Practice Midterm Exam.

1. Complete the following sentences.
   (a) An argument with premises $\phi_1, \ldots, \phi_n$ and conclusion ψ is valid if ...
   (b) A sentence ϕ is inconsistent if ...

2. Short answer: Explain, using words and/or pictures, the v-Elimination rule of inference, including how dependency numbers are tabulated.

3. Translate the following English sentences into sentence logic. Use the suggested letters for elementary sentences.
   (a) Fanny loves Edmund, but Edmund loves Miss Crawford. (F, E)
   (b) Fanny loves Mr. Crawford only if he helps her brother. (F, H)
   (c) If Fanny loves Mr. Crawford then Edmund is happy, unless Miss Crawford doesn't love him. (F, E, M)

4. Prove the validity of the following arguments. You may use any of the rules of inference that we have learned.
   (a) (1) $-P$
   (2) $-Q$ $\quad / \; -(P \vee Q)$

   (b) (1) $(P \rightarrow Q) \vee (P \rightarrow R) \quad / \; P \rightarrow (Q \vee R)$

5. Prove the following theorem. You may use any of the rules of inference that we have learned.

$$// \qquad P \leftrightarrow (P \,\&\, (Q \vee -Q))$$

6. Is the the following sentence a tautology, a contradiction, or a contingency? Justify your answer.

$$(-A \rightarrow A) \rightarrow (B \rightarrow (C \rightarrow (D \rightarrow (E \rightarrow A))))$$

7. True or false (justify your answer): There is an inconsistent sentence of the form $P \rightarrow \phi$ (where P is an elementary sentence, and ϕ is an arbitrary sentence).

8. Does sentence (a) imply sentence (b)? Justify your answer.
   (a) $(P \vee Q) \rightarrow (R \vee S)$
   (b) $(P \rightarrow R) \vee (P \rightarrow S)$

9. Is the English sentence connective "*... because ...*" truth-functional? (e.g., "The glass shattered because it was hit with a baseball.") Justify your answer.

10. Suppose that the sentence connective $\circ$ has the truth table given below:

P	Q	$P \circ Q$
T	T	F
T	F	F
F	T	F
F	F	F

    Find an expression using only $\&, -, P, Q$ that is equivalent to $P \circ Q$.

11. True or False (explain and justify your answer): There could be a correctly written proof with the following line fragments:

$$\begin{array}{ll} 1 & (1)\ (P \rightarrow Q) \rightarrow Q \\ \ldots & \\ 1 & (16)\ -P \rightarrow Q \end{array}$$