

Homework 8 Key

A. For each of the following sentences, give an interpretation with domain $\{1, 2, 3, 4\}$ and nonempty extension of “ Rxy ” that makes the sentence true, and another such interpretation that makes the sentence false.

1. $(\exists x)(y)(Ryx \rightarrow Ryy)$

To make true: $\text{Ext}(Rxy) = \{<1,1>\}$

To make false: $\text{Ext}(Rxy) = \{<1,2>, <2,3>, <3,4>, <4,1>\}$

2. $(x)(y)(Rxy \rightarrow (\exists z)(Rxz \ \& \ Ryz))$

To make true: $\text{Ext}(Rxy) = \{<1,1>\}$

To make false: $\text{Ext}(Rxy) = \{<1,2>\}$

3. $(x)[(y)(Ryx \rightarrow Rxy) \rightarrow (y)(Rxy \rightarrow Ryx)]$

To make true: $\text{Ext}(Rxy) = \{<1,1>\}$

To make false: $\text{Ext}(Rxy) = \{<1,2>\}$

4. $(\exists x)(\exists y)(Rxy \ \& \ Ryx) \ \& \ (x)(y)[(\exists z)(Rxz \ \& \ Rzy) \rightarrow Rxy]$

To make true: $\text{Ext}(Rxy) = \{<1,1>\}$

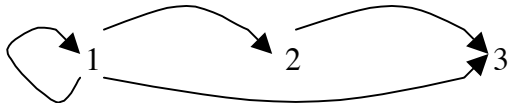
To make false: $\text{Ext}(Rxy) = \{<1,2>\}$

B. For each of the following pairs of sentences, give an interpretation that shows that the first sentence does not imply the second.

1. $(x)(\exists y)(Rxy \vee Ryx)$ $(x)(\exists y)Rxy \vee (x)(\exists y)Ryx$
 Domain = $\{1,2\}$ Ext(Rxy) = $\{<1,2>\}$



2. $(\exists x)(y) \sim Rxy \ \& \ (\exists x)(y)Rxy$ $(x)[(\exists y)Rxy \rightarrow (y)Rxy]$
 Domain = $\{1,2,3\}$ Ext(Rxy) = $\{<1,1>, <1,2>, <1,3>, <2,3>\}$



3. $(y)[(\exists z)Ryz \rightarrow (\exists z)Rzy]$ $(y)[(z)Ryz \rightarrow (z)Rzy]$
 Domain = $\{1,2\}$ Ext(Rxy) = $\{<1,1>, <1,2>\}$



4. $(x)(\exists y)(Rxy \ \& \ \sim Ryx)$ $(x)[(\exists y)Rxy \rightarrow (\exists y)Ryx]$
 Domain = $\{1,2,3,4\}$ Ext(Rxy) = $\{<1,2>, <2,3>, <3,4>, <4,2>\}$

