Practice Final Exam — Key PHI 201

Short Answer

1. To apply EE, we need three lines:

$D(i)$	$(i)\ (\exists x)Fx$	
j	$(j)\ Fa$	A
$D(k)$	$(k)\ C$	

   that satisfy the condition that "a" occurs neither in C nor in the set $\underline{D}(i)$ of dependencies. In this case, EE entitles us to write a line

$D(m)$	$(m)\ C$	i, j, k EE

   where $D(m) = D(i) \cup [D(k) - D(j)]$.

2. True. If $A_1, \ldots, A_n$ are inconsistent, then $A_1, \ldots, A_n \models P \,\&\, -P$ (since no valuation satisfies $A_1, \ldots, A_n$). By the completeness of the predicate calculus, $A_1, \ldots, A_n \vdash P \,\&\, -P$.

3. ...there is no interpretation that makes both A and B true.

4. In line 6, the dependency numbers are tabulated incorrectly. The dependencies on line 6 should be:
$$\underline{D}(1) \cup [\underline{D}(2) - \underline{D}(2)] \cup [\underline{D}(5) - \underline{D}(3)],$$
   which is $1, 2$.

5. Line 2 is not a contradiction, and so lines 1 and 2 are not candidates for RAA.

6. Problem 4: Line 6. Problem 5 has no "bad" lines, despite the fact that line 3 is not warranted by the basic inference rules.

Translation

1. Every man who has a son adores him.
$$(x)(y)[(Mx \,\&\, My \,\&\, Pxy) \rightarrow Axy]$$

2. Every man who has a daughter adores his daughter's mother.
$$(x)(y)(z)[(Mx \,\&\, -My \,\&\, Pxy \,\&\, -Mz \,\&\, Pzy) \rightarrow Axz]$$

3. Everybody adores their own grandchildren.
$$(x)(y)(z)[(Pxy \,\&\, Pyz) \rightarrow Axz]$$

4. Every woman adores her brothers' children.
$$(x)(y)(u)[(-Mx \,\&\, My \,\&\, (\exists z)(Mz \,\&\, Pzx \,\&\, Pzy) \,\&$$

$$(\exists w)(-Mw \,\&\, Pwx \,\&\, Pwy)) \rightarrow (Pyu \rightarrow Axu)]$$

5. No man adores children unless he has his own.

$$(x)[(Mx \,\&\, -(\exists z)Pxz) \rightarrow (y)((\exists t)Pty \rightarrow -Axy)]$$

6. Someone has at most three children.

$$(\exists x)(y)(z)(t)(w)[(Pxy \,\&\, Pxz \,\&\, Pxt \,\&\, Pxw) \rightarrow (Iyz \vee Iyw \vee Iyt \vee Izw \vee Izt \vee Iwt)]$$

Proofs and Counterexamples

1. Prove the following tautology using only basic rules of inference:

$$\vdash -(P \rightarrow Q) \leftrightarrow (P \,\&\, -Q)$$

1	(1) $P \,\&\, -Q$	A
2	(2) $P \rightarrow Q$	A
1	(3) P	1 &E
1,2	(4) Q	2,3 MPP
1	(5) $-Q$	1 &E
1,2	(6) $Q \,\&\, -Q$	4,5 &I
1	(7) $-(P \rightarrow Q)$	2,6 RAA
	(8) $(P \,\&\, -Q) \rightarrow -(P \rightarrow Q)$	1,8 CP
9	(9) $-(P \rightarrow Q)$	A
10	(10) $-(P \,\&\, -Q)$	A
11	(11) P	A
12	(12) $-Q$	A
11,12	(13) $P \,\&\, -Q$	11,12 &I
10,11,12	(14) $(P \,\&\, -Q) \,\&\, -(P \,\&\, -Q)$	10,13 &I
10,11	(15) $--Q$	12,14 RAA
10,11	(16) Q	15 DN
10	(17) $P \rightarrow Q$	11,16 CP
9,10	(18) $-(P \rightarrow Q) \,\&\, (P \rightarrow Q)$	9,17 &I
9	(19) $--(P \,\&\, -Q)$	10,18 RAA
9	(20) $(P \,\&\, -Q)$	19 DN
	(21) $-(P \rightarrow Q) \rightarrow (P \,\&\, -Q)$	9,20 CP
	(22) $(-(P \rightarrow Q) \rightarrow (P \,\&\, -Q)) \,\&\, ((P \,\&\, -Q) \rightarrow -(P \rightarrow Q))$	8,21 &I
	(23) $-(P \rightarrow Q) \leftrightarrow (P \,\&\, -Q)$	22 Dfn$\leftrightarrow$

2. Prove the validity of the following argument using only basic rules of inference.

$$(\exists x)(Fx \,\&\, (y)(Gy \rightarrow Rxy)), (x)(Fx \rightarrow (y)(Hy \rightarrow -Rxy)) \vdash (x)(Gx \rightarrow -Hx)$$

1	(1) $(\exists x)(Fx \,\&\, (y)(Gy \rightarrow Rxy))$	A
2	(2) $(x)(Fx \rightarrow (y)(Hy \rightarrow -Rxy))$	A
3	(3) $Fa \,\&\, (y)(Gy \rightarrow Ray)$	A
3	(4) Fa	3 &E
2	(5) $Fa \rightarrow (y)(Hy \rightarrow -Ray)$	2 UE
2,3	(6) $(y)(Hy \rightarrow -Ray)$	4,5 MPP
3	(7) $(y)(Gy \rightarrow Ray)$	3 &E
2,3	(8) $Hb \rightarrow -Rab$	6 UE
3	(9) $Gb \rightarrow Rab$	7 UE
10	(10) Gb	A
3,10	(11) Rab	9,10 MPP
3,10	(12) $--Rab$	11 DN
2,3,10	(13) $-Hb$	8,12 MTT
2,3	(14) $Gb \rightarrow -Hb$	10,13 CP
2,3	(15) $(x)(Gx \rightarrow -Hx)$	14 UI
1,2	(16) $(x)(Gx \rightarrow -Hx)$	1,3,15 EE