Homework 9.

1. Paraphrase into quantificational notation. For (c)–(f), you may take the domain of discourse to be persons (so you don't have to add an extra predicate symbol for "x is a person").
   (a) There is a river in America that is longer than any river in Europe.
   (b) Not every river in America is longer than every river in Europe.
   (c) Nobody likes all of the people she knows.
   (d) Everyone knows someone to whom she is unknown.
   (e) There is someone who helps all those who help themselves.
   (f) There is someone who helps only those who help themselves.

2. Symbolize, taking the domain of discourse to be the class of persons, and using:
   $Sx \equiv x$ is a soprano. $Tx \equiv x$ is a tenor.
   $Lxy \equiv x$ is louder than y. $Rxy \equiv x$ respects y.
   (a) A soprano who respects all tenors fails to respect herself.
   (b) A tenor who is louder than all sopranos is respected by all sopranos.
   (c) No tenor who is louder than all sopranos respects any soprano.
   (d) A tenor who is louder than some soprano is also louder than some tenor.
   (e) There are sopranos who respect only those tenors who are louder than they.
   (f) If a tenor respects all sopranos who respect him, then that tenor is respected by all sopranos.

3. With domain of discourse and vocabulary as in the previous problem, translate into clear, idiomatic English:
   (a) $(\exists x)(\exists y)(Tx \,\&\, Rxy \,\&\, Sy) \rightarrow (\exists y)(Ty \,\&\, (x)(Sx \rightarrow Rxy))$
   (b) $(\exists x)(Tx \,\&\, (y)(Sy \rightarrow ((\exists z)(Tz \,\&\, Ryz) \rightarrow Ryx)))$

(c) $(x)(Sx \to ((y)(Sy \to Ryx) \to (y)(Ty \to Ryx)))$

4. Symbolize, taking the domain of discourse to be persons, and using:
   $Pxy \equiv x$ is a parent of y. $Mx \equiv x$ is male.
   $Ixy \equiv x$ is identical to y.
   (a) x is y's paternal grandfather.
   (b) x and y are sisters. (Caution: no one is her own sister!)
   (c) x is a nephew of y.
   (d) x is a half-sister of y.
   (e) x is a first cousin of y.

5. For each of the following sentences, find an interpretation with domain $\{1, 2, 3, 4\}$ and nonempty extension of "F" that makes the sentence true, and another such interpretation that makes the sentence false.
   (a) $(\exists x)(y)(Fyx \to Fyy)$
   (b) $(x)(y)(Fxy \to (\exists z)(Fxz \,\&\, Fyz))$
   (c) $(x)((y)(Fyx \to Fxy) \to (y)(Fxy \to Fyx))$
   (d) $(\exists x)(\exists y)(Fxy \,\&\, Fyx) \,\&\, (x)(y)((\exists z)(Fxz \,\&\, Fzy) \to Fxy)$

6. For each of the following pairs of sentences, give an interpretation that shows that the first sentence does not imply the second.

(a)	$(x)(\exists y)(Fxy \lor Fyx)$	$(x)(\exists y)Fxy \lor (x)(\exists y)Fyx$
(b)	$(\exists x)(y) - Fxy \,\&\, (\exists x)(y)Fxy$	$(x)((\exists y)Fxy \to (y)Fxy)$
(c)	$(x)(-Lx \to (\exists y)(Ly \,\&\, Ayx))$	$(x)(-Lx \to (y)(Ly \to Ayx))$
(d)	$(x)(\exists y)(Gxy \,\&\, -Gyx)$	$(x)((\exists y)Gxy \to (\exists y)Gyx)$

7. Prove the validity of the following arguments. (You may use any of the inference rules.)

   (a) (1) $(x)(((\exists y)Gxy \vee (\exists z)Gzx) \rightarrow Gxx)$
   $/ \ (x)(y)(Gxy \rightarrow (Gxx \,\&\, Gyy))$

   (b) (1) $(x)(y)(Gxy \rightarrow (Gxx \,\&\, Gyy))$
   $/ \ (x)(((\exists y)Gxy \vee (\exists z)Gzx) \rightarrow Gxx)$