

PHI 201 Helpful Terms for Midterm

Note: These are some terms it's important to know. I DON'T guarantee that this list is exhaustive – that is, that these are the *only* terms it's important to know.

implies: implication is a relation between sentences. A sentence ϕ *implies* a sentence φ just in case there's no assignment that makes ϕ true and φ false. A set of sentences Δ *implies* a sentence ϕ just in case there's no assignment that makes *all* of the sentences in Δ true that makes ϕ false.

Fun Fact: a sentence ϕ implies a sentence φ if and only if the sentence ' $\phi \rightarrow \varphi$ ' is a tautology.

valid: validity is a property of arguments. An argument is *valid* just in case the premises together imply the conclusion.

sound: we're using 'soundness' in two ways. Soundness can be a property of arguments as well as a property of proof systems.

soundness of an argument: an argument is *sound* just in case it's valid and the premises are all true.

soundness of a proof system: The Soundness Theorem says that our proof system is *sound* in the following sense. If on some line n of a correctly written proof we have:

a,b,c... (n) ϕ some citation numbers, some rule

then the formulas on lines a,b,c... together imply ϕ .

Informally: everything we can prove corresponds to a valid argument.

tautology: a sentence is a *tautology* just in case there's no assignment that makes it false.

consistent: a sentence is *consistent* just in case there's at least one assignment that makes it true. So it's either a tautology or it's contingent.

contingent: a sentence is *contingent* just in case there's at least one assignment that makes it true, and there's at least one assignment that makes it false. So it's consistent, but not a tautology.

inconsistent: a sentence is inconsistent just in case it's not consistent. So no assignment makes it true.

equivalent: two sentences are *equivalent* just in case they have the same truth value under every

assignment.

Fun Fact: sentences ϕ and φ are equivalent if and only if the sentence ' $\phi \leftrightarrow \varphi$ ' is a tautology: each implies the other.

contrary: two sentences are *contrary* if there's no assignment that makes them both true.

Fun Fact: sentences ϕ and φ are contrary if and only if the sentence ' $\neg(\phi \ \& \ \varphi)$ ' is a tautology: ϕ implies $\neg\varphi$

subcontrary: two sentences are *subcontrary* if and only if there's no assignment that makes them both false

Fun Fact: sentences ϕ and φ are subcontrary if and only if the sentence ' $\phi \vee \varphi$ ' is a tautology: $\neg\phi$ implies φ

contradictory: two sentences are *contradictory* if and only if every assignment that makes one true makes the other false.

Fun Fact: sentences ϕ and φ are contradictory if and only if the sentence ' $\neg(\phi \leftrightarrow \varphi)$ ' is a tautology: ϕ implies $\neg\varphi$ and $\neg\phi$ implies φ

completeness: we're using 'completeness' in two ways. A set of connectives can be *truth-functionally complete*, and a proof system can be *complete*.

truth-functional completeness: truth functional completeness is a property of a set of connectives. A set of connectives is *truth-functionally complete* just in case, for every one of the 16 possible connectives listed on Lemmon p. 70, we can come up with some sentence whose only connectives are the connectives in that set that has the same truth table as that connective

proof system: The Completeness Theorem says that our proof system is *complete* in the following sense. Suppose $\alpha, \beta, \delta, \dots$ together imply ϕ . Then there will be a correctly written proof in our system whose final line is:

a,b,c... (n) ϕ some citation numbers, some rule

where a,b,c... are the line numbers of assumptions $\alpha, \beta, \delta, \dots$

Informally: we can prove every valid argument.

assignment: an *assignment* is a correlation of truth values with elementary sentences (P, Q, R, etc.). Every row on a truth table corresponds to a particular assignment.

truth-functional: a connective or operator is *truth-functional* just in case the truth value of a

sentence containing it as the major operator depends *only* on the truth value(s) of the sentence(s) it connects.

Suppose *BLAH* is an operator. So, if you prefix *BLAH* to any sentence, you get another sentence. If *BLAH* is truth-functional, then if *BLAH* ϕ is true, *BLAH* ϕ will be true as well, as long as ϕ and ϕ have the same truth value. And if *BLAH* ϕ is false, *BLAH* ϕ will be false as well, as long as ϕ and ϕ have the same truth value.

Analogously for two-place connectives.

theorem: A sentence is a theorem just in case we can prove it without any assumptions.

By the Soundness Theorem, every theorem is a tautology.

By the Completeness Theorem, every tautology is a theorem.

counterexample: A *counterexample* to an argument form is offered to demonstrate that that argument form is invalid. We have two kinds of counterexample.

informal counterexample: An *informal counterexample* to an argument form is an instance that has OBVIOUSLY true premises and an OBVIOUSLY false conclusion.

You can *make* the truth or falsity of a sentence obvious by *stipulating* the truth values of elementary sentences that make it up. For example,

‘David Gordon was born in NY and snow is white’

is obviously true given the stipulation that David Gordon was born in NY, and *only* given that stipulation is it obviously true.

formal counterexample: a *formal counterexample* to an argument form is an assignment that makes the premises all true and the conclusion false.