Sample Problem for Homework 8.

Problem: Let $\phi_1, \ldots, \phi_n$ and ψ be simple monadic sentences such that all of $\phi_1, \ldots, \phi_n$ are universal. Show that if the instances of $\phi_1, \ldots, \phi_n$ with instantial name a truth-functionally imply the instance of ψ with instantial name a, then $\phi_1, \ldots, \phi_n$ jointly imply ψ.

Solution: Let's use $\phi_1(a), \ldots, \phi_n(a), \psi(a)$ to denote the instances of $\phi_1, \ldots, \phi_n, \psi$ with instantial name a. That is, $\phi_1(a)$ results from taking the quantifier off of ϕ_1 and replacing the quantified variable throughout with a, etc.. Now suppose that $\phi_1(a), \ldots, \phi_n(a)$ truth-functionally imply $\psi(a)$. Then $\phi_1(a), \ldots, \phi_n(a), -\psi(a)$ are truth-functionally inconsistent. Either ψ is existential, or ψ is universal. In either case, Algorithm B would dictate that a consistency test of $\phi_1, \ldots, \phi_n, -\psi$ requires only one instance (with the same name) of all sentences. Thus, if $\phi_1, \ldots, \phi_n, -\psi$ are consistent then $\phi_1(a), \ldots, \phi_n(a), -\psi(a)$ are truth-functionally consistent. Since the latter are not truth-functionally consistent, $\phi_1, \ldots, \phi_n, -\psi$ are not consistent, and so $\phi_1, \ldots, \phi_n$ jointly imply ψ. QED