Homework 8.

Solve any **three** of the following problems.

These problems require you to make arguments about interpretations. Your answer should be a rigorous, but informal, argument — informal, in the sense that your argument will be in prose, and not in the formal deductive system of the predicate calculus. Please resist the temptation to mention anything about proofs, or about rules of inference (especially in problem #2).

1. Let $\phi_1, \ldots, \phi_n$ and ψ be simple monadic sentences such that one of $\phi_1, \ldots, \phi_n$ is existential and ψ is existential. Show that if the instances of $\phi_1, \ldots, \phi_n$ with instantial name "a" truth-functionally imply the instance of ψ with instantial name "a", then $\phi_1, \ldots, \phi_n$ jointly imply ψ.
2. Let $\phi_1, \ldots, \phi_n$ and ψ be simple monadic sentences, with ψ universal. Show that $\phi_1, \ldots, \phi_n$ jointly imply ψ if and only if: either $\phi_1, \ldots, \phi_n$ are inconsistent, or the universal sentences among $\phi_1, \ldots, \phi_n$ jointly imply ψ.
3. Show that if a simple monadic sentence ϕ is true in some interpretation $\mathcal{I}$ of size n, then for every $m > n$, ϕ is true in some interpretation $\mathcal{J}$ of size m. (Hint: Existential sentences never become false by adding more elements to the domain. For universal sentences, add new elements that are "just like" old elements.)
4. Let ϕ be a pure monadic sentence in which only the predicate symbol F occurs. Show that if ϕ is consistent, then there is an interpretation whose domain has two elements in which ϕ is true.