Homework 9 Key

A

1. $(x)(Tx \to ((y)(Sy \to Lxy) \to (z)(Sz \to -Rxz)))$
2. $(x)(Tx \to ((\exists y)(Sy \,\&\, Lxy) \to (\exists z)(Tz \,\&\, Lxz)))$
3. $(\exists x)(Sx \,\&\, (y)(Ty \to (Rxy \to Lyx)))$
4. $(x)(Tx \to ((y)(Sy \to (Ryx \to Rxy)) \to (z)(Sz \to Rzx)))$

B

1. $(\exists u)(\exists v)[(\exists w)(Pwu \,\&\, Pwv \,\&\, -Iuv) \,\&\, Pux \,\&\, Pvy]$
   ... assuming that for x and y to be first cousins, x and y need only have parents (u and v) who are half siblings.
2. $(u)(v)(w)[(-Mu \,\&\, -Mv \,\&\, -Mw \,\&\, Pxu \,\&\, Pxv \,\&\, Pxw) \to (Iuv \vee Iuw \vee Ivw)]$
3. $(\exists u)(\exists v)\{Mu \,\&\, Mv \,\&\, (\exists w)(Puw \,\&\, Pwx) \,\&\, (\exists y)(Pvy \,\&\, Pyx) \,\&\, -Iuv \,\&$
   $(z)[(Mz \,\&\, (\exists z')(Pzz' \,\&\, Pz'x)) \to (Izu \vee Izv)]\}$
4. $Pyx \,\&\, (z)[(Pyz \,\&\, -Ixz) \to Txz]$

C.2 Let **S** be the set of sentences whose only connective is "$\to$." Then **S** is an inductive set generated by the following definition:

1. If A is an atomic sentence then A is in **S**.
2. If A and B are in **S** then $A \to B$ is in **S**.
3. No string of symbols is in **S** unless it is generated in a finite number of steps from clauses 1. and 2.

Claim: No sentence in **S** is logically equivalent to $P \,\&\, -P$. We show this by establishing that every sentence in **S** has T on the first row of its truth table.

Base Case (atomic sentence): By convention, every atomic sentence has T on the first row of its truth table.

Inductive Step ($\to$): We need to establish the following conditional: If A and B have T on the first row of their truth table, then so does $A \to B$. But that is obviously the case since $A \to B$ is true whenever A and B are true.

C.2 (We use notation and definitions from the handout on soundness.) Suppose that line n results from lines i, j, k, l, m via $\vee$-Elimination, and that lines i, j, k, l, m are good. We need to show that line n is good.

Recall that A_i is a disjunction, say $B \vee C$, in which case line j is an assumption of B, and line l is an assumption of C. Furthermore, $A_k = A_m = A_n$, and

$$\underline{D}(n) = \underline{D}(i) \cup (\underline{D}(k) - \{B\}) \cup (\underline{D}(m) - \{A_l\}).$$

Let v be a valuation that satisfies $\underline{D}(n)$. Since line i is good, and $\underline{D}(i) \subseteq \underline{D}(n)$, it follows that $v(B \vee C) = v(A_i) = T$. By truth tables, either $v(B) = T$ or $v(C) = T$. In the first case, v satisfies $\underline{D}(k)$, and since line k is good, $v(A_n) = v(A_k) = T$. In the second case, v satisfies $\underline{D}(m)$, and since line m is good, $v(A_n) = v(A_m) = T$. In either case, $v(A_n) = T$, which means that line n is good.