Homework 7.

1. Prove the following theorem of the predicate calculus.

   $/ \qquad (\exists x)(Fx \rightarrow (y)Fy)$

2. Grade the following "solution" to Problem 1.

1	(1) Fa	A
1	(2) $(y)Fy$	1 UI
	(3) $Fa \rightarrow (y)Fy$	1,2 CP
	(4) $(\exists x)(Fx \rightarrow (y)Fy)$	3 EI

3. Let $\mathcal{I}$ denote the interpretation given by:

   $$\textbf{Domain} = \{1, 2, 3, 4\}$$
   $$m \mapsto 2, \quad n \mapsto 2, \quad o \mapsto 4$$
   $$\text{Ext}(F) = \{1, 2, 3\}, \quad \text{Ext}(G) = \{4\}, \quad \text{Ext}(H) = \emptyset$$

   Determine the truth values of the following sentences relative to interpretation $\mathcal{I}$.

   (a) $(x)(Gx \vee Fx)$

   (b) $(x)(Hx \rightarrow Fx)$

   (c) $(x)((Fx \,\&\, Hx) \rightarrow Gx)$

   (d) $-(x)Fx \rightarrow (\exists x)(Fx \,\&\, Gx)$

   (e) $(\exists x)(Fx \rightarrow (y)Fy)$

   (f) $-Gn \,\&\, (\exists x)(Fx \,\&\, Go)$

4. Transform the following *propositional calculus* sentences into equivalent sentences in disjunctive normal form, showing all steps; then simplify the normal forms as much as possible.

   (a) $(-(Q \,\&\, -R) \rightarrow P) \,\&\, -(Q \vee S)$

   (b) $((-P \,\&\, -Q) \rightarrow R) \leftrightarrow -(Q \vee -R)$

5. Use the algorithm for pure monadic sentences (i.e., "algorithm C") to determine whether the following arguments are valid, showing all steps. If an argument is invalid, give an interpretation relative to which the premises are true and the conclusion is false.

   (a)

   (1) $(x)((Fx \vee Gx) \rightarrow Hx)$ $\quad / -(\exists x)(Fx \,\&\, -Hx)$

   (b)

   (1) $(\exists x)(Fx \rightarrow Gx)$

   (2) $(\exists x)Fx \vee (\exists x)Gx$ $\quad / (\exists x)Fx \rightarrow (\exists x)Gx$