PHI 312 : final exercise

Please read these instructions carefully. You must do problem 1, and either problem 2 or 3, and either problem 4 or 5. In other words, do $1 \wedge (2 \vee 3) \wedge (4 \vee 5)$.

1. Let Σ be a countably infinite signature. Let T be a consistent theory in Σ that has only finitely many axioms. Show that T is equivalent to the empty theory in Σ.

2. Let Σ be a countably infinite signature. Show that there are infinitely many theories in Σ that are pairwise inequivalent.

3. Give an example of two theories, with interpretations $f : T \to T'$ and $g : T' \to T$ such that: (a) f and g are conservative, and (b) T and T' are inequivalent. (You should demonstrate all these facts.)

4. Suppose that T' is a theory with exactly n models, where $0 < n < \infty$. Show that there is a conservative translation $f : T' \to T$, where T is the empty theory in a countably infinite signature.

5. Suppose that T and T' are theories, both of which have exactly n models, where $0 < n < \infty$. Show that T is equivalent to T'.