diagonal elements. Moreover,

$$\begin{aligned} f_{mm}^0 f_{tt}^0 - \left(f_{mt}^0\right)^2 = & \frac{\beta_1(1-\beta_1)\delta^2}{\psi_1^d(m,t)^2 \psi_0^u(m,t)^2}\left[\varphi_1(m) - \tau_0\varphi_2(m)\right]^2 - \frac{\beta_1^2\delta^2\tau_0^2}{\psi_1^d(m,t)^3}\frac{d\varphi_1^d(m)}{dm} \\ & - \frac{\beta_1(1-\beta_1)\delta^2}{\psi_0^u(m,t)^2\psi_1^d(m,t)}\frac{d\varphi_1^d(m)}{dm} > 0. \end{aligned}$$

Therefore the Hessian of $f^0(m,t)$ is negative definite, implying that $f^0(m,t)$ is a concave function. As a result the second order conditions are satisfied.

We show next that the objective function in problem (A.14) is concave for $s+1 < S$. Let

$$f^s(m,t) := \beta_{s+1}\psi_{s+1}^d(m,t) + (1-\beta_{s+1})\psi_s^u(m,t).$$

The first order condition for this problem with respect to m is

$$f_m^s(m,t) = \beta_{s+1}\frac{\varphi_{s+1}^d(m)}{\psi_{s+1}^d(m,t)} - (1-\beta_{s+1})\frac{\varphi_s^u(m)}{\psi_s^u(m,t)} = 0,$$

where

$$\begin{aligned} \varphi_{s+1}^d(m) = & \; \left[D_{s+1}\delta m_{s+1}\right]^{\frac{1}{\gamma_{s+1}}} \frac{1-\gamma_{s+1}}{\gamma_{s+1}}\left[(U_{s+1}-1)\delta m_s^{\alpha_{s+1}} + \delta m^{\alpha_{s+1}}\right]^{\frac{\gamma_{s+1}-1}{\alpha_{s+1}\gamma_{s+1}}-1}\delta m^{\alpha_{s+1}-1}, \\ \varphi_s^u(m) = & \left[\prod_{j=1}^{s}\left[n_j^u\right]^{\frac{-\Gamma_j^s(1-\alpha_j)}{\alpha_j}}\right] C_{m_{s-1}}^{\frac{\gamma_s-1}{\gamma_s}}\frac{1}{\gamma_s}B_s\delta. \end{aligned}$$

Since $\gamma_{s+1} \in (0,1)$ and $\alpha_{s+1} \in (0,1)$,

$$\frac{d\varphi_{s+1}^d(m)}{dm} < 0, \;\; \frac{d\varphi_s^u(m)}{dm} = 0, \;\; \frac{\partial \psi_{s+1}^d(m,t)}{\partial m} > 0, \;\; \frac{\partial \psi_s^u(m,t)}{\partial m} < 0.$$

Therefore

$$f_{mm}^s(m,t) < 0.$$

Moreover,

$$\frac{\partial \psi_{s+1}^d(m,t)}{\partial t} = -\delta\tau_s < 0,\ \frac{\partial \psi_s^u(m,t)}{\partial t} = \delta > 0,$$

and therefore

$$f_{mt}^s(m,t) > 0.$$

Furthermore, the first order condition of (A.14) with respect to t is

$$f_t^s(m,t) = -\beta_{s+1}\frac{\delta\tau_s}{\psi_{s+1}^d(m,t)} + (1-\beta_{s+1})\frac{\delta}{\psi_s^u(m,t)} = 0.$$

Using $\frac{\partial \psi_{s+1}^d}{\partial t} < 0$ and $\frac{\partial \psi_s^u}{\partial t} > 0$, we obtain:

$$f_{tt}^s(m,t) < 0.$$

Thus, the Hessian of $f^s(m,t)$ has negative diagonal elements. Moreover,

$$\begin{aligned} f_{mm}^s f_{tt}^s - (f_{mt}^s)^2 = &\frac{\beta_{s+1}(1-\beta_{s+1})\delta^2}{\psi_{s+1}^d(m,t)^2\psi_s^u(m,s)^2}\left[\varphi_{s+1}^d(m) - \tau_s\varphi_s^u(m)\right]^2 \\ &- \frac{\beta_{s+1}^2\delta^2\tau_s^2}{\psi_{s+1}^d(m,t)^3}\frac{d\varphi_{s+1}^d(m)}{dm} - \frac{\beta_{s+1}(1-\beta_{s+1})\delta^2}{\psi_s^u(m,t)^2\psi_{s+1}^d(m,t)}\frac{d\varphi_{s+1}^d(m)}{dm} > 0. \end{aligned}$$

Therefore the Hessian of $f^s(m,t)$ is negative definite, implying that $f^s(m,t)$ is a concave function. As a result, the second-order conditions of (A.14) are satisfied.

Finally, we show that the second-order conditions are satisfied for problem (A.21). Let

$$f^{S-1}(m,t) := \beta_S\varphi_S^d(m,t) + (1-\beta_S)\psi_{S-1}^u(m,t).$$

The first order condition of this problem with respect to m is

$$f_m^{S-1}(m,t) = \beta_S\frac{\varphi_S^d(m)}{\psi_S^d(m,t)} - (1-\beta_S)\frac{\varphi_{S-1}^u(m)}{\psi_{S-1}^u(m,t)} = 0,$$

where

$$\varphi_S^d(m) = \frac{C_\pi(1-\gamma_S)(\varepsilon-1)}{\varepsilon-\gamma_S(\varepsilon-1)}\left[(U_S-1)\delta m_{S-1}^{\alpha_S}+\delta m^{\alpha_S}\right]^{\frac{(1-\gamma_S)(\varepsilon-1)}{\alpha_S[\varepsilon-\gamma_S(\varepsilon-1)]}-1}\delta m^{\alpha_S-1},$$

$$\varphi_{S-1}^u(m) = \left[\prod_{j=1}^{S-1}\left[n_j^u\right]^{\frac{-\Gamma_j^{S-1}(1-\alpha_j)}{\alpha_j}}\right]C_{m_{S-2}}^{\frac{\gamma_{S-1}-1}{\gamma_{S-1}}}\frac{1}{\gamma_{S-1}}B_{S-1}\delta.$$

Since $\varepsilon > 1$, $\gamma_S \in (0,1)$, $\alpha_S \in (0,1)$ and $\sigma_S > \varepsilon$, we have:

$$\frac{d\varphi_S^d(m)}{dm} < 0,\ \frac{d\varphi_{S-1}^u(m)}{dm} = 0,\ \frac{\partial\psi_S^d(m,t)}{\partial m} > 0,\ \frac{\partial\psi_{S-1}^u(m,t)}{\partial m} < 0.$$

Therefore

$$f_{mm}^{S-1}(m,t) < 0.$$

Moreover,

$$\frac{\partial\psi_S^d(m,t)}{\partial t} = -\delta\tau_{S-1} < 0,\ \frac{\partial\psi_{S-1}^u(m,t)}{\partial t} = \delta > 0,$$

and therefore

$$f_{mt}^{S-1}(m,t) > 0.$$

Next, the first-order condition with respect to t is

$$f_t^{S-1}(m,t) = -\beta_S\frac{\delta\tau_{S-1}}{\psi_S^d(m,t)} + (1-\beta_S)\frac{\delta}{\psi_{S-1}^u(m,t)} = 0.$$

Using $\frac{\partial\psi_S^d}{\partial t} < 0$ and $\frac{\partial\psi_{S-1}^u}{\partial t} > 0$, we obtain

$$f_{tt}^{S-1}(m,t) < 0.$$

Finally,

$$\begin{aligned} f_{mm}^{S-1}f_{tt}^{S-1} - \left(f_{mt}^{S-1}\right)^2 = & \frac{\beta_S(1-\beta_S)\delta^2}{\psi_S^d(m,t)^2\psi_{S-1}^u(m,t)^2}\left[\varphi_S^d(m) - \tau_{S-1}\varphi_{S-1}^u(m)\right]^2 \\ & - \frac{\beta_S^2\delta^2\tau_{S-1}^2}{\psi_S^d(m,t)^3}\frac{d\varphi_S^d}{dm}(m) - \frac{\beta_S(1-\beta_S)\delta^2}{\psi_{S-1}^u(m,t)^2\psi_S^d(m,t)}\frac{d\varphi_S^d}{dm}(m) > 0. \end{aligned}$$

Therefore the Hessian of $f^{S-1}(m,t)$ is negative definite and $f^{S-1}(m,t)$ is a concave function. As a result, the second-order conditions of problem (A.21) are satisfied.

To summarize, we have shown that all the objective functions of the bargaining games, $f_t^s(m,t)$ for $s \in \{0,1,...,S-1\}$, are concave, in which case the first-order conditions for $(\tilde{m}_s, \tilde{t}_s)$ characterize the unique solutions of these bargaining games.

A2 Ex-Post Payoffs and Ex-Ante Investment in Resilience and Links

Given resilience levels $\{r_s\}_{s=0}^{S}$ and search intensities $\{\eta_s\}_{s=1}^{S}$, we derived in the previous section a recursive system from which we can solve the equilibrium values of $\{m_s, t_s\}_{s=0}^{S-1}$. This system is given by equations (A.5), (A.7), (A.18), (A.19), (A.22) and (A.23). We also showed in Section A1.1 that the coefficients $\{C_{m_s}, C_{t_s}\}_{s=1}^{S-1}$ can be solved from the recursive structure (A.6), (A.8), (A.24)-(A.26). In short, we have

$$m_{S-1} = C_{m_{S-1}} \left[n_S^u\right]^{\frac{(1-\alpha_S)(1-\gamma_S)(\varepsilon-1)}{\alpha_S}-1} \prod_{j=1}^{S-1} \left[n_j^u\right]^{\frac{\Gamma_j^{S-1}(1-\alpha_j)[\varepsilon-\gamma_S(\varepsilon-1)]}{\alpha_j}}, \tag{A.36}$$

$$t_{S-1} = C_{t_{S-1}} \left[n_S^u\right]^{\frac{(1-\alpha_S)(1-\gamma_S)(\varepsilon-1)}{\alpha_S}-1} \prod_{j=1}^{S-1} \left[n_j^u\right]^{\frac{\Gamma_j^{S}(1-\alpha_j)(\varepsilon-1)}{\alpha_j}}, \tag{A.37}$$

$$m_{s-1} = C_{m_{s-1}} \left[n_s^u\right]^{\frac{\gamma_s(1-\alpha_s)-1}{\alpha_s}} \left[\prod_{j=1}^{s-1} \left[n_j^u\right]^{\frac{\gamma_s\Gamma_j^{s-1}(1-\alpha_j)}{\alpha_j}}\right] n_s^d m_s, \quad s \in \{2, 3, ..., S-1\}, \tag{A.38}$$

$$t_{s-1} = C_{t_{s-1}} \left[n_s^u\right]^{\frac{\gamma_s(1-\alpha_s)-1}{\alpha_s}} \left[\prod_{j=1}^{s-1} \left[n_j^u\right]^{\frac{-\Gamma_j^{s}(1-\alpha_j)}{\alpha_j}}\right] n_s^d m_s, \quad s \in \{2, 3, ..., S-1\}, \tag{A.39}$$

$$m_0 = C_{m_0} \left[n_1^u\right]^{\frac{\gamma_1(1-\alpha_1)-1}{\alpha_1}} n_1^d m_1, \tag{A.40}$$

$$t_0 = C_{t_0} \left[n_1^u\right]^{\frac{\gamma_1(1-\alpha_1)-1}{\alpha_1}} n_1^d m_1. \tag{A.41}$$

Utilizing the recursive structure, this implies

$$\begin{aligned} m_s = & \left[\prod_{j=s+1}^{S-1} n_j^d\right] \left[\prod_{j=s}^{S-1} C_{m_j}\right] \left[\prod_{j=s+1}^{S} \left[n_j^u\right]^{\frac{(1-\alpha_j)(\varepsilon-1)\Gamma_j^S}{\alpha_j}-1}\right] \\ & \times \left[\prod_{j=1}^{s} \left[n_j^u\right]^{\frac{(1-\alpha_j)\Gamma_j^s\left[1+\Gamma_{s+1}^S(\varepsilon-1)\right]}{\alpha_j}}\right], \quad \text{for } s \in \{0, 1, ..., S-2\}, \end{aligned} \tag{A.42}$$

$$\frac{t_s}{m_s} = \mu_{s+1} \frac{1}{\gamma_s} B_s C_{m_{s-1}}^{\frac{\gamma_s - 1}{\gamma_s}} \prod_{j=1}^{s} \left[n_j^u\right]^{-\frac{(1-\alpha_j)}{\alpha_j}\Gamma_j^s}, \quad \text{for } s \in \{0, 1, ..., S-2\}, \tag{A.43}$$

which together with (A.38)-(A.39) provide closed-form solutions for the transacted quantities and payments, where[16]

$$\prod_{j=1}^{0}\left[n_j^u\right]^{\frac{(1-\alpha_j)\Gamma_j^s\left[1+\Gamma_{s+1}^S(\varepsilon-1)\right]}{\alpha_j}} := 1.$$

We first characterize the ex-post payoffs in a symmetric equilibrium, to which the above transacted quantities and transfers apply. However, in order to identify private incentives for link formation, we need to examine possible deviations of a firm from its equilibrium choices. Therefore, we will also develop payoff functions that admit such off-equilibrium deviations, and we will use them to characterize the ex-ante choices of investment in resilience and link formation.

A2.1 Equilibrium Ex-Post Payoffs under Symmetric Link Formations

In a symmetric equilibrium the payoff of a tier s firm is

$$v_0 = n_0^d t_0 - a n_0^d m_0, \quad \text{for } s = 0, \tag{A.44}$$

$$v_s = n_s^d t_s - n_s^u \tau_{s-1} t_{s-1} - l_s, \quad \text{for } s \in \{1, ..., S-1\}, \tag{A.45}$$

where l_s is a wage bill, equal to the employment level. The wage bill of the tier 0 firm is $a n_0^d m_0$. The employment levels l_s, $s > 0$, satisfy $l_s^{\gamma_s}\left(n_s^u m_{s-1}^{\alpha_s}\right)^{\frac{1-\gamma_s}{\alpha_s}} = n_s^d m_s$, and therefore

$$l_0 = a n_0^d m_0, \tag{A.46}$$

$$l_s = \frac{\left(n_s^d m_s\right)^{\frac{1}{\gamma_s}}}{\left(n_s^u m_{s-1}^{\alpha_s}\right)^{\frac{1-\gamma_s}{\alpha_s \gamma_s}}}, \quad s \in \{1, 2, ..., S-1\}. \tag{A.47}$$

Using (A.38), (A.39) and (A.42), we obtain

$$v_s = \frac{(1-\beta_{s+1})(1-\alpha_{s+1})}{\alpha_{s+1}} \frac{1}{\gamma_s} B_s C_{m_{s-1}}^{\frac{\gamma_s - 1}{\gamma_s}} \left[\prod_{j=s}^{S-1} C_{m_j}\right]\left[\prod_{j=s}^{S-1} n_j^d\right] \tag{A.48}$$
$$\times \left[\prod_{j=s+1}^{S}\left[n_j^u\right]^{\frac{(1-\alpha_j)(\varepsilon-1)\Gamma_j^S}{\alpha_j}-1}\right]\left[\prod_{j=1}^{s}\left[n_j^u\right]^{\frac{(1-\alpha_j)\Gamma_j^S(\varepsilon-1)}{\alpha_j}}\right], \quad \text{for } s \in \{0, 1, ..., S-1\}.$$

For a firm in tier S, the analysis in Section A1 implies that $v_S = \psi_S^d\left(m_{S-1}, t_{S-1}\right) + O_S$. Therefore

$$v_S = C_\pi \left[n_S^u m_{S-1}^{\alpha_S}\right]^{\frac{(1-\gamma_S)(\varepsilon-1)}{\alpha_S\left[\varepsilon-\gamma_S(\varepsilon-1)\right]}} - n_S^u \tau_{S-1} t_{S-1},$$

and using (A.36)-(A.37),

[16] Recall that $n_{s+1}^u = \eta_{s+1}\phi_s(r_s)N_s$ and $n_s^d = \eta_{s+1}\phi(r_{s+1})N_{s+1}$.

$$v_S = C_{t_{S-1}} \frac{\tau_{S-1}}{\mu_S} \left[\frac{\varepsilon - \gamma_S(\varepsilon-1)}{(1-\gamma_S)(\varepsilon-1)} - \frac{1-\beta_S(1-\alpha_S)}{\alpha_S} \right] \prod_{j=1}^{S} \left[n_j^u\right]^{\frac{\Gamma_j^S(1-\alpha_j)(\varepsilon-1)}{\alpha_j}}. \tag{A.49}$$

Now note that $\frac{\varepsilon-\gamma_S(\varepsilon-1)}{(1-\gamma_S)(\varepsilon-1)}$ is an increasing function of γ_S and $\frac{1}{\alpha_S} = \frac{\sigma_S}{\sigma_S - 1}$. Therefore

$$\frac{\varepsilon - \gamma_S(\varepsilon-1)}{(1-\gamma_S)(\varepsilon-1)} - \frac{1-\beta_S(1-\alpha_S)}{\alpha_S} > \frac{\varepsilon}{\varepsilon-1} - \frac{\sigma_S}{\sigma_S-1} + \frac{\beta_S(1-\alpha_S)}{\alpha_S}.$$

It follows that $\sigma_S > \varepsilon$ is a sufficient condition for $v_S > 0$.

A2.2 Ex-Ante Perceived Payoffs

In order to characterize the equilibrium choice of η_s, we need to examine a bargaining problem that allows for a deviation from symmetry. We therefore solve in this section the bargaining games for a firm in a tier s that chooses an $\tilde{\eta}_s$ that is not necessarily the equilibrium value η_s, and therefore may differ from the choices of other firms in tier s. For this purpose we use tildes to denote quantities, transfers and links of a specific firm that is the subject of our analysis.

First, consider a firm in tier $s \in \{1, ..., S-2\}$ that is a potential deviant from $\tilde{\eta}_s = \eta_s$, where η_s is chosen by all other firms in its tier. Moreover, assume that in every tier all firms—other than our potential deviant—have chosen the same investment levels in resilience and in link formation. It then follows that when the potential deviant will bargain with one of its upstream suppliers, it expects the bargaining outcome to be (see (A.38)-(A.41))

$$\tilde{m}_{s-1} = C_{m_{s-1}} \left[\tilde{n}_s^u\right]^{\frac{\gamma_s(1-\alpha_s)-1}{\alpha_s}} \left[\prod_{j=1}^{s-1} \left[n_j^u\right]^{\frac{\gamma_s \Gamma_j^{s-1}(1-\alpha_j)}{\alpha_j}} \right] n_s^d \tilde{m}_s, \tag{A.50}$$

$$\tilde{t}_{s-1} = C_{t_{s-1}} \left[\tilde{n}_s^u\right]^{\frac{\gamma_s(1-\alpha_s)-1}{\alpha_s}} \left[\prod_{j=1}^{s-1} \left[n_j^u\right]^{\frac{-\Gamma_j^{s}(1-\alpha_j)}{\alpha_j}} \right] n_s^d \tilde{m}_s, \tag{A.51}$$

with

$$\prod_{j=1}^{0} \left[\left[n_j^u\right]^{\frac{-\Gamma_j^1(1-\alpha_j)}{\alpha_j}} \right] := 1,$$

where $\tilde{m}_s$ is the commitment it will have to each one of is buyers and $\tilde{n}_s^u$ is its expected measure of upstream suppliers; that is, $\tilde{n}_s^u = \tilde{\eta}_s N_{s-1} \phi_{s-1}(r_{s-1})$, where r_{s-1} is the symmetric investment in resilience of firms in tier $s-1$. Using (A.50) and (A.51), this firm expects its payoff net of the outside option in the bargaining with a firm in tier $s+1$ to be (see (A.29) and (A.33))

$$\psi^u_s\left(\tilde{m}_s,\tilde{t}_s\right) \quad : \quad = \delta\tilde{t}_s + (D_s-1)\delta\hat{t}_s - \left[\tilde{n}^u_s\right]^{-\frac{(1-\alpha_s)(1-\gamma_s)}{\alpha_s}}\left[\prod_{j=1}^{s-1}\left[n^u_j\right]^{\frac{-\Gamma^s_j(1-\alpha_j)}{\alpha_j}}\right] \tag{A.52}$$

$$\times\frac{1}{\gamma_s}B_s C^{\frac{\gamma_s-1}{\gamma_s}}_{m_{s-1}}\left[(D_s-1)\delta\hat{m}_s+\delta\tilde{m}_s\right]-O_s,$$

where

$$O_s = (D_s-1)\delta\hat{t}_s - \left[(D_s-1)\delta\hat{m}_s\right]^{\frac{1}{\gamma_s}}\left[U_s\delta(m^o_{s-1})^{\alpha_s}\right]^{\frac{\gamma_s-1}{\alpha_s\gamma_s}} - U_s\delta\tau_{s-1}t^o_{s-1},$$

$$m^o_{s-1} = C_{m_{s-1}}\left[\tilde{n}^u_s\right]^{\frac{\gamma_s(1-\alpha_s)-1}{\alpha_s}}\left[\prod_{j=1}^{s-1}\left[n^u_j\right]^{\frac{\gamma_s\Gamma^{s-1}_j(1-\alpha_j)}{\alpha_j}}\right]\left[\left(D_s-1\right)\delta\hat{m}_s\right],$$

$$t^o_{s-1} = C_{t_{s-1}}\left[\tilde{n}^u_s\right]^{\frac{\gamma_s(1-\alpha_s)-1}{\alpha_s}}\left[\prod_{j=1}^{s-1}\left[n^u_j\right]^{\frac{-\Gamma^{s}_j(1-\alpha_j)}{\alpha_j}}\right]\left[\left(D_s-1\right)\delta\hat{m}_s\right].$$

In this formulation $\left(\hat{m}_s,\hat{t}_s\right)$ represents the deals this firm has reached with all its buyers other than the one with whom it bargains. When the firm forms its expectations ex-ante, it realizes that it will face the same bargaining game with every buyer, and therefore a change in $\tilde{\eta}_s$ will affect all bargains in similar fashion. For this reason we impose $\left(\hat{m}_s,\hat{t}_s\right)=\left(\tilde{m}_s,\tilde{t}_s\right)$ after solving for $\left(\tilde{m}_s,\tilde{t}_s\right)$.

Next consider the payoff net of the outside option of the downstream firm that bargains with our deviant. It is (see (A.30))

$$\psi^d_{s+1}\left(\tilde{m}_s,\tilde{t}_s\right) \quad : \quad = D_{s+1}\delta t_{s+1} - \left[D_{s+1}\delta m_{s+1}\right]^{\frac{1}{\gamma_{s+1}}}\left[(U_{s+1}-1)\delta m^{\alpha_{s+1}}_s+\delta\tilde{m}^{\alpha_{s+1}}_s\right]^{\frac{\gamma_{s+1}-1}{\alpha_{s+1}\gamma_{s+1}}}$$

$$-(U_{s+1}-1)\delta\tau_s t_s - \delta\tau_s\tilde{t}_s - O_{s+1},$$

where

$$O_{s+1} = D_{s+1}\delta t_{s+1} - \left[D_{s+1}\delta m_{s+1}\right]^{\frac{1}{\gamma_{s+1}}}\left[(U_{s+1}-1)\delta m^{\alpha_{s+1}}_s\right]^{\frac{\gamma_{s+1}-1}{\alpha_{s+1}\gamma_{s+1}}} - (U_{s+1}-1)\delta\tau_s t_s.$$

Here (m_s,t_s) represents the contract of the downstream firm with each one of its suppliers other than the deviant firm. The solution to this bargaining game is

$$(\tilde{m}_s,\tilde{t}_s) = \arg\max_{m,t}\beta_{s+1}\log\psi^d_{s+1}(m,t)+(1-\beta_{s+1})\log\psi^u_s(m,t).$$

Following the procedure discussed in the previous section, the first-order conditions of this maximization problem imply

$$\frac{1-\gamma_{s+1}}{\gamma_{s+1}}\left[n_{s+1}^{d}m_{s+1}\right]^{\frac{1}{\gamma_{s+1}}}\left[(U_{s+1}-1)\delta m_s^{\alpha_{s+1}}+\delta\tilde{m}_s^{\alpha_{s+1}}\right]^{\frac{\gamma_{s+1}-1}{\gamma_{s+1}\alpha_{s+1}}-1}\tilde{m}_s^{\alpha_{s+1}-1}$$
$$=\tau_s\frac{1}{\gamma_s}B_sC_{m_{s-1}}^{\frac{\gamma_s-1}{\gamma_s}}\left[\tilde{n}_s^u\right]^{\frac{-(1-\gamma_s)(1-\alpha_s)}{\alpha_s}}\left[\prod_{j=1}^{s-1}\left[n_j^u\right]^{\frac{-\Gamma_j^s(1-\alpha_j)}{\alpha_j}}\right]. \tag{A.53}$$

In the limit, as $\delta\to 0$, $U_{s+1}\to\infty$, and $(U_{s+1}-1)\delta\to n_{s+1}^u$, this yields

$$\begin{aligned}\tilde{m}_s^{1-\alpha_{s+1}} &= \frac{\gamma_sC_{m_{s-1}}^{\frac{1-\gamma_s}{\gamma_s}}}{B_s\tau_s}\frac{1-\gamma_{s+1}}{\gamma_{s+1}}\left[n_{s+1}^{d}m_{s+1}\right]^{\frac{1}{\gamma_{s+1}}}\left[n_{s+1}^{u}m_s^{\alpha_{s+1}}\right]^{\frac{\gamma_{s+1}-1}{\gamma_{s+1}\alpha_{s+1}}-1} \\ &\quad\times\left[\tilde{n}_s^u\right]^{\frac{(1-\gamma_s)(1-\alpha_s)}{\alpha_s}}\left[\prod_{j=1}^{s-1}\left[n_j^u\right]^{\frac{\Gamma_j^s(1-\alpha_j)}{\alpha_j}}\right],\end{aligned} \tag{A.54}$$

for $\left(\hat{m}_s,\hat{t}_s\right)=\left(\tilde{m}_s,\tilde{t}_s\right)$, which provides a solution to $\tilde{m}_s$ as a function of $\tilde{n}_s^u$. The latter can be affected ex-ante by the deviant firm's choice of $\tilde{\eta}_s$, while all other variables in this formula are beyond its control. Also note that when the downstream firm in tier $s+1$ bargains with any other supplier, it attains an outcome represented by (A.38), which does not depend on the deviant firm's choice of $\tilde{\eta}_s$ (because the deviant firm is of measure zero).

Next, following the procedure from the previous section we solve for $\tilde{t}_s$. For $\left(\hat{m}_s,\hat{t}_s\right)=\left(\tilde{m}_s,\tilde{t}_s\right)$ this yields

$$\begin{aligned}\tau_s\delta\tilde{t}_s &= \tau_s\beta_{s+1}\delta\left[\tilde{n}_s^u\right]^{-\frac{(1-\alpha_s)(1-\gamma_s)}{\alpha_s}}\left[\prod_{j=1}^{s-1}\left[n_j^u\right]^{\frac{-\Gamma_j^s(1-\alpha_j)}{\alpha_j}}\right]\frac{1}{\gamma_s}B_sC_{m_{s-1}}^{\frac{\gamma_s-1}{\gamma_s}}\tilde{m}_s+(1-\beta_{s+1})(n_{s+1}^dm_{s+1})^{\frac{1}{\gamma_{s+1}}} \\ &\quad\times\left\{\left[(U_{s+1}-1)\delta m_s^{\alpha_{s+1}}\right]^{\frac{\gamma_{s+1}-1}{\alpha_{s+1}\gamma_{s+1}}}-\left[(U_{s+1}-1)\delta m_s^{\alpha_{s+1}}+\delta\tilde{m}_s^{\alpha_{s+1}}\right]^{\frac{\gamma_{s+1}-1}{\alpha_{s+1}\gamma_{s+1}}}\right\}.\end{aligned}$$

Dividing by δ and using L'Hôpital's rule, $\delta\to 0$, $U_{s+1}\to\infty$ and $U_{s+1}\delta\to n_{s+1}^u$ imply

$$\begin{aligned}\tilde{t}_s &= \beta_{s+1}\left[\tilde{n}_s^u\right]^{-\frac{(1-\alpha_s)(1-\gamma_s)}{\alpha_s}}\left[\prod_{j=1}^{s-1}\left[n_j^u\right]^{\frac{-\Gamma_j^s(1-\alpha_j)}{\alpha_j}}\right]\frac{1}{\gamma_s}B_sC_{m_{s-1}}^{\frac{\gamma_s-1}{\gamma_s}}\tilde{m}_s \\ &\quad+\frac{1}{\tau_s}(1-\beta_{s+1})(n_{s+1}^dm_{s+1})^{\frac{1}{\gamma_{s+1}}}\left[n_{s+1}^um_s^{\alpha_{s+1}}\right]^{\frac{\gamma_{s+1}-1}{\gamma_{s+1}\alpha_{s+1}}}\frac{1-\gamma_{s+1}}{\alpha_{s+1}\gamma_{s+1}}\frac{1}{n_{s+1}^u}\left(\frac{\tilde{m}_s}{m_s}\right)^{\alpha_{s+1}}.\end{aligned}$$

Finally, substituting (A.54) into this equation we obtain

$$\tilde{t}_s=\mu_{s+1}\left[\tilde{n}_s^u\right]^{-\frac{(1-\alpha_s)(1-\gamma_s)}{\alpha_s}}\left[\prod_{j=1}^{s-1}\left[n_j^u\right]^{\frac{-\Gamma_j^s(1-\alpha_j)}{\alpha_j}}\right]C_{m_{s-1}}^{\frac{\gamma_s-1}{\gamma_s}}\frac{1}{\gamma_s}B_s\tilde{m}_s. \tag{A.55}$$

This equation represents the transfer the deviant firm expects to receive from it bargaining with a downstream firm if it chooses $\tilde{\eta}_s$ that yields $\tilde{n}_s^u$, and its sales to the buyer are $\tilde{m}_s$, implicitly defined in (A.54).

Given $(\tilde{m}_s, \tilde{t}_s)$, we can characterize the ex-ante perceived payoff of a tier s firm when it chooses $\tilde{\eta}_s$. Combining (A.45), (A.47) and (A.55), yields

$$\tilde{v}_s\left(\tilde{\eta}_s\right) = n_s^d \tilde{m}_s \left[(\mu_{s+1}-1) C_{m_{s-1}}^{\frac{\gamma_s - 1}{\gamma_s}} \frac{1}{\gamma_s} B_s\right] \left[\tilde{n}_s^u\right]^{-\frac{(1-\alpha_s)(1-\gamma_s)}{\alpha_s}} \left[\prod_{j=1}^{s-1} \left[n_j^u\right]^{\frac{-\Gamma_j^s (1-\alpha_j)}{\alpha_j}}\right],$$

or, substituting (A.54) into this equation,

$$\tilde{v}_s\left(\tilde{\eta}_s\right) = Q_{v_s}\left(\tilde{\eta}_s\right)^{\frac{(1-\alpha_s)(1-\gamma_s)\alpha_{s+1}}{\alpha_s(1-\alpha_{s+1})}} = Q_{v_s}\left(\tilde{\eta}_s\right)^{\frac{(1-\gamma_s)(\sigma_{s+1}-1)}{\sigma_s - 1}}, \quad \text{for } s \in \{1, 3, ..., S-2\}, \tag{A.56}$$

where Q_{v_s} is an object that is independent of $\tilde{\eta}_s$, which the firm taken as given ex-ante, and

$$\begin{aligned} Q_{v_s} := n_s^d &\left[(\mu_{s+1}-1) C_{m_{s-1}}^{\frac{\gamma_s - 1}{\gamma_s}} \frac{1}{\gamma_s} B_s\right] \left[\prod_{j=1}^{s-1} \left[n_j^u\right]^{\frac{-\Gamma_j^s (1-\alpha_j)}{\alpha_j}}\right] \left[\phi_{s-1}\left(r_{s-1}\right) N_{s-1}\right]^{\frac{(1-\gamma_s)(\sigma_{s+1}-1)}{\sigma_s - 1}} \\ \times &\left\{ \frac{\gamma_s C_{m_{s-1}}^{\frac{1-\gamma_s}{\gamma_s}}}{B_s \tau_s} \frac{1-\gamma_{s+1}}{\gamma_{s+1}} \left[n_{s+1}^d m_{s+1}\right]^{\frac{1}{\gamma_{s+1}}} \left[n_{s+1}^u m_s^{\alpha_{s+1}}\right]^{\frac{\gamma_{s+1}-1}{\gamma_{s+1}\alpha_{s+1}} - 1} \left[\prod_{j=1}^{s-1} \left[n_j^u\right]^{\frac{\Gamma_j^s (1-\alpha_j)}{\alpha_j}}\right] \right\}^{\frac{1}{1-\alpha_{s+1}}}. \end{aligned}$$

It remains to characterize $\tilde{v}_{S-1}$ and $\tilde{v}_S$. First note that, because a tier S firm sells to final consumers and the upstream-looking formulas are the same as in the previous section, we have

$$\tilde{v}_S\left(\tilde{\eta}_S\right) = Q_{v_S}\left(\tilde{\eta}_S\right)^{\frac{(1-\gamma_S)(1-\alpha_S)(\varepsilon-1)}{\alpha_S}}, \tag{A.57}$$

where

$$\begin{aligned} Q_{v_S} \quad : \quad &= C_{t_{S-1}} \frac{\tau_{S-1}}{\mu_S} \left[\frac{\varepsilon - \gamma_S(\varepsilon-1)}{(1-\gamma_S)(\varepsilon-1)} - \frac{1-\beta_S(1-\alpha_S)}{\alpha_S}\right] \\ &\times \left[\phi_{S-1}\left(r_{S-1}\right) N_{S-1}\right]^{\frac{(1-\gamma_S)(1-\alpha_S)(\varepsilon-1)}{\alpha_S}} \prod_{j=1}^{S-1} \left[n_j^u\right]^{\frac{\Gamma_j^S (1-\alpha_j)(\varepsilon-1)}{\alpha_j}}. \end{aligned}$$

To derive $\tilde{v}_{S-1}$, we solve

$$(\tilde{m}_{S-1}, \tilde{t}_{S-1}) := \arg\max_{m,t} \beta_S \log \psi_S^d(m,t) + (1-\beta_S) \log \psi_{S-1}^u(m,t),$$

where $\psi_{S-1}^u(\cdot)$ satisfies (A.52) and $\psi_S^d(\cdot)$ satisfies (A.31) with $S+1$ replaced by S and S replaced by $S-1$, i.e.,

$$\psi_S^d(\tilde{m}_{S-1}, \tilde{t}_{S-1}) = C_\pi \left[(U_S - 1)\delta m_{S-1}^{\alpha_S} + \delta \tilde{m}_{S-1}^{\alpha_S}\right]^{\frac{(1-\gamma_S)(\varepsilon-1)}{\alpha_S[\varepsilon - \gamma_S(\varepsilon-1)]}} - (U_S - 1)\delta\tau_{S-1} - \delta\tau_{S-1}\tilde{t}_{S-1} - O_S,$$

$$O_S = \psi_S^d(\tilde{m}_{S-1}, \tilde{t}_{S-1}) = C_\pi \left[(U_S - 1)\delta m_{S-1}^{\alpha_S}\right]^{\frac{(1-\gamma_S)(\varepsilon-1)}{\alpha_S[\varepsilon-\gamma_S(\varepsilon-1)]}} - (U_S - 1)\delta\tau_{S-1}.$$

Following an analysis similar to the above, we obtain

$$\begin{aligned} \tilde{m}_{S-1}^{1-\alpha_S} &= \frac{C_\pi}{\tau_{S-1}} \frac{(1-\gamma_S)(\varepsilon-1)}{\varepsilon-\gamma_S(\varepsilon-1)} \left[n_S^u m_{S-1}^{\alpha_S}\right]^{\frac{(1-\gamma_S)(\varepsilon-1)}{\alpha_S[\varepsilon-\gamma_S(\varepsilon-1)]}-1} \frac{1}{\gamma_{S-1}} B_{S-1} C_{m_{S-2}}^{\frac{\gamma_{S-1}-1}{\gamma_{S-1}}} \\ &\times \left[\tilde{n}_{S-1}^u\right]^{\frac{(1-\gamma_{S-1})(1-\alpha_{S-1})}{\alpha_{S-1}}} \left[\prod_{j=1}^{S-2} \left[n_j^u\right]^{\frac{\Gamma_j^{S-1}(1-\alpha_j)}{\alpha_j}}\right], \end{aligned} \tag{A.58}$$

where

$$m_{S-1} = C_{m_{S-1}} \left[n_S^u\right]^{\frac{(1-\alpha_S)(1-\gamma_S)(\varepsilon-1)}{\alpha_S}-1} \prod_{j=1}^{S-1} \left[n_j^u\right]^{\frac{\Gamma_j^{S-1}(1-\alpha_j)[\varepsilon-\gamma_S(\varepsilon-1)]}{\alpha_j}}.$$

Note that $\tilde{m}_{S-1}$ depends only on objects that the deviant firm in tier $S-1$ takes as given ex-ante, except for $\tilde{\eta}_{S-1}$. Our first-order conditions and the limit argument for $\delta \to 0$, $U_j \to \infty$ and $U_j\delta \to n_j^u$ also imply

$$\tilde{t}_{S-1} = \mu_S \left[\tilde{n}_{S-1}^u\right]^{-\frac{(1-\alpha_{S-1})(1-\gamma_{S-1})}{\alpha_{S-1}}} \left[\prod_{j=1}^{S-2} \left[n_j^u\right]^{\frac{-\Gamma_j^{S-1}(1-\alpha_j)}{\alpha_j}}\right] C_{m_{S-2}}^{\frac{\gamma_{S-1}-1}{\gamma_{S-1}}} \frac{1}{\gamma_{S-1}} B_{S-1} \tilde{m}_{S-1}.$$

Therefore, using this equation together with (A.45), (A.47) and (A.58), yields

$$\tilde{v}_{S-1}\left(\tilde{\eta}_{S-1}\right) = Q_{v_{S-1}} \left(\tilde{\eta}_{S-1}\right)^{\frac{(1-\gamma_{S-1})(\sigma_S-1)}{\sigma_{S-1}-1}} \tag{A.59}$$

where $Q_{v_{S-1}}$ is an object that, ex-ante, is taken as given by a firm in tier $S-1$, and

$$\begin{aligned} Q_{v_{S-1}} &:= n_{S-1}^d \left[(\mu_S - 1) C_{m_{S-2}}^{\frac{\gamma_{S-1}-1}{\gamma_{S-1}}} \frac{1}{\gamma_{S-1}} B_{S-1}\right] \left[\prod_{j=1}^{S-2} \left[n_j^u\right]^{\frac{-\Gamma_j^{S-1}(1-\alpha_j)}{\alpha_j}}\right] \left[\phi_{S-2}\left(r_{S-2}\right) N_{S-2}\right]^{\frac{(1-\gamma_{S-1})(\sigma_S-1)}{\sigma_{S-1}-1}} \\ &\times \left[\frac{C_\pi}{\tau_{S-1}} \frac{(1-\gamma_S)(\varepsilon-1)}{\varepsilon-\gamma_S(\varepsilon-1)} \left[n_S^u m_{S-1}^{\alpha_S}\right]^{\frac{(1-\gamma_S)(\varepsilon-1)}{\alpha_S[\varepsilon-\gamma_S(\varepsilon-1)]}-1} \frac{1}{\gamma_{S-1}} B_{S-1} C_{m_{S-2}}^{\frac{\gamma_{S-1}-1}{\gamma_{S-1}}} \left[\prod_{j=1}^{S-2} \left[n_j^u\right]^{\frac{\Gamma_j^{S-1}(1-\alpha_j)}{\alpha_j}}\right]\right]^{\frac{1}{1-\alpha_S}}. \end{aligned}$$

Thus, we have derived $\tilde{v}_s\left(\tilde{\eta}_s\right)$ for $s \in \{1, ..., S\}$. Note that tier 0 firms make no link formation decisions, since they have no upstream suppliers. Therefore, $\tilde{v}_0 := v_0$ is viewed as a constant ex-ante.

A2.3 Ex-Ante Choice of Resilience and Link Formation

Because $\tilde{v}_s$ is not a function of r_s, due to the fact that every firm in tier s is of measure zero, the solution to a firm's ex-ante investment levels satisfies

$$r_0 = \arg\max_r \phi_0(r)v_0 - \theta_0 r \quad \text{for } s = 0,$$

$$(r_s, \eta_s) = \arg\max_{r,\tilde{\eta}} \phi_s(r)\tilde{v}_s(\tilde{\eta}) - \theta_s r - k\vartheta_s N_{s-1}\tilde{\eta} \quad \text{for } s \in \{1, 2, ..., S\},$$

where $\theta_s r_s$ and $k\vartheta_s N_{s-1}\tilde{\eta}_s$ are investment costs in resilience and link formation, respectively. We assume that the government provides a subsidy to investment in resilience at the rate $1-\theta_s$ and a subsidy to investment in link formation at the rate $1-\vartheta_s$. Since $\phi_s(r)$ is a concave function, the first-order condition for an interior solution to the investment level,

$$\phi_s'(r_s)\tilde{v}_s(\tilde{\eta}_s) = \theta_s, \quad s \in \{0, 1, ..., S\},$$

characterizes the value of r_s, where $\tilde{v}_0 := v_0$. The first-order condition for the choice of $\tilde{\eta}_s$ by a firm in tier $s \in \{1, ..., S\}$ satisfies

$$\phi_s(r_s)\tilde{v}_s'(\tilde{\eta}_s) = k\vartheta_s N_{s-1}, \quad s \in \{1, 2, ..., S\}.$$

In a symmetric equilibrium $\tilde{\eta}_s = \eta_s$. For this condition to secure an interior solution, $\tilde{v}_s(\tilde{\eta}_s)$—which is a power function for all s—has to be concave. Assumption 1, which states that $\sigma_1 \geq \sigma_2 \geq ... \geq \sigma_S > \varepsilon > 1$, ensures that these power functions are concave, because under this assumption the exponents of these power functions are smaller than one. That is,

$$\frac{(1-\gamma_s)(\sigma_{s+1}-1)}{\sigma_s - 1} < 1, \quad \text{for } s \in \{1, 2, ..., S-1\},$$

$$\frac{(1-\gamma_S)(\varepsilon-1)}{\sigma_S - 1} < 1.$$

We conclude that in these circumstance a symmetric equilibrium is characterized by[17]

$$\phi_0'(r_0)v_0 = \theta_0, \tag{A.60}$$

$$\phi_s'(r_s)\tilde{v}_s(\eta_s) = \theta_s, \quad s \in \{1, ..., S\}, \tag{A.61}$$

$$\frac{(1-\gamma_s)(\sigma_{s+1}-1)}{\sigma_s - 1}\phi_s(r_s)\tilde{v}_s(\eta_s) = \vartheta_s k\eta_s N_{s-1}, \quad s \in \{1, 2, ..., S-1\}, \tag{A.62}$$

[17] We have used (A.56) to obtain (A.62) and (A.57) to obtain (A.63).

$$\frac{(1-\gamma_S)(1-\alpha_S)(\varepsilon-1)}{\alpha_S}\phi_S(r_S)\tilde{v}_S(\eta_S)=\vartheta_S k\eta_S N_{S-1}, \tag{A.63}$$

where $\tilde{v}_s = v_s$ in such a symmetric equilibrium.

A3 Labor Demand

Labor demand for investment in resilience is $\sum_{s=0}^{S} N_s r_s$ while labor demand for the formation of links is $k\sum_{s=1}^{S}\eta_s^u N_{s-1}N_s$, where k is the cost per link and each firm in tier s forms $\eta_s^u N_{s-1}$ links. If the number of links is exogenous and there is no investment in link formation, then $k=0$. The final component of labor demand is for manufacturing purposes. This depends on the quantities $\{m_s\}_{s=0}^{S-1}$. We have seen in the bargaining game that labor demand of the final good producer satisfies

$$l_S=\left[\frac{\gamma_S(\varepsilon-1)}{\varepsilon}\right]^{\frac{\varepsilon}{\varepsilon-\gamma_S(\varepsilon-1)}}A^{\frac{1}{\varepsilon-\gamma_S(\varepsilon-1)}}\left(n_S^u m_{S-1}^{\alpha_S}\right)^{\frac{(1-\gamma_S)(\varepsilon-1)}{\alpha_S[\varepsilon-\gamma_S(\varepsilon-1)]}}m_{S-1}.$$

In upper tiers we have

$$l_s^{\gamma_s}\left(n_s^u m_{s-1}^{\alpha_s}\right)^{\frac{1-\gamma_s}{\alpha_s}}=n_s^d m_s,$$

and therefore

$$l_s=\frac{\left(n_s^d m_s\right)^{\frac{1}{\gamma_s}}}{\left(n_s^u m_{s-1}^{\alpha_s}\right)^{\frac{1-\gamma_s}{\alpha_s\gamma_s}}},\quad s\in\{1,2,...,S-1\}. \tag{A.64}$$

In tier 0,

$$l_0=am_0. \tag{A.65}$$

Manufacturing labor demand is therefore $\sum_{s=0}^{S}\phi_s(r_s)N_s l_s$. Denoting by $L_{s,m}:=\phi_s(r_s)N_s l_s$ aggregate manufacturing labor demand in tier s, and using the iterative relationships (A.5), (A.18) and (A.22), we obtain

$$L_{S,m}=C_{L_S,m}\phi_S(r_S)N_S\prod_{j=1}^{S}\left[\eta_j^u\phi_{j-1}(r_{j-1})N_{j-1}\right]^{\frac{\Gamma_j^S(1-\alpha_j)(\varepsilon-1)}{\alpha_j}} \tag{A.66}$$

where, using (A.25),

$$\begin{aligned}C_{L_S,m} &= \left[\frac{\gamma_S(\varepsilon-1)}{\varepsilon}\right]^{\frac{\varepsilon}{\varepsilon-\gamma_S(\varepsilon-1)}}C_{m_{S-1}}^{\frac{(1-\gamma_S)(\varepsilon-1)}{\varepsilon-\gamma_S(\varepsilon-1)}}A^{\frac{1}{\varepsilon-\gamma_S(\varepsilon-1)}} \\ &= \left[\frac{\gamma_S(\varepsilon-1)}{\varepsilon}\right]^{\varepsilon}\left(\frac{\gamma_{S-1}C_{m_{S-2}}^{\frac{1-\gamma_{S-1}}{\gamma_{S-1}}}}{\tau_{S-1}B_{S-1}}\right)^{(1-\gamma_S)(\varepsilon-1)}\left(\frac{1-\gamma_S}{\gamma_S}\right)^{(1-\gamma_S)(\varepsilon-1)}A,\end{aligned} \tag{A.67}$$

$$L_{s,m} = C_{m_{s-1}}^{-\frac{1}{\gamma_s}} \left[\prod_{j=s-1}^{S-1} C_{m_j} \right] \phi_S(r_S) N_S \prod_{j=1}^{S} \left[\eta_j^u \phi_{j-1}(r_{j-1}) N_{j-1} \right]^{\frac{\Gamma_j^S (1-\alpha_j)(\varepsilon-1)}{\alpha_j}}, \tag{A.68}$$

$$s \in \{1, 2, ..., S-1\},$$

$$L_{0,m} = a \left[\prod_{j=0}^{S-1} C_{m_j} \right] \phi_S(r_S) N_S \prod_{j=1}^{S} \left[\eta_j^u \phi_{j-1}(r_{j-1}) N_{j-1} \right]^{\frac{\Gamma_j^S (1-\alpha_j)(\varepsilon-1)}{\alpha_j}}. \tag{A.69}$$

The labor market clearing condition requires

$$L = \sum_{s=0}^{S} N_s r_s + k \sum_{s=1}^{S} \eta_s^u N_{s-1} N_s + \sum_{s=0}^{S} L_{s,m},$$

and it can be expressed as

$$L = \sum_{s=0}^{S} N_s r_s + k \sum_{s=1}^{S} \eta_s^u N_{s-1} N_s + C_{L_m} \phi_S(r_S) N_S \prod_{j=1}^{S} \left[\eta_j^u \phi_{j-1}(r_{j-1}) N_{j-1} \right]^{\frac{\Gamma_j^S (1-\alpha_j)(\varepsilon-1)}{\alpha_j}}, \tag{A.70}$$

where

$$C_{L_m} = a \prod_{j=0}^{S-1} C_{m_j} + \sum_{j=1}^{S-1} C_{m_{j-1}}^{-\frac{1}{\gamma_j}} \prod_{z=j-1}^{S-1} C_{m_z} + C_{L_S,m}.$$

For given investment levels in resilience and in the formation of links, this labor market clearing condition provides a solution to the demand shifter A, because $C_{m_{S-1}}$ and C_{L_m} are proportional to A; see (A.25) and (A.67). The other coefficients C_{m_j} do not depend on A; see (A.6) and the recursive equation (A.24). Therefore C_{L_m} is proportional to A and A is uniquely determined by the labor market clearing condition. The use of labor for manufacturing purposes can be expressed as

$$L_m := C_{L_m} \phi_S(r_S) N_S \prod_{s=1}^{S} \left[\eta_s^u \phi_{s-1}(r_{s-1}) N_{s-1} \right]^{\frac{\Gamma_s^S (1-\alpha_s)(\varepsilon-1)}{\alpha_s}}. \tag{A.71}$$

A4 Optimal Allocation

The the social planner maximizes utility subject to the resource constraint. The utility is

$$W = \left[\phi_S(r_S) N_S x^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where $\phi_S(r_S) N_S$ is the number of final goods available for consumption and x is consumption per product. Consumption per product is $x = l_S^{\gamma_S} \left[\phi_{S-1}(r_{S-1}) \eta_S^u N_{S-1} m_{S-1}^{\alpha_S} \right]^{\frac{1-\gamma_S}{\alpha_S}}$. Therefore welfare can be expressed as

$$W=\left[\phi_S(r_S)N_S\right]^{\frac{\varepsilon-\gamma_S(\varepsilon-1)}{\varepsilon-1}}L_{S,m}^{\gamma_S}\left[\phi_{S-1}(r_{S-1})\eta_S N_{S-1}m_{S-1}^{\alpha_S}\right]^{\frac{1-\gamma_S}{\alpha_S}},$$

where

$$L_{S,m}=\phi_S(r_S)N_S l_S$$

is aggregate manufacturing labor employed by tier S firms. Manufacturing employment in tier s for $s<S$ is $\phi_s(r_s)N_s l_s$, where the values of l_s are given by (A.47) and (A.46) and $n_s^d=\eta_{s+1}\phi_{s+1}(r_{s+1})N_{s+1}$. Therefore the resource constraint of the planner is

$$\begin{aligned} L \;=\;& L_{S,m}+\sum_{s=1}^{S-1}L_{s,m}\left(r_{s-1},r_s,\eta_s,\eta_{s+1},m_{s-1},m_s\right)+\phi_0(r_0)N_0 a m_0\eta_1 N_1 \\ &+\sum_{s=0}^{S}N_s r_s+k\sum_{s=1}^{S}\eta_s N_{s-1}N_s, \end{aligned} \tag{A.72}$$

where

$$\begin{aligned} L_{s,m}\left(r_{s-1},r_s,\eta_s,\eta_{s+1},m_{s-1},m_s\right) \;:&=\phi_s(r_s)N_s\left[\eta_{s+1}\phi_{s+1}(r_{s+1})N_{s+1}m_s\right]^{\frac{1}{\gamma_s}} \\ \times\left[\eta_s\phi_{s-1}(r_{s-1})N_{s-1}m_{s-1}^{\alpha_s}\right]^{-\frac{1-\gamma_s}{\alpha_s\gamma_s}},\quad s\;&\in\;\{1,2,...,S-1\}, \end{aligned}$$

$$L_{0,m}(r_0,m_0):=\phi_0(r_0)N_0 a m_0,$$

and $L_{s,m}(\cdot)$ is aggregate labor employment in tier s. Labor employment in tier s, $s\in\{1,2,...,S-1\}$, is a function of investment in resilience in tiers $s-1$ and s, the number of upstream links in tiers s and $s+1$, and the number of intermediate products per user in tiers $s-1$ and s. In tier 0 aggregate labor use depends on the resilience level in this tier and on its output of intermediate inputs per user.

Maximizing W subject to (A.72), the first order conditions with respect to $L_{S,m}$ and m_{S-1} yield[18]

$$\gamma_S L_{S-1,m}^{*}=\left(1-\gamma_S\right)\gamma_{S-1}L_{S,m}^{*},$$

where asterisks represent optimal quantities of labor in the tiers. Further differentiating with respect to m_s, $s<S-1$, we obtain

[18] These first-order conditions are

$$\gamma_S W=\omega L_{S,m},$$
$$\left(1-\gamma_S\right)W=\omega\frac{1}{\gamma_{S-1}}L_{S-1,m},$$

where ω is the Lagrangian multiplier of (A.72), and therefore represents wages in utility terms. Together these equations yield

$$\gamma_S L_{S-1,m}=\left(1-\gamma_S\right)\gamma_{S-1}L_{S,m}.$$

$$\gamma_1 L^*_{0,m} = (1-\gamma_1) L^*_{1,m}, \tag{A.73}$$

$$\gamma_{s+1} L^*_{s,m} = (1-\gamma_{s+1})\gamma_s L^*_{s+1,m}, \quad s \in \{1,2,...,S-1\}. \tag{A.74}$$

Together with (A.72), these recursive equations of labor quantities imply

$$L^*_{S,m} = \gamma_S \left(L - \sum_{s=0}^{S} N_s r_s - k \sum_{s=1}^{S} \eta_s N_{s-1} N_s \right). \tag{A.75}$$

That is, optimal manufacturing employment of labor by final good producers is a fraction γ_S of aggregate labor employed in manufacturing. Using this optimal choice of employment in tier S, the recursive equations (A.73) and (A.74) provide solutions to optimal manufacturing employment in all the remaining tiers. These employment levels do not depend on the elasticities of substitution σ_s; they only depend on the Cobb-Douglas labor shares $\{\gamma_s\}$ and the labor available for manufacturing. If γ_s is the same in all tiers, i.e., $\gamma_s = \gamma$ for all s, these employment levels are lower in s than in $s+1$ for all $s \in \{1,2,...,S-1\}$. In tier 0 it is lower than in tier 1 if and only if $\gamma > 1/2$.

Next, consider the optimal choice of resilience, using the recursive structure of the first-order conditions (A.73) and (A.74). The first-order conditions for r_S and $L_{S,m}$ are

$$\begin{aligned} \frac{\varepsilon - \gamma_S(\varepsilon-1)}{\varepsilon-1} \frac{\phi'_S(r_S)}{\phi_S(r_S)} W &= \omega \left[N_S + \frac{1}{\gamma_{S-1}} L_{S-1,m} \frac{\phi'_S(r_S)}{\phi_S(r_S)} \right], \\ \gamma_S \frac{1}{L_{S,m}} W &= \omega, \end{aligned}$$

where ω is the Lagrangian multiplier of the labor constraint. Using (A.74) and (A.75) these conditions yield[19]

$$\frac{1}{\varepsilon-1} \frac{\phi'_S(r^*_S) r^*_S}{\phi_S(r^*_S)} = \frac{N_S r^*_S}{L - \sum_{s=0}^{S} N_s r^*_S - k \sum_{s=1}^{S} \eta^*_s N_{s-1} N_s}. \tag{A.76}$$

Optimizing with respect to r_{S-1}, using $\omega = \gamma_S W / L^*_{S,m}$, delivers the first-order condition

$$\frac{1-\gamma_S}{\alpha_S} \frac{\phi'_{S-1}(r^*_{S-1})}{\phi_{S-1}(r^*_{S-1})} = \frac{\gamma_S}{L^*_{S,m}} \left[N_{S-1} + \left(L^*_{S-1,m} + \frac{1}{\gamma_{S-2}} L^*_{S-2,m} \right) \frac{\phi'_{S-1}(r^*_{S-1})}{\phi_{S-1}(r^*_{S-1})} \right].$$

Using (A.74) and (A.75), this yields

$$\frac{(1-\gamma_S)(1-\alpha_S)}{\alpha_S} \frac{\phi'_{S-1}(r^*_{S-1})}{\phi_{S-1}(r^*_{S-1})} = \frac{\gamma_S}{L^*_{S,m}} N_{S-1}.$$

Continuing this analysis for optimal resilience in tiers $s < S-1$ we conclude that the first-order

[19] We use here the optimal values of $\{\eta_s\}$, $\{\eta^*_s\}$, although the conditions for the optimal investment in resilience apply to arbitrary $\{\eta_s\}$.

conditions for r_s, $s \leq S-1$, satisfy

$$\frac{\Gamma_{s+1}^{S}(1-\alpha_{s+1})}{\alpha_{s+1}} \frac{\phi_s'(r_s^*)r_s^*}{\phi_s(r_s^*)} = \frac{N_s r_s^*}{L - \sum_{s=0}^{S} N_s r_s^* - k \sum_{s=1}^{S} \eta_s^* N_{s-1} N_s}, \quad s \in \{0, 2, ..., S-1\}. \quad \text{(A.77)}$$

Next we examine the optimal number of links, $\{\eta_s\}_{s=1}^{S}$. Using $\omega = \gamma_S W / L_{S,m}^*$ and (A.74), the first-order condition with respect to η_S yields

$$\frac{(1-\gamma_S)(1-\alpha_S)}{\alpha_S} = \frac{k N_{S-1} N_S \eta_S^*}{L - \sum_{s=0}^{S} N_s r_s^* - k \sum_{s=1}^{S} \eta_s^* N_{s-1} N_s}. \quad \text{(A.78)}$$

Continuing with the first order conditions for η_s, $s < S$, we obtain

$$\frac{\Gamma_{s+1}^{S}(1-\alpha_{s+1})}{\alpha_{s+1}} = \frac{k N_s N_{s+1} \eta_{s+1}^*}{L - \sum_{s=0}^{S} N_s r_s^* - k \sum_{s=1}^{S} \eta_s^* N_{s-1} N_s}, \quad s \in \{0, 1, ..., S-1\}. \quad \text{(A.79)}$$

A5 Optimal Policies

In this section, we derive optimal policies. For this, it will be useful to derive expressions for the equilibrium ex-post payoffs at each tier s in general equilibrium. Combining (A.48) and (A.49) with the labor market clearing condition (A.70) yields

$$v_s = \frac{L_m}{\phi_s(r_s) N_s} \frac{\Gamma_{s+1}^{S-1}(1-\beta_{s+1}) \frac{1-\alpha_{s+1}}{\alpha_{s+1}} \frac{1}{\tau_s \prod_{j=s+1}^{S-1} \tau_j B_j}}{\frac{1}{\tau_0} \frac{\Gamma_1^{S-1}}{\prod_{j=1}^{S-1} \tau_j B_j} + \sum_{j=1}^{S-1} \frac{\gamma_j}{1-\gamma_j} \Gamma_j^{S-1} \prod_{z=j}^{S-1} \frac{1}{\tau_z B_z} + \frac{\gamma_S}{1-\gamma_S}}, \quad \text{for } s \in \{0, 1, ..., S-1\}, \quad \text{(A.80)}$$

and

$$v_S = \frac{L_m}{\phi_S(r_S) N_S} \frac{\frac{\varepsilon - \gamma_S(\varepsilon-1)}{(1-\gamma_S)(\varepsilon-1)} - \frac{1-\beta_S+\beta_S\alpha_S}{\alpha_S}}{\frac{1}{\tau_0} \frac{\Gamma_1^{S-1}}{\prod_{j=1}^{S-1} \tau_j B_j} + \sum_{j=1}^{S-1} \frac{\gamma_j}{1-\gamma_j} \Gamma_j^{S-1} \prod_{z=j}^{S-1} \frac{1}{\tau_z B_z} + \frac{\gamma_S}{1-\gamma_S}}, \quad \text{(A.81)}$$

where

$$\prod_{j=S}^{S-1} \tau_j B_j := 1 \quad \text{and} \quad \Gamma_S^{S-1} := 1,$$

$$L_m := L - \sum_{s=0}^{S} N_s r_s - k \sum_{s=1}^{S} \eta_s N_{s-1} N_s.$$

Note that in these equations r_s represents the ex-ante symmetrically chosen resilience levels of all firms in tier s, $s \in \{0, 1, ..., S\}$, which determines the fraction of active firms in the tier. Moreover, (A.80)-(A.81) hold in the market equilibrium for arbitrary policies $\{\tau_s\}_{s=0}^{S-1}$, not only for optimal policies, and that Assumption 1 ensures $v_S > 0$. Finally note that if $\tau_0 = 1$ and $\tau_s B_s = 1$ for

$s \in \{1, 2, ..., S-1\}$, then (A.80) and (A.81) imply

$$v_s = \frac{L_m}{\phi_s(r_s)N_s}\Gamma^S_{s+1}(1-\beta_{s+1})\frac{1-\alpha_{s+1}}{\alpha_{s+1}}B_s, \quad \text{for } s \in \{0, 1, ..., S-1\},$$

and

$$v_S = \frac{L_m}{\phi_S(r_S)N_S}\left[\frac{\varepsilon - \gamma_S(\varepsilon-1)}{(1-\gamma_S)(\varepsilon-1)} - \frac{1-\beta_S+\beta_S\alpha_S}{\alpha_S}\right].$$

A5.1 First-Best Policies

We start with the case in which the government can implement first-best subsidies to input purchases, $\{\tau^*_s\}^{S-1}_{s=0}$, first-best subsidies to investment in resilience, $\{\theta^*_s\}^S_{s=0}$, and first-best subsidies to investment in link formation, $\{\vartheta^*_s\}^S_{s=1}$. Combining (A.73) and (A.74) (the optimal allocation of labor to tiers 0 and 1, respectively) with (A.68) and (A.69) (the equilibrium demand for labor in tiers 1 and 0, respectively), using (A.24) for C_{m_s}, yields

$$\tau^*_0 = 1.$$

Repeating this procedure for $s \in \{1, ..., S-1\}$ with the aid of (A.68) and (A.74), we obtain

$$\tau^*_s = \frac{1}{B_s} = \frac{1}{\gamma_s + (1-\gamma_s)\left[(1-\beta_s)\frac{\sigma_s}{\sigma_s-1} + \beta_s\right]} < 1 \quad \text{for } s \in \{1, ..., S-1\}.$$

These imply no intervention in purchases of inputs from tier 0 and subsidies to purchases of inputs (i.e., $\tau^*_s < 1$) from all the other tiers. In tiers $s > 0$ the purchase subsidy is larger the smaller is the bargaining weight β_s, the smaller is the elasticity of substitution σ_s, and the smaller is the labor share γ_s. If the labor share and the bargaining weight are the same in all tiers, Assumption 1 implies that the optimal subsidy is increasing the closer a tier is to the final stage of production.

Next, consider the first-best policies for investments in resilience and link formation. First, examine a tier $s \neq S$. Combining (A.60)-(A.61) with (A.80), evaluated at the optimal policies $\{\tau^*_s\}^{S-1}_{s=0}$ (which imply $B_s\tau^*_s =$ for all s, using $B_0 := 1$), we obtain

$$\frac{\phi'_s(r^*_s)r^*_s}{\phi_s(r^*_s)}\frac{L_m}{N_s}\Gamma^S_{s+1}\frac{1-\beta_{s+1}}{\tau^*_s}\frac{1-\alpha_{s+1}}{\alpha_{s+1}} = \theta^*_s r_s.$$

Combining this with (A.77) then yields

$$\theta^*_s = \frac{1-\beta_{s+1}}{\tau^*_s} \quad \text{for } s \in \{0, ..., S-1\}. \tag{A.82}$$

Next combine (A.61) with (A.81) and evaluate the result at the optimal policies $\{\tau^*_s\}^{S-1}_{s=0}$ to obtain

$$\frac{\phi'_S(r^*_S)r^*_S}{\phi_S(r^*_S)}\frac{L_m}{N_S}(1-\gamma_S)\left[\frac{\varepsilon-\gamma_S(\varepsilon-1)}{(1-\gamma_S)(\varepsilon-1)} - \frac{1-\beta_S+\beta_S\alpha_S}{\alpha_S}\right] = \theta^*_S r_S.$$

Finally, combining this equation with (A.76) yields

$$\theta_S^* = 1 - \frac{(1-\beta_S)(\varepsilon-1)(1-\gamma_S)}{\sigma_S - 1} \in (0,1)\,. \tag{A.83}$$

This variable is between zero and one due to Assumption 1. It follows that investment in resilience is subsidized at the earliest and latest stages of production, but it may be subsidized or taxed, i.e., $\theta_s^* \lesseqgtr 1$, at intermediate stages of production. If the labor share and the bargaining weight are the same in all tiers, Assumption 1 implies that the optimal subsidy is decreasing the closer a tier is to the final stage of production. In other words, investment in resilience should be more subsidized (or less taxed) at earlier stages of production. Note that in this case θ_s^* is declining in the common value β, and $\theta_s^* \to \gamma + (1-\gamma)\frac{\sigma_s}{\sigma_s - 1} > 1$ for $\beta \to 0$ and $\theta_s^* \to 0 < 1$ for $\beta \to 1$. Therefore the optimal policy can be a tax or a subsidy and if there is a mix of both, the taxes are for tiers closer to the final stage of production while there are subsidies to tiers closer to the initial stage of production.

We next derive optimal policies for the formation of links. Recall that only firms in tiers $s \in \{1, ..., S\}$ make such decisions. From (A.61)-(A.63), the optimal policies satisfy

$$\frac{(1-\gamma_s)(\sigma_{s+1}-1)}{\sigma_s - 1} = \frac{\phi_s'(r_s^*) r_s^*}{\phi_s(r_s^*)} \frac{\vartheta_s^* k \eta_s^* N_{s-1}}{\theta_s^* r_s^*}, \quad s \in \{1, 2, ..., S-1\}\,,$$

$$\frac{(1-\gamma_S)(1-\alpha_S)(\varepsilon-1)}{\alpha_S} = \frac{\phi_S'(r_S^*) r_S^*}{\phi_S(r_S^*)} \frac{\vartheta_S^* k \eta_S^* N_{S-1}}{\theta_S^* r_S^*},$$

while (A.78)-(A.79) imply

$$\frac{(1-\gamma_s)(\sigma_{s+1}-1)}{\sigma_s - 1} = \frac{\phi_s'(r_s^*) r_s^*}{\phi_s(r_s^*)} \frac{k \eta_s^* N_{s-1}}{r_s^*}, \quad s \in \{1, ..., S-1\}\,,$$

$$\frac{(1-\gamma_S)(1-\alpha_S)(\varepsilon-1)}{\alpha_S} = \frac{\phi_S'(r_S^*) r_S^*}{\phi_S(r_S^*)} \frac{k \eta_S^* N_{S-1}}{r_S^*}.$$

It therefore follows that

$$\vartheta_s^* = \theta_s^*, \quad \text{for } s \in \{1, 2, ..., S\}\,. \tag{A.84}$$

In summary, to support the optimal allocation, the policy maker has to subsidize investments in link formation at the same rate as investments in resilience in the same tier.

A5.2 Second-Best Policies

We consider second-best policies as policies that subsidize or tax investments in resilience or link formation, with no subsidies or taxes on purchases of intermediate inputs. To this end the analysis proceeds in the footsteps of Section A5.1, assuming $\tau_s = 1$ for all s. It turns out that in this case the social planner's first-order conditions for the optimal choices of r_s and η_s are the same as the first-order conditions for the first-best allocation. And moreover, the individual firms' first-order conditions for the choices of r_s and η_s are the same in both cases.

First, consider the second-best optimal resilience policies $\{\theta_s^\circ\}_{s=0}^S$, where $1 - \theta_s^\circ$ is the rate of subsidy to investment in resilience in tier s. Combining the first-order conditions for the choices of resilience levels by firms, (A.60)-(A.61), with the equilibrium ex-post payoffs (A.80), we obtain

$$\frac{\phi_s'(r_s^\circ)r_s^\circ}{\phi_s(r_s^\circ)}\frac{L_m^\circ}{N_s}\Gamma_{s+1}^S(1-\beta_{s+1})\frac{1}{J\prod_{j=s+1}^{S-1}B_j}\frac{1-\alpha_{s+1}}{\alpha_{s+1}} = \theta_s^\circ r_s^\circ, \quad s\in\{0,...,S-1\},$$

where r_s° is the second-best optimal investment in resilience, L_m° is labor employment in manufacturing in the second-best optimal allocation, and

$$J := \frac{\Gamma_1^S}{\prod_{j=1}^{S-1}B_j} + \sum_{j=1}^{S-1}\frac{\gamma_j}{1-\gamma_j}\Gamma_j^S\prod_{z=j}^{S-1}\frac{1}{B_z} + \gamma_S.$$

Since $B_j > 1$ for $j \in \{1,2,...,S-1\}$, this implies

$$J < \Gamma_1^S + \sum_{j=1}^{S-1}\frac{\gamma_j}{1-\gamma_j}\Gamma_j^S + \gamma_S = 1.$$

In the general case B_j is replaced with $\tau_j B_j$ (see (A.80)), which becomes B_j in the second-best case and $\tau_j B_j = 1$ in the first-best case.[20] Combining this equation with the policy maker's first-order condition (A.77), this time applied to the second-best choices, yields

$$\theta_s^\circ = \frac{1-\beta_{s+1}}{J\prod_{j=s+1}^{S-1}B_j} \quad \text{for } s\in\{0,1,...,S-1\}. \tag{A.85}$$

Next, (A.61) together with (A.81) imply

$$\frac{\phi_S'(r_S^\circ)r_S^\circ}{\phi_S(r_S^\circ)}\frac{L_m^\circ}{N_S}(1-\gamma_S)\frac{1}{J}\left[\frac{\varepsilon-\gamma_S(\varepsilon-1)}{(1-\gamma_S)(\varepsilon-1)} - \frac{1-\beta_S+\beta_S\alpha_S}{\alpha_S}\right] = \theta_S^\circ r_S^\circ,$$

and in combination with (A.76)

$$\theta_S^\circ = \frac{1}{J}\left[1-\frac{(1-\beta_S)(\varepsilon-1)(1-\gamma_S)}{\sigma_S-1}\right]. \tag{A.86}$$

Unlike the first-best policies, these second-best policies depend not only on technologies in adjacent tiers, but rather on technologies in all tiers.

Now note that (A.83) and (A.86) imply

$$\frac{\theta_S^\circ}{\theta_S^*} = \frac{1}{J} > 1.$$

It follows that the subsidy to investment in resilience to final good producers is smaller in the second-best than in the first-best allocation. In other tiers (A.82) and (A.85) imply

$$\frac{\theta_s^\circ}{\theta_s^*} = \frac{1}{J\prod_{j=s}^{S-1}B_j} \quad \text{for } s\in\{0,1,...,S-1\}.$$

[20] We can obtained from these equations the formulas for the first-best case, by replacing B_j with $\tau_j B_j = 1$.

Therefore the ratio $\theta_s^\circ/\theta_s^*$ is larger the closer tier s is to the production of final goods (because $B_j > 1$), which means that the second-best subsidy to investment in resilience relative to the first-best subsidy is declining the closer tier s is to the production of final goods. But, as we have argued in the previous section, it may be optimal to tax investment in resilience in some tiers in order to attain the first best (i.e., $\theta_s^* > 1$ is possible). And indeed, it may be optimal to tax resilience in some tiers in the second best as well (i.e., $\theta_s^\circ > 1$ is possible). From (A.85) we see that the latter possibility is more likely to arise for tiers closer to the production of final goods. Finally note that (A.85) implies

$$\frac{\theta_s^\circ}{\theta_{s-1}^\circ} = \frac{1-\beta_{s+1}}{1-\beta_s} B_s \quad \text{for } s \in \{1, 2, ..., S-1\}.$$

Therefore $\theta_s^\circ > \theta_{s-1}^\circ$ if $\beta_{s+1} \leq \beta_s$. In other words, if the bargaining power of sellers does not rise as we get closer in the supply chain to the production of final goods, the second-best subsidy to investment in resilience declines.

Next, consider second-best subsidies to link formation $\{\vartheta_s^\circ\}_{s=1}^S$. Following the procedure we used in Section A5.1 to derive the first-best policies, we find now that the second-best subsidy to investment in link formation is the same as the second-best subsidy to investment in resilience in the same tier. That is,

$$\vartheta_s^\circ = \theta_s^\circ \quad \text{for } s \in \{1, ..., S\}.$$

A5.3 Alternative Derivation of $\{\tau_s^*\}_{s=0}^{S-1}$

In this section, we discuss the approach followed in Section 3 of the main body of the paper for the derivation of the first-best policies $\{\tau_s^*\}_{s=0}^{S-1}$. The planner's first-order conditions for the pairs $\{l_s, m_{s-1}\}$ are (using $\gamma_0 = 1$)

$$\rho_0 = \omega a, \tag{A.87}$$

$$\rho_s \left(l_s^*\right)^{\gamma_s} \left[\phi(r_{s-1}^*)\eta_s^* N_{s-1} \left(m_{s-1}^*\right)^{\alpha_s}\right]^{\frac{1-\gamma_s}{\alpha_s}} \gamma_s = \omega l_s^*, \quad s = 0, 1, ..., S-1, \tag{A.88}$$

$$\begin{aligned} & \rho_s \left(l_s^*\right)^{\gamma_s} \left[\phi(r_{s-1}^*)\eta_s^* N_{s-1} \left(m_{s-1}^*\right)^{\alpha_s}\right]^{\frac{1-\gamma_s}{\alpha_s}} (1-\gamma_s) \\ = \; & \rho_{s-1} m_{s-1}^* \phi(r_{s-1}^*)\eta_s^* N_{s-1}, \quad s = 1, 2, ..., S-2. \end{aligned} \tag{A.89}$$

Condition (A.87) equates the shadow price of tier 0 intermediates, ρ_0, to the marginal cost of production, where ω is the shadow price of labor. More generally, the shadow price of intermediates in tier s is ρ_s. Condition (A.88) states that the value of labor employed in manufacturing tier s inputs equals the fraction γ_s of the value of their output. Condition (A.89) states that the value of intermediate inputs of tier $s-1$ equals fraction $1-\gamma_s$ of the value of output in tier s. The first-order conditions for $\{l_S, m_{S-1}\}$ are

$$\rho_S = \left[\phi(r_S^*) N_S\right]^{\frac{1}{\varepsilon-1}}, \tag{A.90}$$

$$\rho_S \left(l_S^*\right)^{\gamma_S} \left[\phi(r_{S-1}^*)\eta_S^* N_{S-1} \left(m_{S-1}^*\right)^{\alpha_S}\right]^{\frac{1-\gamma_S}{\alpha_S}} \gamma_S = \omega l_S^*, \tag{A.91}$$

$$\left(1-\gamma_S\right) \left(l_S^*\right)^{\gamma_S} \left[\phi(r_{S-1}^*)\eta_S^* N_{S-1} \left(m_{S-1}^*\right)^{\alpha_S}\right]^{\frac{1-\gamma_S}{\alpha_S}} \rho_S = \mu_{S-1}\phi(r_{S-1}^*)\eta_S^* N_{S-1} m_{S-1}^*. \tag{A.92}$$

We now provide details for the arguments in Section 3. Recall that inverting the production function (A.1), we obtain

$$\frac{l_1^*}{m_0^*} = \left[n_1^{d,*} m_1^*\right]^{\frac{1}{\gamma_1}} \left[n_1^{u,*}\right]^{\frac{\gamma_1 - 1}{\alpha_1 \gamma_1}} \left[m_0^*\right]^{-\frac{1}{\gamma_1}}. \tag{A.93}$$

The asterisks represent variables in the first-best allocation. However, this relationship is technological and must hold also in the market equilibrium. The transaction size m_0 that maximizes the joint surplus of a tier 1 and a tier 0 supplier for given pre-determined downstream quantities m_s, and a policy τ_0, $s > 0$, is (see (A.5))

$$m_0 = \left(\frac{1-\gamma_1}{a\gamma_1\tau_0}\right)^{\gamma_1} \left[n_1^u\right]^{\frac{\gamma_1(1-\alpha_1)-1}{\alpha_1}} n_1^d m_1. \tag{A.94}$$

Together with (A.93) this implies that in the optimal allocation

$$\frac{l_1^*}{n_1^{u,*} m_0^*} = \frac{\tau_0^* a \gamma_1}{1-\gamma_1}.$$

Also note that (A.87)-(A.89) imply

$$\frac{l_1^*}{n_1^{u,*} m_0^*} = \frac{\gamma_1 a}{1-\gamma_1},$$

and the last two equations yield

$$\tau_0^* = 1.$$

Next, consider $s = 2$. For this tier the technological relationship is

$$\frac{l_2}{m_1} = \left[n_2^d m_2\right]^{\frac{1}{\gamma_2}} \left[n_2^u\right]^{\frac{\gamma_2 - 1}{\alpha_2 \gamma_2}} \left[m_1\right]^{-\frac{1}{\gamma_2}}.$$

In the market equilibrium, (A.33) and (A.38) imply

$$m_1 = \left[\frac{(1-\gamma_2)\gamma_1 C_{m_0}^{\frac{1-\gamma_1}{\gamma_1}}}{\gamma_2 \tau_1 B_1}\right]^{\gamma_2} \left[n_2^u\right]^{\frac{\gamma_2(1-\alpha_2)-1}{\alpha_2}} \left[n_1^u\right]^{\frac{\gamma_2(1-\gamma_1)(1-\alpha_1)}{\alpha_1}} n_2^d m_2.$$

Together with the above technological relationship this yields

$$\frac{l_2}{n_2^u m_1} = \left(n_1^u\right)^{-\frac{(1-\gamma_1)(1-\alpha_1)}{\alpha_1}} \frac{\gamma_2 \tau_1}{1-\gamma_2} \frac{1}{\gamma_1} B_1 \left[C_{m_0}\right]^{\frac{\gamma_1 - 1}{\gamma_1}}, \tag{A.95}$$

while (A.88) and (A.89) imply

$$\frac{\omega l_2^*}{\rho_1 n_2^{u,*} m_1^*} = \frac{\gamma_2}{1-\gamma_2}.$$

From (A.87) and (A.89), we we also have

$$(1-\gamma_1)\frac{\rho_1}{\omega a} = \frac{n_1^{u,*} m_0^*}{n_1^{d,*} m_1^*},$$

and the last two equations yield

$$\frac{l_2^*}{n_2^{u,*} m_1^*} = \frac{\gamma_2}{1-\gamma_2} \frac{a m_0^* n_1^{u,*}}{n_1^{d,*} m_1^* (1-\gamma_1)}.$$

Using (A.94) then delivers

$$\frac{l_2^*}{n_2^{u,*} m_1^*} = \frac{\gamma_2}{1-\gamma_2} a^{1-\gamma_1} \left(\frac{1-\gamma_1}{\gamma_1}\right)^{\gamma_1} \left[n_1^{u,*}\right]^{-\frac{(1-\gamma_1)(1-\alpha_1)}{\alpha_1}} \frac{1}{1-\gamma_1}.$$

Comparing this equation to (A.95) evaluated at the optimal allocation, using (A.6), then implies

$$\tau_1^* = \frac{1}{\gamma_1 + (1-\gamma_1)\left[\frac{1-\beta_1+\beta_1\alpha_1}{\alpha_1}\right]} = \frac{1}{B_1}.$$

A5.3.1 Extension to Tiers $s > 1$

We can extend this analysis to tiers $s \in \{2, 3, ..., S-1\}$. Using (A.88) and (A.89), the social planner sets

$$\frac{\omega l_s^*}{\rho_{s-1} n_s^{u,*} m_{s-1}^*} = \frac{\gamma_s}{1-\gamma_s}. \tag{A.96}$$

To obtain a similar equation from the bargaining game, we start with the technological relationship

$$\frac{l_s}{m_{s-1}} = \left[n_s^d m_s\right]^{\frac{1}{\gamma_s}} \left[n_s^u\right]^{\frac{\gamma_s - 1}{\alpha_s \gamma_s}} \left[m_{s-1}\right]^{-\frac{1}{\gamma_s}}. \tag{A.97}$$

Using (A.38) from the bargaining equilibrium, i.e.,

$$m_{s-1} = C_{m_{s-1}} \left[n_s^u\right]^{\frac{\gamma_s(1-\alpha_s)-1}{\alpha_s}} \left[\prod_{j=1}^{s-1} \left[n_j^u\right]^{\frac{\gamma_s \Gamma_j^{s-1}(1-\alpha_j)}{\alpha_j}}\right] n_s^d m_s, \tag{A.98}$$

together with (A.33), the technological relationship implies

$$\frac{l_s^*}{n_s^{u,*} m_{s-1}^*} = \frac{\gamma_s}{1-\gamma_s} \tau_{s-1}^* \frac{1}{\gamma_{s-1}} B_{s-1} \left(C_{m_{s-2}}^*\right)^{\frac{\gamma_{s-1}-1}{\gamma_{s-1}}} \prod_{j=1}^{s-1} \left[n_j^{u,*}\right]^{\frac{-\Gamma_j^{s-1}(1-\alpha_j)}{\alpha_j}}, \tag{A.99}$$

or

$$\frac{l_s}{n_s^u m_{s-1}} = \frac{\gamma_s}{1-\gamma_s}\tau_{s-1}^* q_{s-1}^*,$$

where

$$q_{s-1}^* := \frac{1}{\gamma_{s-1}} B_{s-1}\left(C_{m_{s-2}}^*\right)^{\frac{\gamma_{s-1}-1}{\gamma_{s-1}}} \prod_{j=1}^{s-1}\left[n_j^{u,*}\right]^{\frac{-\Gamma_j^{s-1}(1-\alpha_j)}{\alpha_j}}$$

is the marginal cost of production in the first-best allocation. Using again (A.89), we obtain

$$\frac{\rho_s}{\rho_{s-1}} = \frac{1}{1-\gamma_s}\frac{n_s^{u,*} m_{s-1}^*}{n_s^{d,*} m_s^*}. \tag{A.100}$$

In the optimal allocation (A.98) implies

$$m_{s-1}^* = C_{m_{s-1}}^*\left[n_s^{u,*}\right]^{\frac{\gamma_s(1-\alpha_s)-1}{\alpha_s}}\left[\prod_{j=1}^{s-1}\left[n_j^{u,*}\right]^{\frac{\gamma_s\Gamma_j^{s-1}(1-\alpha_j)}{\alpha_j}}\right] n_s^{d,*} m_s^*,$$

which together with (A.100) yield

$$\frac{\rho_s}{\rho_{s-1}} = \frac{1}{1-\gamma_s} C_{m_{s-1}}^*\left[n_s^{u,*}\right]^{-\frac{(1-\gamma_s)(1-\alpha_s)}{\alpha_s}} \prod_{j=1}^{s-1}\left[n_j^{u,*}\right]^{\frac{\gamma_s\Gamma_j^{s-1}(1-\alpha_j)}{\alpha_j}}.$$

But since $\rho_0 = \omega a$ (see (A.87)),

$$\frac{\rho_{s-1}}{\omega} = \frac{\rho_{s-1}}{\rho_{s-2}}\cdots\frac{\rho_1}{\rho_0}a,$$

and therefore

$$\frac{\rho_{s-1}}{\omega} = \frac{1}{\Gamma_1^{s-1}} a\left[\prod_{j=0}^{s-2} C_{m_j}^*\right]\prod_{j=1}^{s-1}\left[n_j^{u,*}\right]^{-\frac{\Gamma_j^{s-1}(1-\alpha_j)}{\alpha_j}}.$$

Combining (A.96) and (A.99) implies

$$\frac{\rho_{s-1}}{\omega} = \tau_{s-1}^*\frac{1}{\gamma_{s-1}} B_{s-1}\left(C_{m_{s-2}}^*\right)^{\frac{\gamma_{s-1}-1}{\gamma_{s-1}}} \prod_{j=1}^{s-1}\left[n_j^{u,*}\right]^{\frac{-\Gamma_j^{s-1}(1-\alpha_j)}{\alpha_j}},$$

which in combination with the previous equation yields

$$\tau_{s-1}^* B_{s-1} = \frac{a\gamma_{s-1}}{\Gamma_1^{s-1}\left(C_{m_{s-2}}^*\right)^{\frac{\gamma_{s-1}-1}{\gamma_{s-1}}}}\left[\prod_{j=0}^{s-2} C_{m_j}^*\right]. \tag{A.101}$$

Now recall from (A.33) that

$$C^*_{m_s} = \left[\frac{\left(1-\gamma_{s+1}\right)\gamma_s\left(C^*_{m_{s-1}}\right)^{\frac{1-\gamma_s}{\gamma_s}}}{\gamma_{s+1}\tau^*_s B_s}\right]^{\gamma_{s+1}}, \quad s \in \{1,2,...,S-2\},$$

and from (A.6) that

$$C^*_{m_0} = \left(\frac{1-\gamma_1}{\gamma_1 a \tau^*_0}\right)^{\gamma_1}.$$

Using $\tau^*_0 = 1$, and substituting these equations for $s = 2$ into (A.101) yields

$$\tau^*_1 B_1 = 1,$$

which is what we showed above. Repeat this process for $s = 3$, accounting for $\tau^*_0 = 1$ and $\tau^*_1 B_1 = 1$, to obtain

$$\tau^*_2 B_2 = 1.$$

And continuing for $s = 4, 5, ..., S$, we conclude that

$$\tau^*_{s-1} B_{s-1} = 1, \quad \text{for } s \in \{2,3,...,S\}.$$

In other words,

$$\tau^*_{s-1} = \frac{1}{B_{s-1}} \quad \text{for } s \in \{2,3,...,S\}.$$

A6 The Markup Factor

We derive in this section the equations for Section 2.4 in the main text, which are used to interpret the markup factor. To this end we define the unit cost variable

$$q_s := \frac{\tau_{s-1} n^u_s t_{s-1} + l_s}{n^d_s m_s},$$

where the numerator on the right-hand side represents intermediate input and labor costs of a firm in tier s and the denominator represents its output volume. From the technological relationship (A.97) we obtain

$$l_s = \left(n^d_s m_s\right)^{\frac{1}{\gamma_s}} \left(n^u_s\right)^{\frac{\gamma_s - 1}{\alpha_s \gamma_s}} \left(m_{s-1}\right)^{\frac{\gamma_s - 1}{\gamma_s}}.$$

Substituting the recursive equation (A.38) into this relationship yields

$$l_s = n^d_s m_s C_{m_{s-1}}^{\frac{\gamma_s - 1}{\gamma_s}} \prod_{j=1}^{s} \left[n^u_j\right]^{\frac{-\Gamma^s_j (1-\alpha_j)}{\alpha_j}}.$$

Next, we obtain from (A.39)

$$\tau_{s-1} n_s^u t_{s-1} = n_s^d m_s \tau_{s-1} C_{t_{s-1}} \prod_{j=1}^{s} \left[n_j^u\right]^{\frac{-\Gamma_j^s(1-\alpha_j)}{\alpha_j}}, \quad \text{for } s \in \{1, ..., S\}.$$

Therefore, this equation together with the equations for l_s and q_s yield

$$q_s := \left(C_{m_{s-1}}^{\frac{\gamma_s - 1}{\gamma_s}} + \tau_{s-1} C_{t_{s-1}} \right) \prod_{j=1}^{s} \left[n_j^u\right]^{\frac{-\Gamma_j^s(1-\alpha_j)}{\alpha_j}}. \tag{A.102}$$

Now recall (A.24) and (A.26), which are

$$C_{m_s} = \left[\frac{(1-\gamma_{s+1}) \gamma_s C_{m_{s-1}}^{\frac{1-\gamma_s}{\gamma_s}}}{\gamma_{s+1} \tau_s B_s} \right]^{\gamma_{s+1}}, \quad s \in \{1, 2, ..., S-2\}$$

and

$$C_{t_s} = C_{m_s} \frac{1}{\gamma_s} C_{m_{s-1}}^{\frac{\gamma_s - 1}{\gamma_s}} B_s \mu_{s+1}, \quad s \in \{1, 2, ..., S-1\},$$

respectively. They imply

$$\begin{aligned}
C_{m_{s-1}}^{\frac{\gamma_s - 1}{\gamma_s}} + \tau_{s-1} C_{t_{s-1}} &= C_{m_{s-1}} \left[C_{m_{s-1}}^{-\frac{1}{\gamma_s}} + \frac{\tau_{s-1}}{\gamma_{s-1}} C_{m_{s-2}}^{\frac{\gamma_{s-1}-1}{\gamma_{s-1}}} B_{s-1} \mu_s \right] \\
&= C_{m_{s-1}} \left[\frac{\gamma_s}{1-\gamma_s} \frac{\tau_{s-1}}{\gamma_{s-1}} B_{s-1} C_{m_{s-2}}^{\frac{\gamma_{s-1}-1}{\gamma_{s-1}}} + \frac{\tau_{s-1}}{\gamma_{s-1}} C_{m_{s-2}}^{\frac{\gamma_{s-1}-1}{\gamma_{s-1}}} B_{s-1} \mu_s \right] \\
&= C_{m_{s-1}} B_{s-1} C_{m_{s-2}}^{\frac{\gamma_{s-1}-1}{\gamma_{s-1}}} \frac{\tau_{s-1}}{\gamma_{s-1}} \frac{\gamma_s}{1-\gamma_s} \left[\gamma_s + (1-\gamma_s)\mu_s\right] \frac{1}{\gamma_s} \\
&= C_{m_{s-1}}^{\frac{\gamma_s - 1}{\gamma_s}} \frac{B_s}{\gamma_s},
\end{aligned}$$

where the last equality follows from using the expression of $C_{m_{s-1}}$ from (A.24). Together with (A.102), this yields

$$q_s = \frac{B_s}{\gamma_s} C_{m_{s-1}}^{\frac{\gamma_s - 1}{\gamma_s}} \prod_{j=1}^{s} \left[n_j^u\right]^{\frac{-\Gamma_j^s(1-\alpha_j)}{\alpha_j}}. \tag{A.103}$$

Next note that, using again the expression of $C_{m_{s-1}}$ from (A.24),

$$\begin{aligned}
\frac{B_s}{\gamma_s} C_{m_{s-1}}^{\frac{\gamma_s - 1}{\gamma_s}} &= \frac{B_s}{\gamma_s} C_{m_{s-1}} \left[\frac{\gamma_s}{1-\gamma_s} \frac{\tau_{s-1}}{\gamma_{s-1}} B_{s-1} C_{m_{s-2}}^{\frac{\gamma_{s-1}-1}{\gamma_{s-1}}} \right] \\
&= C_{m_{s-1}} \tau_{s-1} \frac{1}{1-\gamma_s} B_s \times \frac{B_{s-1}}{\gamma_{s-1}} C_{m_{s-2}}^{\frac{\gamma_{s-1}-1}{\gamma_{s-1}}}, \quad \text{for } s \in \{2, ..., S\}.
\end{aligned} \tag{A.104}$$

Using this recursion and (A.103), it follows that

$$q_s = q_{s-1}\left[n_s^u\right]^{\frac{-(1-\gamma_s)(1-\alpha_s)}{\alpha_s}}\left[\prod_{j=1}^{s-1}\left[n_j^u\right]^{\frac{\gamma_s\Gamma_j^{s-1}(1-\alpha_j)}{\alpha_j}}\right]C_{m_{s-1}}\tau_{s-1}\frac{1}{1-\gamma_s}B_s.$$

Together with (A.38) this implies

$$q_s = q_{s-1}n_s^u m_{s-1}\frac{1}{n_s^d m_s}\tau_{s-1}\frac{1}{1-\gamma_s}B_s,$$

or,

$$(1-\gamma_s)n_s^d q_s m_s = q_{s-1}n_s^u m_{s-1}\tau_{s-1}B_s.$$

This is equation (7) in the main text. Finally, note that dividing (A.11) by (A.10), using (A.26), implies

$$\frac{t_s}{m_s} = \frac{B_s}{\gamma_s}C_{m_{s-1}}^{\frac{\gamma_s-1}{\gamma_s}}\mu_{s+1}\prod_{j=1}^{s}\left[n_j^u\right]^{\frac{-\Gamma_j^s(1-\alpha_j)}{\alpha_j}}.$$

Together with (A.103), this yields

$$t_s = q_s\mu_{s+1}m_s,$$

which is the equation that follows (8) in the main text.