Homework 4.

1. Prove the following theorem. You may use of any the rules of inference that we have learned.

$$// \qquad (P \rightarrow Q) \vee (Q \rightarrow P)$$

2. Write out a full truth table following sentence. Highlight in some way (e.g., draw a circle around) the column under the major operator of the sentence.

$$-(P \vee R) \,\&\, (-Q \rightarrow (P \,\&\, R))$$

3. Determine whether the following arguments are valid. If an argument isn't valid, give a truth-assignment that witnesses this fact.

   (a) (1) $(P \rightarrow Q) \vee (Q \rightarrow R)$
   (2) $-R \rightarrow -(P \,\&\, Q) \qquad / \; Q \rightarrow -P$

   (b) (1) $(P \vee Q) \rightarrow (R \vee S)$
   (2) $P \leftrightarrow -(R \,\&\, S)$
   (3) $Q \leftrightarrow -(P \,\&\, R) \qquad / \; (S \,\&\, P) \rightarrow -(P \vee R)$

4. Determine whether each of the following sentences is consistent. If a sentence is consistent, give an assignment of truth values to its elementary sentences relative to which the sentence is true.

   (a) $(P \vee -Q) \rightarrow (P \leftrightarrow (Q \,\&\, R))$
   (b) $(-P \vee (-Q \rightarrow R)) \rightarrow ((P \,\&\, R) \rightarrow -Q)$

5. For each of the following pairs of sentences, determine whether the first sentence implies the second. If the implication fails to hold, give a truth-assignment that witnesses this fact.

   (a) $(P \,\&\, Q) \leftrightarrow (Q \,\&\, R)$ $\qquad$ $P \leftrightarrow Q$

   (b) $P \leftrightarrow (Q \vee R)$ $\qquad$ $-P \rightarrow (Q \leftrightarrow R)$

6. Show that for any sentences ϕ, ψ, the sentence $-(\phi \rightarrow \psi)$ is logically equivalent to the sentence $\phi \,\&\, -\psi$.

7. Is logical implication symmetric? That is, if ϕ implies ψ then does ψ imply ϕ? Justify your answer.

8. Translate the following into sentence logic form. Choose a (distinct) capital letter for each elementary component sentence, and clearly designate your choices. (You might have to paraphrase.)

   (a) Alice goes to law school only if she is admitted to Yale or Harvard.

   (b) Unless we reduce the incidence of child abuse, future crime rates will increase.

   (c) Plasma televisions are a technological marvel, but they are expensive.

   (d) A necessary condition for a successful business venture is good planning.

   (e) Ozone depletion in the atmosphere is a sufficient condition for increased cancer rates.

9. Is the sentence connective "It was true that . . . " truth-functional? Justify your answer.

10. Suppose that the sentence connective $\circ$ has the truth table given below.

P	Q	$P \circ Q$
T	T	F
T	F	T
F	T	T
F	F	F

Do "$\circ$" and "$-$" form a truth-functionally complete set of connectives? Justify your answer.