PHI 312 : pset 4

1. Let Σ be a propositional signature, and let X be the set of valuations of Σ. Show that there is no surjection $f : \Sigma \to X$.

2. Let X be a set, and let $a \in X$. Show that if X is uncountably infinite, then $X \backslash \{a\}$ is also uncountably infinite.

3. Let Σ be a countably infinite propositional signature, and let X be the set of valuations of Σ. For any sentence ϕ of Σ, let $C_\phi \subseteq X$ be the subset of valuations on which ϕ is true.

   (a) Show that for any sentences θ and ϕ, $\theta \vDash \phi$ if and only if $C_\theta \subseteq C_\phi$.

   (b) Let p be a propositional constant. Show that both C_p and $C_{\neg p}$ are uncountably infinite.

   (c) Suppose that $\theta = \gamma_1 \wedge \cdots \wedge \gamma_n$, where each γ_i is either a propositional constant or a negated propositional constant. Show that if no propositional constant occurs twice in θ, then C_θ is uncountably infinite.

   (d) Now let ϕ be an arbitrary sentence. Show that C_ϕ is either empty or uncountably infinite. [Hint: if ϕ is consistent, then there is some θ as above such that $\theta \vDash \phi$.]