Homework 2

A. Prove that the following arguments are valid. You may use only the following rules: MPP, MTT, DN, &I, &E, ∨I, the Rule of Assumptions (A), and Conditional Proof (CP). You must list dependency numbers for each line of your proof. [*None of these proofs requires RAA. You will get no points for a proof that uses RAA.*]

1. Suffixing

$$P \to Q \ \vdash \ (Q \to R) \to (P \to R)$$

2. Contraction

$$P \to (P \to Q) \ \vdash \ P \to Q$$

3. [No name]

$$P \to (Q \to R) \ \vdash \ (P \to Q) \to (P \to R)$$

B. Prove that the following arguments are valid. You may use only the following rules: MPP, MTT, DN, &I, &E, ∨I, the Rule of Assumptions (A), Conditional Proof (CP), and ∨-Elimination (∨E) . You must list dependency numbers for each line of your proof. [*None of these proofs requires RAA. You will get no points for a proof that uses RAA.*]

1. Distribution

$$(P\&Q) \vee (P\&R) \ \vdash \ P\&(Q \vee R)$$

2. [No name]

$$P \to Q, R \to S \ \vdash \ (P \vee R) \to (Q \vee S)$$

C. Prove that the following arguments are valid. You may use any of the rules of inference in Chapter 1 of Lemmon's book, including RAA. You must list dependency numbers for each line of your proof.

1. Negative Paradox

$$-P \vdash P \to Q$$

2. Material Implication A

$$P \to Q \vdash -P \vee Q$$

3. Material Implication B

$$-P \vee Q \vdash P \to Q$$

4. DeMorgan's Law

$$-P \& -Q \vdash -(P \vee Q)$$

5. Peirce's Law

$$\vdash ((P \to Q) \to P) \to P$$